Universität Siegen

Extracting $B_s \to D_s^* \ell \nu_\ell$ Form Factors

Anastasia Boushmelev, Matthew Black, Oliver Witzel

Theoretical Particle Physics, Center for Particle Physics Siegen, University of Siegen

The RBC & UKQCD collaborations

BNL & RBRC
Peter Boyle
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Patrick Oare
Amarjit Soni (emeritus)
Xin-Yu Tuo
Tianle Wang
Shuhei Yamamoto

CERN
Matteo Di Carlo
Felix Erben
Andreas Jüttner (Southampton)

Columbia University
Ryan Abbott
En-Hung Chao (MIT)
Norman Christ
Jonah Eick
Rajiv Eranki
Sarah Fields
Ceran Hu
Yikai Huo
Erik Lundstrum
Robert Mawhinney

University of Connecticut
Thomas Blum
Jonas Hildebrand
Luchang Jin
Vaishakhi Moningi
Anton Shcherbakov
Douglas Stewart
Joshua Swain
Masaaki Tomii (BNL)

DESY Zeuthen
Raoul Hodgson

University of Edinburgh
Matthew Black
Luigi Del Debbio
Vera Gülpers
Maxwell T. Hansen
Nils Hermansson Truedsson
Ryan Hill
Rajnandini Mukherjee
Antonin Portelli
Teseo San Jose

Johannes Gutenberg University of Mainz
Alessandro Barone

KEK
Shigemi Ohta (emeritus)

Liverpool Hope
Nicolas Garron (Liverpool)

University of Liverpool
Tobias Tsang
Aurelia Zuchanke

University of Milano Bicocca
Francesca A. Bresciani
Mattia Bruno
Gabriele Morandi

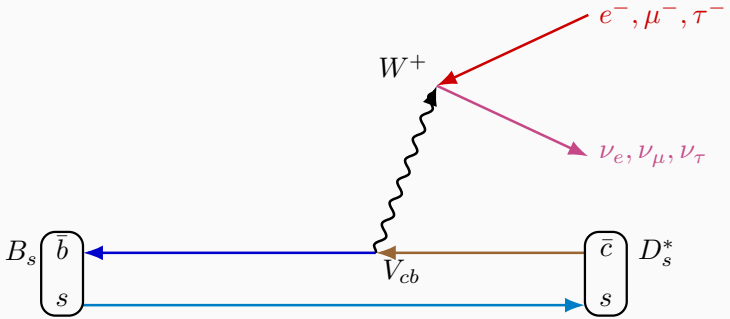
University of Regensburg
Daniel Knüttel
Christoph Lehner
Julian Parrino
Andreas Völklein

University of Siegen
Anastasia Boushmelev
Oliver Witzel

University of Southampton
Bipasha Chakraborty
Ahmed Elgaziari
Jonathan Flynn
Ollie Heald
Joe McKeon
Chris Sachrajda

Stony Brook University
Sergey Syritsyn (RBRC)

$B_s \to D_s^*$



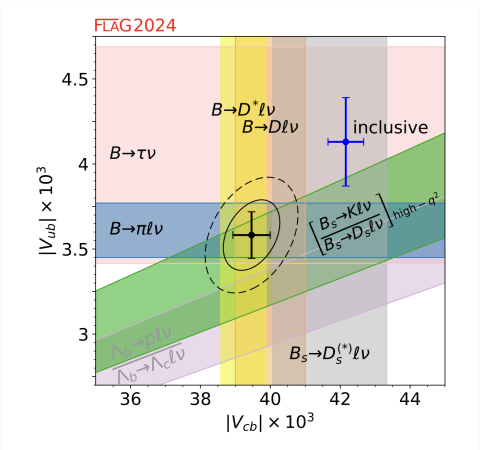
- $B \to D^* \ell \nu_\ell$ similar, just different spectator
- Narrow width approximation for D_s^*
  $\Rightarrow$ Assume D_s^* to be stable on the lattice

