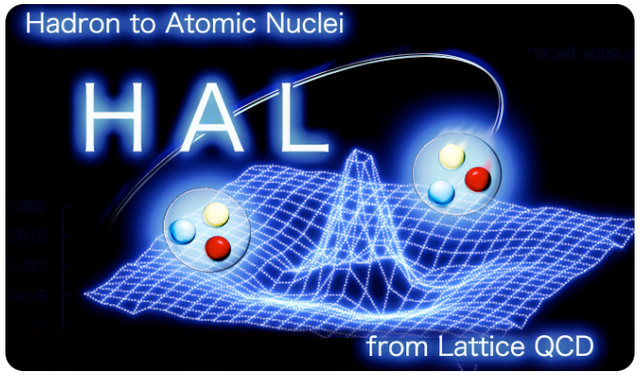
Quark mass dependence of the nuclear LS force

Takuya Sugiura (Rissho Univ., RIKEN),
for HAL QCD Collaboration

Based on an upcoming paper



Lattice2025, Mumbai, 7 Nov 2025

NN force in P-wave

General form of NN potential

$$V(r) = V_c(r) + V_t(r)S_{12} + V_{ls}(r)(\boldsymbol{L}\cdot\boldsymbol{S}) + \mathcal{O}(p^2)$$

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Spin-orbit force (LS force) plays a role in S=I=1 P-wave channels

- σ, ω, ρ exchanges give negative contribution $V_{ls}^{(\sigma/\omega/\rho),I=1} < 0$
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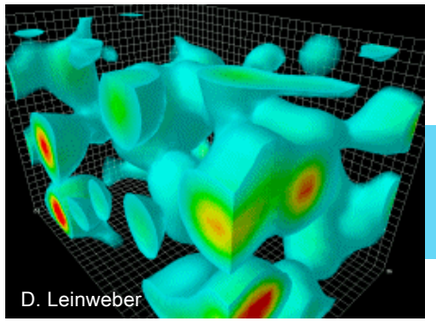
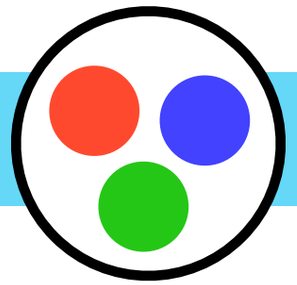
Lattice QCD evaluation?

