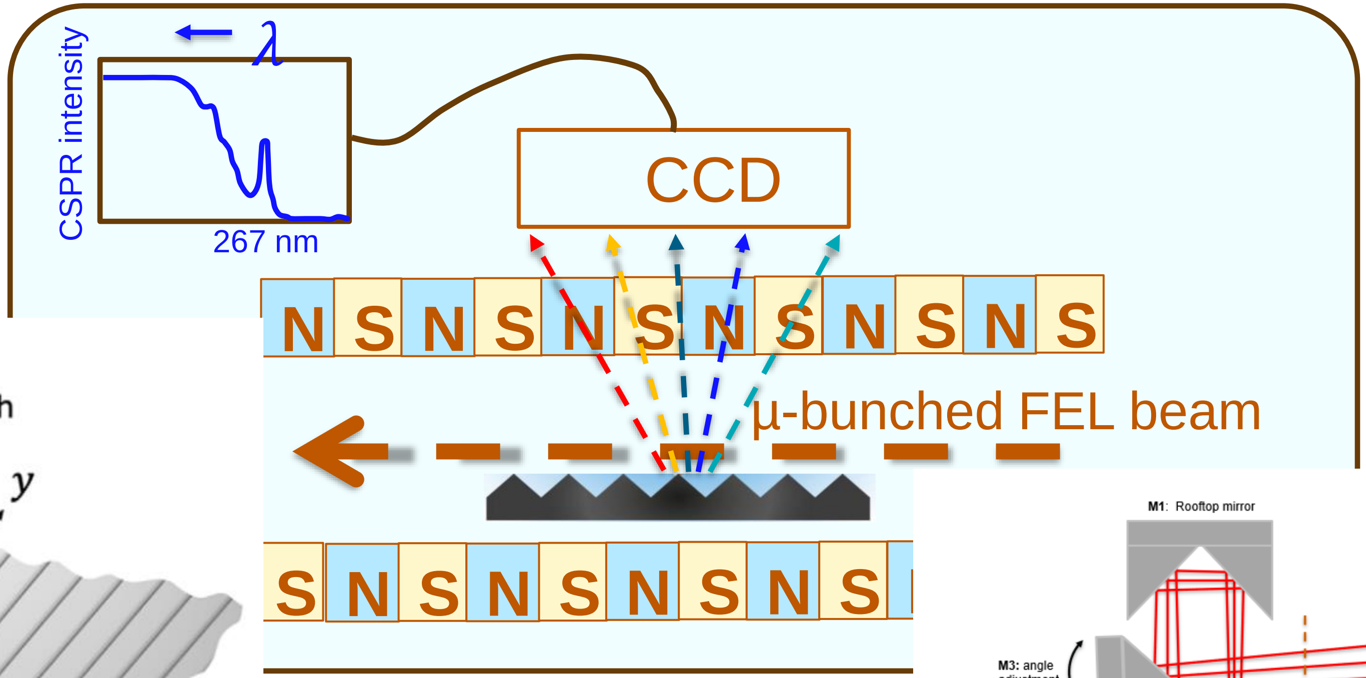
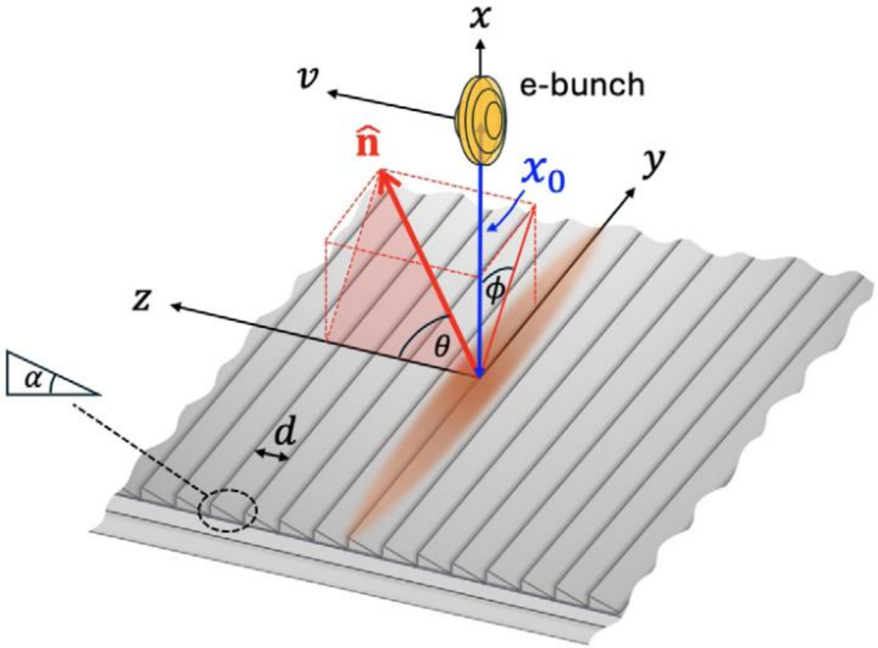
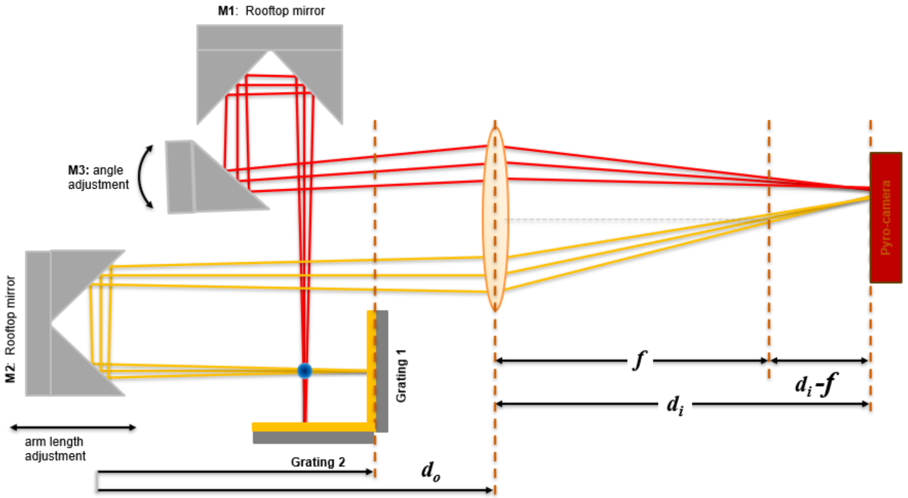
The role of coherent Smith-Purcell radiation for LWFA e-beam diagnosis







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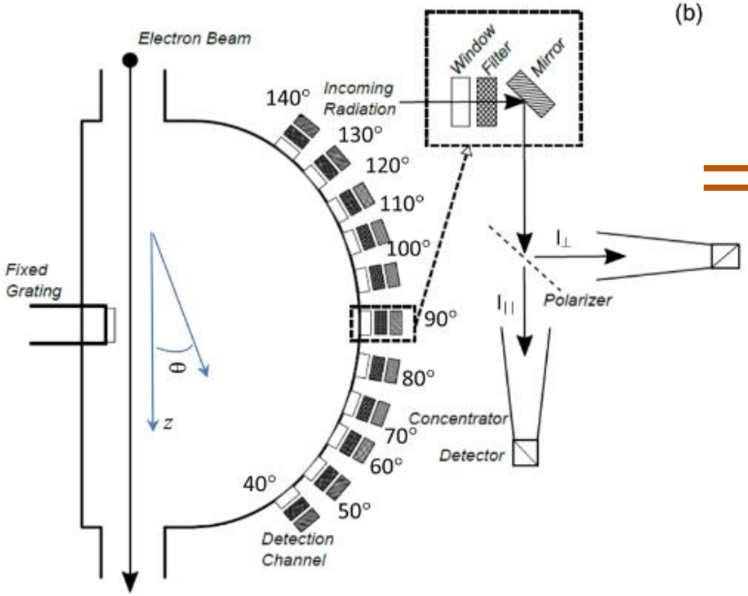
Previous work on bunch reconstruction

