Kaon radiative leptonic decay rates from Twisted Mass simulations at the physical point

Francesco Sanfilippo, INFN Roma Tre

Lattice@Mumbai, 7 November 2025

Extended Twisted Mass Collaboration

INFN

ICSC
Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

Based on

Kaon radiative leptonic decay rates from lattice QCD simulations at the physical point

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RM123-SOTON collaboration

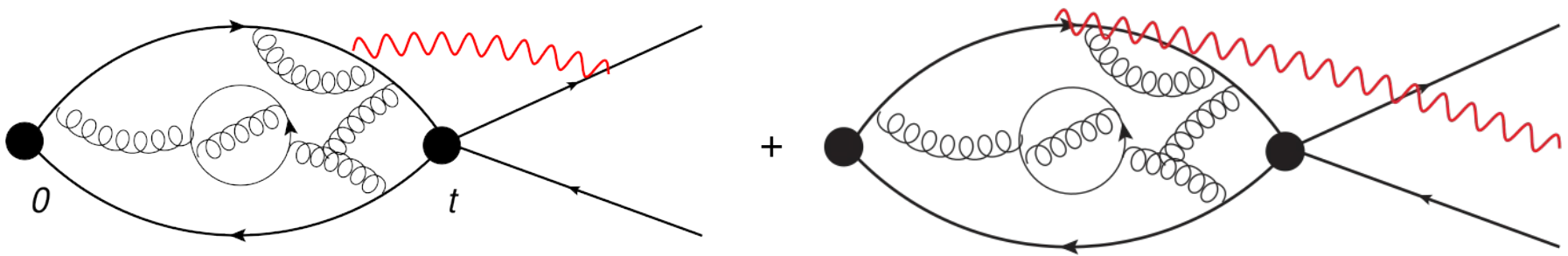
Phys.Rev.D 111 (2025) 11, 114523 arXiv: arXiv:2504.08680

A long time ago, in a different universe

Computing Pion & Kaon decay into leptons beyond percent accuracy

"*QED Corrections to Hadronic Processes in Lattice QCD*," N.Carrasco et al, PRD 91 (2015)

→ QED correction to the $K \to \ell_2$ process are infrared divergent
→ Need to combine virtual correction with (unobserved) soft photon $K \to \ell\nu\gamma(E_\gamma), \;\; E_\gamma < \Delta$



Numerical results

"*First lattice calculation of QED corrections to leptonic decay rates*", D.Giusti et al, PRL 120 (2018)

"*Light-meson leptonic decay rates in lattice QCD+QED*", M.Di Carlo et al, PRD 100 (2019)

Phenomenological interest

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \ (?)$$

$|V_{ud}|$ from superallowed nuclear beta decays: 0.03%

$|V_{ud}|$ from semileptonic $\pi \to \pi^0 e \nu_e$ beta decay: 0.3%

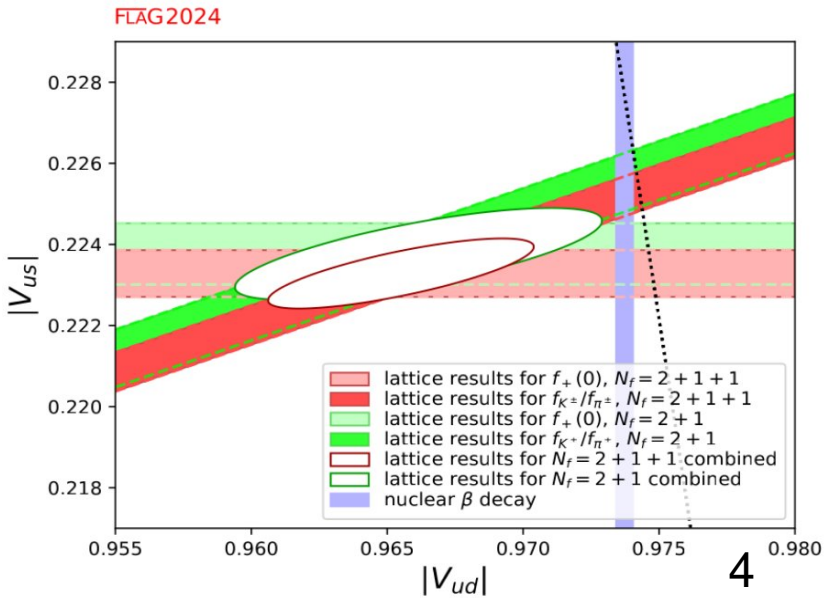
$|V_{ub}|$ is very small and we can drop it

$|V_{us}|$ from semiletponic decay $K \to \pi \ell \nu$: 0.2%

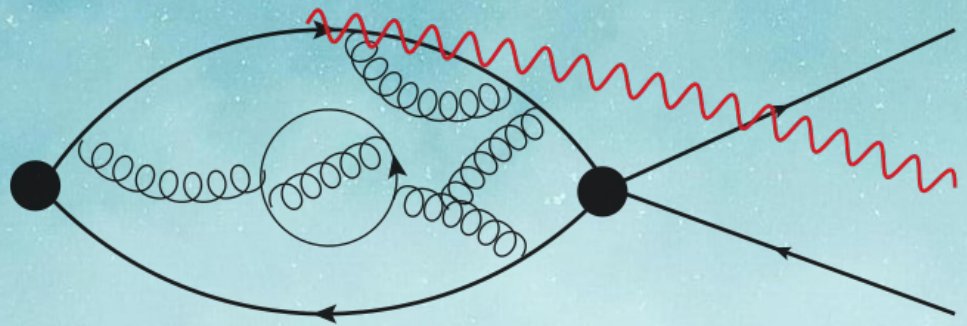
$\dfrac{|V_{us}|}{|V_{ud}|}$ from the ratio of letponic decay $K/\pi \to \ell \nu(\gamma)$: 0.3%

Tension between the value of $|V_{us}|$ from leptonic and semileptonic decays vs. $|V_{us}^{uni}|$

CFR Gregorio overview this morning



Radiative decay process



$\Gamma[K \to \ell\nu\gamma(E_\gamma)]$ at finite energy is infrared finite! $x_\gamma = 2E_\gamma/M_K$ $x_\ell = 2E_\ell/M_K$

And, experimentally measured, indeed:

- Suppressed by $\alpha_{\rm em}$
- But helicity suppression is removed!

$\Gamma(K^+ \to e^+\nu_e\gamma)/\Gamma(K^+ \to \mu^+\nu_\mu)$ Γ_{17}/Γ_2

VALUE (10^{-5})	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
1.62 ± 0.22 OUR AVERAGE Error includes scale factor of 3.9.						
1.98 ±0.11	95	[1] KOBAYASHI	2023	E36		E_γ in $10-250$ MeV, $p_e > 200$ MeV/c
1.483 ±0.066 ±0.013	1.4k	[2] AMBROSINO	2009E	KLOE	±	E_γ in $10-250$ MeV, $p_e > 200$ MeV/c

Can we earn an additional determination of the CKM matrix elements?

