Kaon leptonic and semileptonic decays with $N_f = 2 + 1 + 1$ HISQ fermions

Ramón Merino Rojas

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Fermilab Lattice and MILC collaboration
Universidad de Granada

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Urs Heller, Andreas Kronfeld, William Jay

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First row CKM unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

We will explore the unitarity in the first row

$$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \sim 0 - 1$$

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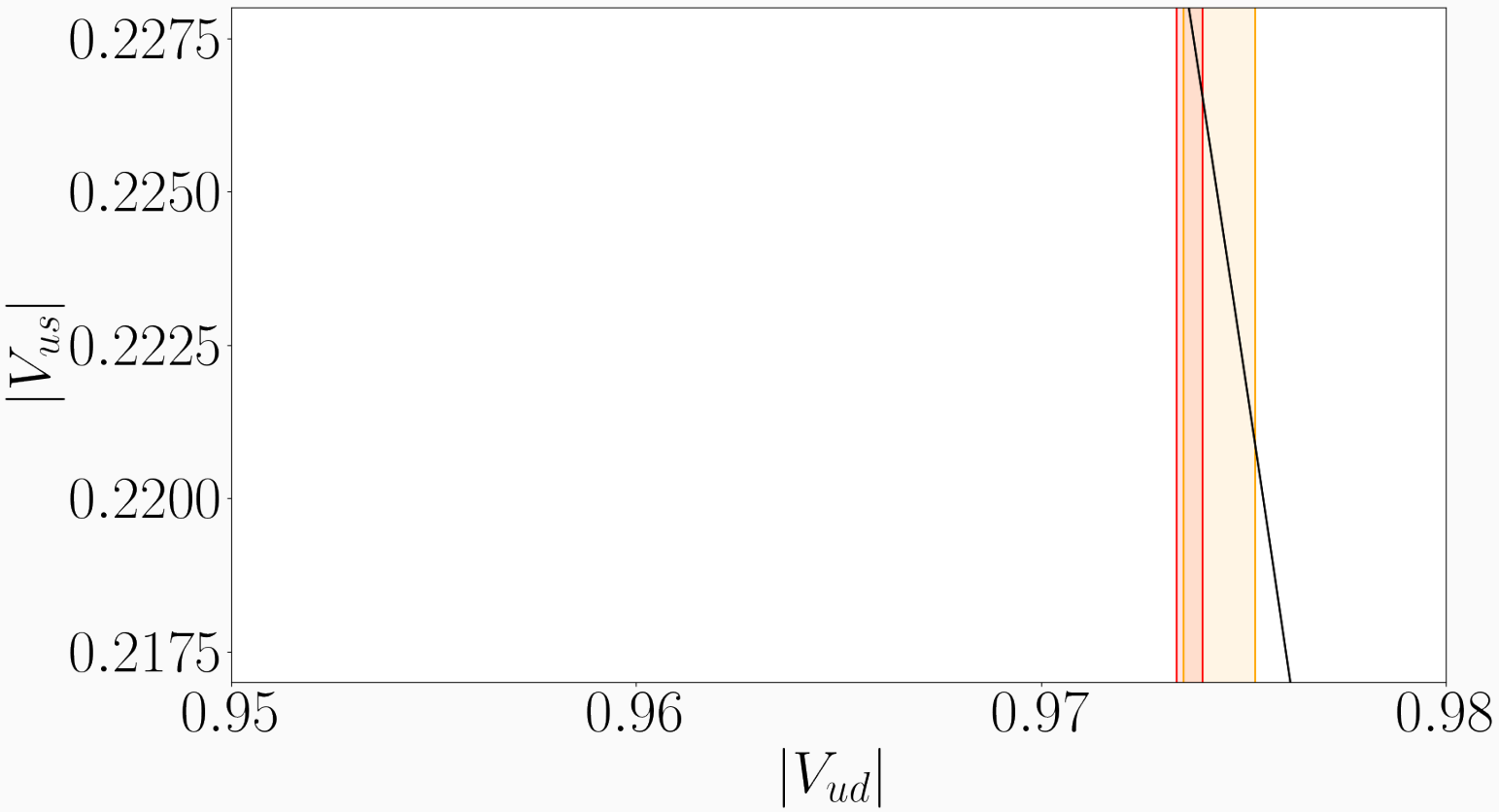
$K_{\ell 3} \equiv K \to \pi \ell \nu_\ell \qquad K_{\ell 2}(\pi_{\ell 2}) \equiv K(\pi) \to \ell \nu_\ell$

*QED corrections needed to match to experiment: Progress on replacing ChPT estimates by lattice results, see Matteo Di Carlo talk at KAON2025

Current status (using 2208.11707, 1712.09262, 1809.02827)



• Unitarity band • $0^+ \to 0^+$ beta decays • Neutron decay

2208.11707 Cirigliano *et al.*, Physics Letters B, Volume 838

1712.09262 FNAL/MILC, PRD98. 1809.02827 FNAL/MILC, PRD99

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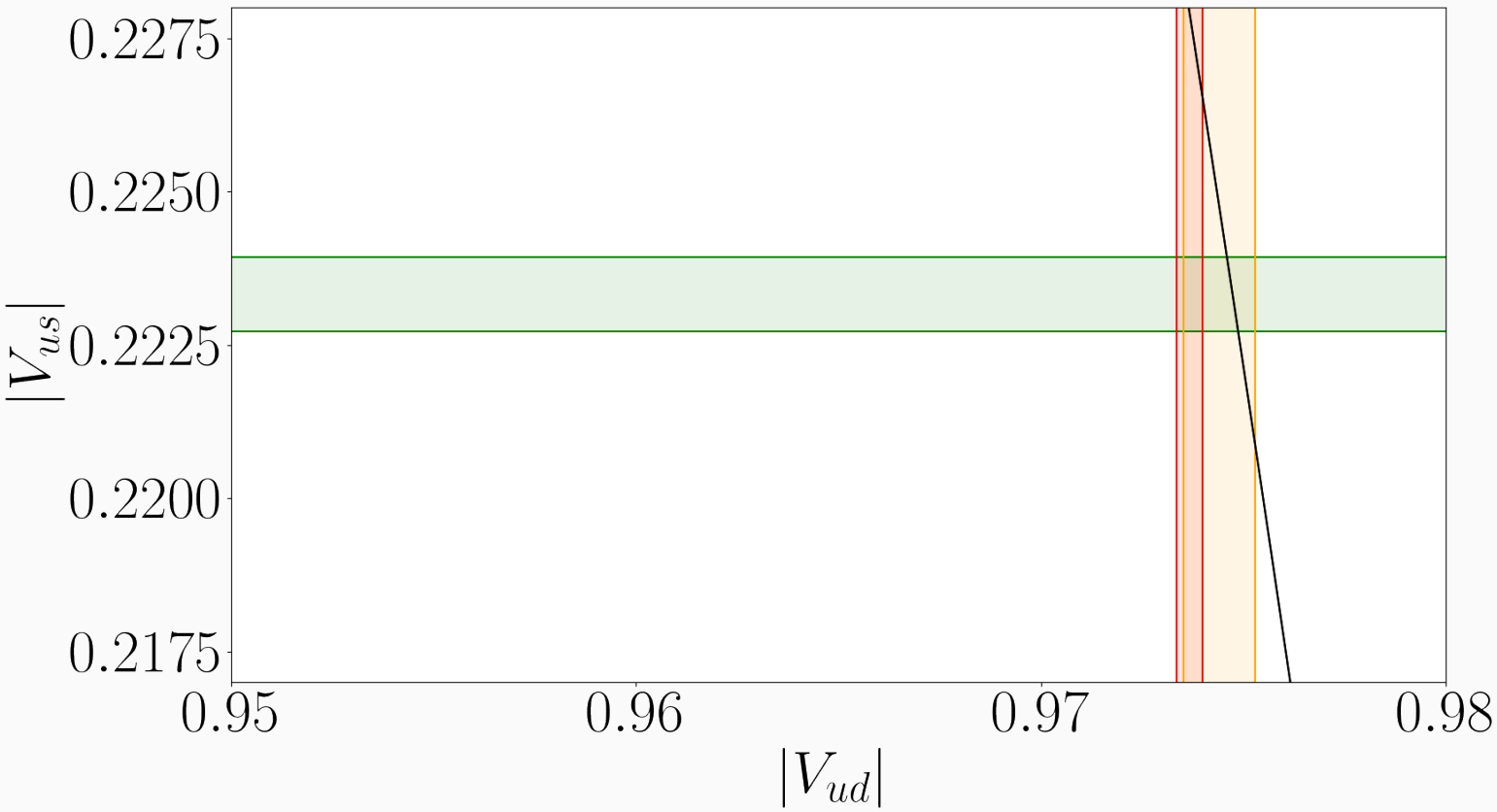

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- $K_{\ell 3}$

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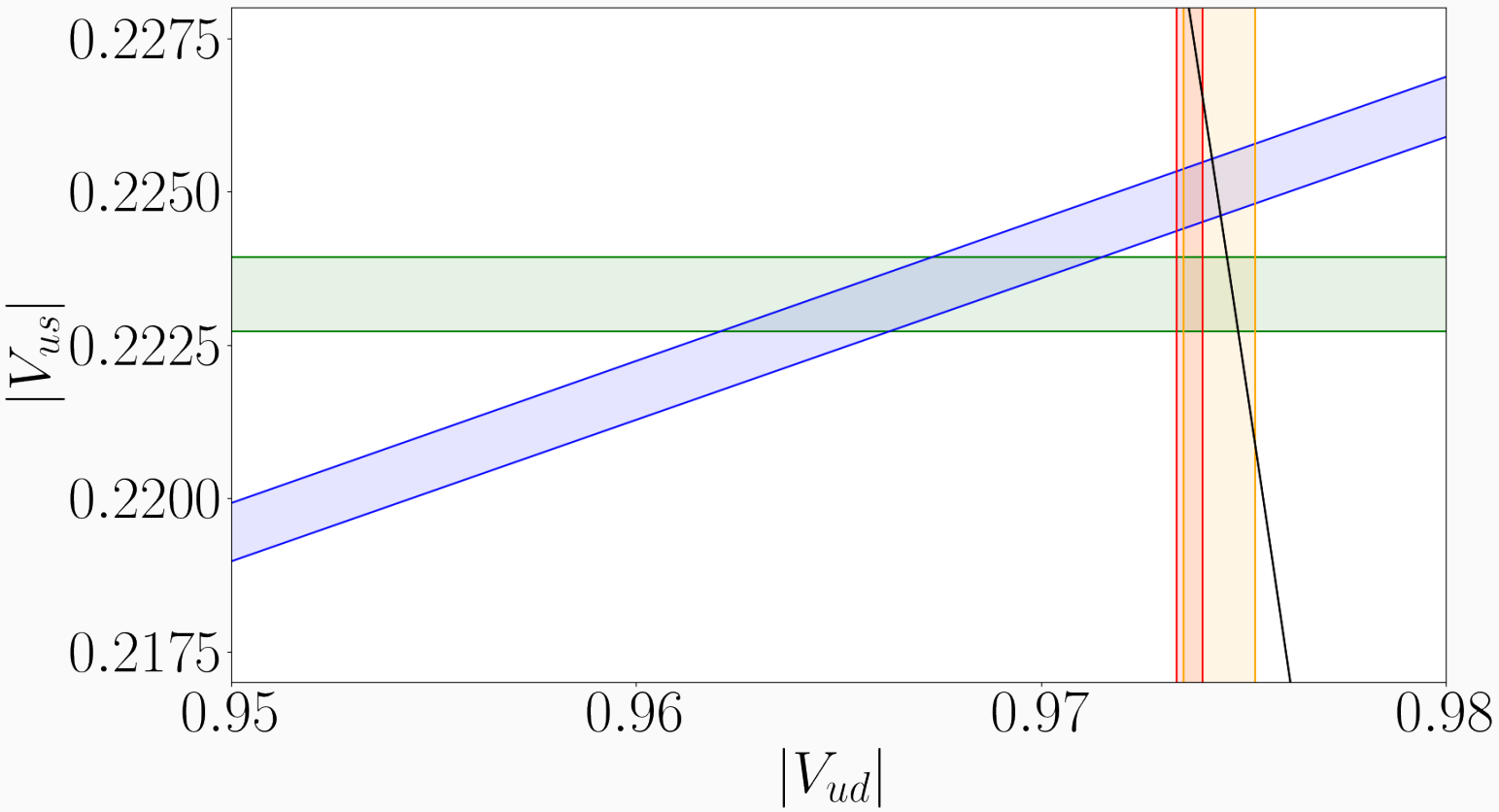