- Used conventional RF linacs, with bunch durations **~2.5 ps**
- Electron bunches in MeV to GeV range
- mm-wave regime



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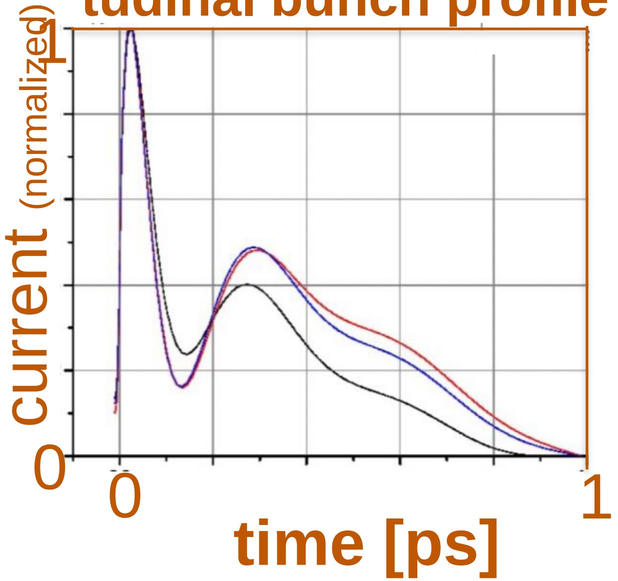
Reconstructed longitudinal bunch profile



1.) Blackmore V., et. al., "*First measurements of the longitudinal bunch profile of a 28.5 GeV beam using coherent Smith-Purcell radiation*", Phys. Rev. Spec. Topics – Acc. And Beams, **12**, 032803, (2009)

2.) Andrews H. L., et. al., "*Reconstruction of the time profile of 20.35 GeV, subpicosecond long electron bunches by means of coherent Smith-Purcell radiation",* Phys. Rev. Spec. Topics – Acc. And Beams, **17**, 052802, (2014)

3.) Konoplev I. V., et. al., *"Single shot, nondestructive monitor for longitudinal subpicosecond bunch profile measurements with femtosecond resolution",* Phys. Rev. Acc. And Beams, **24**, 022801, (2021)

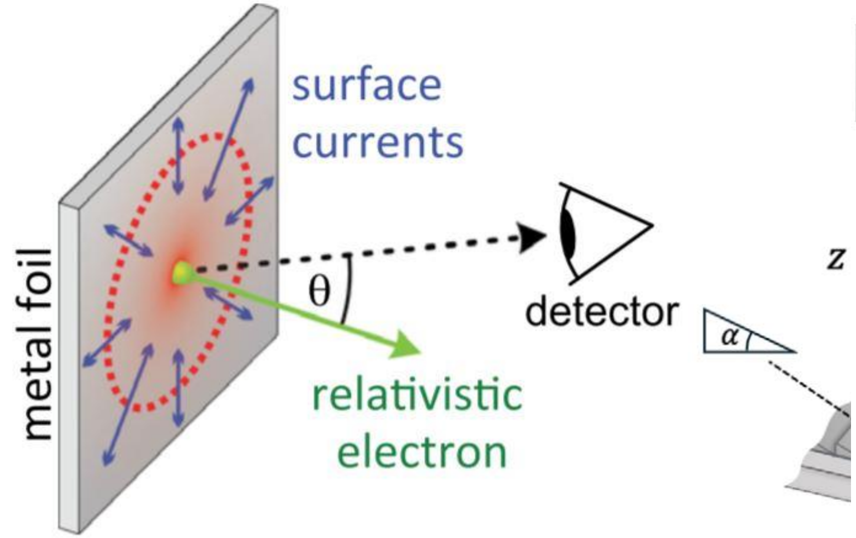
State of the art LWFA e-beam diagnostics

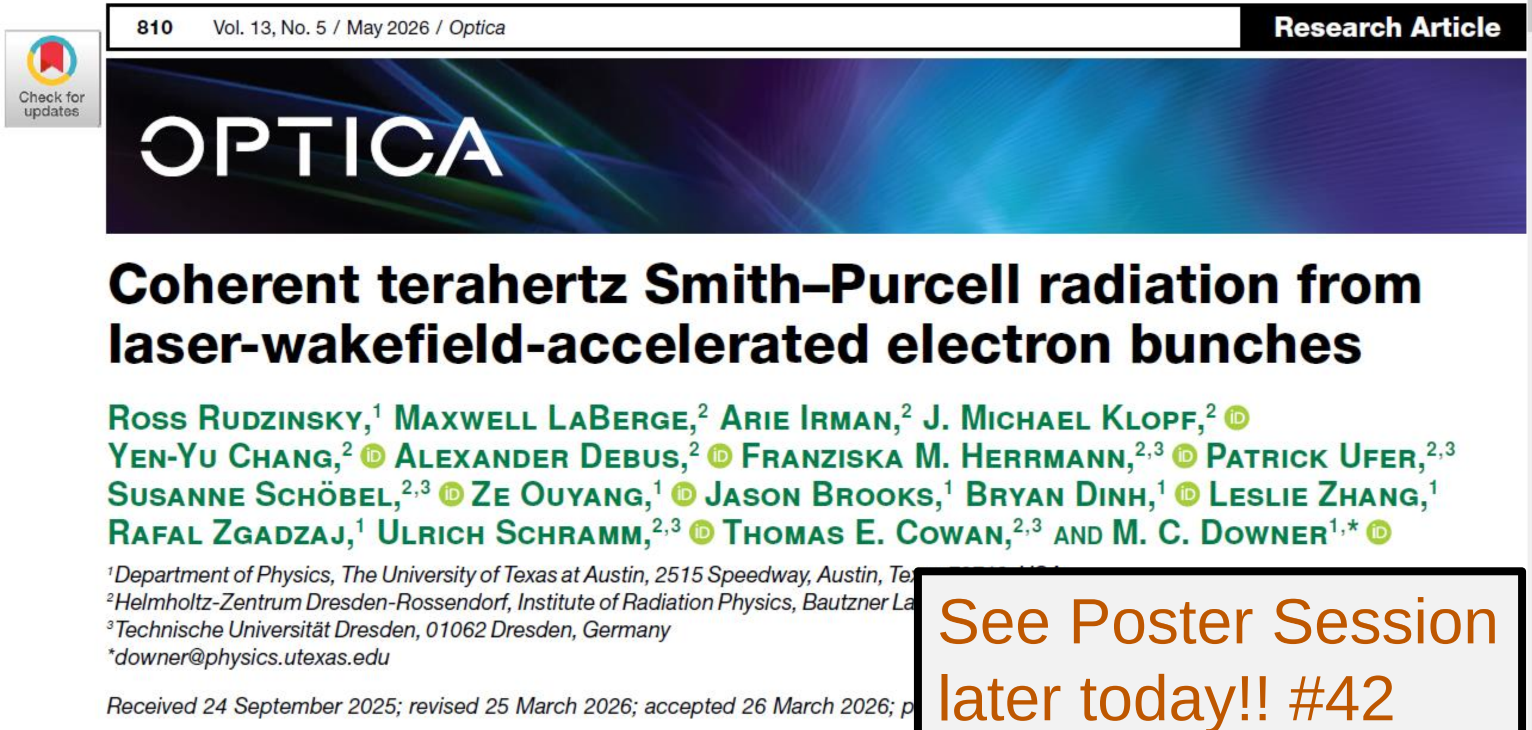
- Electron bunches from LWFA: ***fs-level*** bunch duration
  - Requires diagnostics that are sensitive to this ultrashort structure

And/or talk on Thursday: Mike Downer, 5:20 PM, Legacy A (Luskin)

Coherent Transition Radiation (CTR)

Coherent Smith-Purcell Radiation (CSPR)

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Research Article
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OPTICA

Coherent terahertz Smith–Purcell radiation from laser-wakefield-accelerated electron bunches