Radiative decay process – exp. controversy

Unfortunately the experimental results are inconsistent with each other

$\Gamma(K^+ \to e^+\nu_e\gamma)/\Gamma(K^+ \to \mu^+\nu_\mu)$ Γ_{17}/Γ_2

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… and more inconsistencies with ISTRA, OKA, E787

→ Lattice can play a role at clarifying the experimental inconsistencies

Computational framework

Method & first results for a range of mesons presented in:

"First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons", A. Desiderio et al, PRD 103 (2021)

Several volumes/lattice spacings but unphysical M_π, limited accuracy.

Rate $\Gamma[P \to \ell\nu\gamma(E_\gamma)]$ computed in terms of f_P and two form factors f_V, f_A

$$\frac{d^2\Gamma(P^+ \to \ell^+\nu_\ell\gamma)}{dx_\gamma dx_\ell} \equiv \frac{d^2\Gamma_1}{dx_\gamma dx_\ell} = \frac{\alpha_{\rm em}}{4\pi}\Gamma^{(0)} \left[\frac{d^2R_1^{\rm pt}}{dx_\gamma dx_\ell} + \frac{d^2R_1^{\rm SD}}{dx_\gamma dx_\ell} + \frac{d^2R_1^{\rm INT}}{dx_\gamma dx_\ell}\right]$$

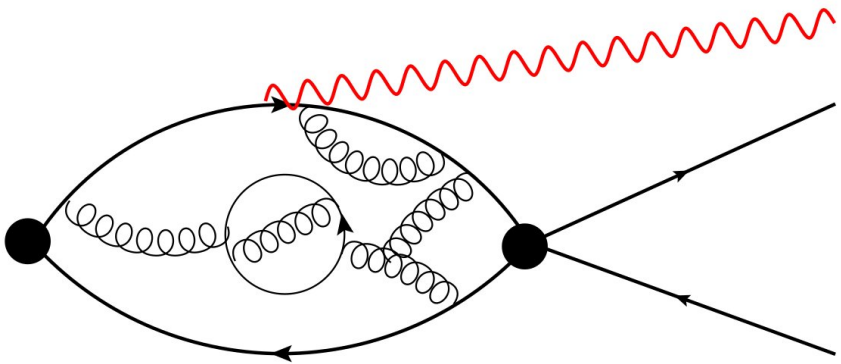
Point like *(pt)* contribution related to f_P, structure dependent *(SD)* related to f_V, f_A

Determine the form factors from a suitable decomposition of hadronic tensor

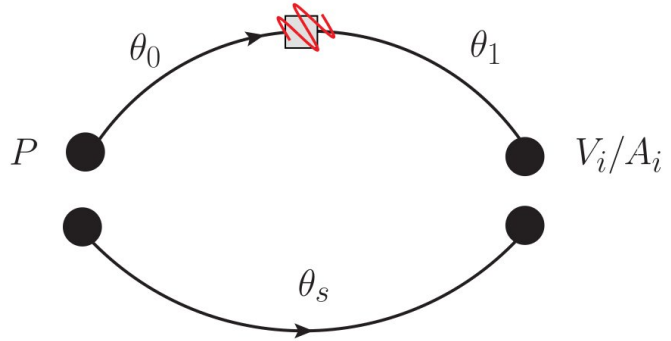
Photon emission ...without a photon

- Photon: external particle, gets completely amputated in the rate
- Only role: to project to the correct polarization the vector vertex

→ We can omit the photon propagator from the beginning



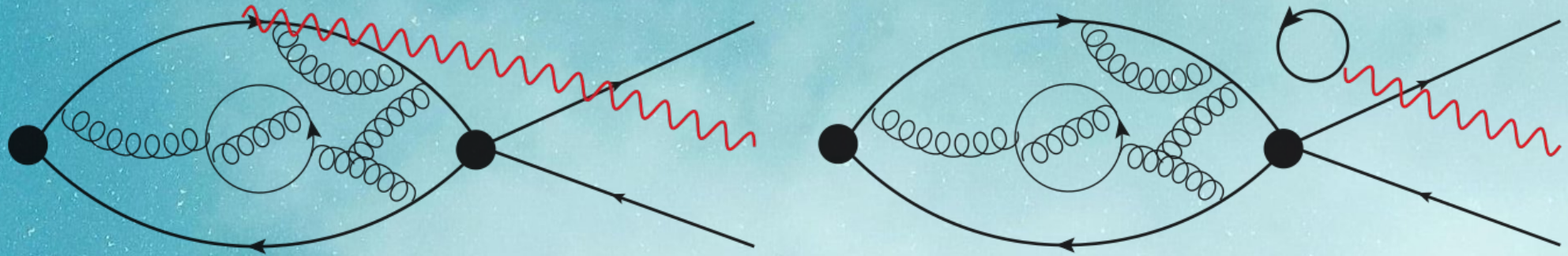
→



= conserved current vertex

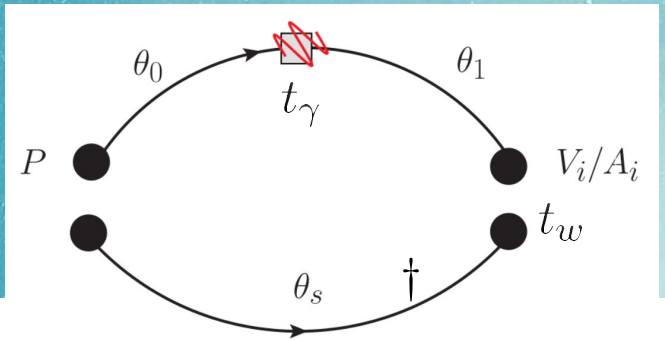
No large finite volume effect (no photon at all is present on the lattice)

Sea emission



- First time that both the contributions at $\mathcal{O}(\alpha_{\rm em})$ are computed
- Requires to compute sea loops with momentum, as a function of t_γ
- Sea emission expected to be suppressed w.r.t valence
- Computed on a single ensemble, the largest volume one

Sequential props & Momentum choice

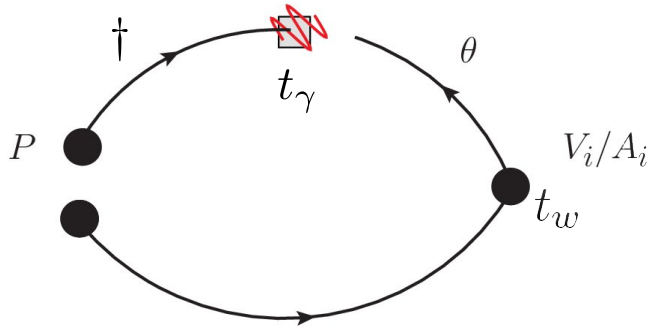


First-results paper PRD 103 (2021)