Inclusive vs. Exclusive $|V_{cb}|$

Tension excl. vs. incl.: $\sim 3\sigma$

- $|V_{cb}|_{\text{excl.}} = 39.8(6) \times 10^{-3}$ [PDG, 2024]
- $|V_{cb}|_{\text{incl.}} = 42.2(5) \times 10^{-3}$ [PDG, 2024]
- $|V_{cb}|_{\text{avg.}} = 41.1(1.2) \times 10^{-3}$ [PDG, 2024]

- No 'good' new physics case
- Additional uncertainty for CKM SM Tests

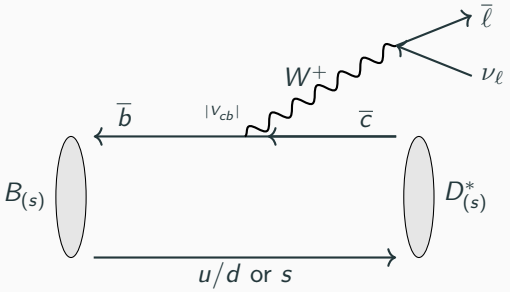


Exclusive $|V_{cb}|$ from $B_{(s)} \to D_{(s)}^{(*)} \ell\nu$

Exclusive:

- Experimentally preferred channel
  $B \to D^* \ell\nu$
- Form factors (nonperturbative) from **Lattice QCD** or sum rules

  For tree-level decays we have:
  - Pseudoscalar $\to$ Pseudoscalar:
    2 Form factors
  - Pseudoscalar $\to$ Vector:
    4 Form factors
- Result: $|V_{cb}|_{\text{excl}} = 39.8(6) \times 10^{-3}$ [PDG, 2024]



Existing Results $B \to D^*$

- $B \to D^*$ calculation by FNAL/MILC, HPQCD, JLQCD
- $B_s \to D_s^*$ calculation by HPQCD
- Two 2+1, one 2+1+1 determinations
- Covered q^2 ranges before extrapolation
  - FNAL/MILC: $\approx 7.2 - 11.0\text{GeV}^2$
  - HPQCD: $\approx 4.2 - 11.0\text{GeV}^2$
  - JLQCD: $\approx 8.8 - 11.0\text{GeV}^2$
- HPQCD pushing to larger recoil
- Resolving the slope is challenging

Plots: [M. Bordone, A. Jüttner arxiv:2406.10074 (2024)]

- $V_{cb}^{\text{excl}} = 38.40(78) \times 10^{-3}$

  [FNAL-MILC, Bazavov et al. (2022), EPJC 81.1141]
- $V_{cb}^{\text{excl}} = 39.19(91) \times 10^{-3}$

  [JLQCD, Aoki et al. (2023), PRD 109.074503]
- $V_{cb}^{\text{excl}} = 39.03(87) \times 10^{-3}$

  [HPQCD, Harison et al. (2024), PRD 109.094515]
  For $B_s \to D_s^* \ell \nu_\ell$ see [HPQCD, Harrison et al. (2017), PRD 97.054502],[HPQCD, Harrison et al. (2021), PRD 105.094506]

- $V_{cb}^{\text{incl}} = 42.2(5) \times 10^{-3}$

  [PDG, 2024]

Our Work

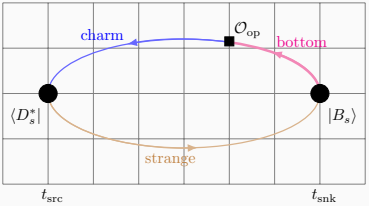
Lattice Setup for calculating Exclusive Semileptonic Decays

- Lattice QCD provides nonperturbative method to calculate matrix elements

$$\frac{d\Gamma(B_s \to D_s^* \ell \nu_\ell)}{dq^2} = \mathcal{K}_{D_s^*}(q^2, m_\ell) \cdot |\mathcal{F}(q^2)|^2 \cdot |V_{cb}|^2$$

From Experiment	Known Factors	Nonpert. Input	CKM

- Heavy quarks require very small lattice spacings $a \cdot m_Q < 1$
- Light quarks desire large physical boxes $M_\pi \cdot L \to$ large
- Or use effective action



Our Lattice Set Up

- RBC/UKQCD's 2+1 flavour gauge field ensembles
- Dynamical up/down and strange quarks in the sea and light sector using chiral domain-wall fermions
- Specifically optimized heavy domain-wall fermions for charm
- Relativistic heavy quark (RHQ) action for bottom
- Bottom, charm and strange close to physical