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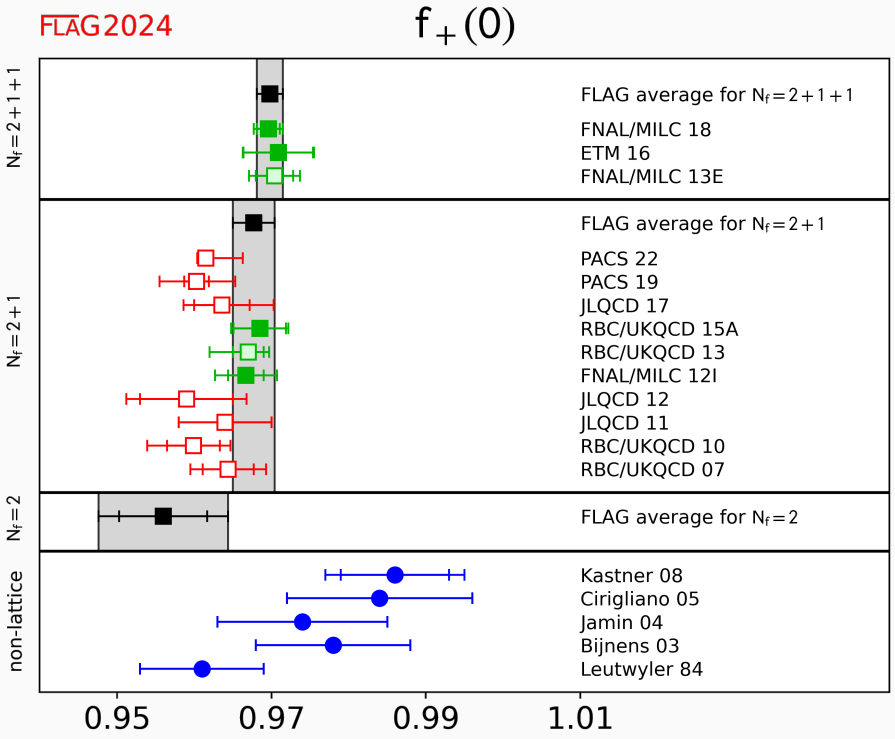
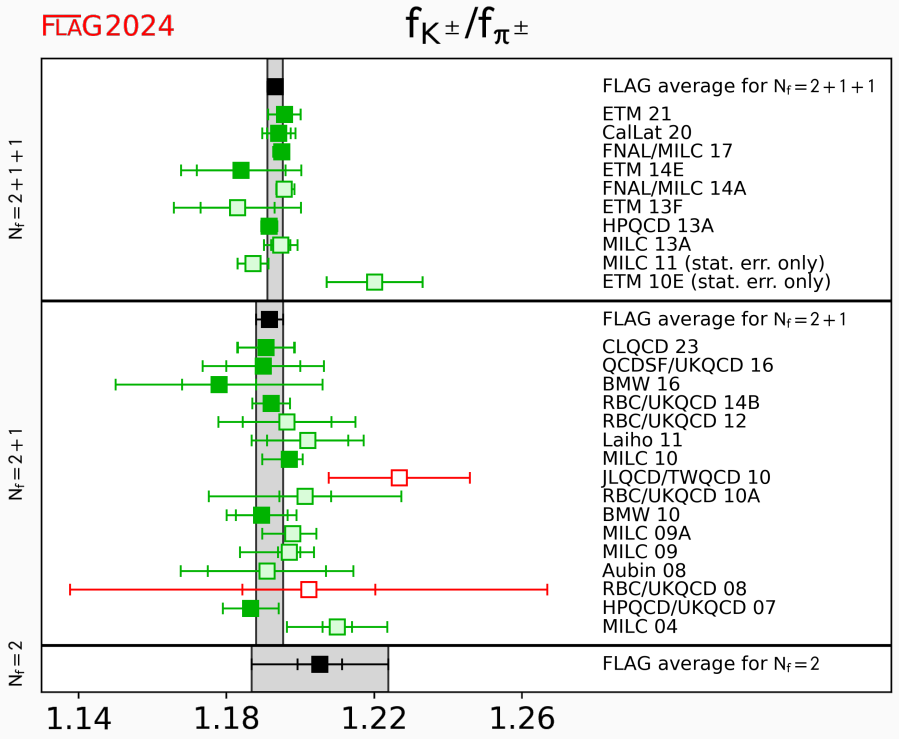
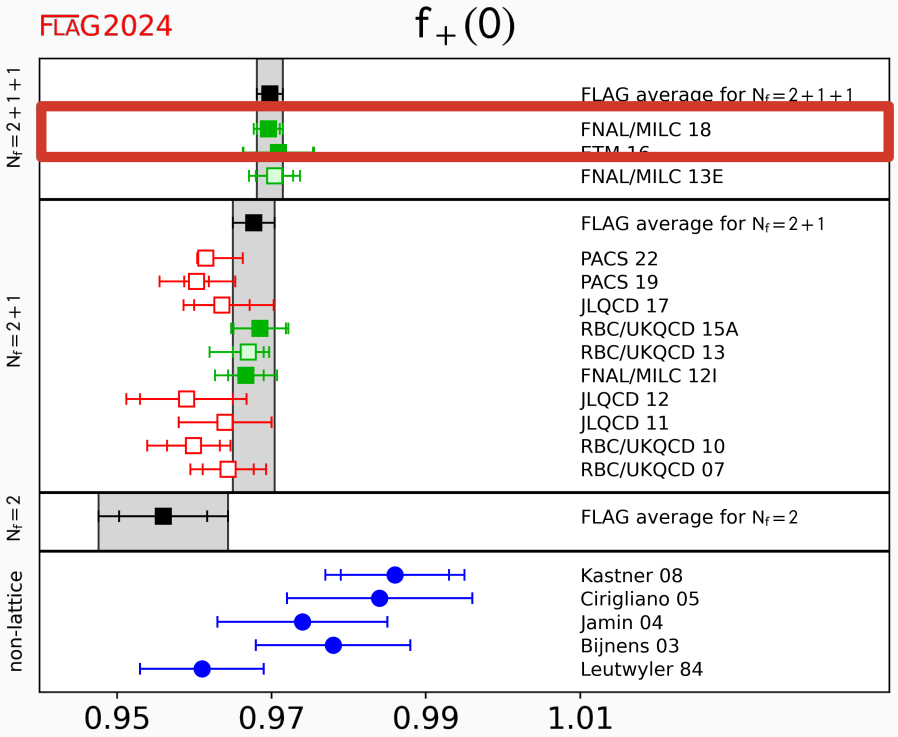
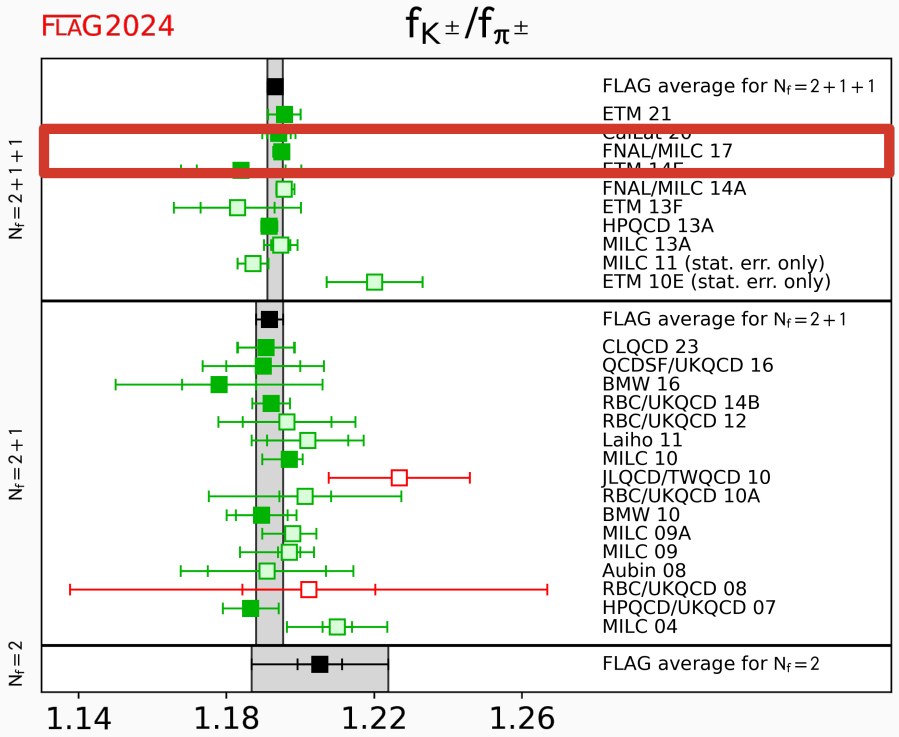




Current status: FNAL/MILC Collaboration





$$f_+(0) = 0.9696(15)(12)$$

1809.02827

FNAL/MILC, PRD99

$$f_K/f_\pi = 1.1980(12)(^{+5}_{-15})$$

1712.09262

FNAL/MILC, PRD98

Semileptonic form factor

Ward-Takahashi Identity

$$f_+^{K\pi}(0) = f_0^{K\pi}(0) = \frac{m_s - m_\ell}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle_{q^2=0}$$

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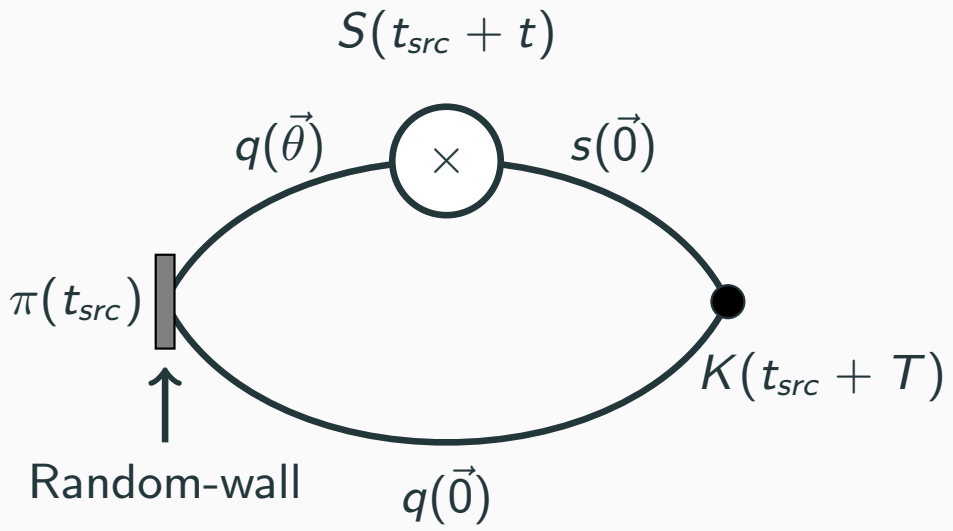

Twisted Boundary Conditions:

tune θ to get $q^2 = 0$

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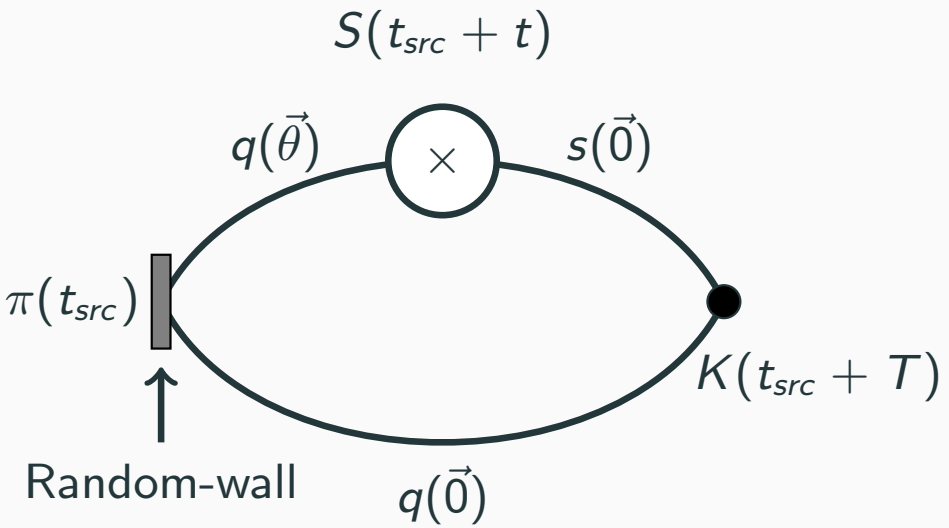
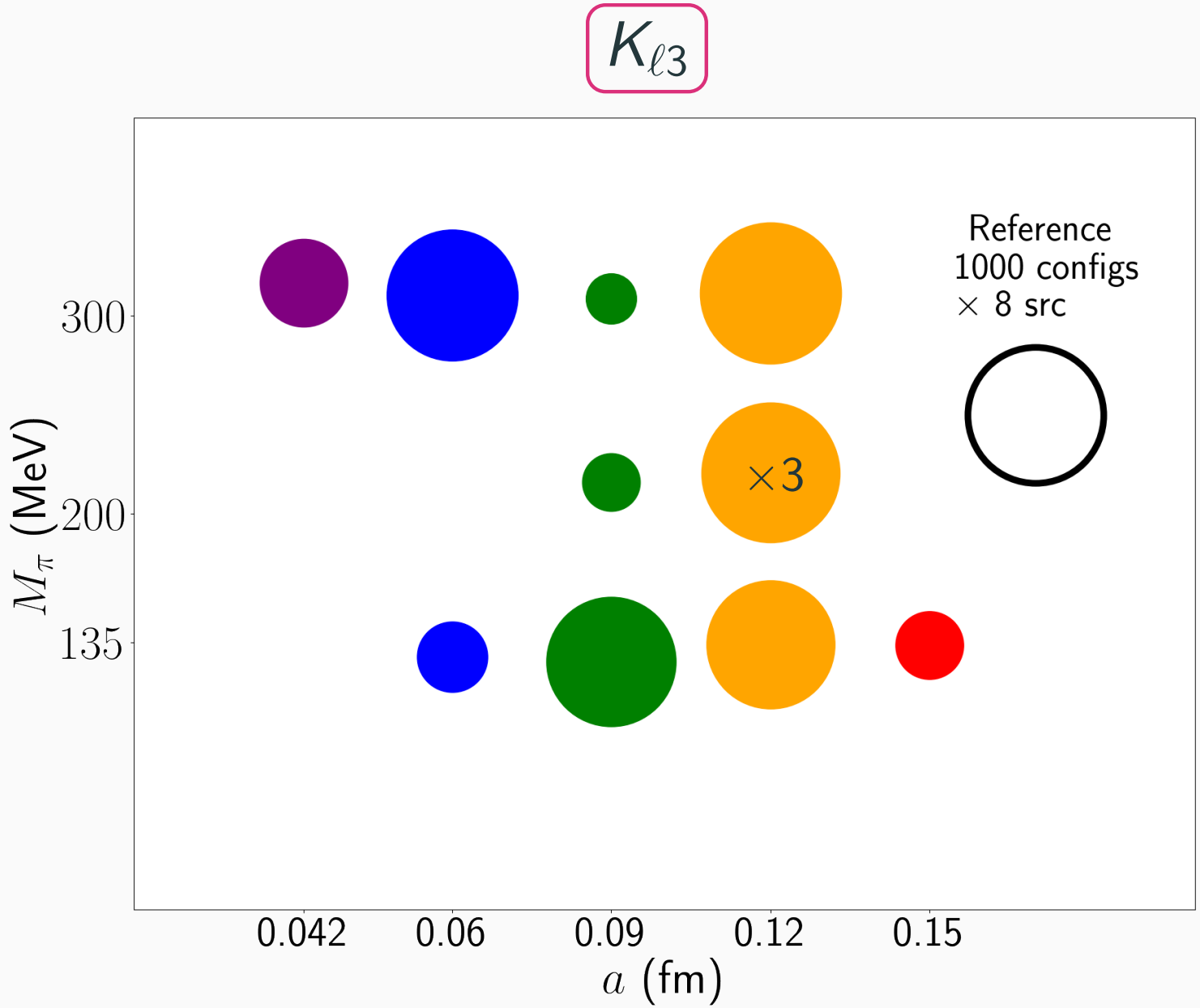