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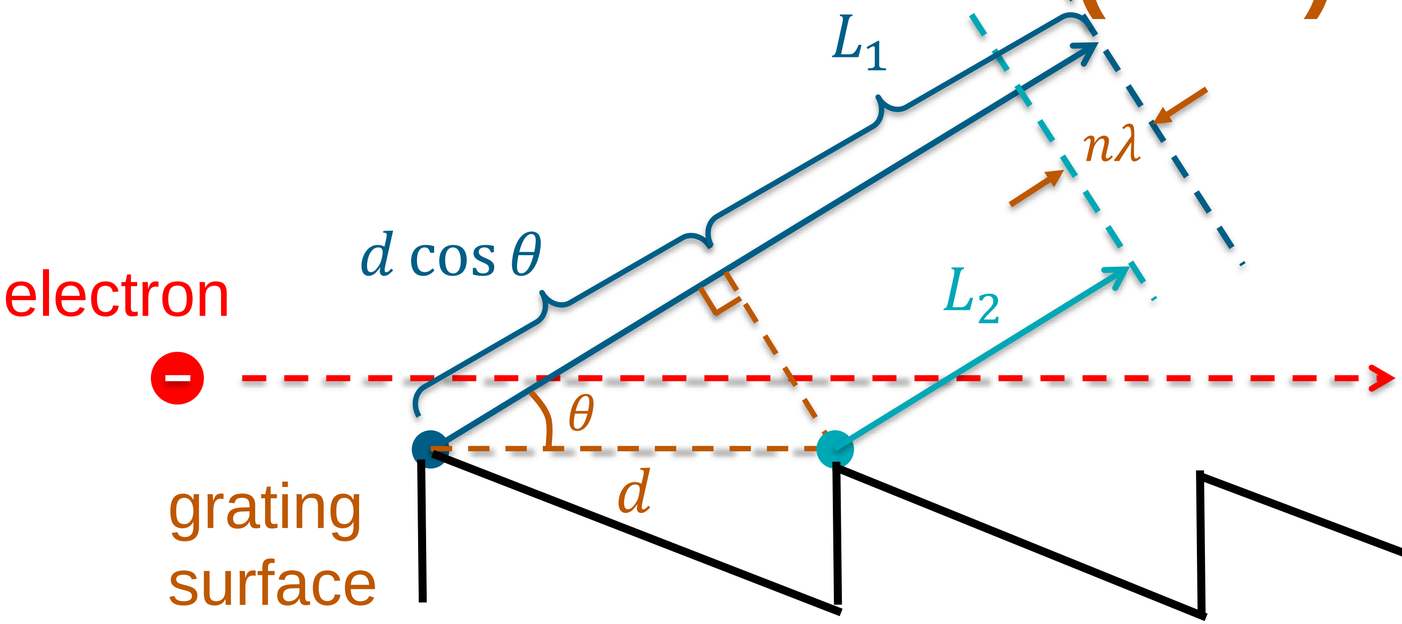
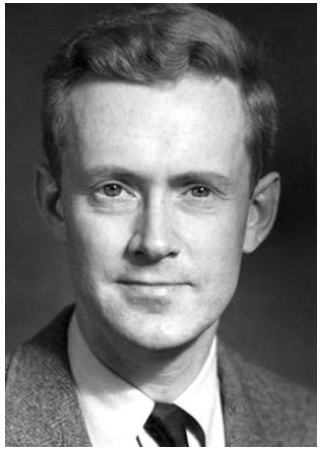
See Poster Session later today!! #42

Smith-Purcell Radiation (SPR) – Tunability



$$T_1 = \frac{1}{c}(d\cos\theta + L_1)$$

$$T_2 = \frac{d}{\beta c} + \frac{L_2}{c} = T_1$$



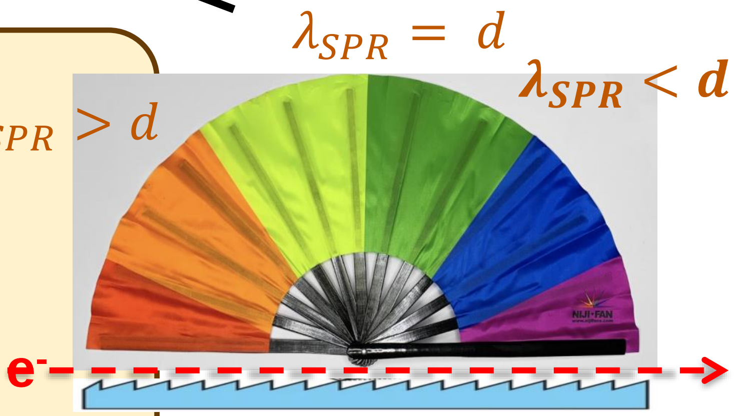
Edward M. Purcell
1912-1997
(Nobel Prize 1952)

For constructive interference:

$$L_1 - L_2 = n\lambda$$

$$c(T_1 - T_2) = d\cos\theta - \frac{d}{\beta} + L_1 - L_2$$

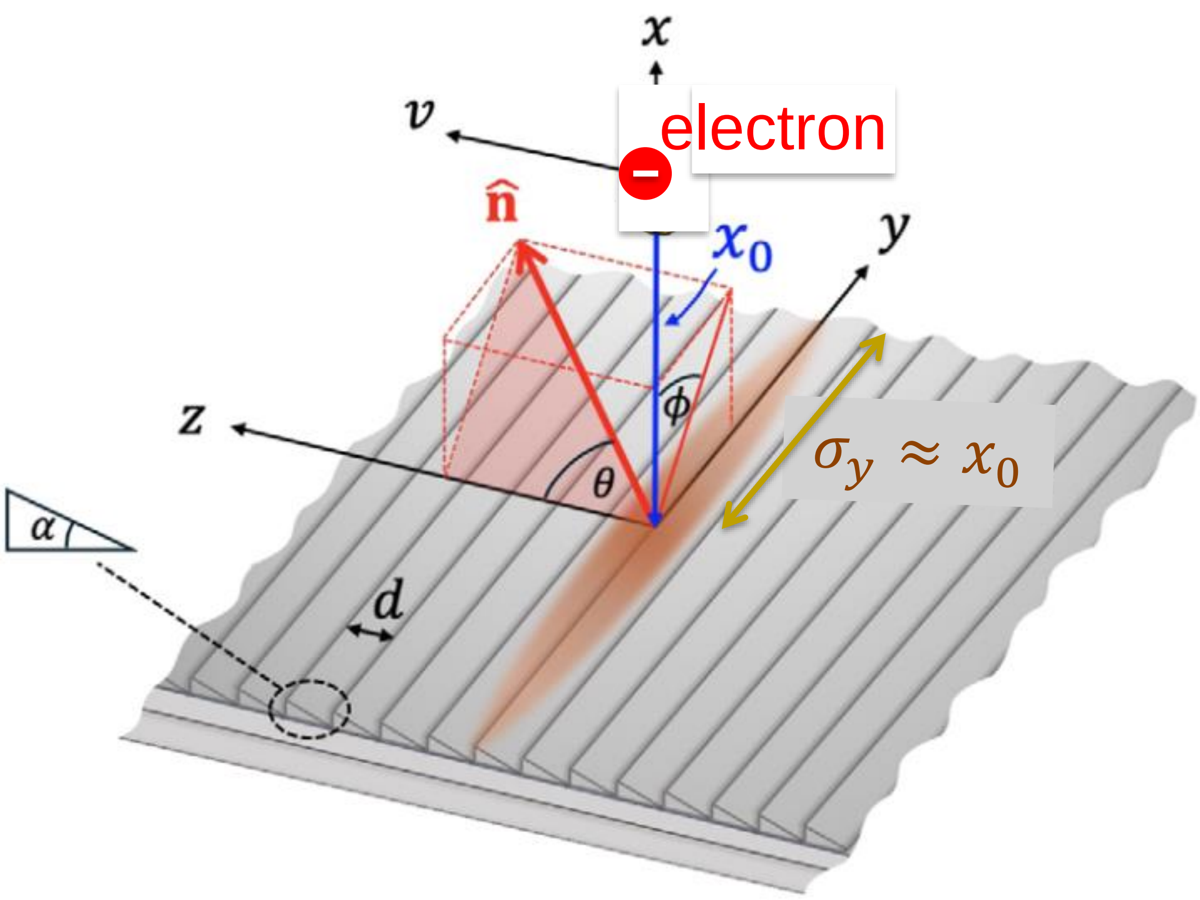
$$\lambda = \frac{d}{n}\left(\frac{1}{\beta} - \cos\theta\right)$$