	L	T	a^{-1} GeV	am_l^{sea}	am_s^{sea}	M_π/ MeV	srcs $\times$ N_{conf}
C1	24	64	1.7848	0.005	0.040	340	1×1636
C2	24	64	1.7848	0.010	0.040	433	1×1419
M1	32	64	2.3833	0.004	0.030	302	2×628
M2	32	64	2.3833	0.006	0.030	362	2×889
M3	32	64	2.3833	0.008	0.030	411	2×544
F1S	48	96	2.785	0.002144	0.02144	268	24×98

Vector Final State Form Factors

$$\langle D_s^*(k,\varepsilon)|\bar{c}\gamma^\mu b|B_s(p)\rangle = V(q^2)\frac{2\mathrm{i}\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu^* k_\rho p_\sigma}{M_{B_s}+M_{D_s^*}},$$

$$\begin{aligned}\langle D_s^*(k,\varepsilon)|\bar{c}\gamma^\mu\gamma_5 b|B_s(p)\rangle &= A_0(q^2)\frac{2M_{D_s^*}\varepsilon^*\cdot q}{q^2}q^\mu \\ &+ A_1(q^2)(M_{B_s}+M_{D_s^*})\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ &- A_2(q^2)\frac{\varepsilon^*\cdot q}{M_{B_s}+M_{D_s^*}}\left[k^\mu+p^\mu-\frac{M_{B_s}^2-M_{D_s^*}^2}{q^2}q^\mu\right]\end{aligned}$$

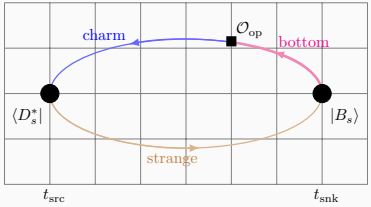
- Alternative to these relativistic form factors are HQET convention $f, \mathcal{F}_1, \mathcal{F}_2$ and g
- Consider combinations of directions of the momentum and polarization of the D_s^* to extract specific form factors
- A_2 especially complicated as we need a linear combination of matrix element, A_0 and A_1

Lattice Correlation Functions

$$R^{\Gamma,\mu}_{B_s\to D_s^*}(t, t_{\text{snk}}) = \frac{C^{3pt,\Gamma,\mu}_{B_s\to D_s^*}(t, t_{\text{snk}}, k)}{\sqrt{C^{2pt}_{D_s^*}(t,k) C^{2pt}_{B_s}(t_{snk}-t, p)}} \sqrt{\frac{4E_{D_s^*} M_{B_s} \sum_j \varepsilon_j(k)\varepsilon^{*j}(k)}{e^{-E_{D_s^*}} e^{-M_{B_s}(t_{\text{snk}}-t)}}}$$

$$\xrightarrow[t_{\text{snk}}-t\to\infty]{t\to\infty} \varepsilon^{\mu}(k)\langle D_s^*(k,\varepsilon)|\bar{c}\Gamma b|B_s(p)\rangle$$

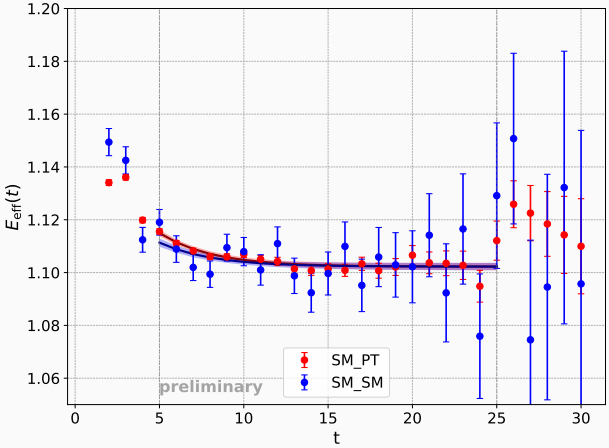
- Matrix elements extracted from ratios of 3pt- and 2pt-functions
- Lattice form factors $\widetilde{V}, \widetilde{A}_0, \widetilde{A}_1, \widetilde{A}_2$ still to be renormalized



