Radiative decays of pseudoscalar mesons

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Introduction

Radiative leptonic decays of pseudoscalar mesons

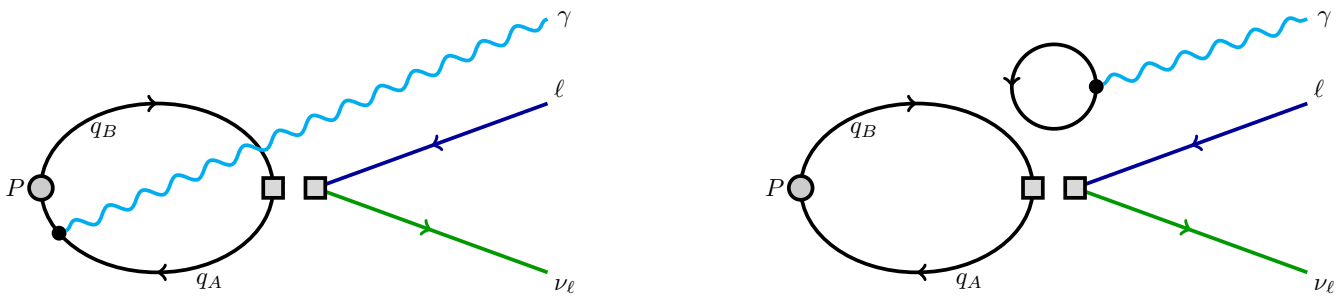
$$\Gamma(P^+ \to \ell^+ \nu_\ell \gamma) \quad \text{with} \quad P = D_{(s)}, B_{(s,c)}$$

They are interesting because...

- Probe new physics (LFU, new vector bosons)
- Reveal the meson structure...

  Parametrized by f_P, F_V, F_A (hard-photon emission)
- Give access to $|V_{cd}|$, $|V_{cs}|$, $|V_{ub}|$...

  Complement the leptonic decays $\Gamma(P \to \ell \nu_\ell)$



We aim to compute both diagrams

Motivation

Leptonic decays are flavor changing processes

$$\Gamma^{(0)} \equiv \Gamma(P \to \ell\nu_\ell) = \frac{G_F^2}{8\pi}\left|V_{q_1 q_2}\right|^2 f_P^2 m_\ell^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2$$

with $\Gamma^{(0)}$ the isospin-symmetric rate
Current determinations reach $\sim 1\%$ precision [5] for

$$f_D|V_{cd}| = 45.82(1.10)\,\text{MeV} \qquad f_{D_s}|V_{cs}| = 243.5(2.7)\,\text{MeV}$$

At this level, we need the electromagnetic corrections δR_P [1]

$$\Gamma(P^+ \to \ell^+\nu_\ell[\gamma])\Big|_{E_\gamma < E_\gamma^{\text{max}}} = \Gamma^{(0)}(1 + \delta R_P)$$

$\Gamma(P^+ \to \ell^+\nu_\ell\gamma)$ provides...

The structure-dependent contributions F_V and F_A

What about the $B_{(s,c)}$?

B leptonic decays much less understood...

but something stirs at Belle II and LHCb

Figure taken from [4]