4D method: sequential propagator on t_γ

- Automatic integration over t_γ, two helicities needed
- Study as a function of t_w to isolate the ground state
- Get for free all components of the weak current

→ error grows with t_γ when in integrating over it



Dedicated *K* paper (this work) PRD 111 (2025)

3D method [Tuo et al., Giusti et al.] seq prop at fixed t_w

- Preliminary study to fix t_γ to isolate the ground state
- Repeat for all components of the weak current
- Manually integrate over t_w

→ 3D method wins as it allows to control the integration

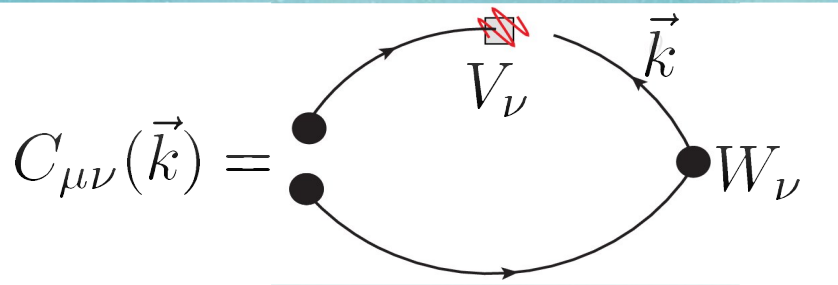
Rewired the momentum choice to ease the calculation & reduce computing cost

Numerical setup

ID	L/a	M_π [GeV]	a [fm]
B48	48	140.3(3)	0.07948(11)
B64	64	140.2(3)	0.07948(11)
B96	96	140.1(3)	0.07948(11)
C80	80	136.7(3)	0.06819(14)
D96	96	140.8(3)	0.056850(90)

- N_f=2+1+1 simulations of Twisted Mass at maximal twist
- Pion mass close to its physical value
- Three lattice spacing (on a fixed volume) – automatic $\mathcal{O}(a)$ improvement
- Three volumes (on a fixed lattice spacing)

Kinematic & Form Factor Estimators



Choosing $\vec{k} = (0, 0, k)$

one needs only $\mu, \nu \in (1, 2)$

Evaluate the emission from both lines

Form factors estimators

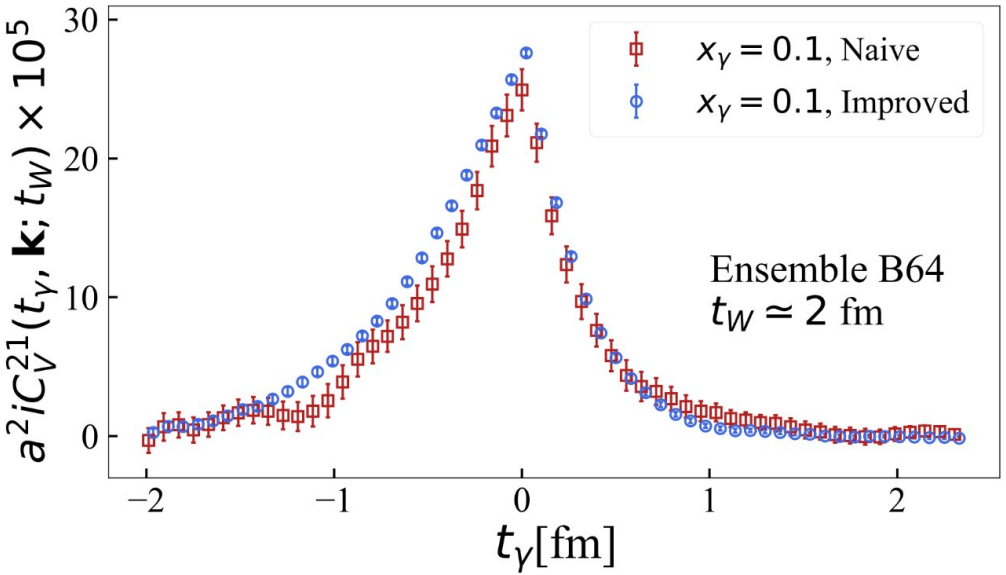
$$F_V(x_\gamma; t_W) = \frac{iH_V^{21}(\boldsymbol{k}; t_W)}{k_z} = \frac{i}{k_z} \int_{-\infty}^{\infty} dt_\gamma \; C_V^{21}(t_\gamma, \boldsymbol{k}; t_W) \; ,$$

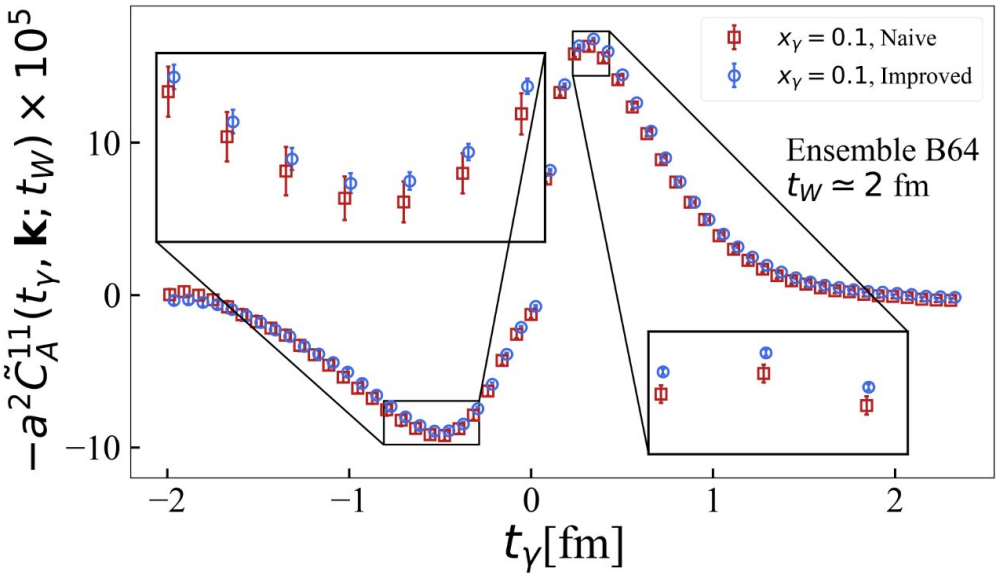
$$F_A(x_\gamma; t_W) = \; - \frac{1}{E_\gamma} \left[H_A^{11}(\boldsymbol{k}; t_W) - H_A^{11}(\boldsymbol{0}; t_W) \right] = - \frac{1}{E_\gamma} \int_{-\infty}^{\infty} dt_\gamma \; \left[C_A^{11}(t_\gamma, \boldsymbol{k}; t_W) - C_A^{11}(t_\gamma, \boldsymbol{0}; t_W) \right]$$

$$F_A(x_\gamma = 0; t_W) = \lim_{E_\gamma \to 0} - \int_{-\infty}^{\infty} dt_\gamma \frac{e^{E_\gamma t_\gamma} - 1}{E_\gamma} \; C_A^{11}(t_\gamma, \boldsymbol{0}; t_W)$$