Wavelength tunability

Stephen J. Smith
1925-2017
(EMP PhD student)

J. H. Brownell, J. Walsh, & G. Doucas, "Spontaneous Smith-Purcell radiation described through induced surface currents," *Phys. Rev. E* **57**, 1975 (1998)



Flat conducting surface ($\alpha = 0$)

Induced charge: $\Sigma(y,\zeta) = \dfrac{e\gamma x_0}{2\pi(\gamma^2\zeta^2 + y^2 + x_0^2)^{3/2}}$

$\zeta \equiv z - vt$

"Footprint": $\sigma_y \approx x_0$, $\sigma_z \approx x_0/\gamma$

Induced current: $\mathbf{K}(y,\zeta) = \Sigma(y,\zeta)\,\underbrace{\mathbf{v}(y,\zeta)}_{v_y\,\hat{\mathbf{y}} + v\,\hat{\mathbf{z}}}$
(non-radiating)

Grating surface ($\alpha > 0$)

Induced current: $\mathbf{J}(\mathbf{r},t) = \Sigma(\mathbf{r},t)\,\mathbf{v}(\mathbf{r},t)$

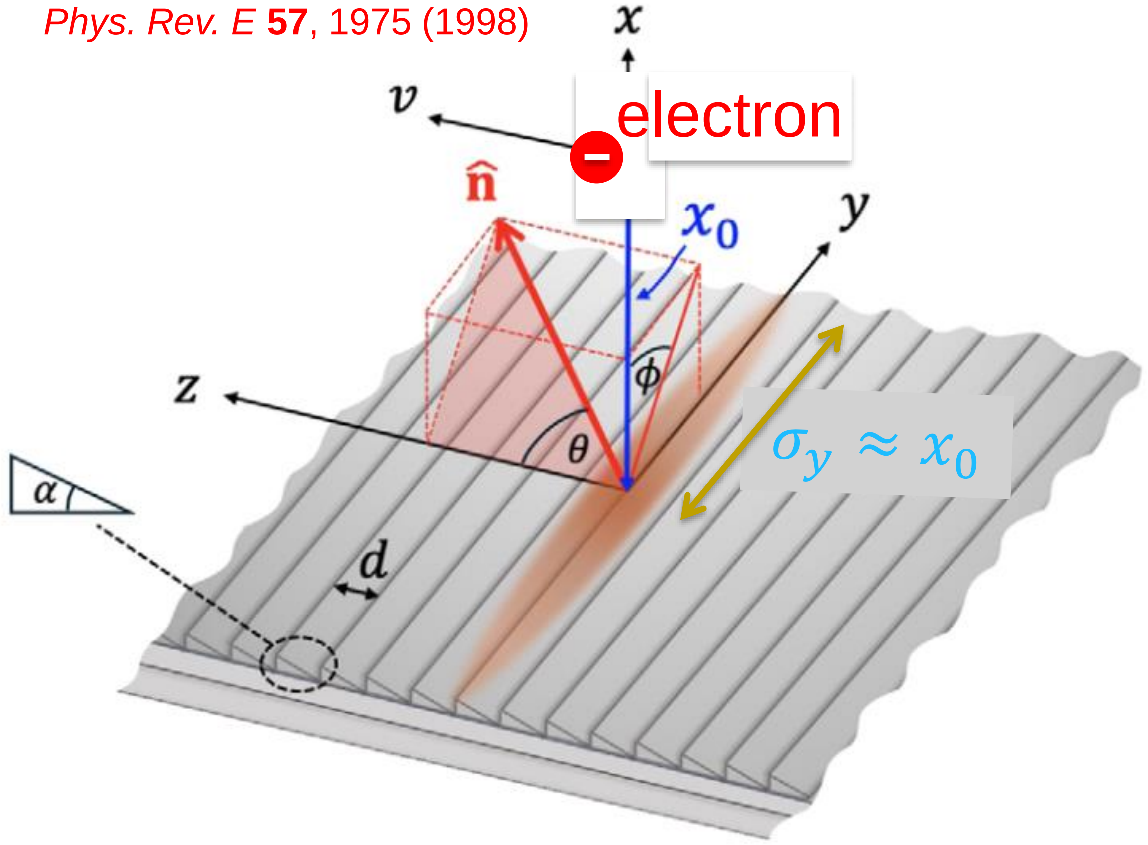
Energy radiated into far field per unit ω per unit Ω:

Jackson, 3rd ed., Eq. 14.67

$$\frac{\partial^2 W_e}{\partial\omega\partial\Omega} = \frac{\omega^2}{4\pi^2 c^3}\left|\int dt \int d^3x\,\hat{\mathbf{n}} \times \hat{\mathbf{n}} \times \mathbf{J}(\mathbf{r},t)e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}\right|^2$$

$$= \left(\frac{dW_e}{d\Omega}\right)_1 \underbrace{\delta(\omega - \omega_n)}_{\text{SP condition}} \quad \frac{2\pi nc}{d(\beta^{-1} - \cos\theta)}$$

J. H. Brownell, J. Walsh, & G. Doucas, "Spontaneous Smith-Purcell radiation described through induced surface currents," *Phys. Rev. E* **57**, 1975 (1998)



Ex. For highly relativistic electrons $(\gamma \gg 1)$ and near-normal emission $(\sin\theta \approx 1)$:

$$\left(\frac{dW_e}{d\Omega}\right)_1 \propto exp\left[\frac{-2x_o}{\lambda_e}\right] R^2$$

Evanescent wavelength (λ_e)

Decaying exponential

Grating efficiency (R^2)

$$\lambda_e(\theta,\phi) = \frac{\gamma\lambda}{2\pi\sqrt{1+\gamma^2\sin^2\theta\sin^2\phi}}$$

$x_0 \lesssim \lambda_e \Rightarrow$ strong SPR

Decaying exponential =1/e → $x_o = \lambda_e/2$

460 MeV

$$\lambda_e^{(max)}(\phi = 0) = \frac{\gamma\lambda}{2\pi} = \begin{cases} 1.4\ \text{mm}\ (\lambda = 10\ \mu\text{m}, \gamma = 900) \\ 3.1\ \text{cm}\ (\lambda = 10\ \mu\text{m}, \gamma = 2000) \end{cases}$$

1 GeV

$$\lambda_e\left(\phi = \frac{\pi}{180}\right) = \frac{\lambda}{2\pi\phi} = \begin{cases} 0.09\ \text{mm}\ (\lambda = 10\ \mu\text{m}, \gamma = 900) \\ 0.09\ \text{mm}\ (\lambda = 10\ \mu\text{m}, \gamma = 2000) \end{cases}$$

Example:

detected wavelength: $\lambda_{SPR} \approx 10\ \mu m$

electron bunch:

$\gamma \approx 900\ \ (E_e \approx 450\ \text{MeV})$

$N \approx 3 \times 10^9\ (Q \approx 0.5\ \text{nC})$

$\sigma_z \approx 3\ \mu m\ \ (\sigma_z/c \approx 10\ \text{fs})$

$\bar{x}_0 \approx 200\ \mu m$

grating:

$d = 10\ \mu m$

$R^2 = 0.42$

$\alpha = 27^\circ$

$L = 2.5\ \text{cm}$

Detection geometry:

$\theta \approx 90^\circ$

$\phi \approx 0$

$\Omega_{det} = 0.7 \times 10^{-4}\ \text{sr}$

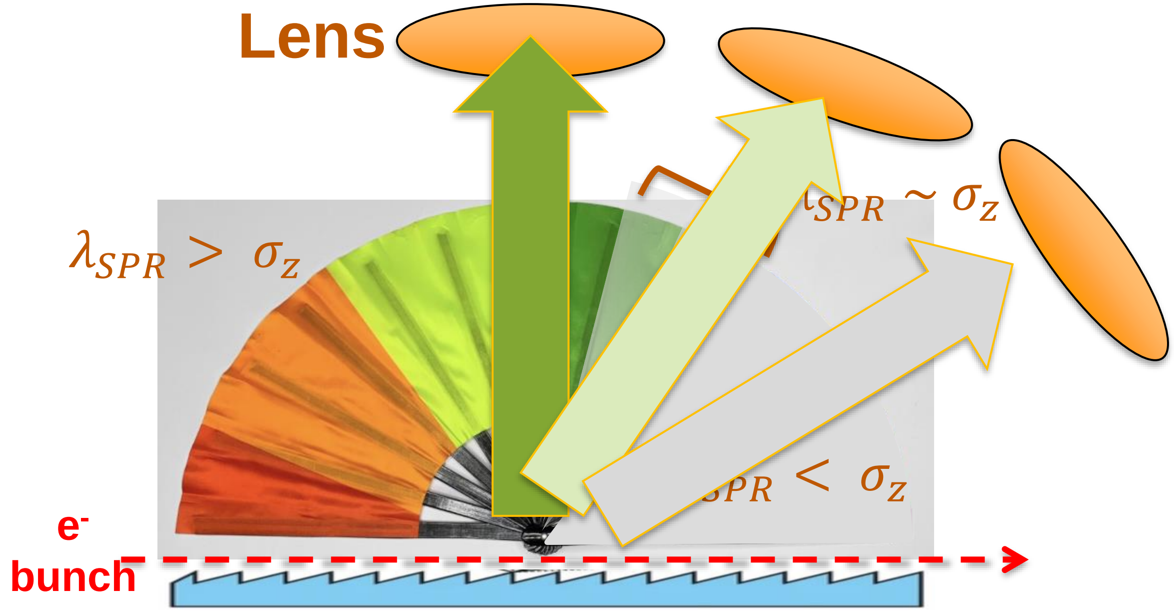
derived quantities:

$\lambda_e = 1.4\ \text{mm}$

$S_{inc} \approx S_{coh} \approx 0.6$

$$W_{SPR} = N \underbrace{\int_{\Omega_{det}} \overbrace{S_{inc} \left(\frac{dW}{d\Omega}\right)_1^{(\bar{x}_0)} d\Omega}^{\sim 10^{-24}\ \text{J (1 electron)}}}_{\sim 3 \times 10^{-15}\ \text{J}} + N^2 \underbrace{\int_{\Omega_{det}} \overbrace{S_{coh} \left(\frac{dW}{d\Omega}\right)_1^{(\bar{x}_0)} d\Omega}^{\sim 10^{-24}\ \text{J}}}_{\sim 10^{-5}\ \text{J}}$$

(N electrons)



"Cutoff" in SPR spectrum:

Simulation Results for various simple bunch shapes

Example:

detected wavelength: $\lambda_{SPR} \approx 10\ \mu m$

electron bunch:

$\gamma \approx 900\ \ (E_e \approx 450\ \text{MeV})$
$N \approx 3 \times 10^9\ (Q \approx 0.5\ \text{nC})$
$\sigma_z \approx 3\ \mu m\ \ (\sigma_z/c \approx 10\ \text{fs})$
$\bar{x}_0 \approx 200\ \mu m$

grating:

$d = 10\ \mu m$
$R^2 = 0.42$
$\alpha = 27^\circ$
$L = 2.5\ \text{cm}$

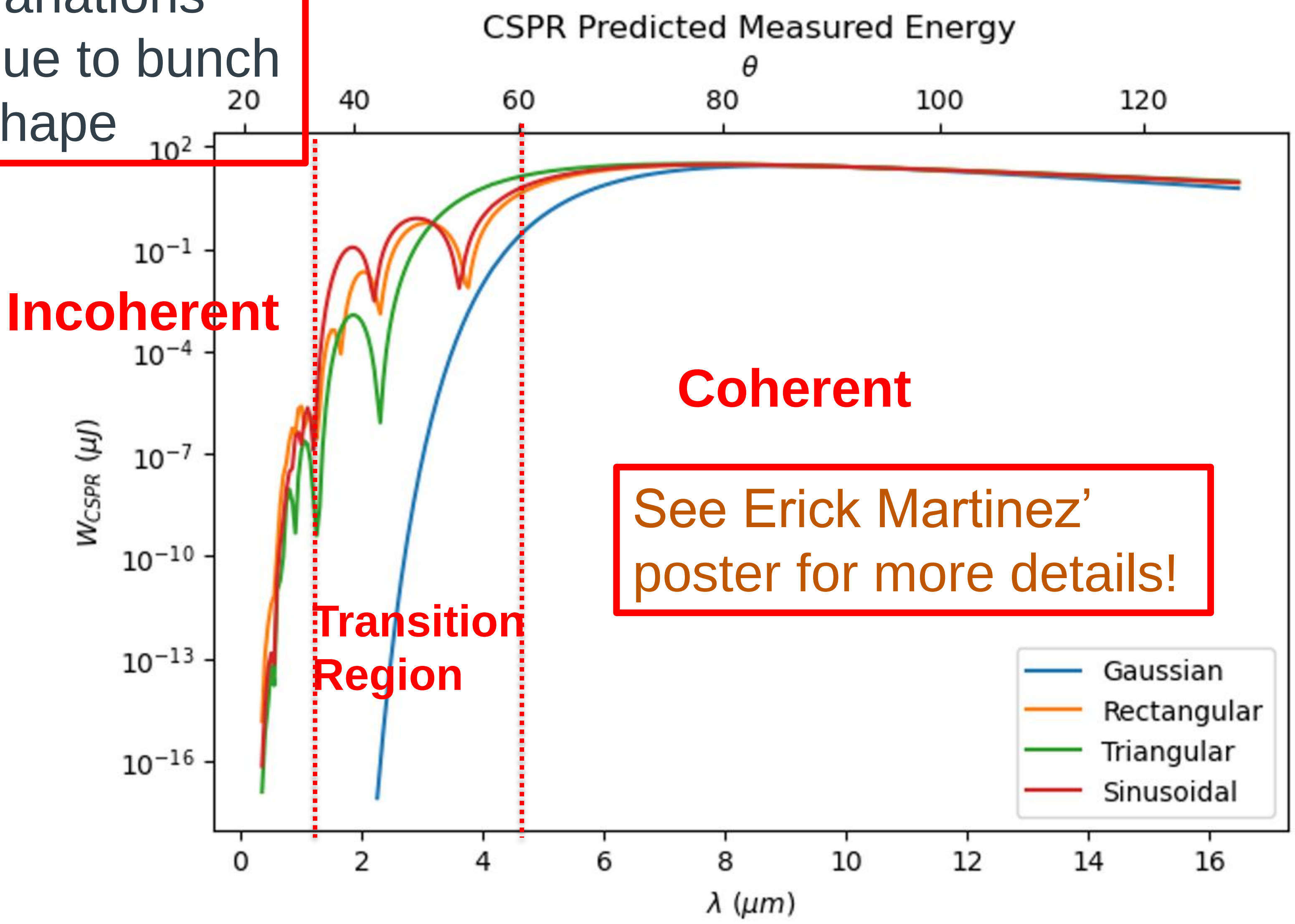
Detection geometry:

$\theta \approx varied$
$\phi \approx 0$
$\Omega_{det} = 0.7 \times 10^{-4}\ \text{sr}$

derived quantities:

$\lambda_e = 1.4\ \text{mm}$
$S_{inc} \approx S_{coh} \approx 0.6$

Variations due to bunch shape

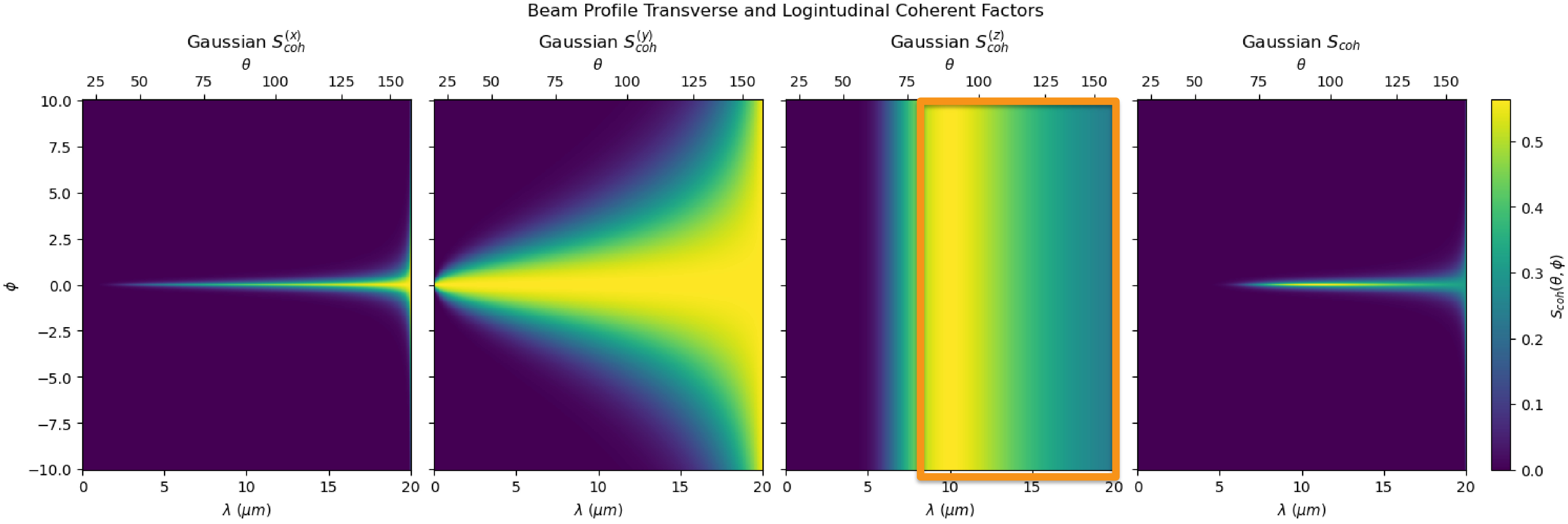


See Erick Martinez' poster for more details!

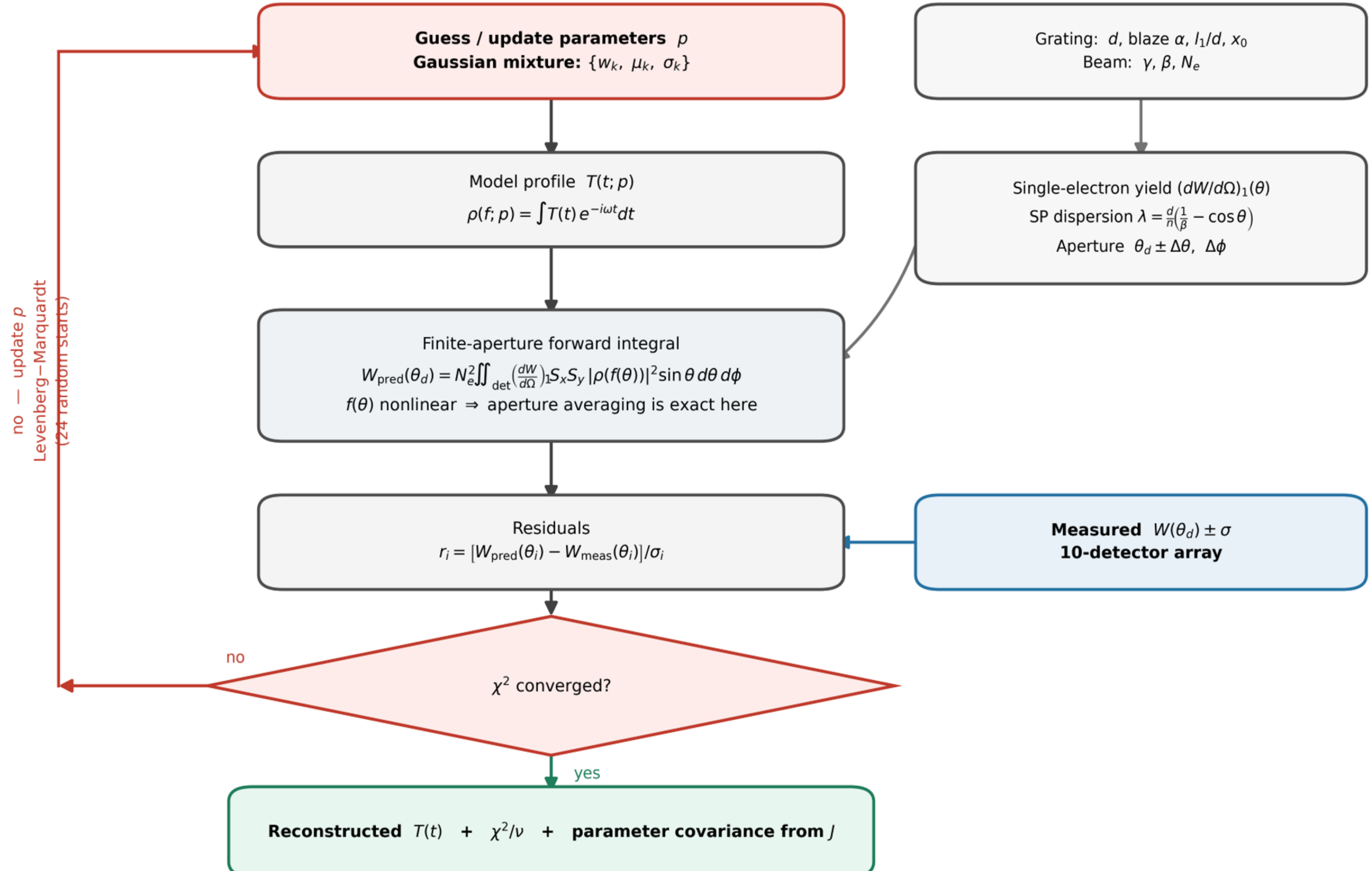
How would one extract bunch information from this curve?

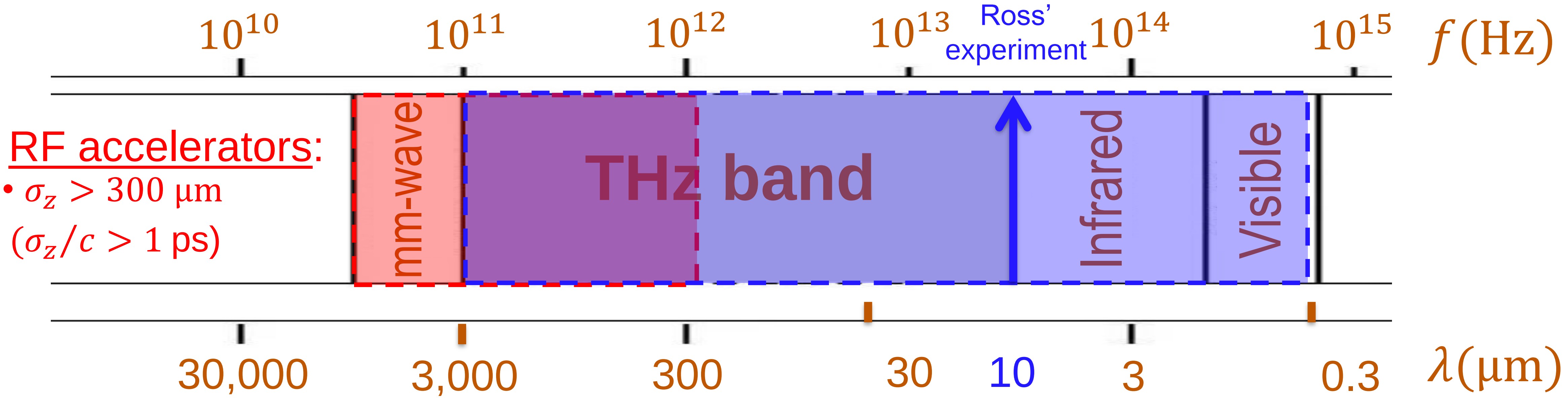
$$W_{SPR} = N^2 \int_{\Omega_{det}} S_{coh} \left(\frac{dW}{d\Omega}\right)_1^{(\bar{x}_0)} d\Omega$$