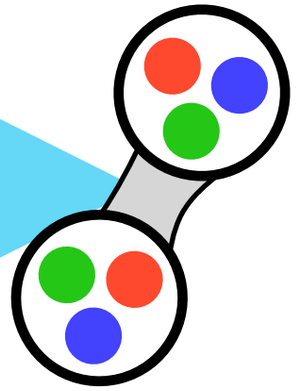
HAL QCD method

QCD

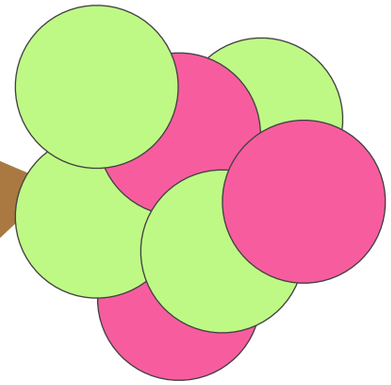


Hadrons

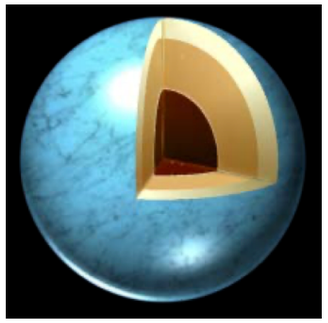




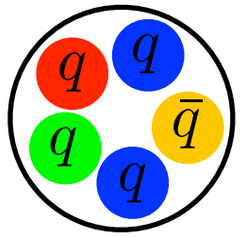
Nuclei



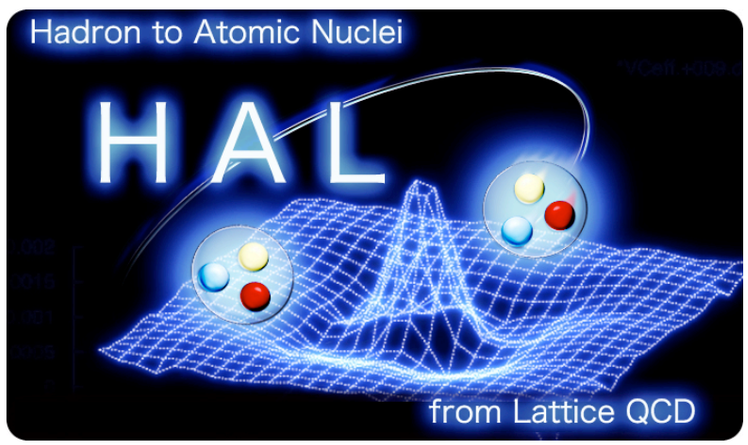
Nuclear matter



Hadron Forces



Exotic Hadrons



- Equivalent to Lüscher's method
- Correct S-matrix
- Applicable to any hadron forces

HAL QCD method

$$R(\boldsymbol{r},t) \equiv \frac{C_{NN}(\boldsymbol{r},t)}{C_N(t)^2} = \sum_n A_n \psi_n(\boldsymbol{r}) e^{-W_n t}$$

If R is dominated by the elastic scattering states at $t \gg W_{th}^{-1}$,

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\boldsymbol{r},t) = \int d^3\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

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$$V(r) = V_c(r) + V_t(r) S_{12} + V_{ls}(r)(\boldsymbol{L}\cdot\boldsymbol{S}) + \mathcal{O}(p^2)$$

3 functions $V_c(r), V_t(r), V_{ls}(r)$ are determined from
3 independent channels A_1^-, T_1^-, T_2^- ($\sim {}^3P_0, {}^3P_1, {}^3P_2$) :

$$\begin{pmatrix} R(\boldsymbol{r},t;A_1^-) & S_{12}R(\boldsymbol{r},t;A_1^-) & (\boldsymbol{L}\cdot\boldsymbol{S})R(\boldsymbol{r},t;A_1^-) \\ R(\boldsymbol{r},t;T_1^-) & S_{12}R(\boldsymbol{r},t;T_1^-) & (\boldsymbol{L}\cdot\boldsymbol{S})R(\boldsymbol{r},t;T_1^-) \\ R(\boldsymbol{r},t;T_2^-) & S_{12}R(\boldsymbol{r},t;T_2^-) & (\boldsymbol{L}\cdot\boldsymbol{S})R(\boldsymbol{r},t;T_2^-) \end{pmatrix} \begin{pmatrix} V_c(r) \\ V_t(r) \\ V_{ls}(r) \end{pmatrix} = \begin{pmatrix} \mathcal{D}R(\boldsymbol{r},t;A_1^-) \\ \mathcal{D}R(\boldsymbol{r},t;T_1^-) \\ \mathcal{D}R(\boldsymbol{r},t;T_2^-) \end{pmatrix}$$

fLapH distillation

Covariant LapH

Peardon et al., Phys. Rev. D **80** (2009) 054506

- Projection to the "*LapH subspace*" spanned by low modes of the gauge-covariant Laplacian operator
- Essentially introduces momentum cutoff: $\theta(\sigma_s^2 + \tilde{\Delta}^{ab}_{x,y}(t))$

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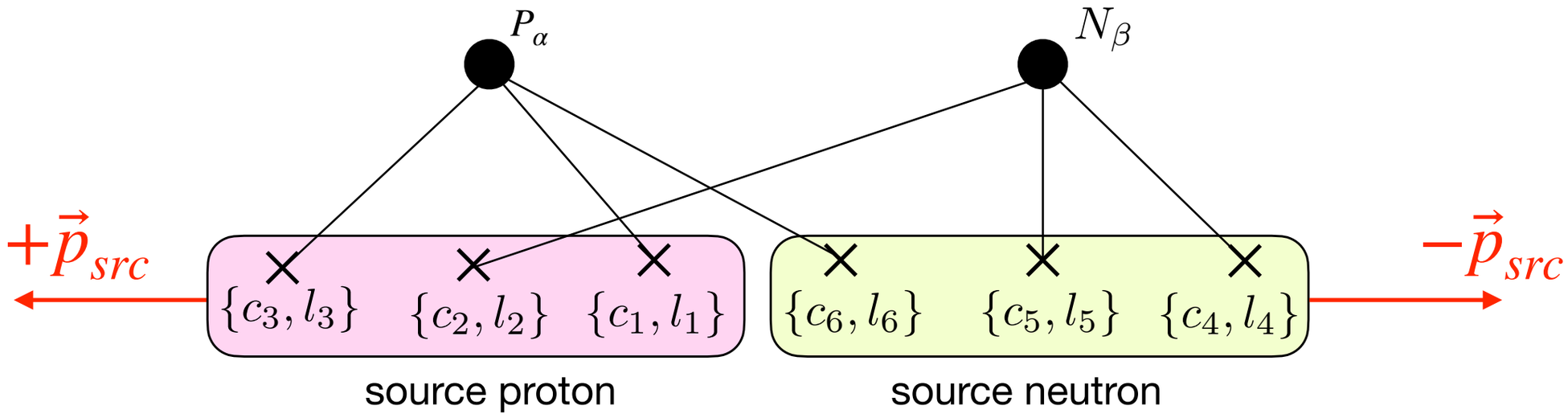
Free LapH (fLapH)