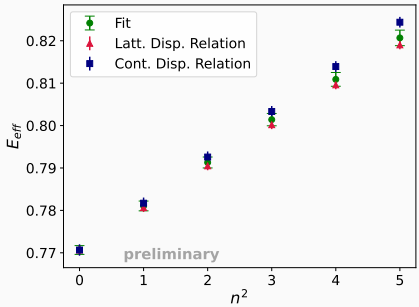
Effective Energy of D_s^*

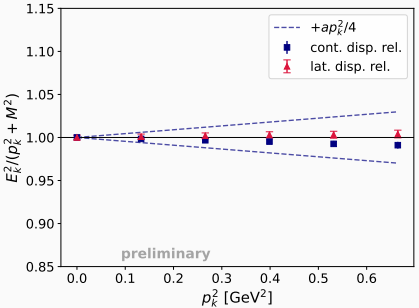
- Combined correlated fit of smeared-smeared and smeared-point data
- Include excited state effects
- Example: $M_{D_s^*} = E_{\text{eff}}(n^2 = 0)$ for C1, $am_c = 0.400$

- Fit Result 1.1023(12)
- In physical units: $M_{D_s^*} = 1.9674(21)$ GeV
- pvalue=0.73



Effective Energy of D_s^*: Dispersion Relation on F1S, am_c : 0.275

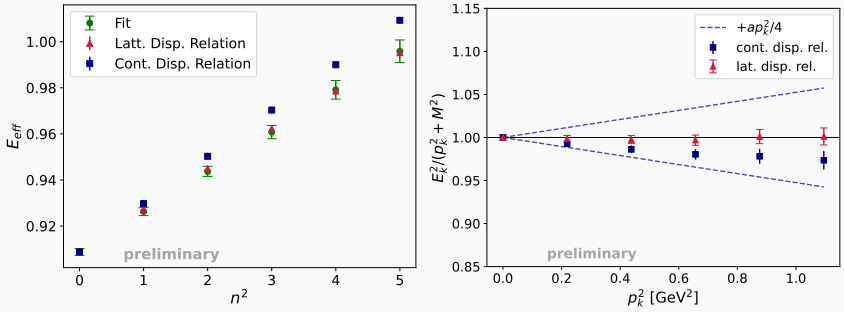




$$E = 2a^{-1}\sinh^{-1}\sqrt{\sinh^2\left(\frac{am}{2}\right) + \sum_{i=1}^{3}\sin^2\left(\frac{ap_i}{2}\right)}$$

Excellent agreement between measured values and lattice dispersion relation

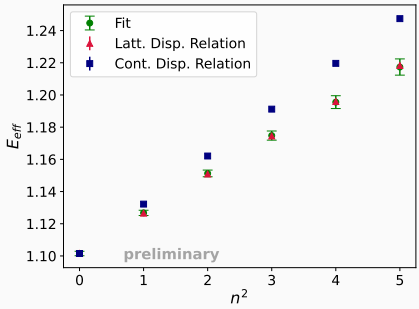
Effective Energy of D_s^*: Dispersion Relation on M1, am_c : 0.340

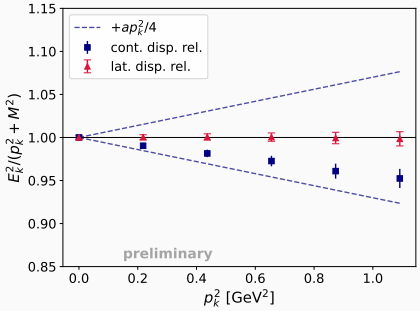


$$E = 2a^{-1}\sinh^{-1}\sqrt{\sinh^2\left(\frac{am}{2}\right)+\sum_{i=1}^{3}\sin^2\left(\frac{ap_i}{2}\right)}$$