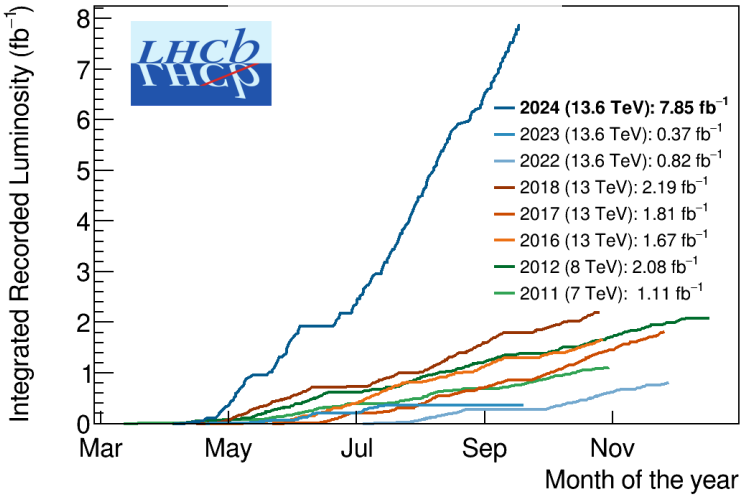
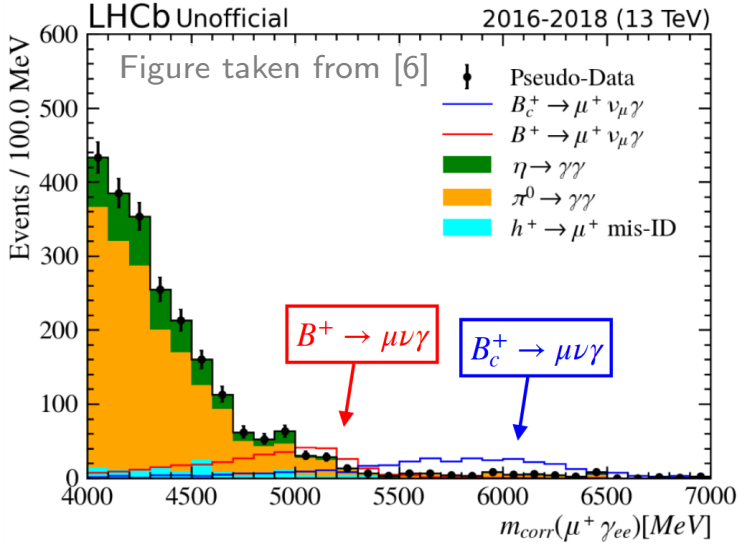


Figure taken from [6]



The experimental constraints will be pushed further than expected

Belle II uncertainties [9]			
	current	2030	2040
$\mathcal{B}(B^+ \to \mu^+ \nu_\mu)$	41%	11%	5%
$\mathcal{B}(B^+ \to \tau^+ \nu_\tau)$	34%	10%	6%

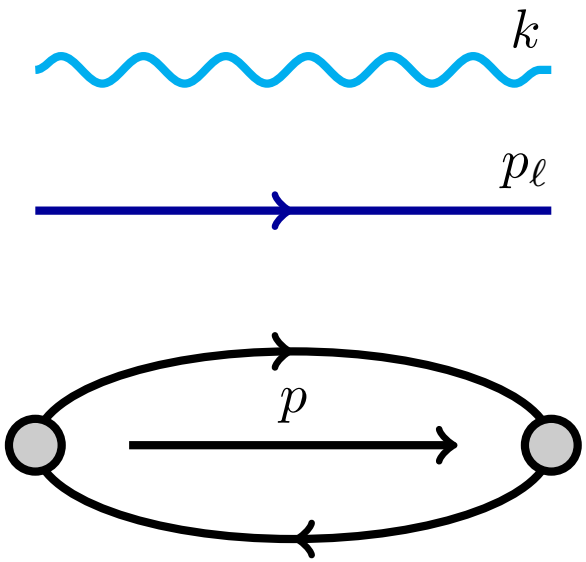
Parametrization

The differential cross section for $\Gamma \equiv \Gamma(P^+ \to \ell^+ \nu_\ell \gamma)\Big|_{E_\gamma < E_\gamma^{\max}}$ [2]

$$\frac{\partial^2 \Gamma}{\partial x_\gamma \partial x_\ell} = \frac{\alpha_{\text{em}}}{4\pi} \Gamma^{(0)} \left\{ \frac{\partial^2 \Gamma_{\text{pt}}}{\partial x_\gamma \partial x_\ell} + \frac{\partial^2 \Gamma_{\text{SD}}}{\partial x_\gamma \partial x_\ell} + \frac{\partial^2 \Gamma_{\text{INT}}}{\partial x_\gamma \partial x_\ell} \right\}$$

The two Lorentz scalars are

$$x_\gamma = \frac{2p \cdot k}{m_P^2} \qquad x_\ell = \frac{2p \cdot p_\ell - m_\ell^2}{m_P^2}$$