- Utilizes eigenmodes of the free Laplacian, *i.e.*, plain waves
- Coulomb gauge fixing to create spatially coherent states
- Hadron-level momentum conservation allows for substantial speedup!

fLapH distillation

Example diagram:



fLapH distillation

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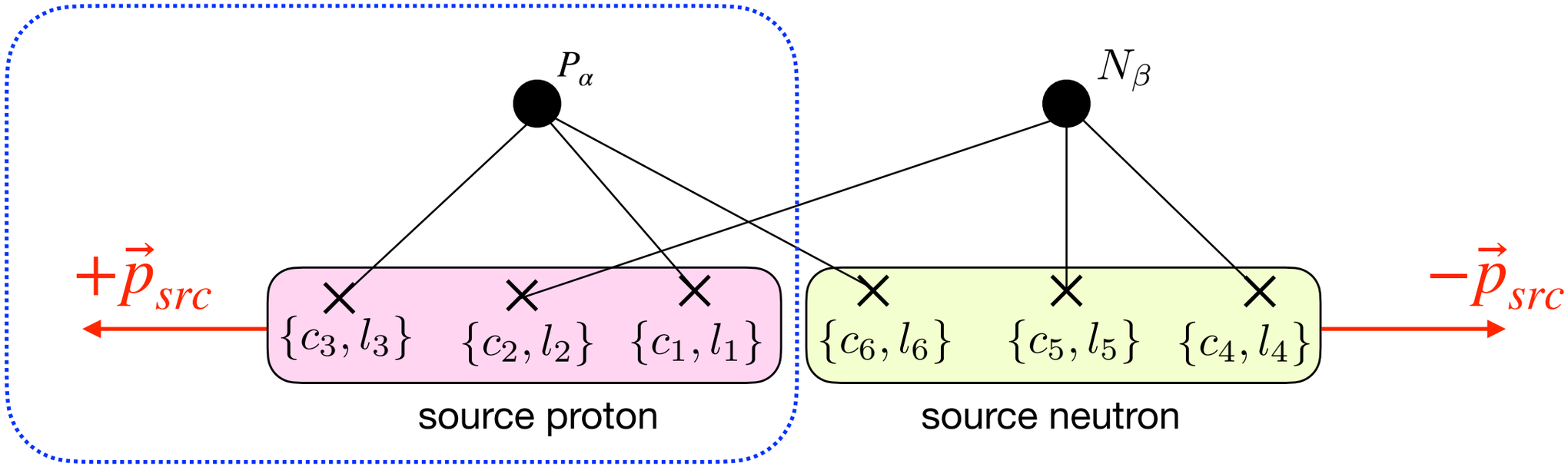