← Specific for axial ff at $x_\gamma = 0$

Example of integration



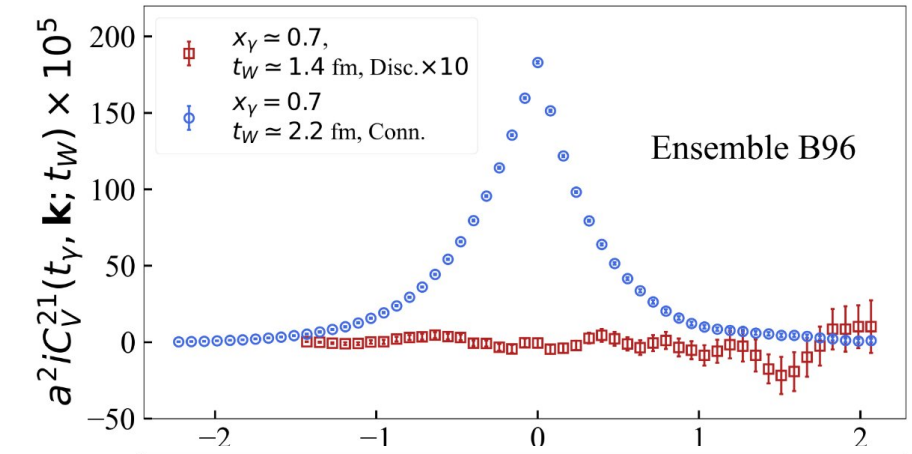


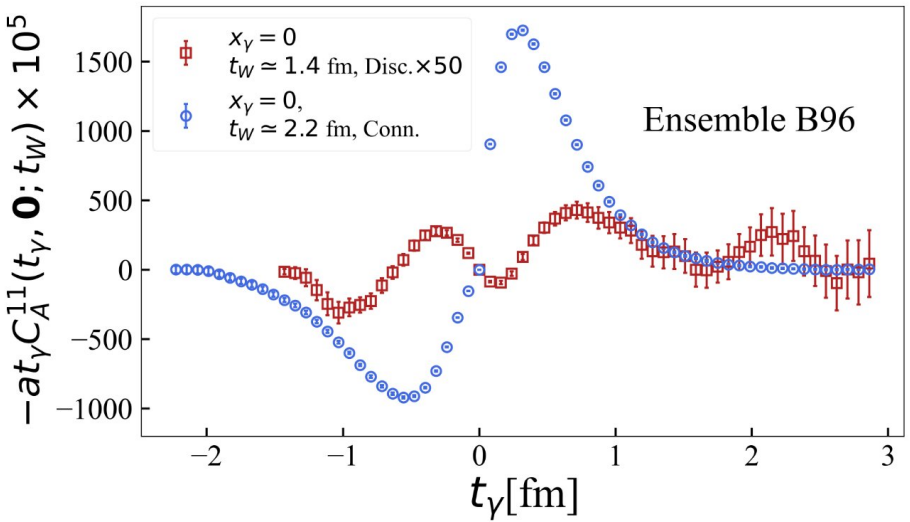
Improved estimators useful at small x_γ

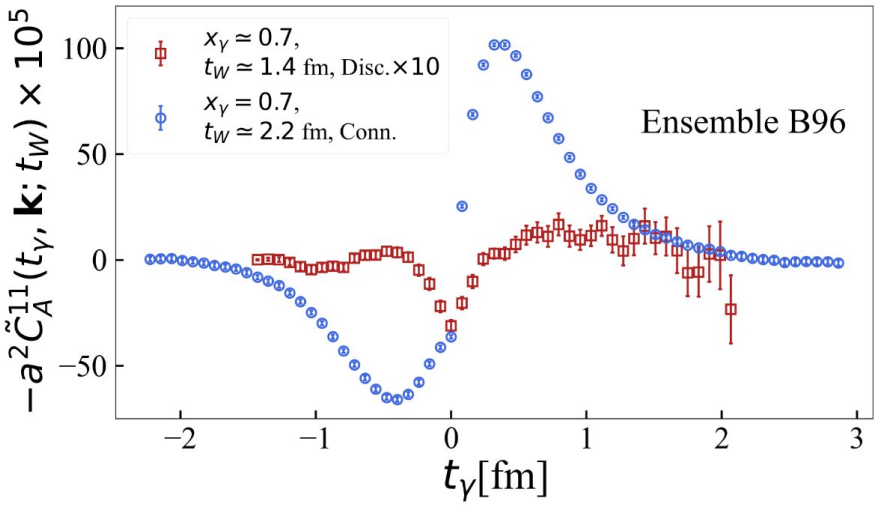
$$\widetilde{C}_A^{11}(t_\gamma, \boldsymbol{k}; t_W) \rightarrow \frac{1}{2}\left[\widetilde{C}_A^{11}(t_\gamma, \boldsymbol{k}; t_W) + \widetilde{C}_A^{11}(t_\gamma, -\boldsymbol{k}; t_W)\right]$$ Enforcing parity

$$C_V^{21}(t_\gamma, \boldsymbol{k}; t_W) \rightarrow C_V^{21}(t_\gamma, \boldsymbol{k}; t_W) - C_V^{21}(t_\gamma, \boldsymbol{0}; t_W) e^{E_\gamma t_\gamma}$$ Subtracting "zero"