Γ_{pt}, Γ_{SD}, Γ_{INT} depend on f_P, F_V, F_A [2]

$$F_V(x_\gamma) \qquad F_A(x_\gamma)$$

Lattice calculation

Consider the CM frame ($\boldsymbol{p} = (0,0,0)$) and compute

$$C^{\mu\nu}_{V-A} = \sum_{\boldsymbol{z},\boldsymbol{x}} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \langle 0|\hat{\mathrm{T}}\left[j^{\nu}_{V-A}(t_W,\boldsymbol{0}) j^{\mu}_{\mathrm{em}}(t_\gamma,\boldsymbol{x})\phi^{\dagger}_P(0,\boldsymbol{z})\right]|0\rangle$$

ROME123 proved [2] the Laplace transform yields the hadronic tensor

$$H^{\mu\nu}_{V-A}(\boldsymbol{k};t_W) = \int_{-\infty}^{\infty} dt_\gamma 2m_P \frac{e^{E_\gamma t_\gamma} C^{\mu\nu}_{V-A}(t_\gamma,\boldsymbol{k};t_W)}{e^{-m_P t_W}\langle P(\boldsymbol{0})|\phi^{\dagger}_P(0)|0\rangle}$$

As an example choose $\boldsymbol{k} = (0,0,k)$

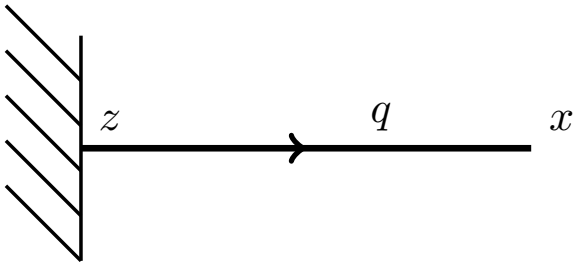
$$F_V(x_\gamma;t_W) = iH^{21}_V(\boldsymbol{k};t_W)/k$$
$$F_A(x_\gamma;t_W) = \left(H^{11}_A(\boldsymbol{0};t_W) - H^{11}_A(\boldsymbol{k};t_W)\right)/E_\gamma$$

with the Lorentz scalar

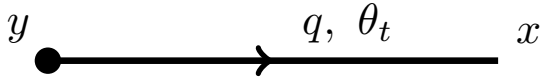
$$x_\gamma \equiv 2p\cdot k/m_P^2 = 2E_\gamma/m_P$$

Quark-connected contribution

Compute each point-to-all propagator individually

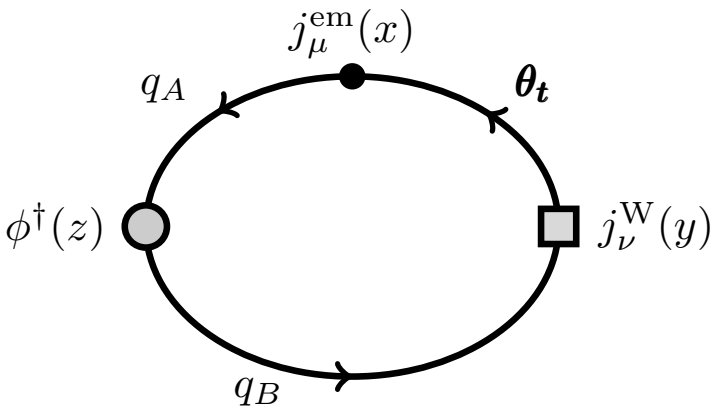


$$z: \begin{cases} N_{\text{src}} \text{ stochastic time-wall sources} \\ \text{Jacobi smearing} \end{cases}$$



$$y: \begin{cases} N_{\text{snk}} \text{ point sources} \\ \text{Twist angles } 0, \pm\theta_t \end{cases}$$