fLapH distillation

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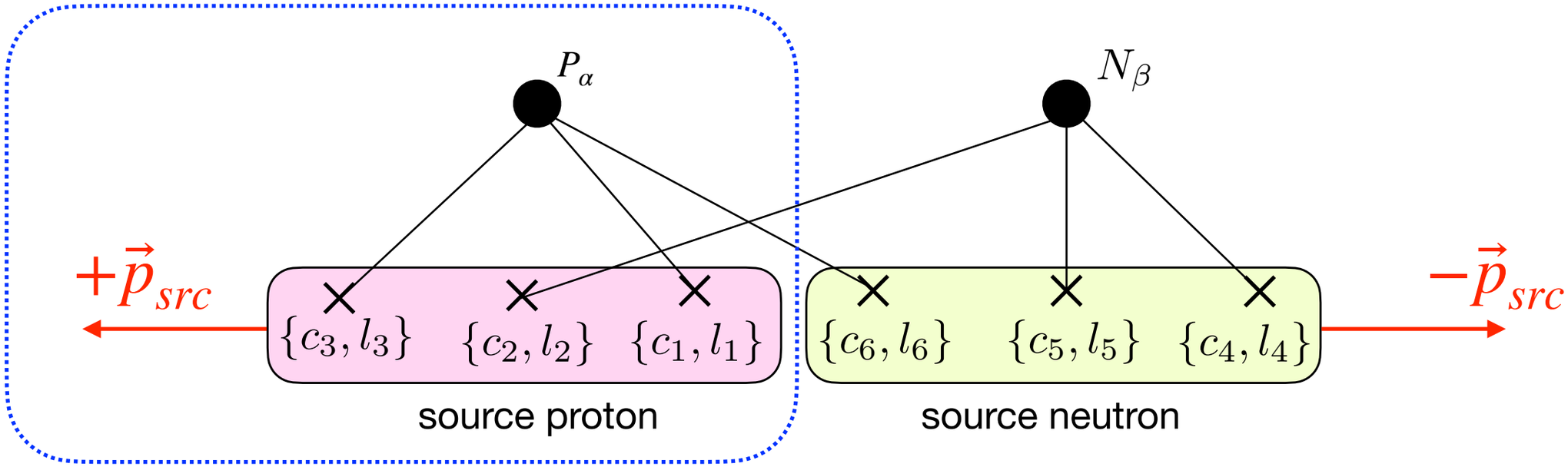


- Proton sub-diagram contains a factor

$$\bar{V}(l_1, l_2, l_3) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}_{\mathrm{src}}\cdot\boldsymbol{x}}\, v^\dagger_{l'_1}(\boldsymbol{x}) v^\dagger_{l'_2}(\boldsymbol{x}) v^\dagger_{l'_3}(\boldsymbol{x})$$

- Since $v_{l_i}(\vec{x}) = \exp(i\vec{p}_{l_i}\cdot\vec{x})$ in fLapH,

$$\bar{V}(l_1, l_2, l_3) = \sum_{\boldsymbol{x}} \exp\left[i(\boldsymbol{p}_{\mathrm{src}} - \boldsymbol{p}_{l_1} - \boldsymbol{p}_{l_2} - \boldsymbol{p}_{l_3})\cdot\boldsymbol{x}\right]$$

$$= \begin{cases} L^3 & (\boldsymbol{p}_{l_1} + \boldsymbol{p}_{l_2} + \boldsymbol{p}_{l_3} = \boldsymbol{p}_{\mathrm{src}}) \\ 0 & (\boldsymbol{p}_{l_1} + \boldsymbol{p}_{l_2} + \boldsymbol{p}_{l_3} \neq \boldsymbol{p}_{\mathrm{src}}) \end{cases}$$