Twisted Boundary Conditions:

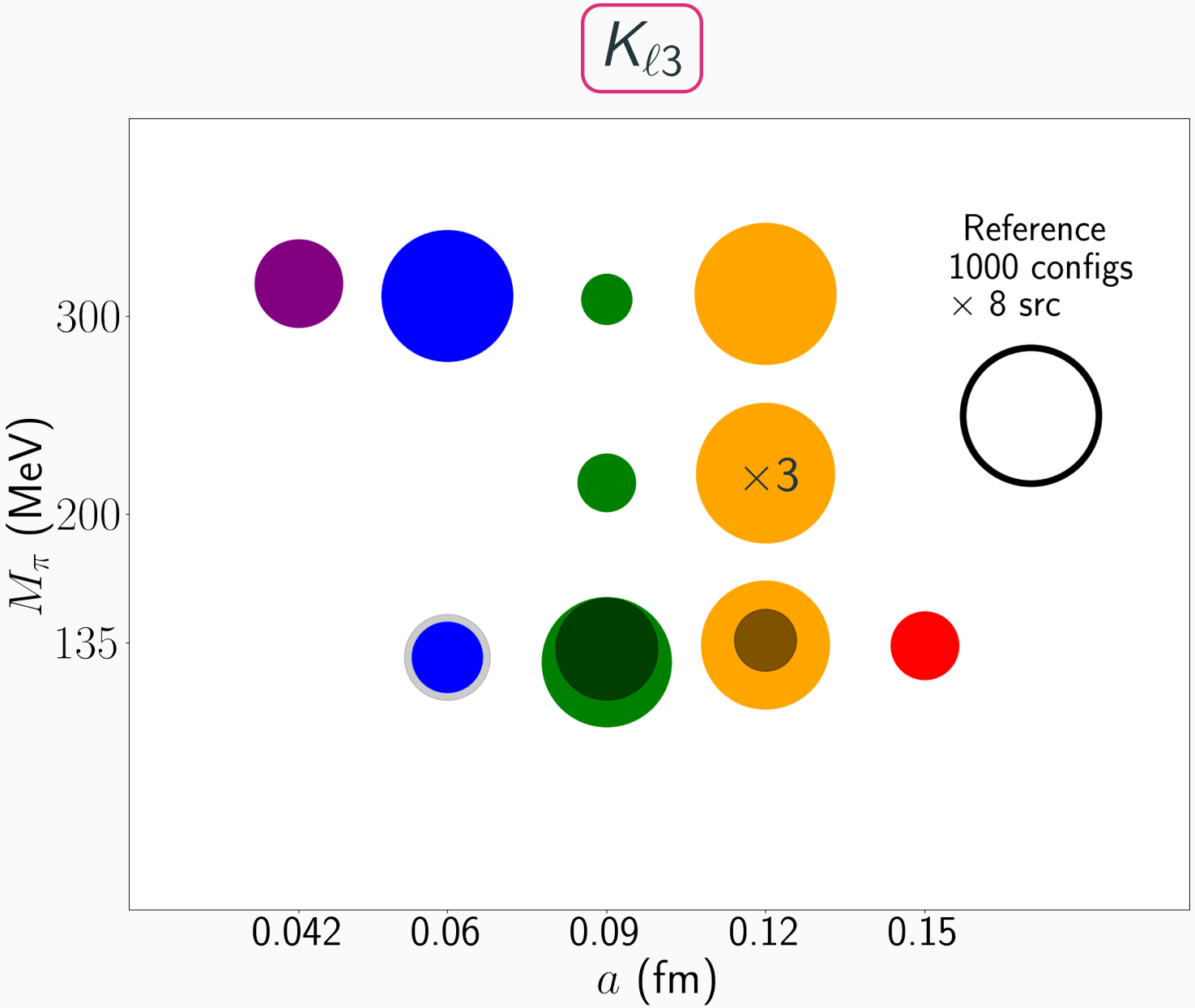
tune θ to get $q^2 = 0$

Local scalar density: avoid renormalization factors

FNAL/MILC 2018: Ensembles HISQ 2+1+1



FNAL/MILC 2025: Ensembles HISQ 2+1+1



Semileptonic form factor: Data analysis

Autocorrelations

- Re-scaling of the covariance matrix with binsize $= 4$
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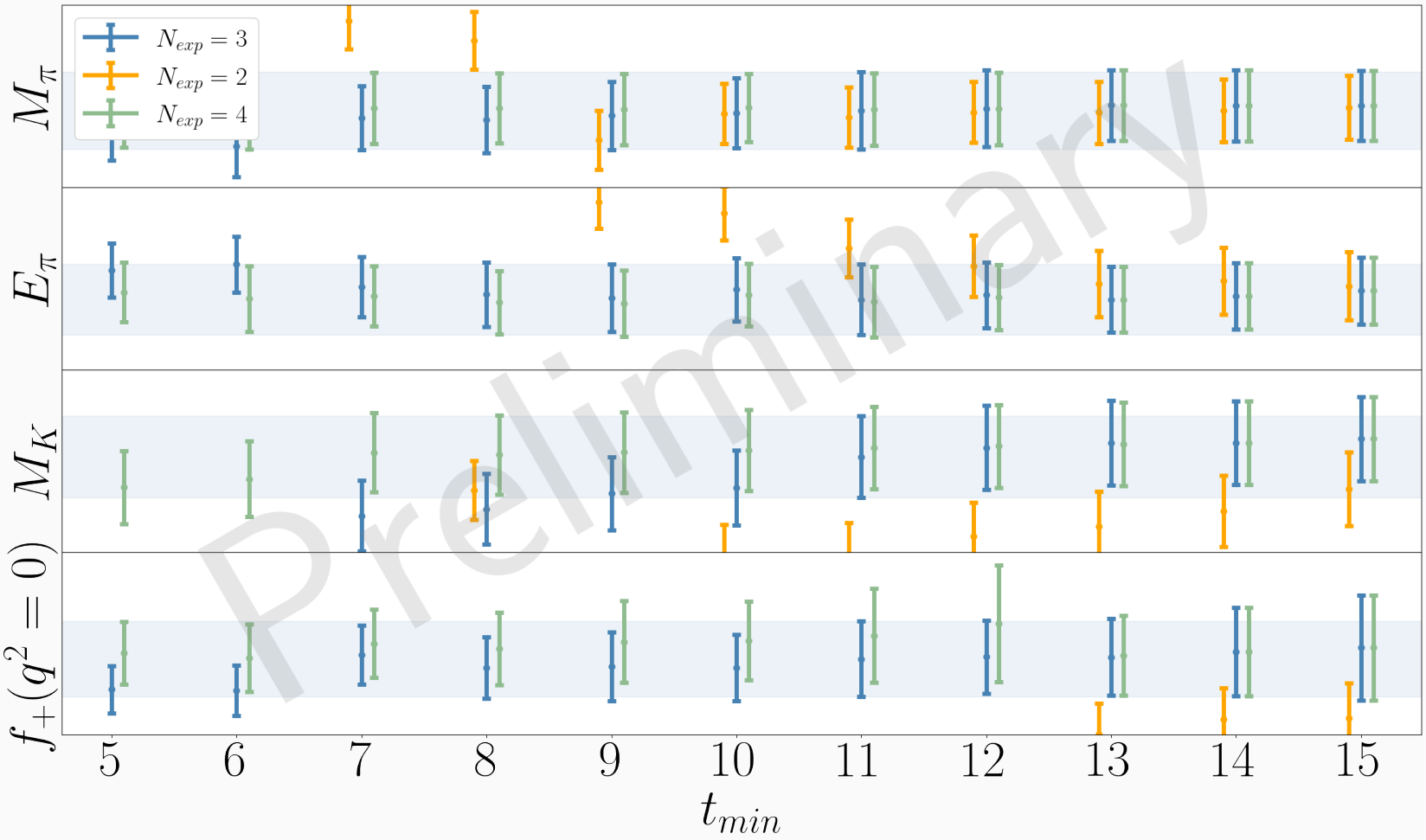
Scale setting

- $r_1 \to w_0$ with w_0^{phys} from M_Ω (2509.14367 Bazavov *et al.*)

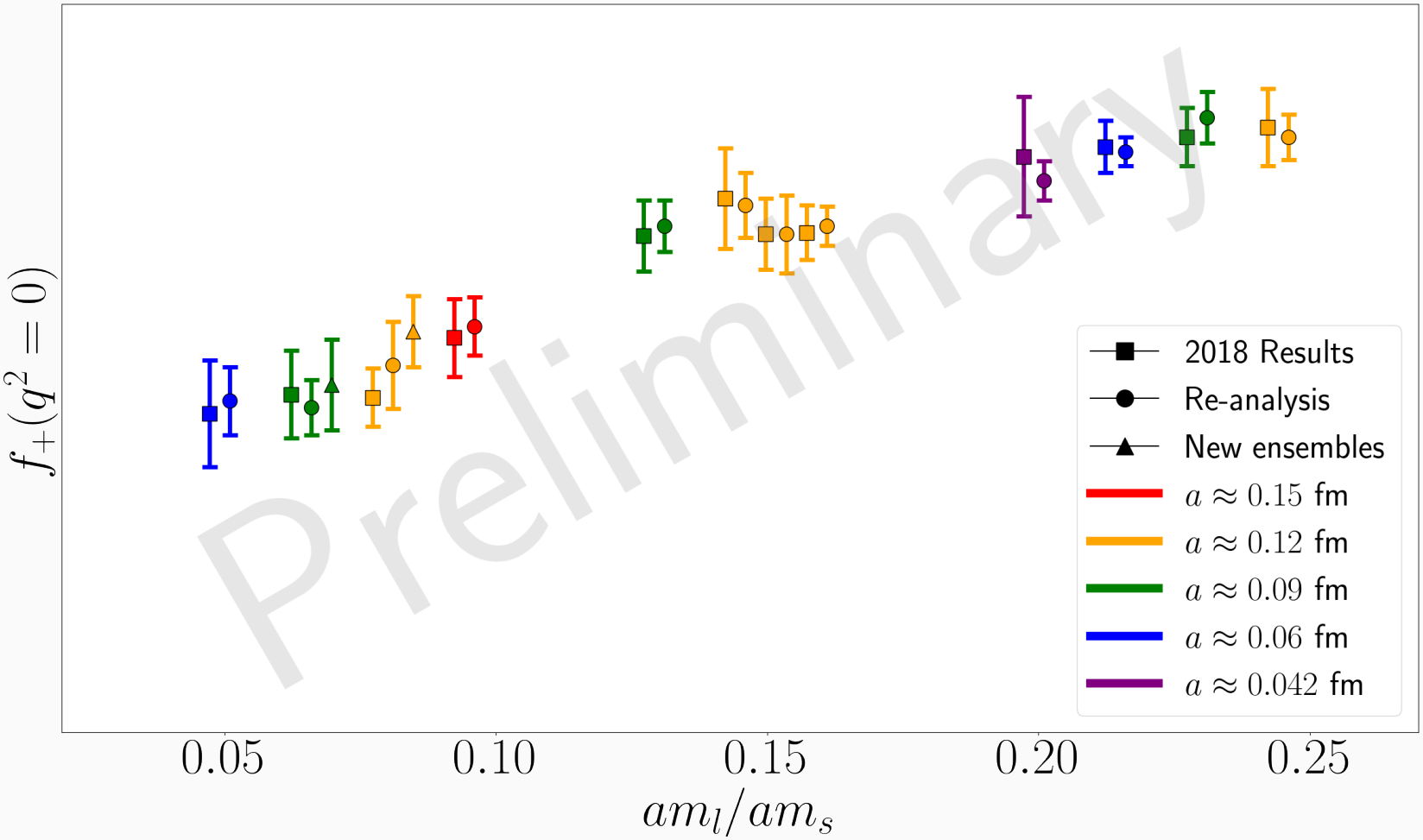
Semileptonic form factor: Correlator fits stability