Form the quark-connected Wick contraction



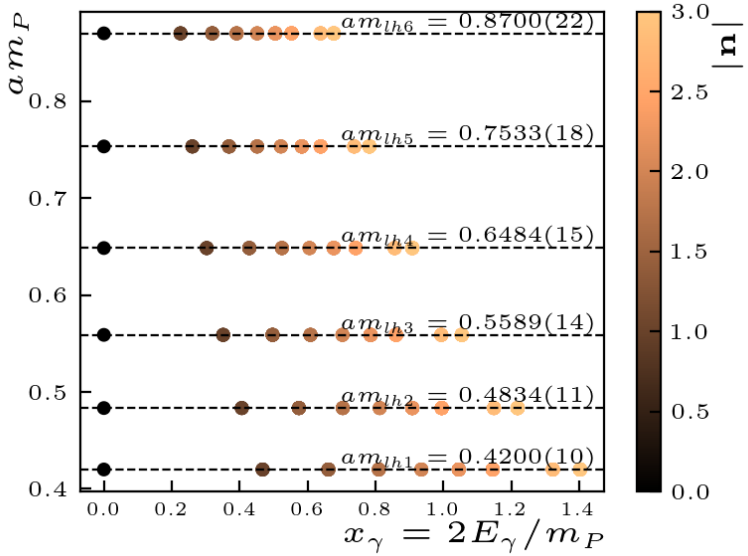
- Meson momentum $\boldsymbol{p} = 0$
- Photon momenta
  $\boldsymbol{k} = 2\pi/L \cdot (\boldsymbol{n} - \boldsymbol{\theta_t})$
  - Fourier modes with $|\boldsymbol{n}| \leq 3$
  - Twist angles $\boldsymbol{\theta_t} = (0, 0, \pm 0.2762)$
- Set $\boldsymbol{y} = 0$

Accessible values of x_γ

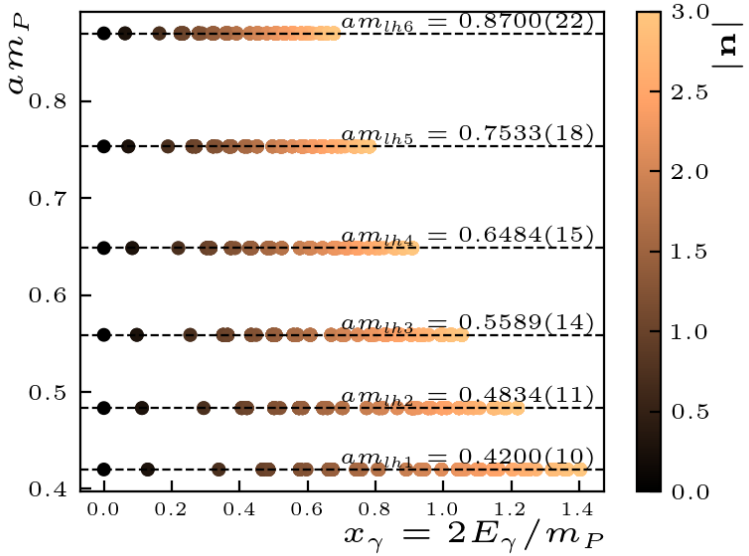
We employ one **JLQCD** domain-wall fermion ensemble [3]

Label	β	a [fm]	a^{-1} [GeV]	$L^3 \times T \times L_s$	m_π	$m_\pi L$	am_{ud}	am_s
F-ud3-sa	4.47	**0.0439(1)**	4.494(2)	$64^3 \times 128 \times 8$	**284(1)**	4.0	0.003	0.015

Interpolate the D and B mesons with 6 heavy-quark masses



Fourier $|\boldsymbol{n}| \leq 3$

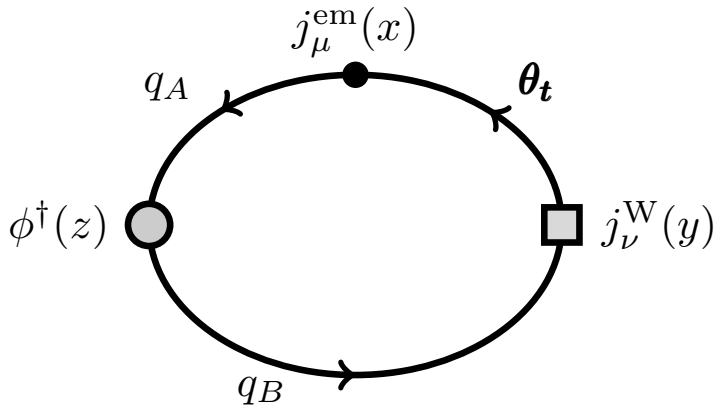


Fourier $|\boldsymbol{n}| \leq 3$ and $\pm\theta_t$

Comparison to other methods

Consider

$N_l = 2 \quad N_h = 6 \quad N_q = N_l + N_h$
$N_{\text{snk}} = 32 \quad N_{\text{src}} = 128 \quad N_{\theta_t} = 3$