Momentum conservation: given l_1 and l_2, surviving l_3 value is unique

$\rightarrow \times \mathcal{O}(N_l)$ speedup

Lattice Setup

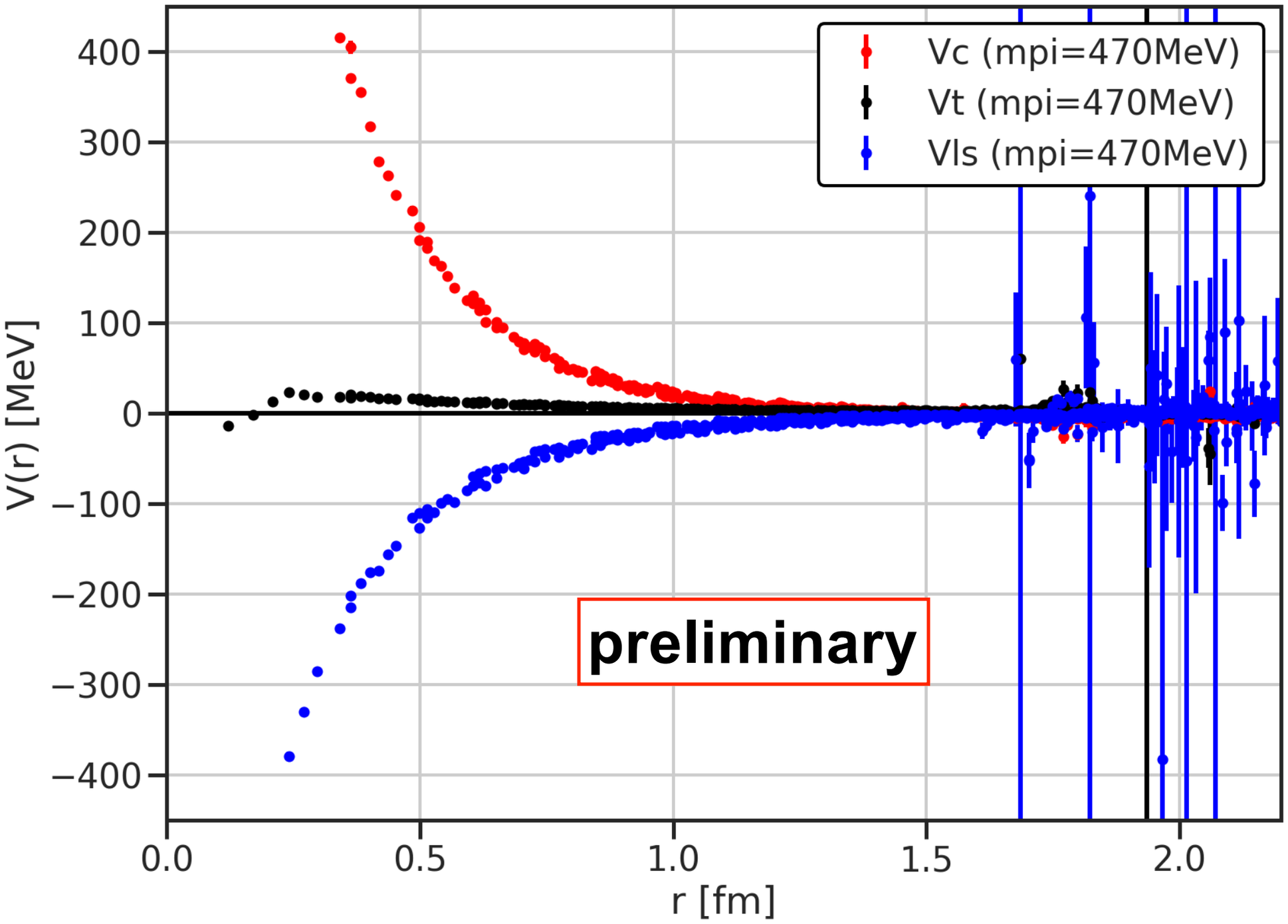
T.Inoue *et al.*, PRL**106** (2011) 162002.

- RG improved Iwasaki gauge action
- non-perturbatively O(a) improved Wilson quarks at flavor SU(3) symmetric point
  1. $m_\pi = 470\,\mathrm{MeV}, m_N = 1165\,\mathrm{MeV}$
  2. $m_\pi = 670\,\mathrm{MeV}, m_N = 1480\,\mathrm{MeV}$
  3. $m_\pi = 840\,\mathrm{MeV}, m_N = 1740\,\mathrm{MeV}$
- $L^3T = 32^4, a = 0.121(2)\,\mathrm{fm}$
- Number of fLapH modes: $N_l = 1,7,19$