Physical mass ensemble at $a \simeq 0.06$ fm



$N_{exp} = 2N = N_{\text{regular}} + N_{\text{oscillating}}$

Semileptonic form factor: Preliminary results



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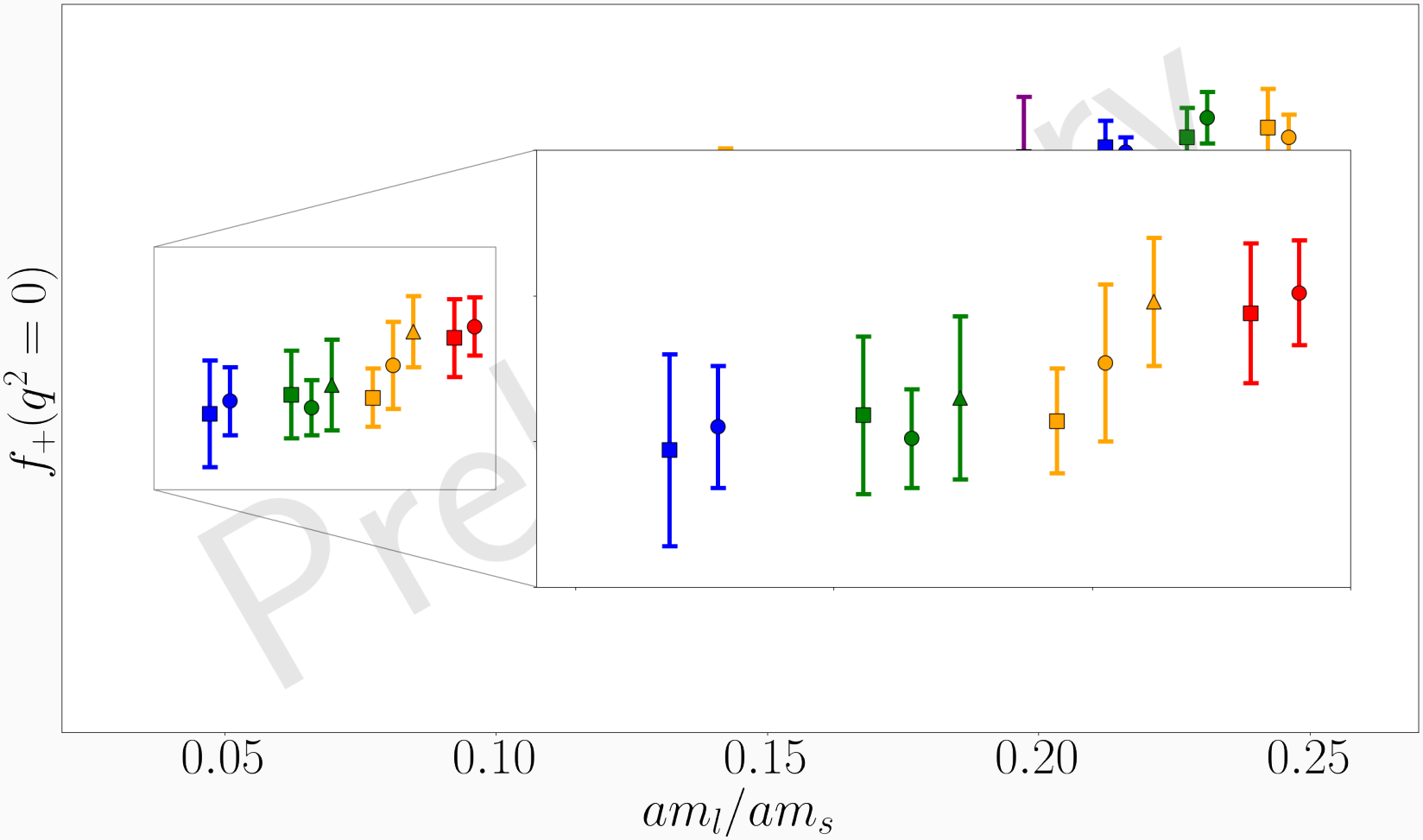

Staggered Chiral Perturbation Theory (SChPT)

$$f_+^{K\pi}(0) = 1 + f_2^{\mathrm{SChPT}} + f_4^{\mathrm{cont}} + g_{1,a} + (m_\pi^2 - m_K^2)^2 \left[C_4 + g_{2,a} + h_{m_\pi} \right]$$

Staggered Chiral Perturbation Theory (SChPT)

$(\mu,\ \delta_V,\ \delta_A)$

NLO

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$\delta_{A,V}$: hairpin parameters

$$M_{xy} = \mu(m_x + m_y)$$

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NNLO

(L_{1-8})

L_{1-8}: $\mathcal{O}(p^4)$ LECs

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Higher order discretization effects

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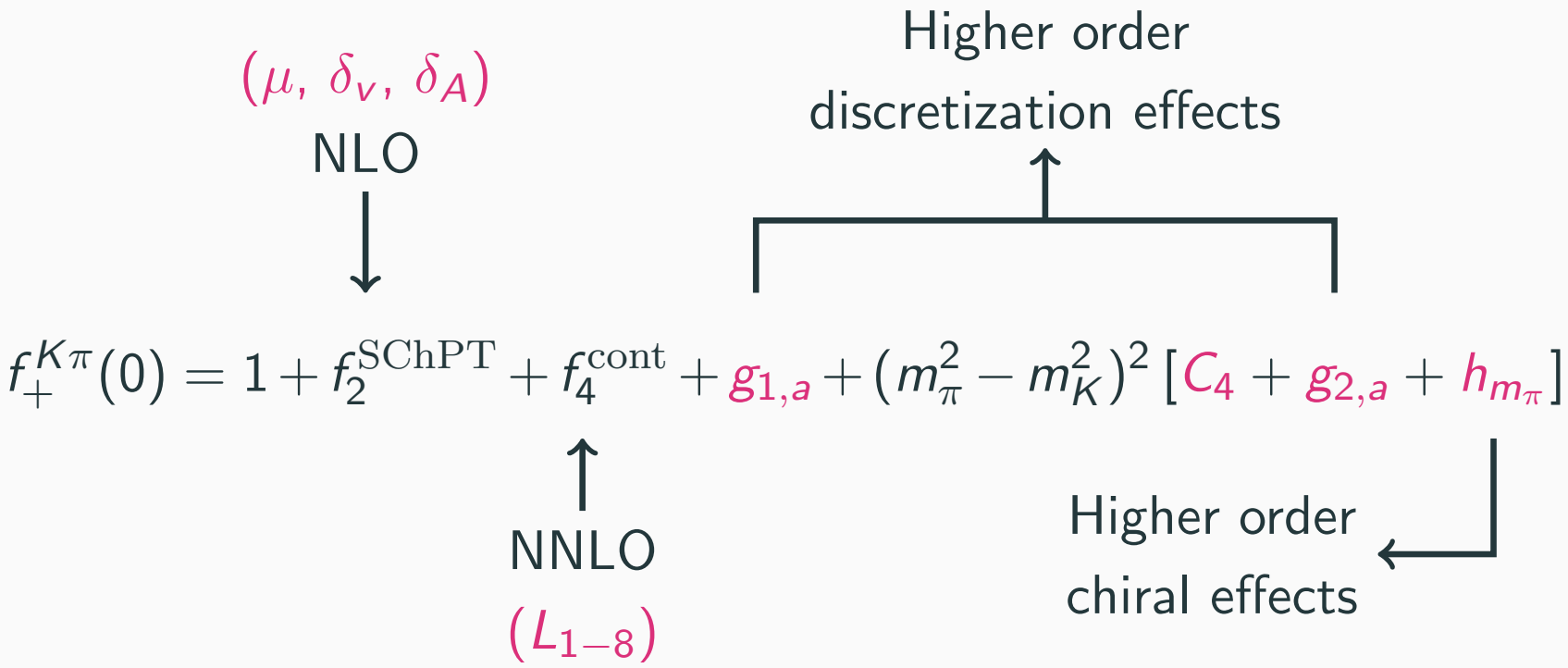

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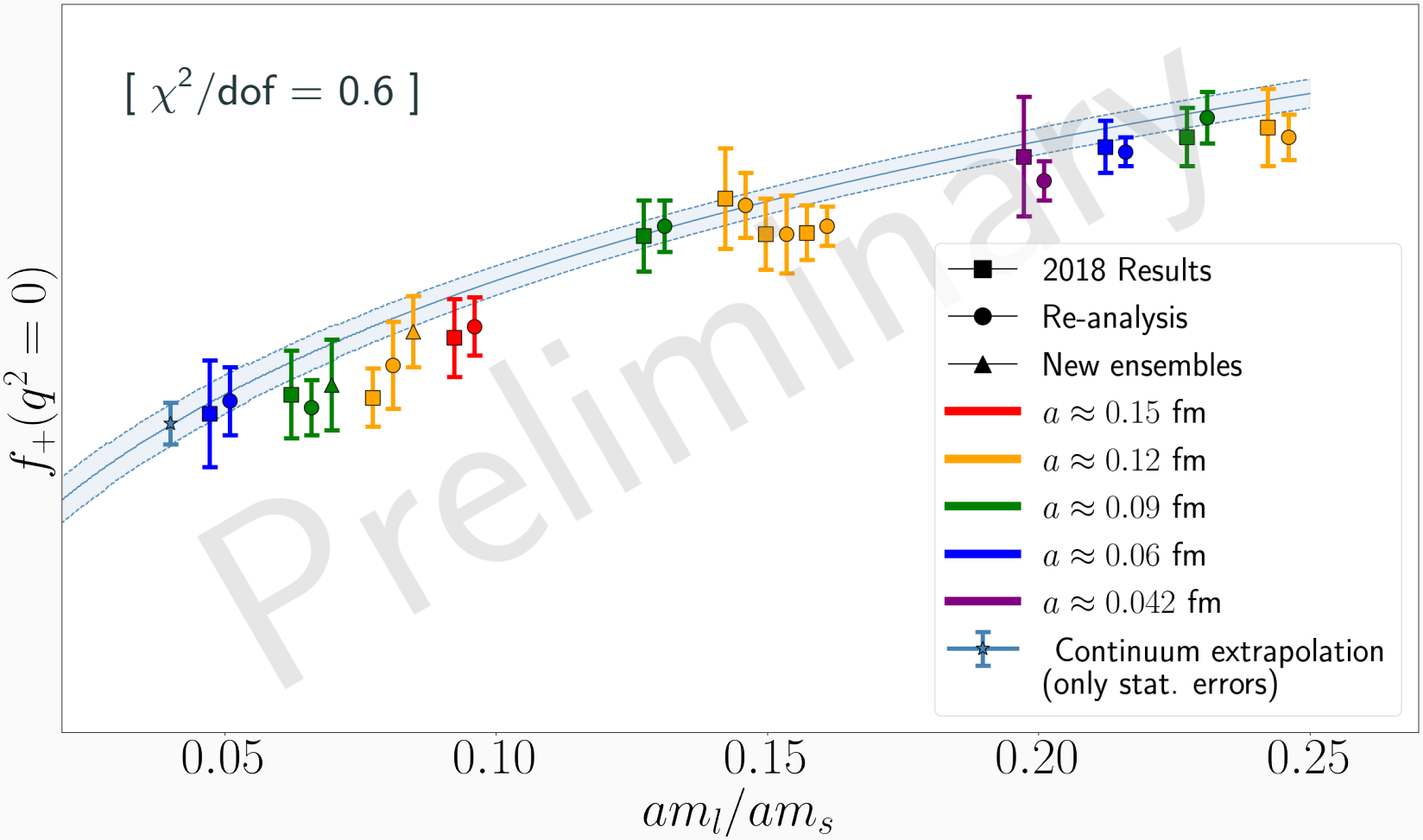
NNLO

(L_{1-8})