Point-to-all method (This work)
# of propagators N_{prop} grows linearly with # of sources

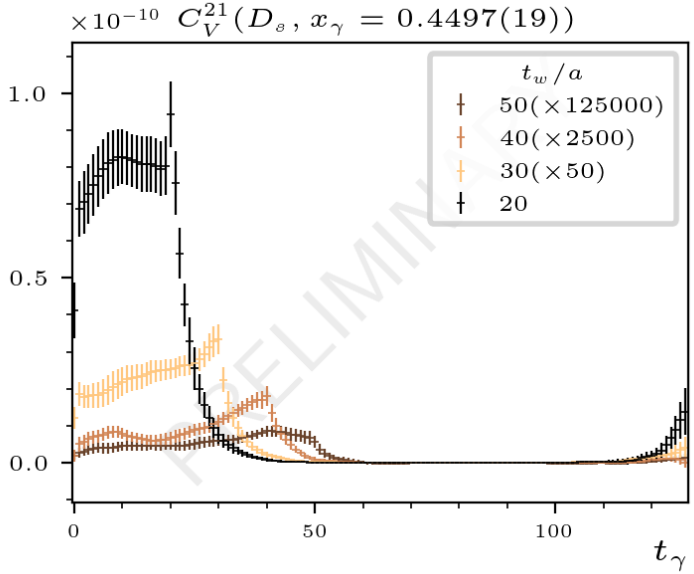
$$N_{\text{prop}} = N_q \cdot (N_{\text{src}} + N_{\text{snk}} \cdot N_{\theta_t}) = 1792$$

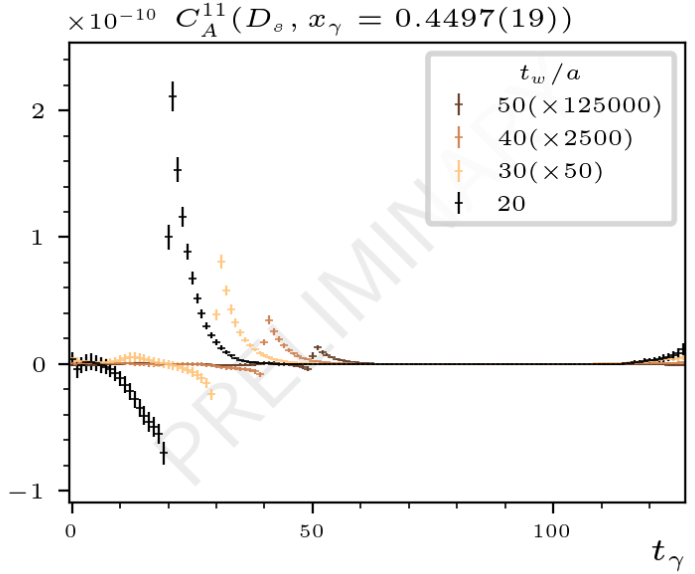
Sequential method [7, 8]
# of propagators N_{prop} grows quadratically with # of sources