  corresponding to $\sigma = 2\pi/L \times (0,1,2)$

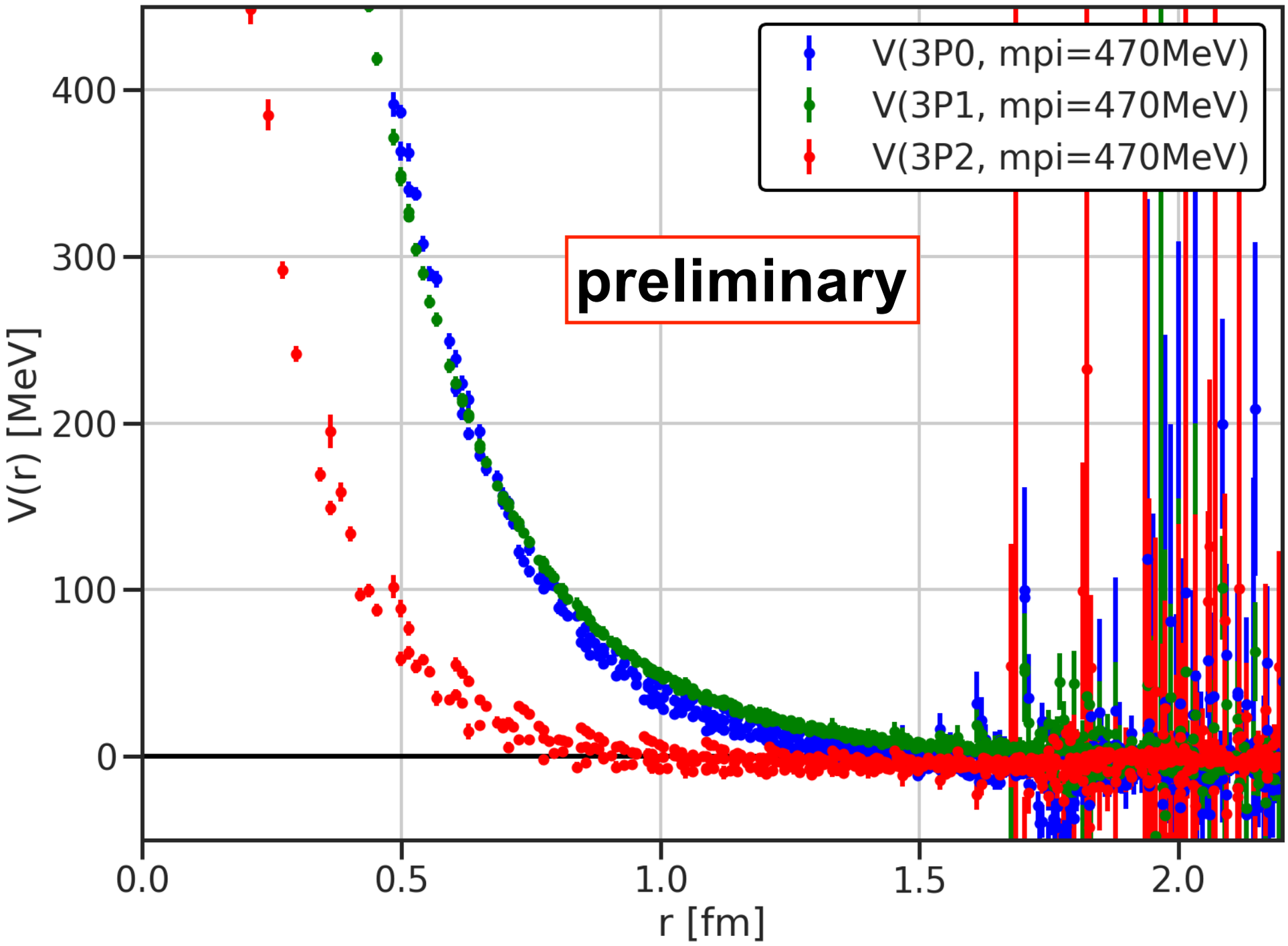
P-wave potentials, operator basis (m_π=470 MeV)

Red: Vc(r) black: Vt(r) blue: Vls(r)