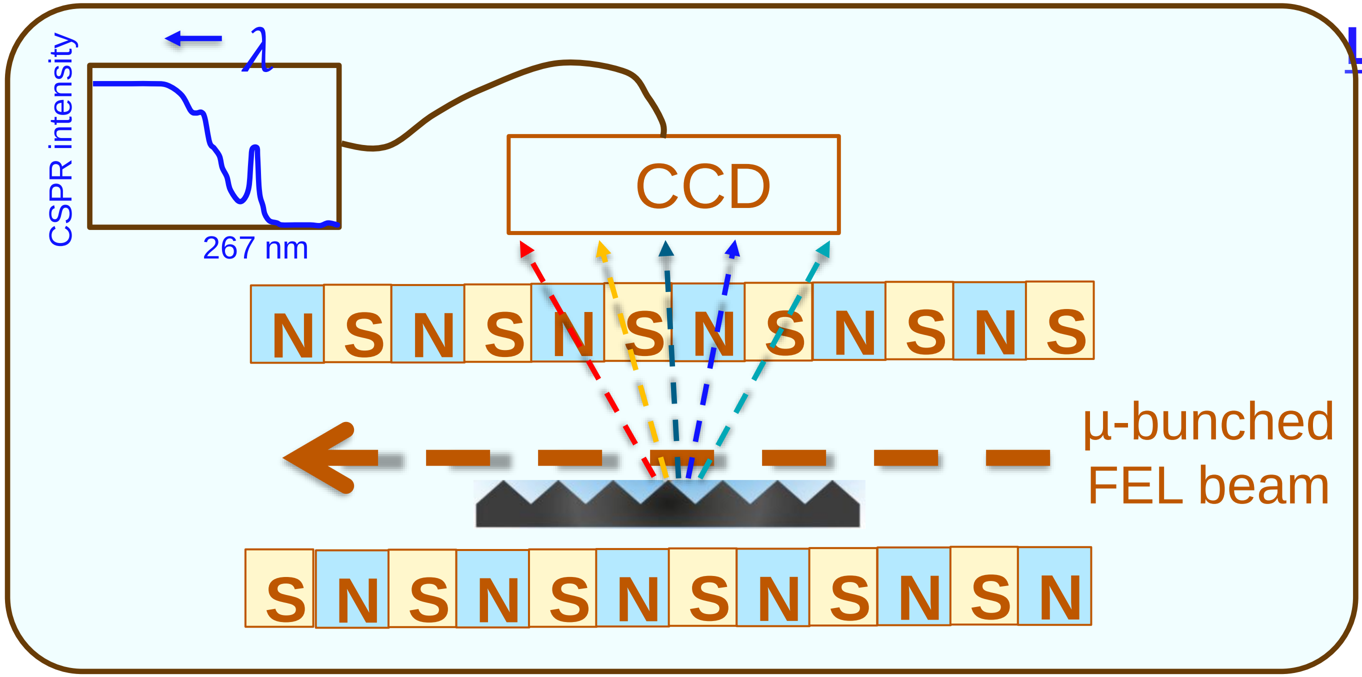
<u>For the Gaussian e-bunch case:</u>



Forward-fitting workflow for CSPR longitudinal-profile reconstruction





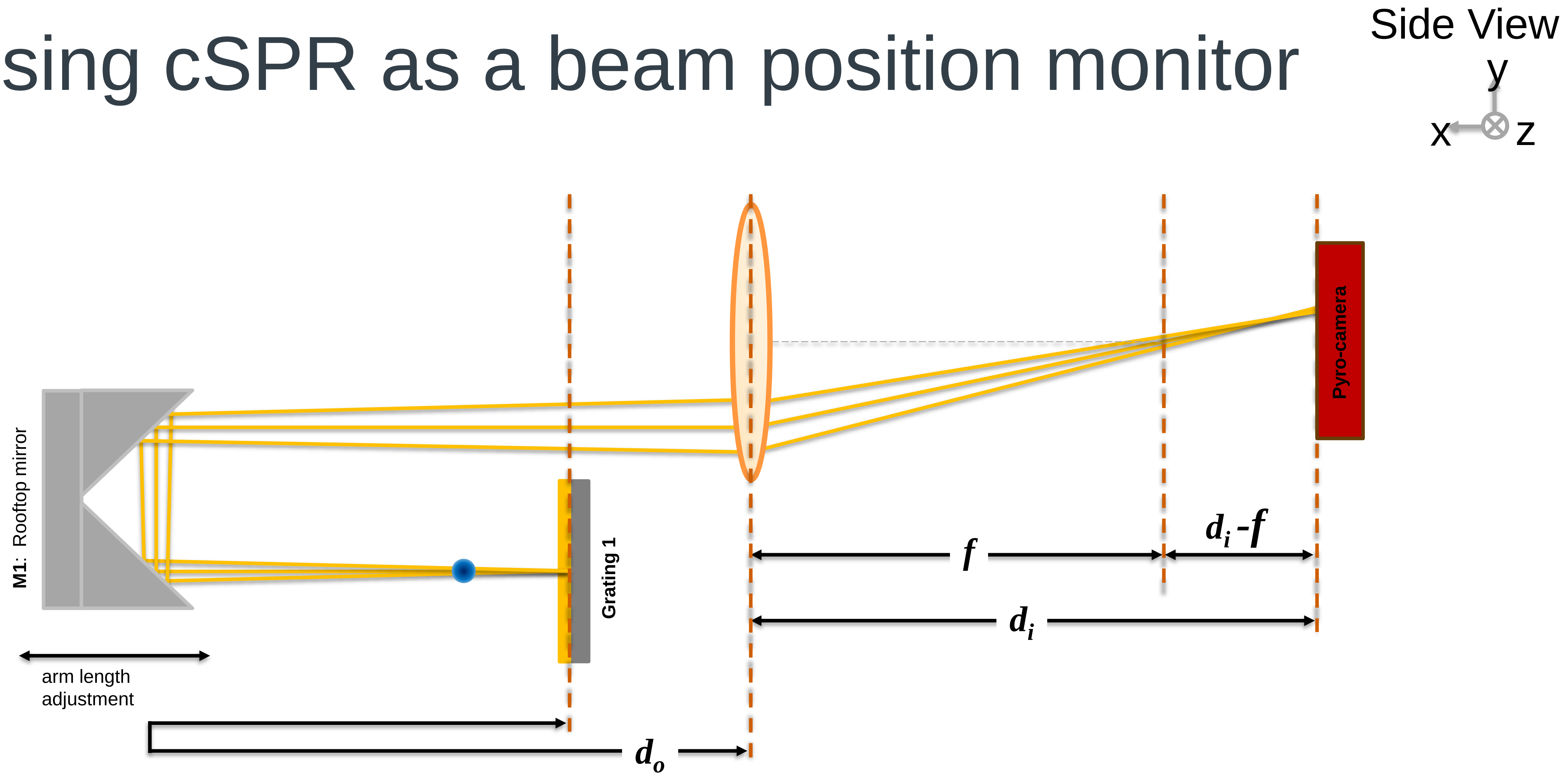


Laser-driven wakefield accelerators

- $0.3 < \sigma_z < 3$ μm $(1 < \sigma_z/c < 10$ fs)