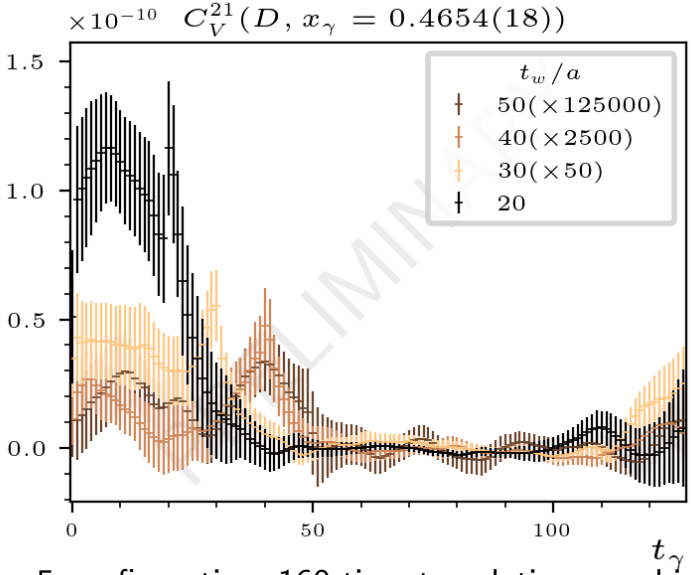
$$N_{\text{prop}} = N_{\text{snk}} \cdot (2 \cdot N_h \cdot N_l \cdot N_{src} + N_q \cdot N_{\theta_t}) = 99072$$

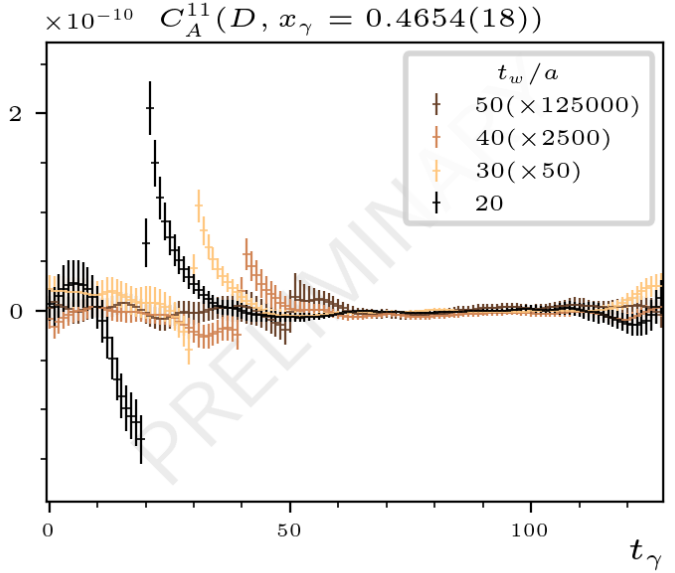
where the factor 2 comes from two possible Feynman diagrams

Numerical results









Statistics: 5 configuration, 160 time translations, no binning

Data crosschecks/symmetries

We have all $t_W \rightarrow$ **Time reversal symmetry**

$$C_A^{\mu\nu}(t_\gamma, \boldsymbol{k}; t_W) = C_A^{\mu\nu}(-t_\gamma, \boldsymbol{k}; T - t_W)$$
$$C_V^{\mu\nu}(t_\gamma, \boldsymbol{k}; t_W) = -C_V^{\mu\nu}(-t_\gamma, \boldsymbol{k}; T - t_W)$$

We have all Fourier modes $\boldsymbol{k}$

In the simpler case $\boldsymbol{k} = (0, 0, k) = 2\pi/L \cdot \boldsymbol{n} = 2\pi/L \cdot (0, 0, n)$

$$C_A^{11}(\boldsymbol{n} = (0, 0, 1)) = C_A^{11}(\boldsymbol{n} = (0, 0, -1))$$
$$= C_A^{11}(\boldsymbol{n} = (0, 1, 0)) = C_A^{11}(\boldsymbol{n} = (0, -1, 0))$$
$$C_V^{21}(t_\gamma, \boldsymbol{k}; t_W) = -C_V^{21}(t_\gamma, -\boldsymbol{k}; t_W)$$

In general, look at O_h group irreps and Lorentz structure

We have all non-zero $C_{V-A}^{\mu\nu} \rightarrow$ **Global fit** of all $\mu\nu$ entries

Summary and outlook

We present an **alternative approach** to compute $C^{\mu\nu}_{V-A}$

- Access to **all source-sink separations** and non-zero $C^{\mu\nu}_{V-A}$
- Improve the signal using all symmetries

In the near future

- **Increase statistics**
- Extend analysis to **other JLQCD ensembles**
- Run **quark-disconnected loops** (developed by Ryan Hill)

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