Disconnected contribution







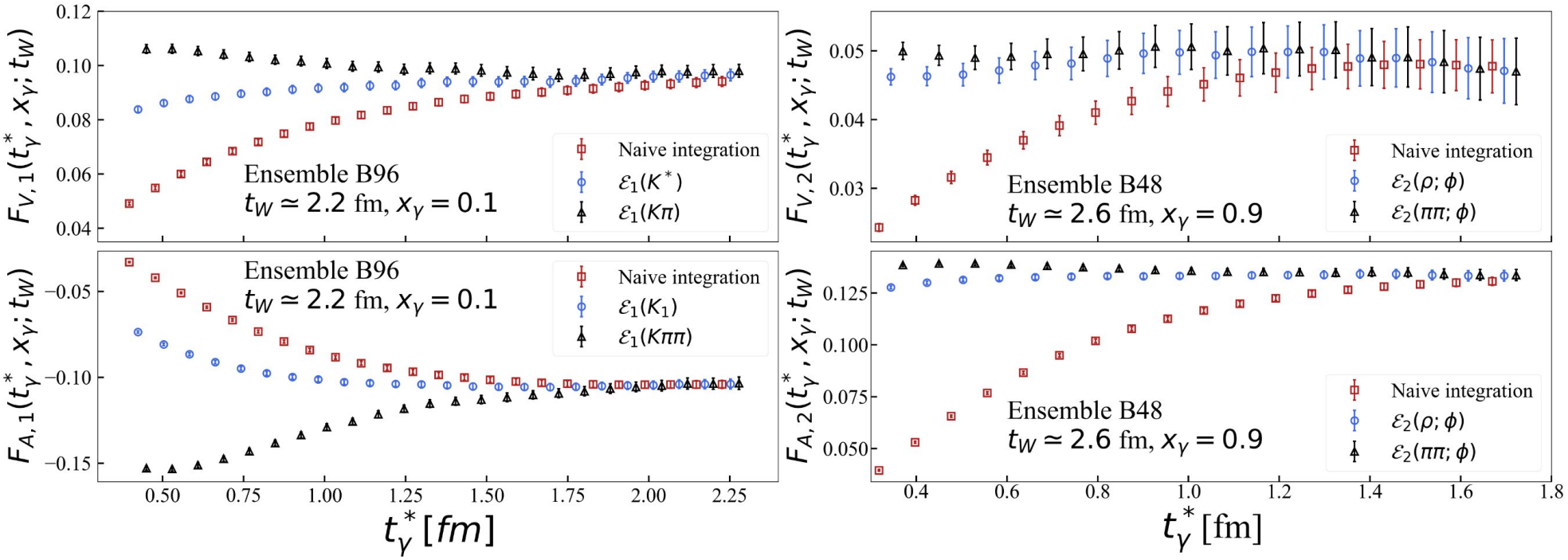
<u>Disconnected emission:</u>

Tested on given x_γ~0.7, B96 ensemble

One-end trick to improve evaluation

(magnification by a factor of 10-50)

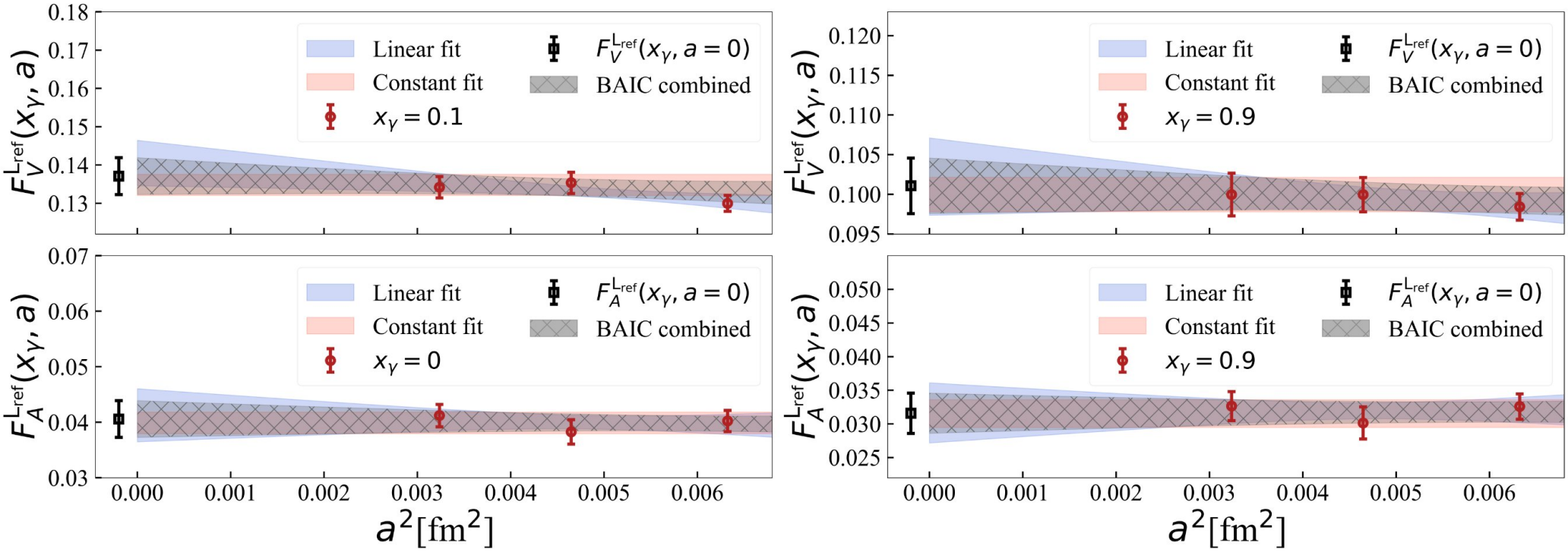
Analytic integration on t_γ



- Allows to speed up the integration convergence [Tuo et al]
- Projecting on physical on-shell helicity simplifies the continuation