Higher order chiral effects

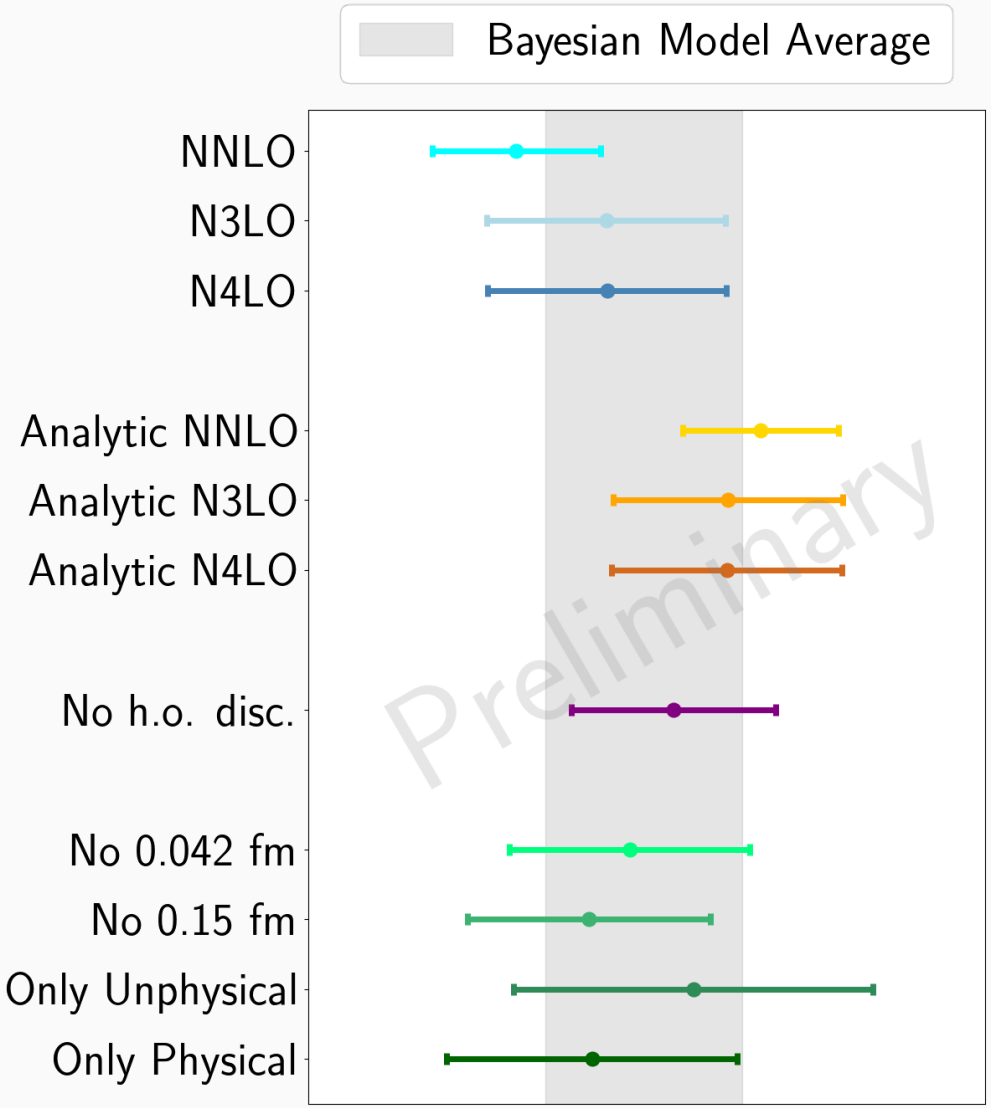
Some Low Energy Constants (LECs) are also present in the decay constants analysis

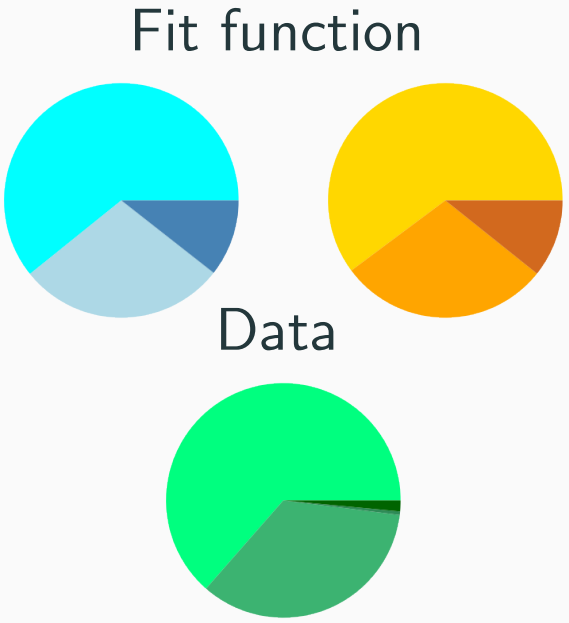
Semileptonic form factor: Central fit

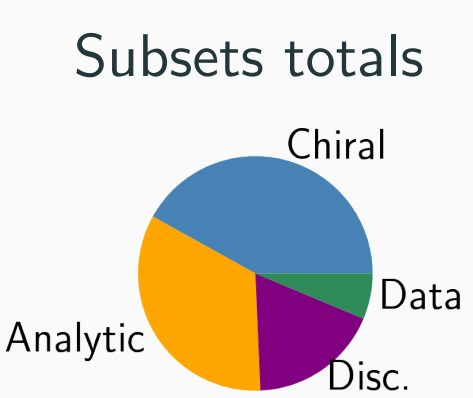


Includes N3LO, N4LO chiral terms $+\mathcal{O}(\alpha_s a^2, \alpha_s^2 a^2, a^4)$

Semileptonic Form Factor: Bayesian Model Average







Decay constants: 2017 analysis

Data

- Partially quenched pseudoscalar meson masses and decay constants
- $>$ 20 ensembles with 6 lattice spacings $a \in [0.15, 0.03]$ fm

Continuum fit for f_K/f_π

- Continuum extrapolation of only physical mass ensembles
- Corrections due to quark mass mistunings

Scale-setting

- p4s method and f_π

Decay constants:

Data

New Analysis

- New light-m_s ensembles at $a \simeq 0.12$, 0.09 and 0.06 fm
- New physical mass ensembles at $a \simeq 0.12$ and 0.09 fm

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- NLO SU(3) SChPT + NNLO ChPT + higher order chiral and disc.
- We can use the whole set of ensembles

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Continuum fit for f_K/f_π

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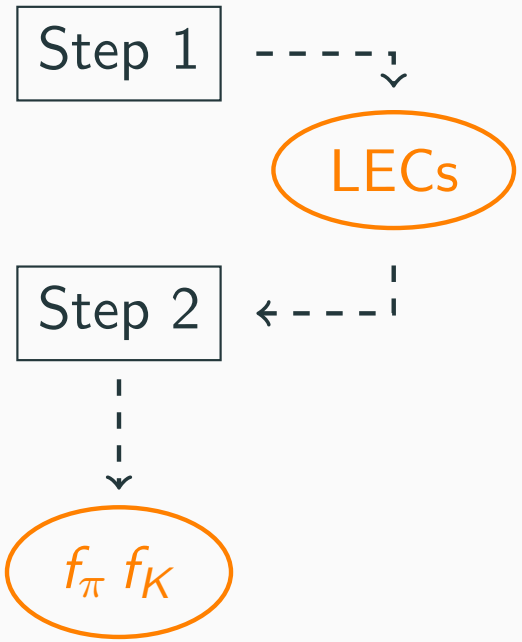
Scale-setting

- We plan on updating to w_0 and M_Ω (2509.14367 Bazavov *et al.*)

What's the plan?

Leptonic analysis



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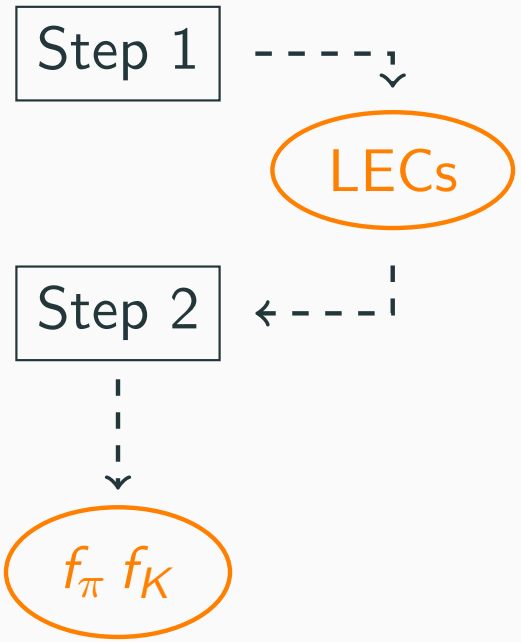
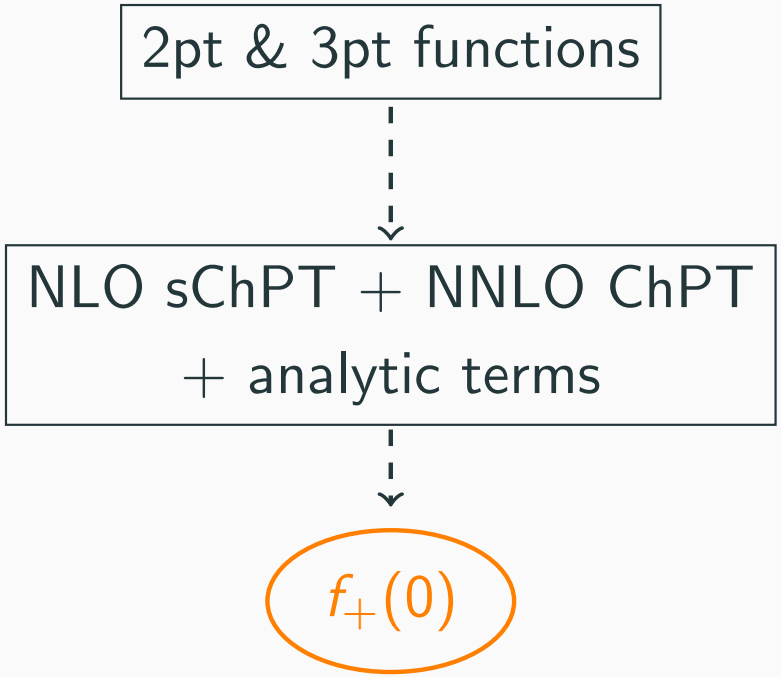


Semileptonic analysis



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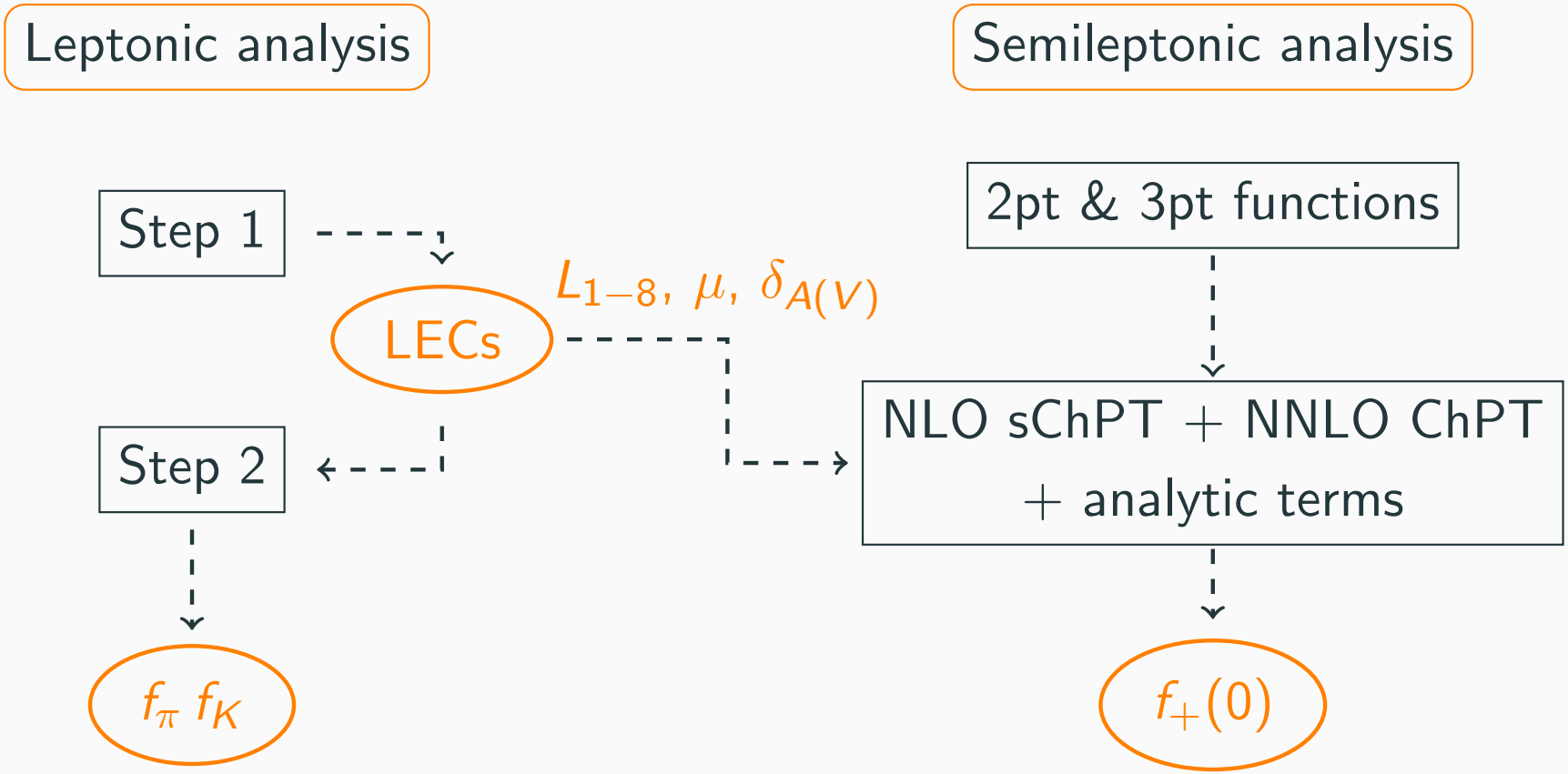