Excellent agreement between measured values and lattice dispersion relation

Effective Energy of D_s^*: Dispersion Relation on C1, am_c : 0.400

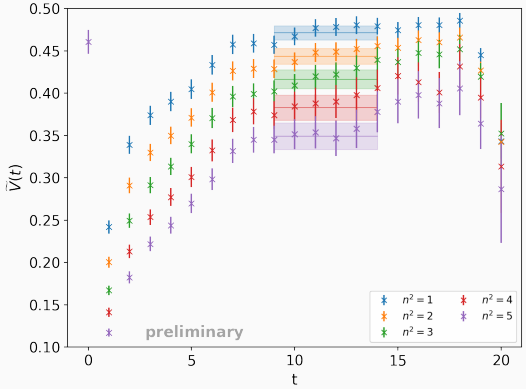




$$E = 2a^{-1}\sinh^{-1}\sqrt{\sinh^2\left(\frac{am}{2}\right) + \sum_{i=1}^{3}\sin^2\left(\frac{ap_i}{2}\right)}$$

Excellent agreement between measured values and lattice dispersion relation

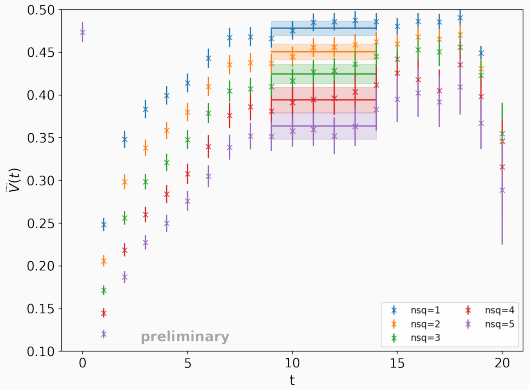
Form Factor Analysis: Simple Ground State Fit



- Simple plateau fit on individual form factors
- pvalue=0.34-0.83

- $\widetilde{V}(n^2 = 1)$ =0.4714(86)
- $\widetilde{V}(n^2 = 2)$ =0.4437(93)
- $\widetilde{V}(n^2 = 3)$ =0.417(11)
- $\widetilde{V}(n^2 = 4)$ =0.383(15)
- $\widetilde{V}(n^2 = 5)$ =0.349(16)

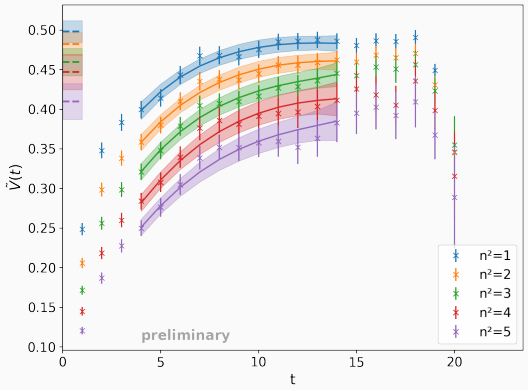
Form Factor Analysis: Combined Ground State Fit



- Combined Plateau Fit
- pvalue=0.65

- $\widetilde{V}(n^2 = 1)$ =0.4783(85)
- $\widetilde{V}(n^2 = 2)$ =0.4505(92)
- $\widetilde{V}(n^2 = 3)$ =0.425(11)
- $\widetilde{V}(n^2 = 4)$ =0.394(15)
- $\widetilde{V}(n^2 = 5)$ =0.364(15)

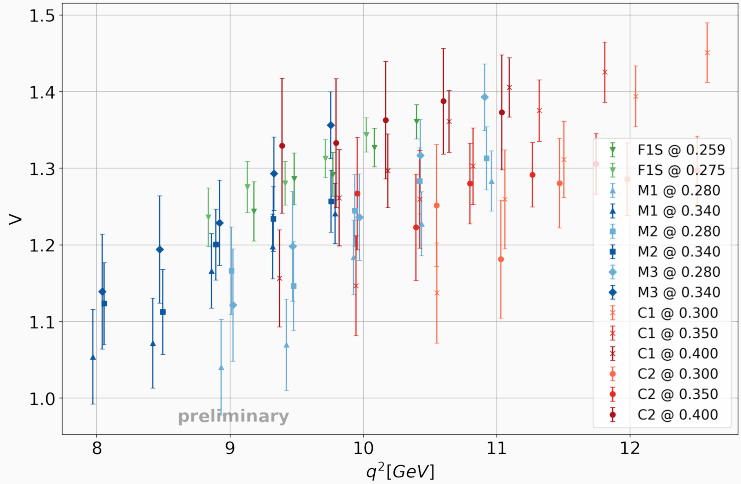
Form Factor Analysis: Combined Excited State Fit