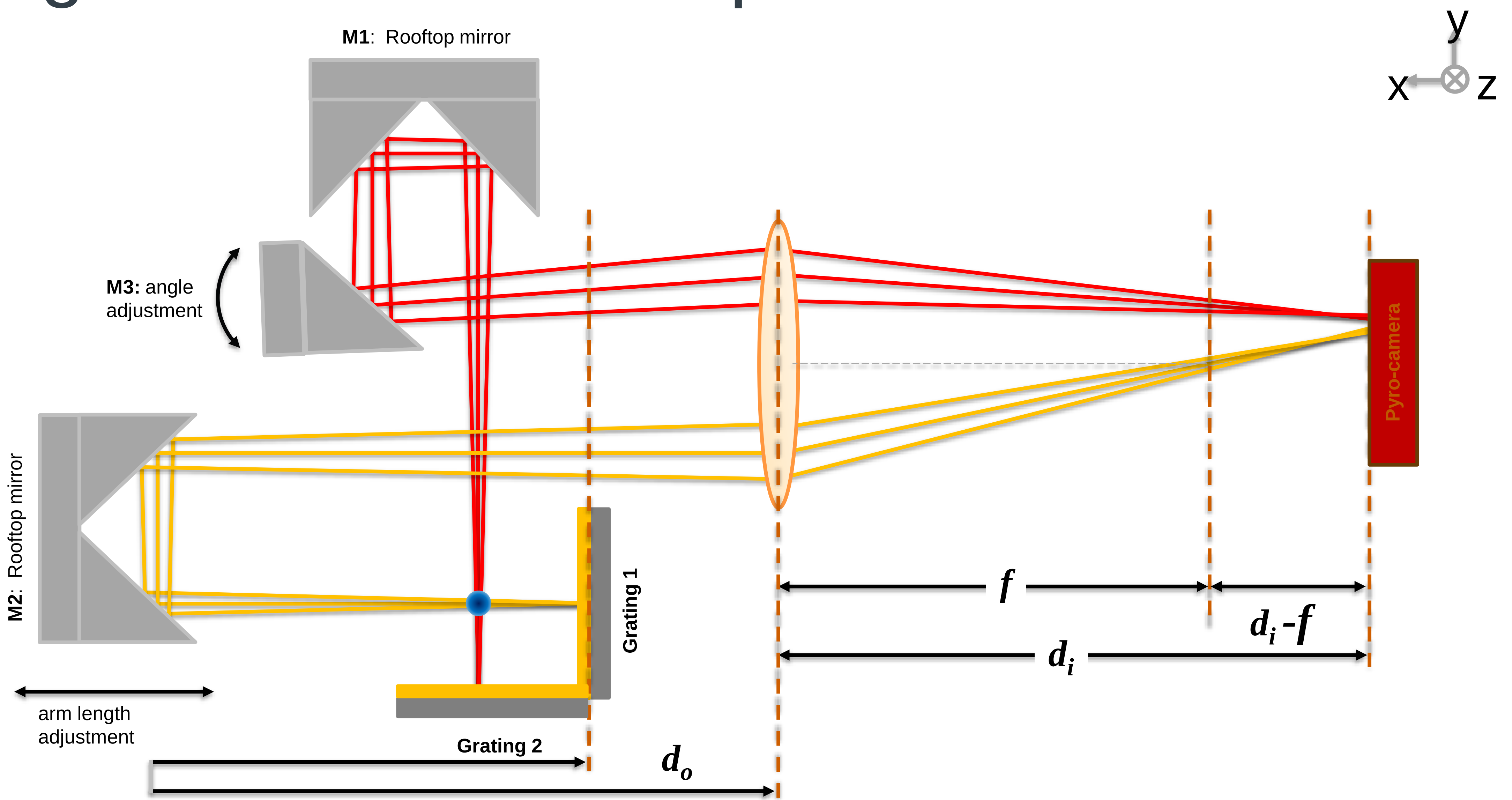
- **noninvasive diagnostic of fs e-bunches**
  - intra-FEL operation in real time
  - observe UV signatures of microbunching
  - incorporate into COTR-based diagnostic suit
  - distribute SPR gratings along e-beam path

Using cSPR as a beam position monitor

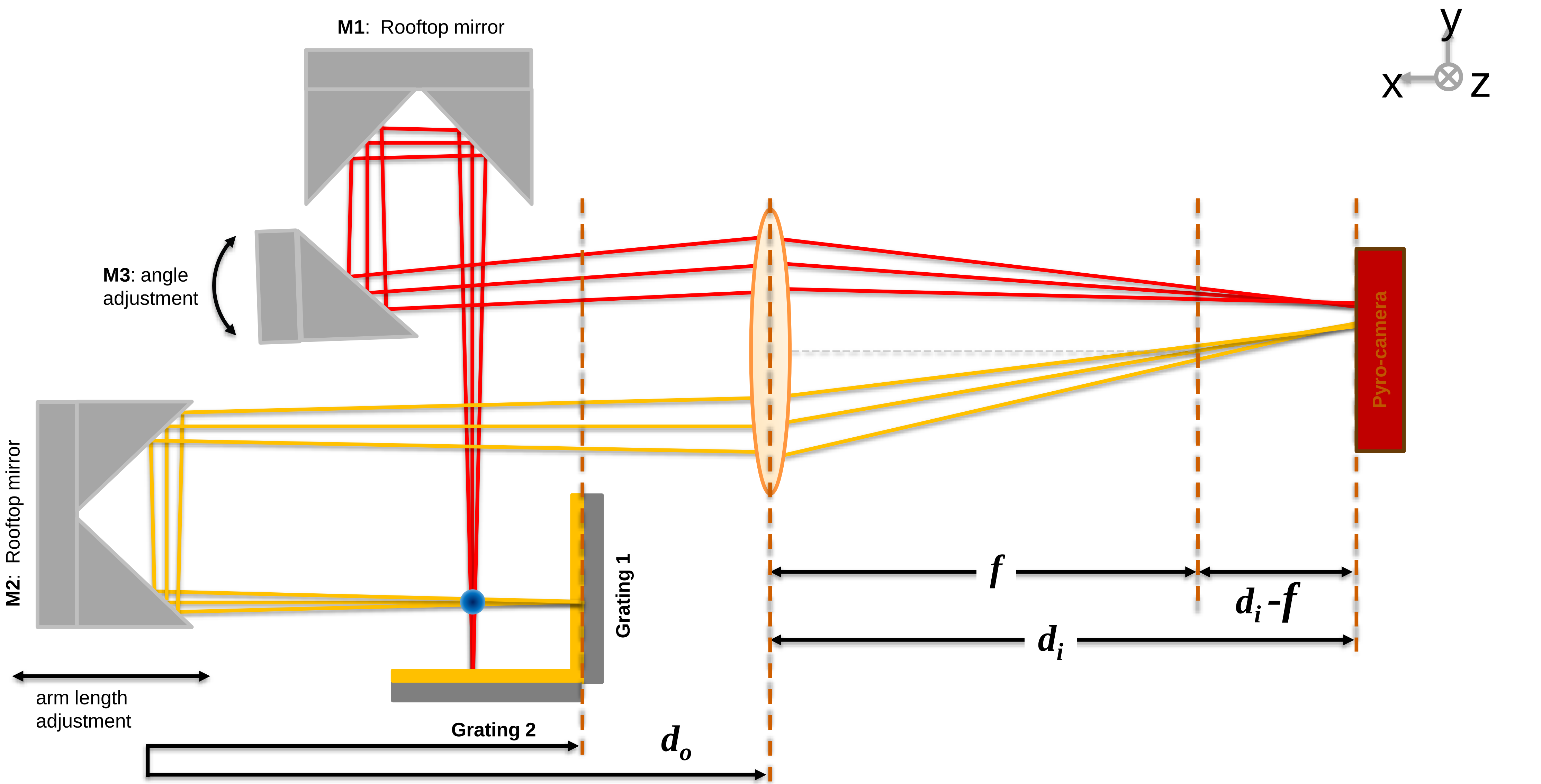


Using cSPR as a beam position monitor

Side View



Using cSPR as a beam position monitor



Summary – Thank you!

- cSPR is a promising mechanism for which a LWFA electron bunch can be probed non-invasively

Funding. "U. S. Department of Energy, Office of Science, Office of High Energy Physics (DE SC0011617); Helmholtz Association; Alexander von Humboldt-Stiftung.

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Appendix Slide – R^2 + S_coh

$$R^2 \approx \left[\frac{1}{\sin\theta - \tan\alpha_1\,(1+\cos\theta)}\right]^2 \text{x}$$

$$tan^2\alpha_1 sin^2\theta\,\frac{sin^2\left[\frac{kh\sin\theta}{2}\right]}{\left[\frac{kh\sin\theta}{2}\right]^2} \qquad (4)$$