→ remove $K\gamma$ state, spectral decomposition starts with K^* and K_1

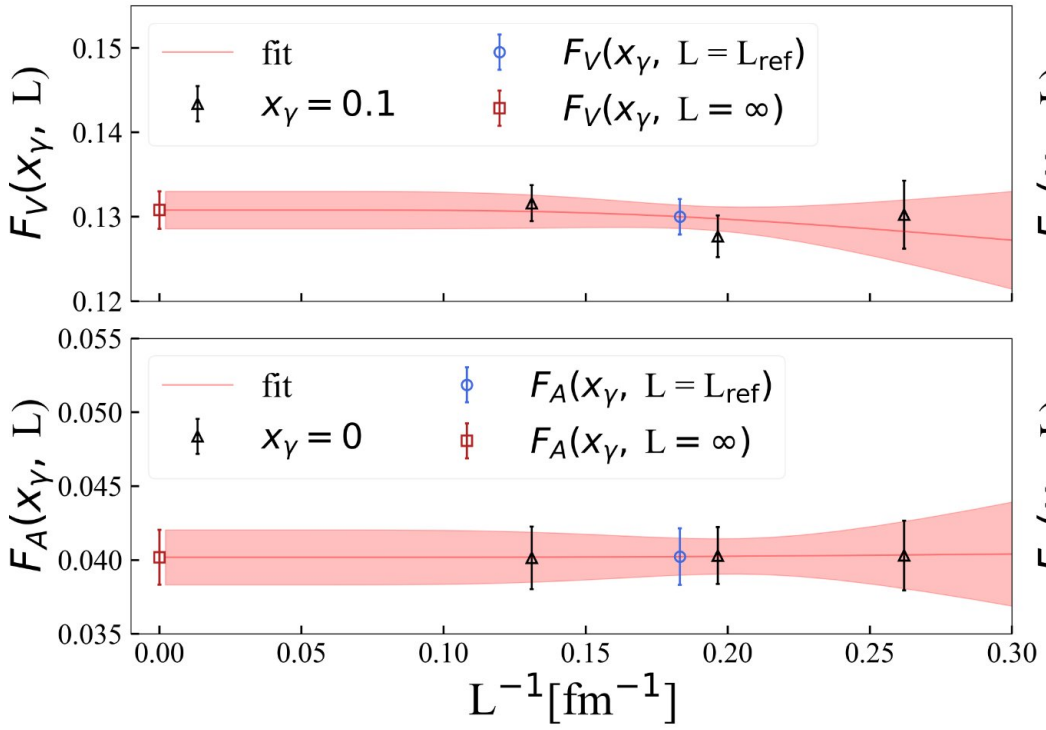
Continuum limit extrapolation

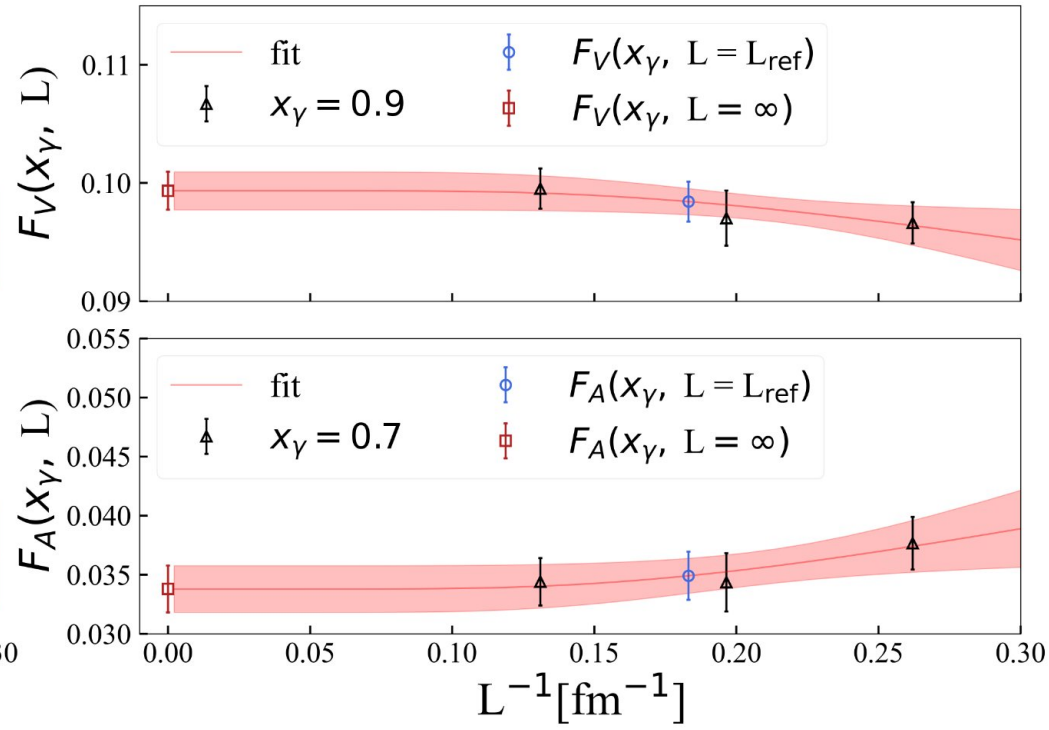


Selection of extrapolation for a few values of x_γ and for *V*, *A*

Constant and a^2 ansatze combined with BAIC, mild effects observed

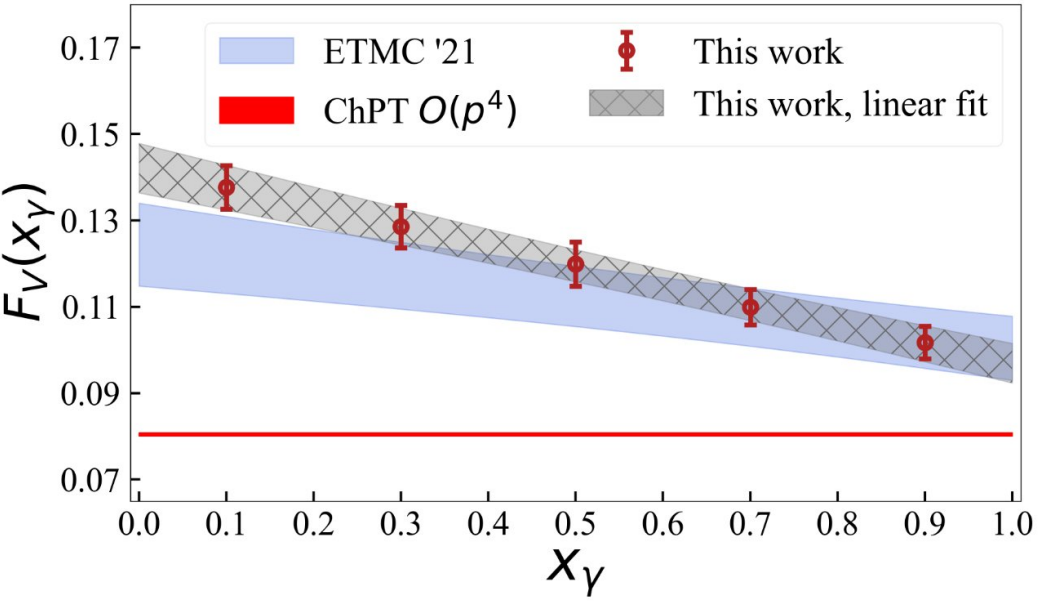
Infinite volume limit extrapolation

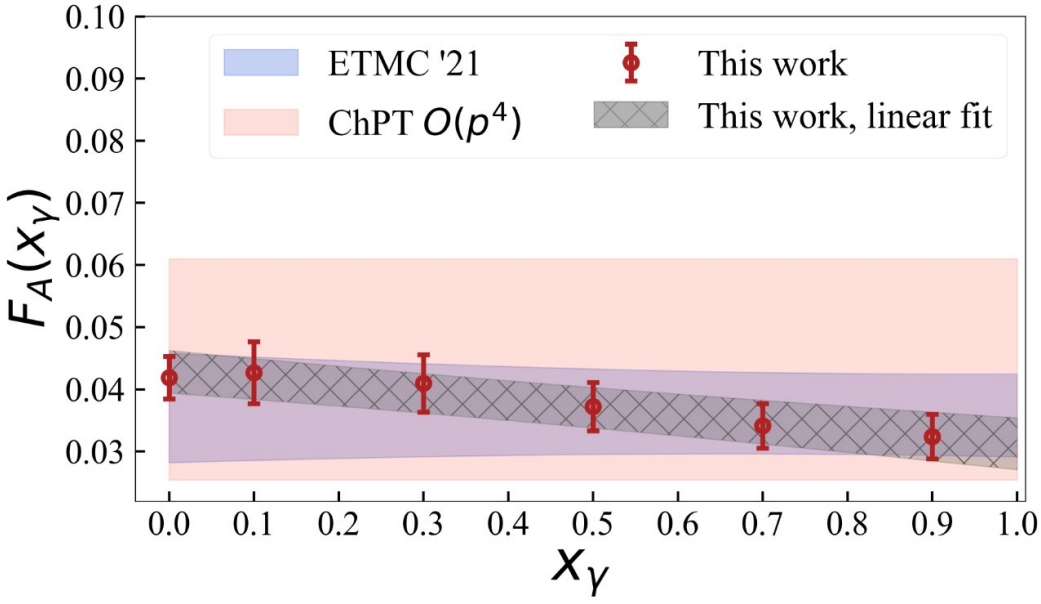




- No photon present in the calculation, no power correction expected
- Indeed mild exponential effect observed
- Fit ansatz $\exp(-M_\pi L)$

Final results





- Substantially more precise than previous ‘21 study
- Well compatible for *A*, reasonably compatible for *V*
- Corrections to the LO ChPT prediction clearly visible

Pheno comparisons $\Gamma[K \to e\nu_e\gamma]$

KLOE has performed a binned analysis of [*EPJC 64 (2009) 627-636*]