- Combined fit including B_s and D_s^* excited states
- pvalue=0.35

- $\widetilde{V}(n^2 = 1) = 0.498(14)$
- $\widetilde{V}(n^2 = 2) = 0.482(14)$
- $\widetilde{V}(n^2 = 3) = 0.450(17)$
- $\widetilde{V}(n^2 = 4) = 0.447(22)$
- $\widetilde{V}(n^2 = 5) = 0.410(22)$

Form Factor q^2 Range



- Using mostly non-perturbative renormalization
- Multiplied blinding factor to renormalization

Summary & Next Steps

- Lattice QCD provides non-perturbative form factors for CKM extraction.
- Our calculation for $B_s \to D_s^* \ell\nu$: first results, continuum & mass extrapolation in progress.
- Combined theory + experiment key for CKM tests and NP sensitivity.

Next Steps:

- Combined c mass and chiral continuum extrapolation
- Estimate systematic uncertainties
- Kinematic extrapolation (z-expansion)

Backup Slides

Form Factor Analysis: Combined Excited State Fit

$$R_\mu^{3\text{pt}}(t, t_{\text{snk}}, \mathbf{p}) = \sqrt{\frac{4E_{D_s}^{(0)} M_{B_s}^{(0)}}{e^{-E_{D_s}^{(0)} t - M_{B_s}^{(0)}(t_{\text{snk}}-t)}}} \left\{ \frac{Z_{B_s}^{(0)} \mathcal{O}^{00} Z_{D_s}^{(0)}(\mathbf{p}) e^{-E_{D_s}^{(0)}(\mathbf{p})t - M_{B_s}^{(0)}(t_{\text{snk}}-t)}}{4E_{D_s}^{(0)} M_{B_s}^{(0)}} \right.$$
$$+ \frac{Z_{B_s}^{(1)} \mathcal{O}^{10} Z_{D_s}^{(0)}(\mathbf{p}) e^{-E_{D_s}^{(0)}(\mathbf{p})t - M_{B_s}^{(1)}(t_{\text{snk}}-t)}}{4E_{D_s}^{(0)} M_{B_s}^{(1)}} + \left. \frac{Z_{B_s}^{(0)} \mathcal{O}^{01} Z_{D_s}^{(1)}(\mathbf{p}) e^{-E_{D_s}^{(1)}(\mathbf{p})t - M_{B_s}^{(0)}(t_{\text{snk}}-t)}}{4E_{D_s}^{(1)} M_{B_s}^{(0)}} \right\}$$
$$\times \Bigg/ \sqrt{\left(\frac{(Z_{B_s}^{(0)})^2 e^{-M_{B_s}^{(0)}(t_{\text{snk}}-t)}}{2M_{B_s}^{(0)}} + \frac{(Z_{B_s}^{(1)})^2 e^{-M_{B_s}^{(1)}(t_{\text{snk}}-t)}}{2M_{B_s}^{(1)}} \right)}$$
$$\times \left(\frac{(Z_{D_s}^{(0)}(\mathbf{p}))^2 e^{-E_{D_s}^{(0)}(\mathbf{p})t}}{2E_{D_s}^{(0)}(\mathbf{p})} + \frac{(Z_{D_s}^{(1)}(\mathbf{p}))^2 e^{-E_{D_s}^{(1)}(\mathbf{p})t}}{2E_{D_s}^{(1)}(\mathbf{p})} \right)$$

with

$$C_{\Gamma_I \Gamma_J}^{2\text{pt,g.s.}}(t, \mathbf{p}) = \frac{Z_{0,J} Z_{0,I}^*}{2E_0(\mathbf{p})} e^{-E_0(\mathbf{p})t}, \qquad C_{\Gamma_I \Gamma_J}^{2\text{pt,g.+e.s.}}(t, \mathbf{p}) = \frac{Z_{0,J} Z_{0,I}^*}{2E_0(\mathbf{p})} e^{-E_0(\mathbf{p})t} + \frac{Z_{1,J} Z_{1,I}^*}{2E_1(\mathbf{p})} e^{-E_1(\mathbf{p})t}.$$