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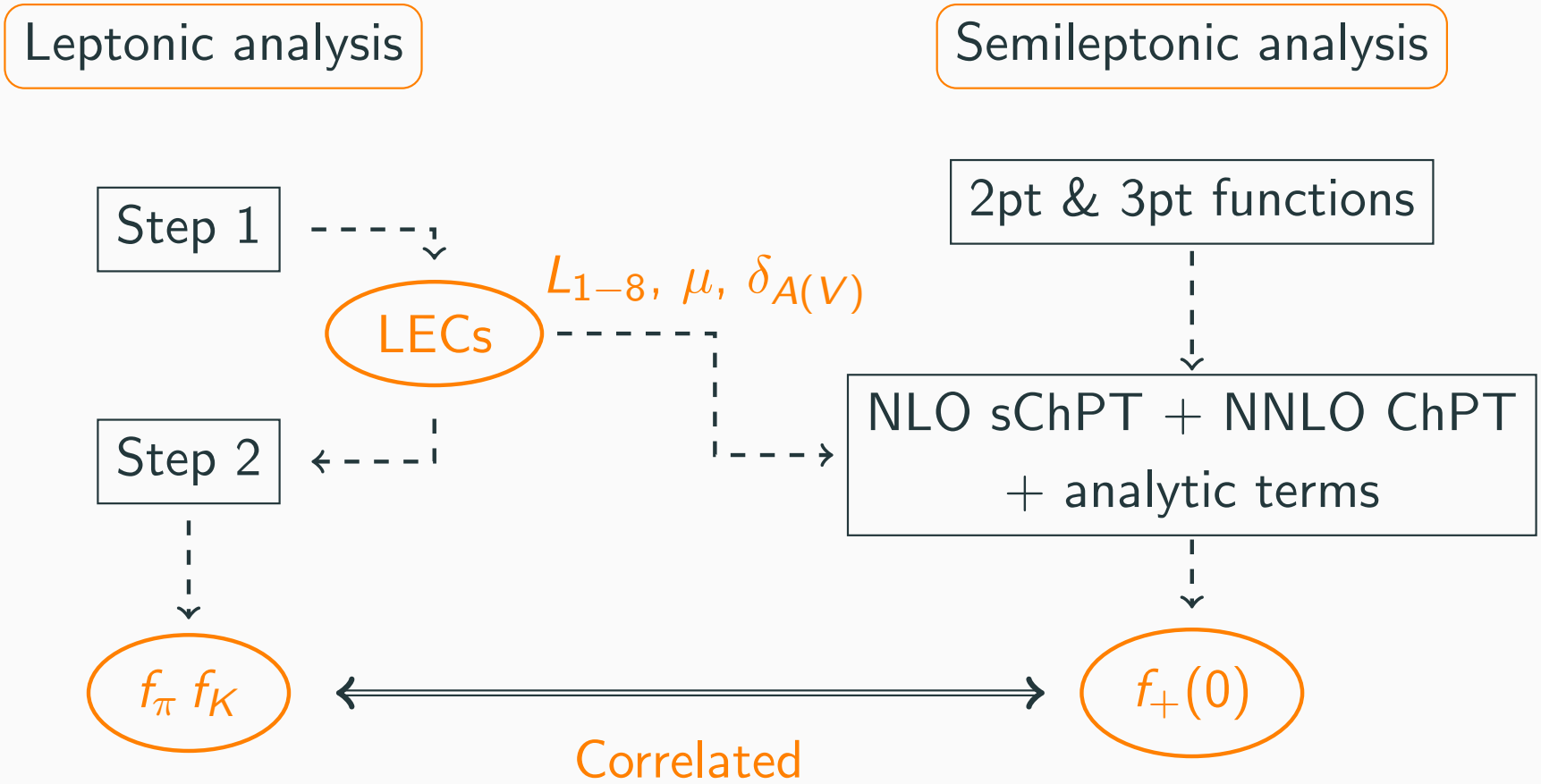
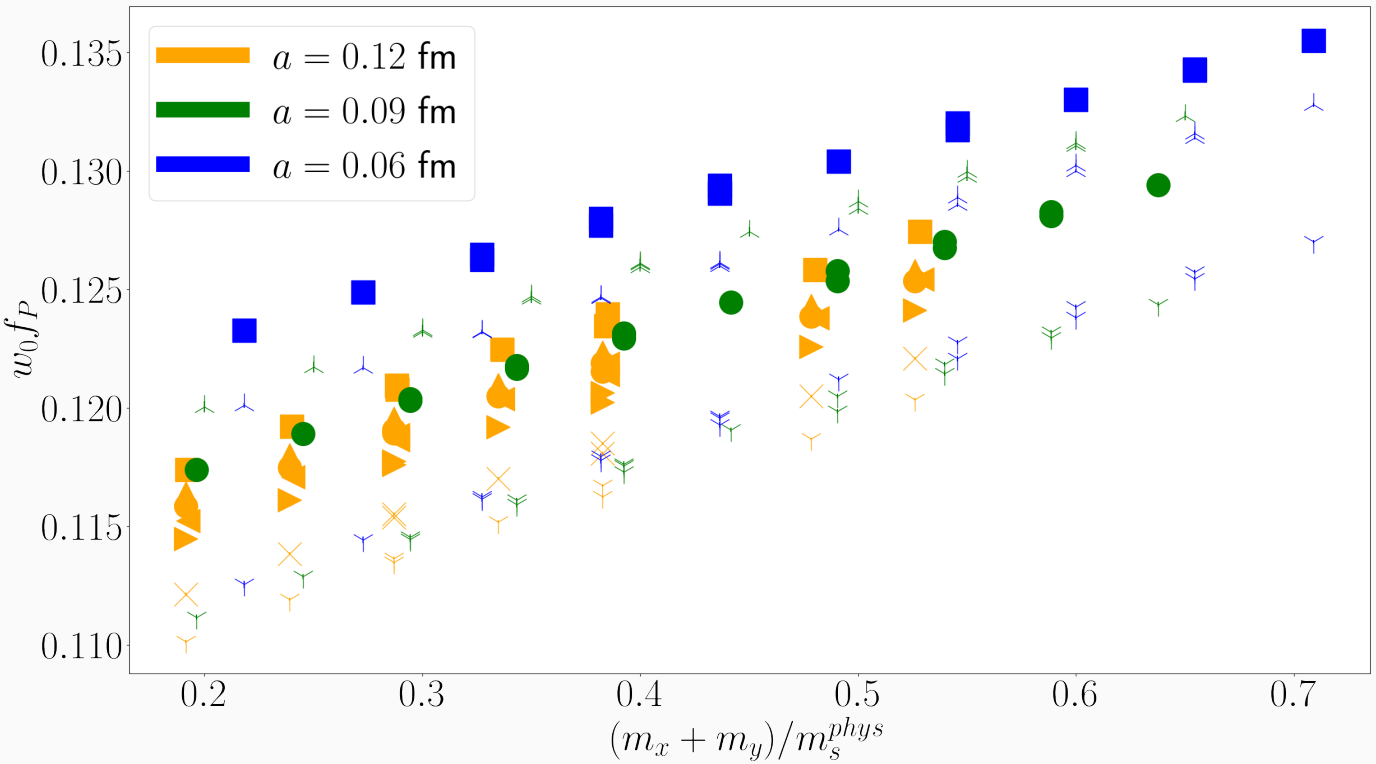
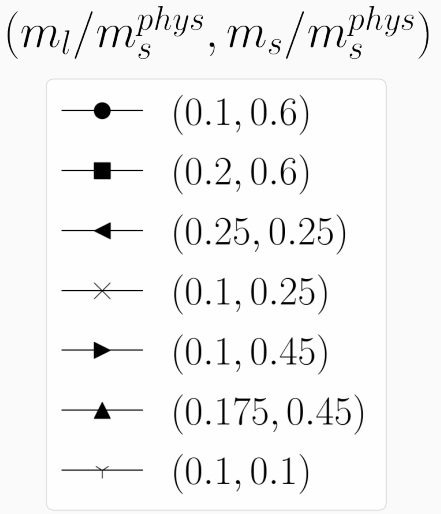