$$\Delta R_\gamma^i = \int_{E_\gamma^i}^{E_\gamma^{i+1}} dE_\gamma \, \frac{1}{\Gamma(K^- \to \mu^- \bar{\nu}_\mu(\gamma))} \frac{d\Gamma(K^- \to e^- \bar{\nu}_e \gamma)}{dE_\gamma}$$

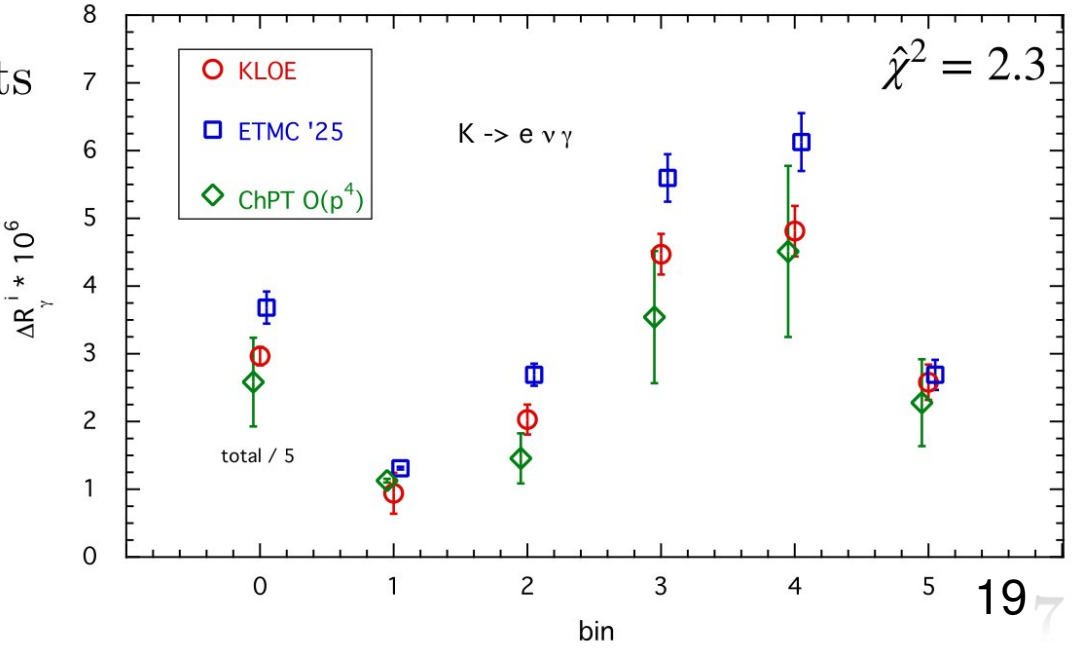
- They impose kinematical constraints

  $|p_e| > 200$ MeV,

  $E_\gamma \in [10,250]$ MeV

- Dominant contribution

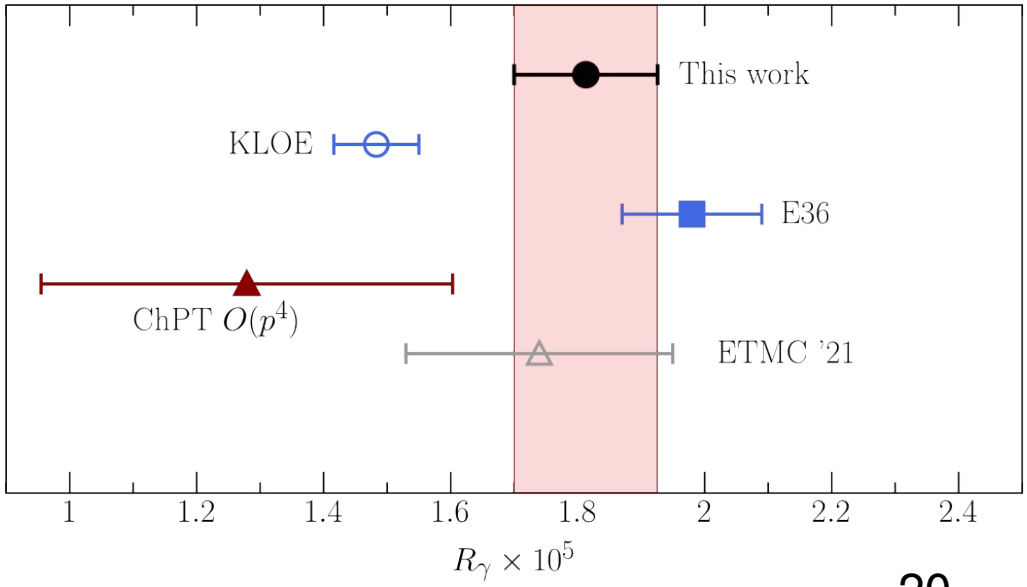
  $\propto F^+ = F_V + F_A$



Pheno comparisons, integrated $\Gamma[K \to e\nu_e\gamma]$

- KLOE and the E36 experiment at J-PARC have provided an integrated analysis [*P.Lett.B 843 (2023) 138020*]
- Our estimate of R_γ include all the contributions at $O(\alpha_{em})$

$$R_\gamma = \int_{10\ \mathrm{MeV}}^{250\ \mathrm{MeV}} dE_\gamma \, \frac{1}{\Gamma(K^- \to \mu^- \bar{\nu}_\mu(\gamma))} \frac{d\Gamma(K^- \to e^- \bar{\nu}_e \gamma)}{dE_\gamma}$$



[No time to discuss discrepancies with E787, ISTRA+ and OKA in the muon channel]

Final state radiation?

arXiv:2510.26993v1 [hep-lat] 30 Oct 2025

Lattice Calculation of Light Meson Radiative Leptonic Decays

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(Dated: November 3, 2025)

Abstract

In this work, we perform a lattice QCD calculation of the branching ratios and the form factors of radiative leptonic decays $P \to \ell\nu_\ell\gamma$ ($P = \pi, K$) using $N_f = 2+1$ domain wall fermion ensembles generated by the RBC and UKQCD collaborations at the physical pion mass. We adopt the infinite-volume reconstruction (IVR) method, which extends lattice data to infinite volume and effectively controls the finite-volume effects. This study represents a first step toward a complete calculation of radiative corrections to leptonic decays using the IVR method, including both real photon emissions and virtual photon loops. For decays involving a final-state electron, collinear radiative corrections, enhanced by the large logarithmic factors such as $\ln(m_\pi^2/m_e^2)$ and $\ln(m_K^2/m_e^2)$, can reach the level of $O(10\%)$ and are essential at the current level of theoretical and experimental precision. After including these corrections, our result for $\pi \to e\nu_e\gamma$ agrees with the PIBETA measurement; for $K \to e\nu_e\gamma$, our results are consistent with the KLOE data and exhibit a 1.7σ tension with E36; and for $K \to \mu\nu_\mu\gamma$, where radiative corrections are negligible, our results confirm the previously observed discrepancies between lattice results and the ISTRA/OKA measurements at large photon energies, and with the E787 results at large muon–photon angles.

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Recent calculation based on RBC/UKQCD configurations

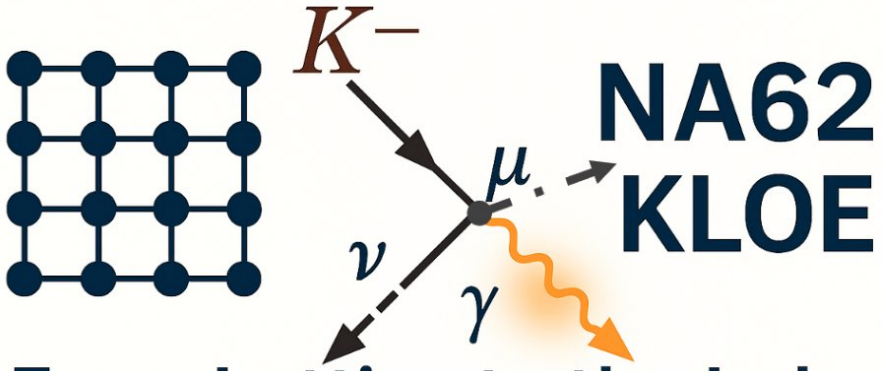
[P.Boyle et al, arXiv:2510.26993, 30/10/2025]

- Good compatibility with our work
- Suggested issues in the treatment of final-state electron radiation
- Might contribute to explain the discrepancy between experiments

Advertisement: Topical workshop @LNF

Bring together theorists & experimentalists on radiative kaon decays

@Frascati INFN National Laboratory (LNF) 24-25 March 2026



From Lattice to the Lab:
Illuminating Kaon Radiative Decays

Conclusions & Outlook

Radiative Kaon decay into leptons

- Room for novel determination of V_{us} CKM matrix element
- Experimental inconsistencies
- Preliminary LQCD calculation at non-physical pion mass, PRD 2021

This work

- Physical pion mass, disconnected contribution included
- Continuum & infinite volume limit extrapolation, higher statistics
- Results in tension with KLOE measurements

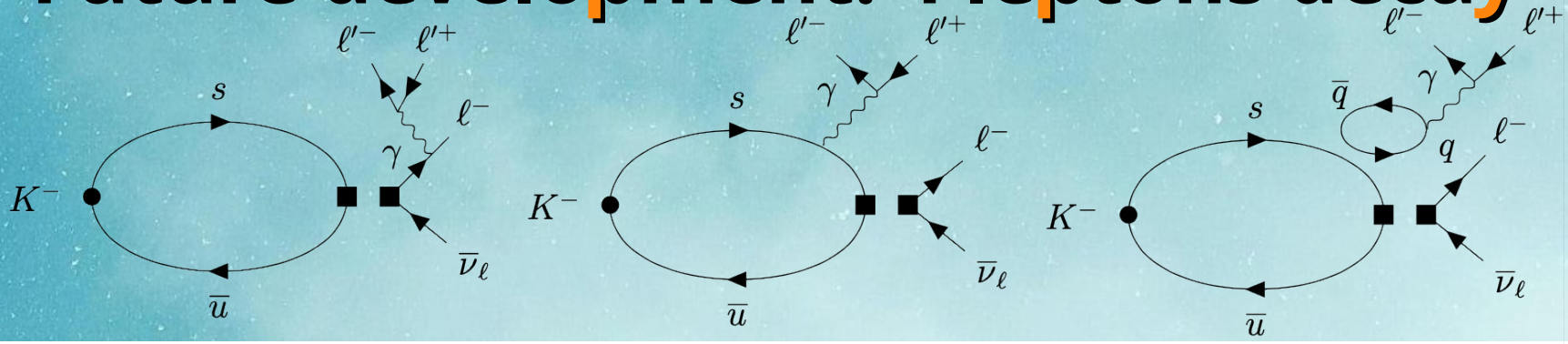
Perspectives

- Heavy meson decay (cfr Teseo contribution for JLQCD/UKQCD yesterday)
- Virtual decay into 4leptons at physical pion mass via HLT method

THANK YOU FOR YOU ATTENTION!

BACKUP SLIDES

Future development: 4 leptons decay



Suppressed by α_{em}^2 in SM, probe for NP (heavy neutral leptons, non universal gauge bosons...)

- No further correlators are needed, by using the 3D method
- Two more form factors to be extracted, $H_1,\ H_2$
- Calculation proceeds easily if the photon virtuality $k_\gamma^2 < (2M_\pi)^2$
- Analytic continuation issues if $(2M_\pi)^2 < k_\gamma^2 < M_K^2$! Integration in x_γ does not converge
- Preliminary works with single volume and lattice spacing, heavy pion mass ~ 300 *MeV*
  RM123-Soton [G.Gagliardi et al, PRD 105 (2022)], RBC/UKQCD [X.Tuo et al, PRD 105 (2022)]
- In previous work no issue present, since $2M_\pi > M_K$