Decay constants: Step 1

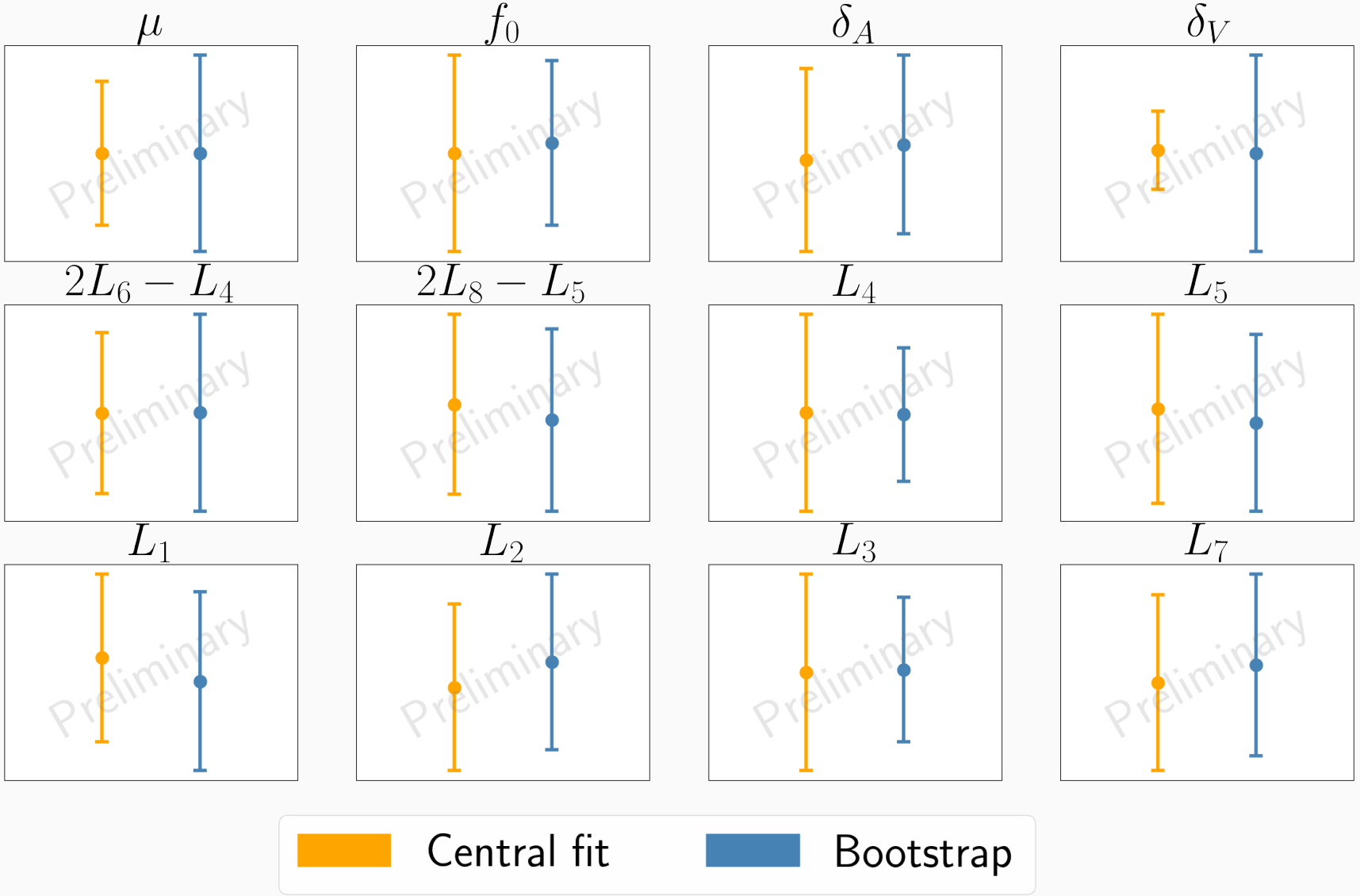




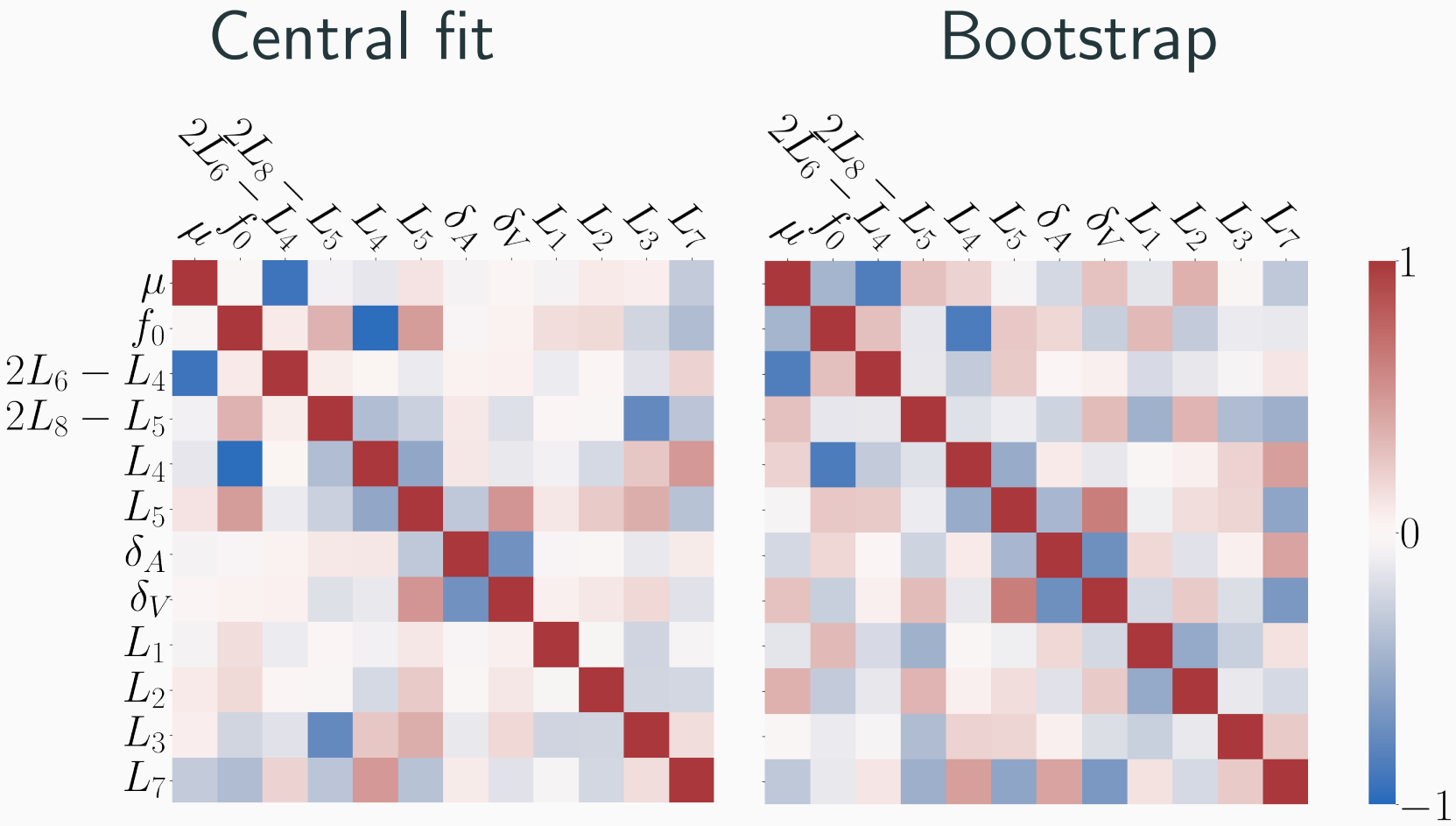
Step 1: 13 ensembles

- 123 masses + 123 decay constants
- Only data with $m_s < m_s^{\text{phys}}$
- NLO SU(3) SChPT + NNLO ChPT + $\mathcal{O}(\alpha_s a^2)$
- 47 parameters

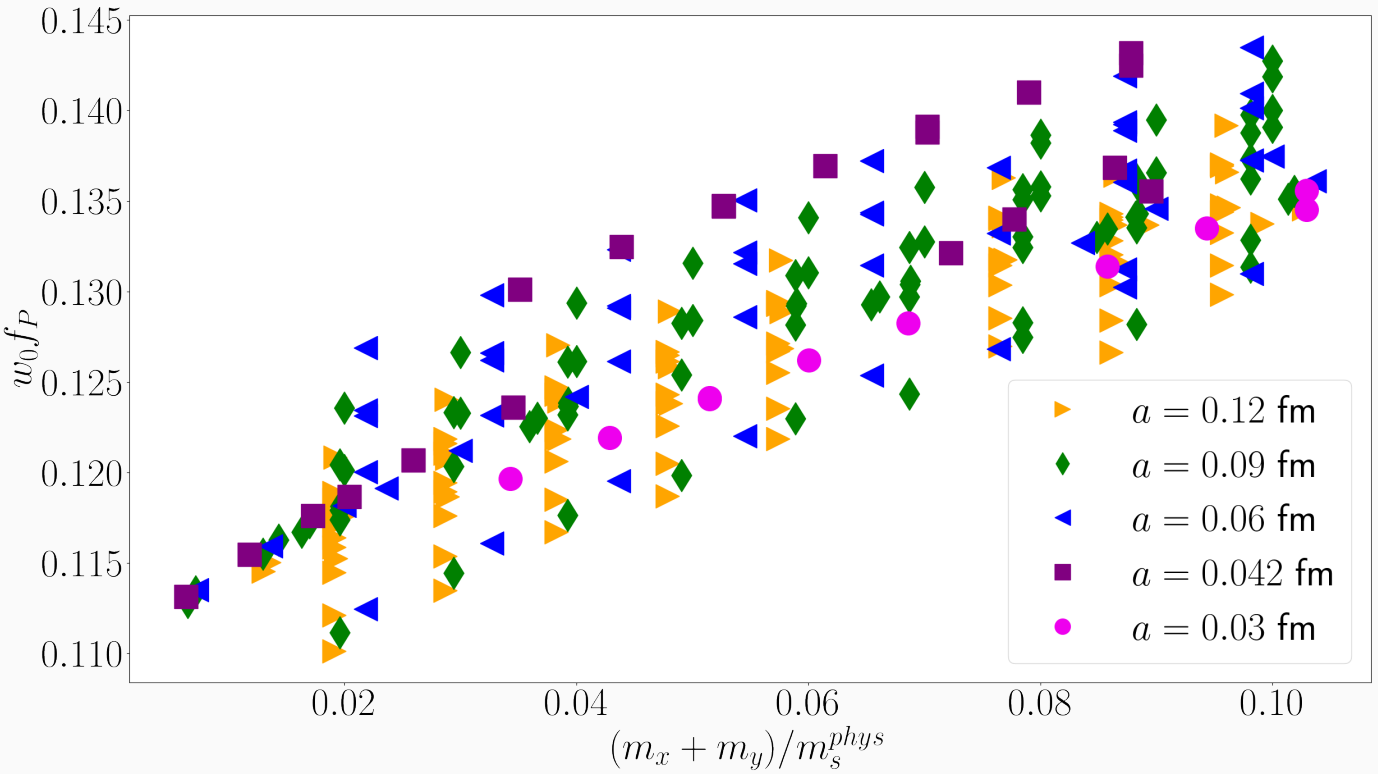
Decay constants: Step 1 LECs



Decay constants: Step 1 LECs correlation matrix



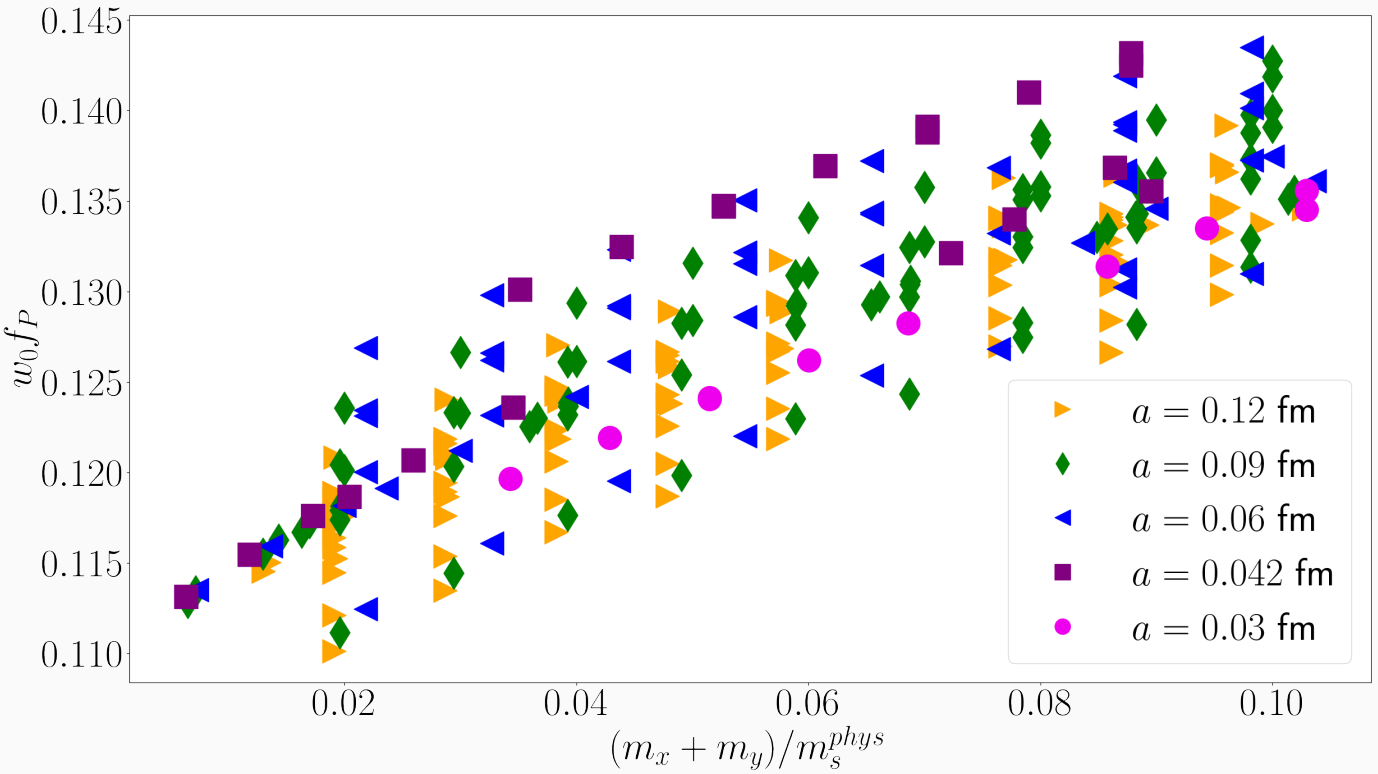
Decay constants: Step 2



Step 2: 29 ensembles

- 258 masses + 258 decay constants
- Physical and unphysical data
- Add N^{3}LO + N^{4}LO + $\mathcal{O}(a^4)$ + $\mathcal{O}(\alpha_s a^2)$
- 131 parameters

Decay constants: Step 2



Working on best strategy to perform bootstrap

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- 258 masses + 258 decay constants
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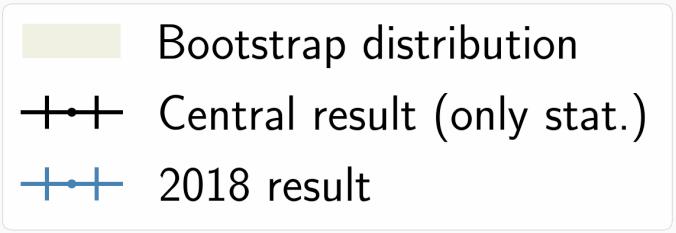
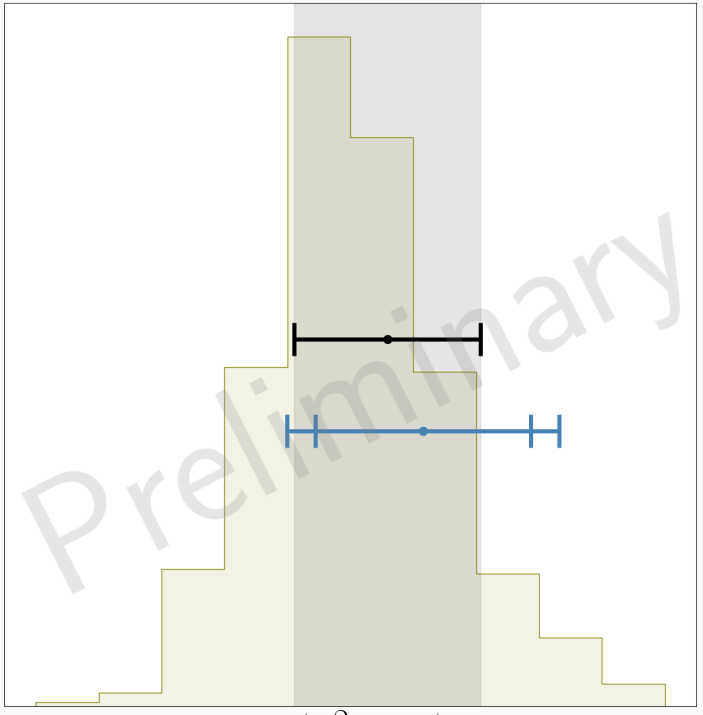
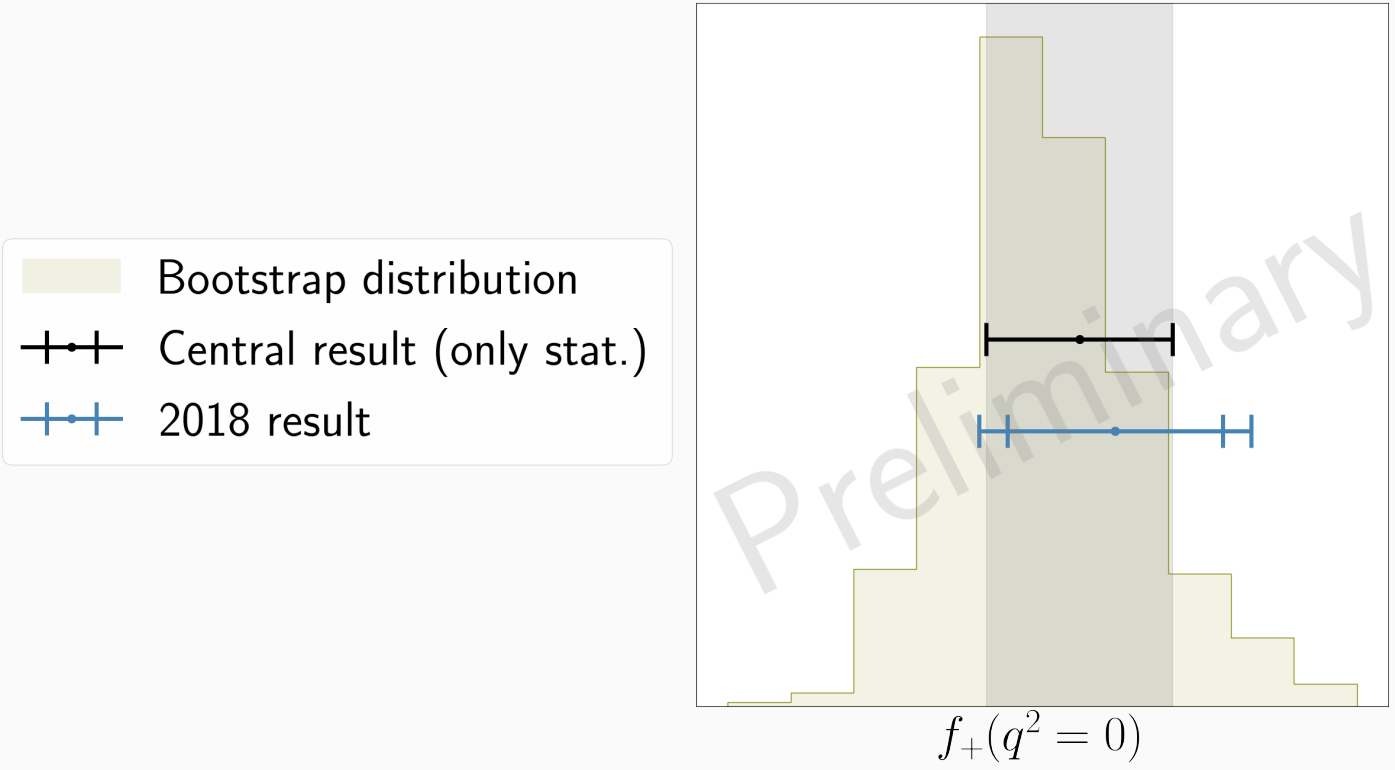
- Add $\mathrm{N^3LO} + \mathrm{N^4LO} + \mathcal{O}(a^4) + \mathcal{O}(\alpha_s a^2)$
- 131 parameters

Semileptonic bootstrap with leptonic inputs

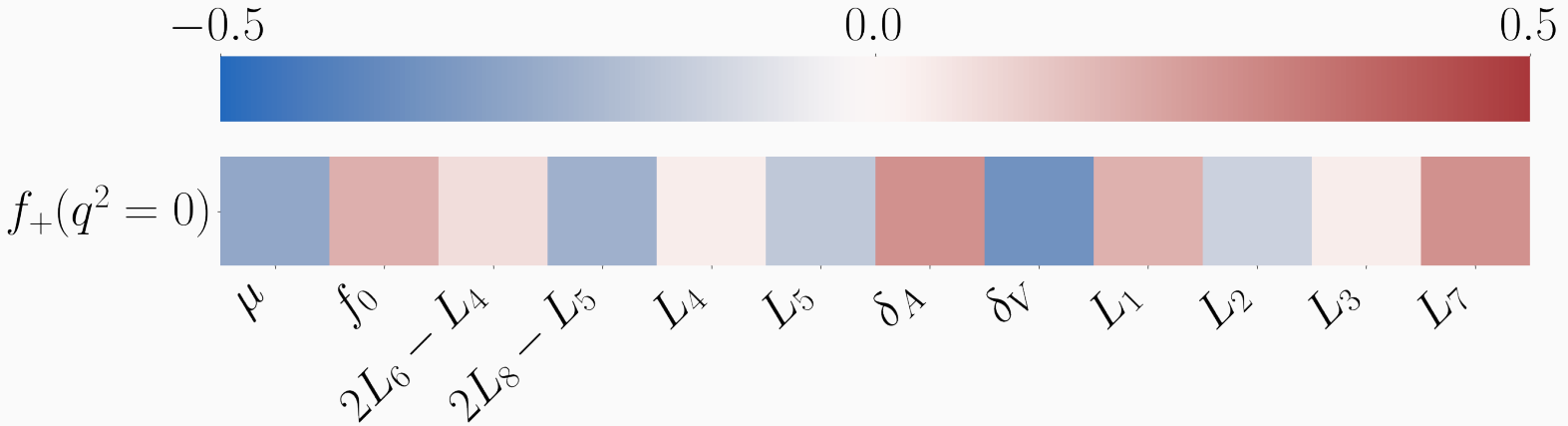




Semileptonic bootstrap with leptonic inputs



Correlation between $f_+(q^2=0)$ and SChPT parameters



Conclusions. Correlated boostrap analysis of f_K/f_π & $f_+(0)$

Decay constants

- Use SChPT in the chiral+continuum fits:
- Include physical and unphysical quark mass ensembles
- Determine LECs
- More robust estimate of systematic error of continuum extrapolation

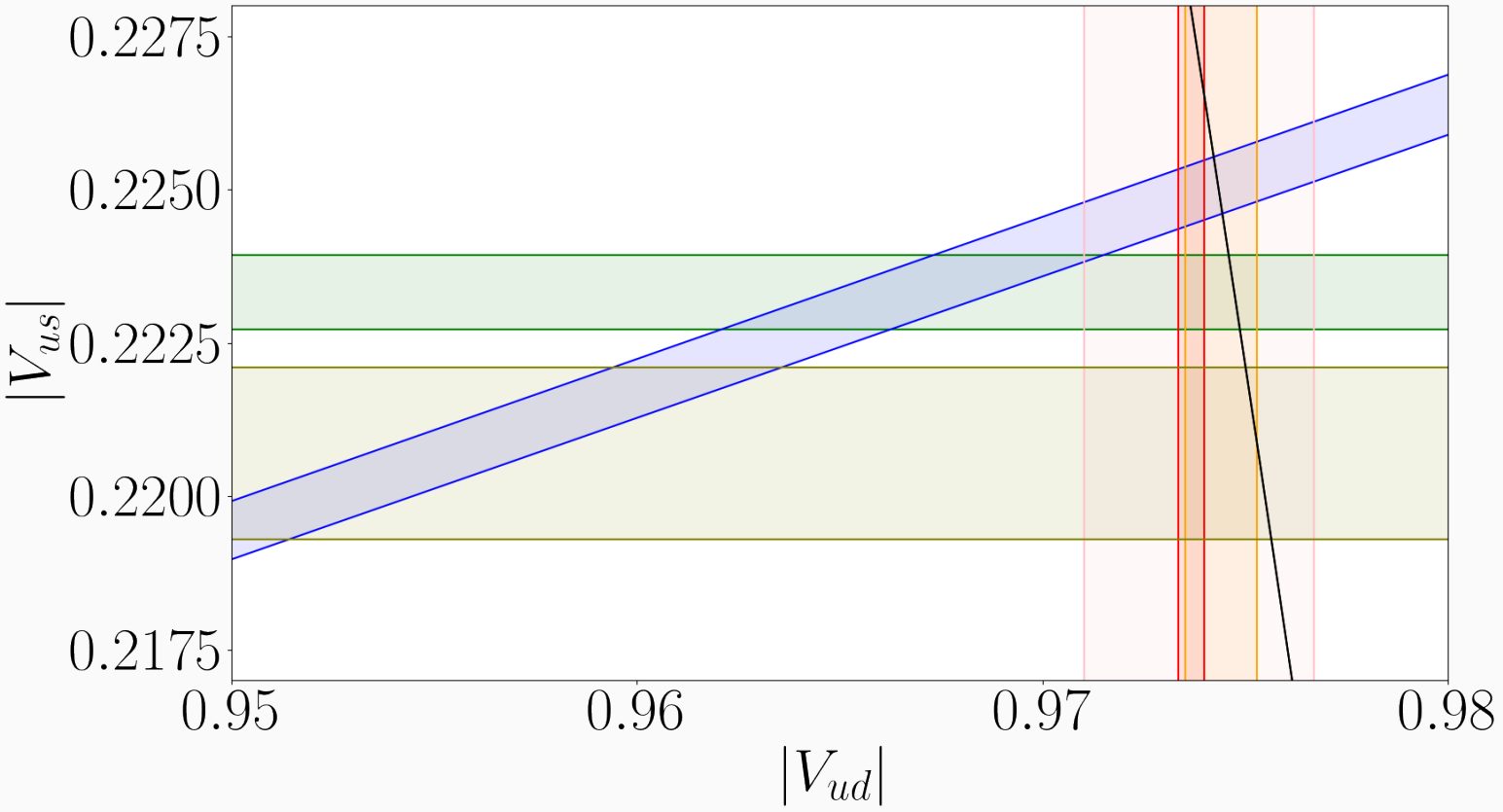
Semileptonic form factor

- Redo $f_+(0)$ analysis: more data and new fitting techniques $\Rightarrow$ slightly smaller (and more robust) statistical errors
- LECs from decay constants analysis including their correlations

Expect to reduce both statistical and systematic uncertainties in both quantities, and obtain correlations

Thanks for your attention !

Current status (2208.11707, 1712.09262, 1809.02827)



- Unitarity band
- $0^+ \to 0^+$ β decays
- Neutron decay
- $K_{\ell 3}$
- $K_{\ell 2}/\pi_{\ell 2}$
- τ decays
- π β decays

Extraction of $|V_{ud}|$ and $|V_{us}|$

$K \to \pi \ell \nu_\ell$

$$\Gamma^{K\ell 3} = \frac{G_F^2 m_K^5}{128\pi^3} S_{EW} \left| V_{us} f_+(q^2 = 0) \right|^2 I^{(0)} (1 + \delta_{\mathrm{EM}} + \delta_{\mathrm{SU}(2)})$$

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$\Gamma^{K\ell 3}$ ↓

Experimentally measured

Extraction of $|V_{ud}|$ and $|V_{us}|$

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Known factors

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Experimentally measured

From LQCD

$K/\pi \to \ell \nu$

$$\frac{\Gamma_K}{\Gamma_\pi} = \left| \frac{f_{K^+}}{f_{\pi^+}} \frac{V_{us}}{V_{ud}} \right|^2 \frac{m_K \left(1 - \frac{m_\ell^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2} (1 + \delta_{EW})$$

hep-ph/0402299

QED corrections

$$\frac{\Gamma(K^+ \to \ell^+ \nu_\ell(\gamma))}{\Gamma(\pi^+ \to \ell^+ \nu_\ell(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_{K^\pm}}{f_{\pi^\pm}} \frac{(1 + \delta_{\mathrm{EM,K}})}{(1 + \delta_{\mathrm{EM},\pi})}$$

Some QED references:

- RM123+SOTON 2006.05358: $\mathrm{QED_L}$
- RBC/QCD 2211.12865: $\mathrm{QED_L}$

Real γ emission

- Desiderio *et al.* 2006.05358
- Frezzotti *et al.* 2306.05904
- Giusti *et al.* 2302.01298

Past Monday appeared in arxiv:

- Boyle *et al.* 2510.26993

BMA

We use the framework shown in Ethan T. Neil and Jacob W. Sitison 2208.14983

$$pr(M_\mu|y) \propto \exp\left[\chi^2 + 2k + 2d_c\right]$$

where k is the number of parameters and d_c the number of cut data points.