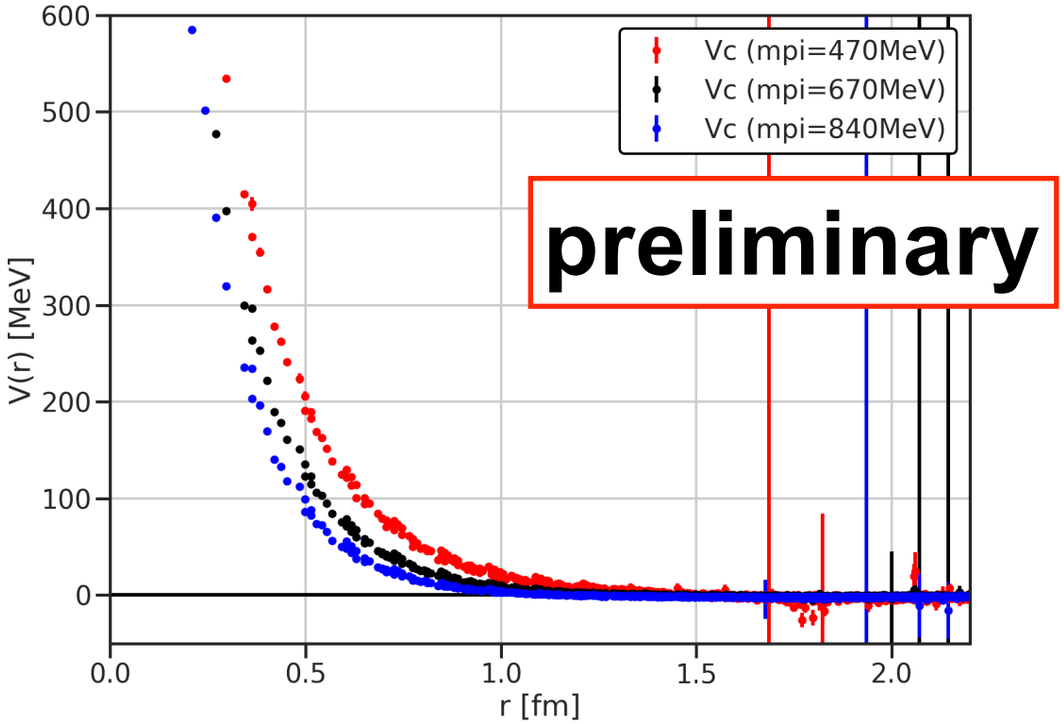
P-wave potentials, channel basis (m_π=470 MeV)

blue: $^3P_0$ green: $^3P_1$ red: $^3P_2$

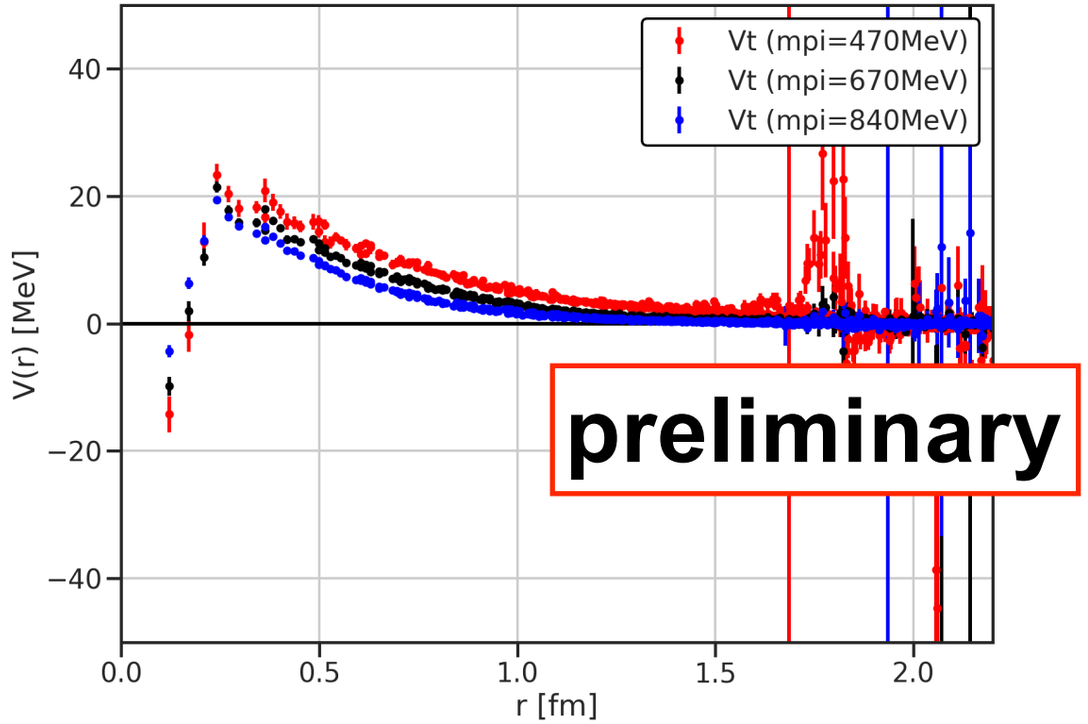


m_π dependence

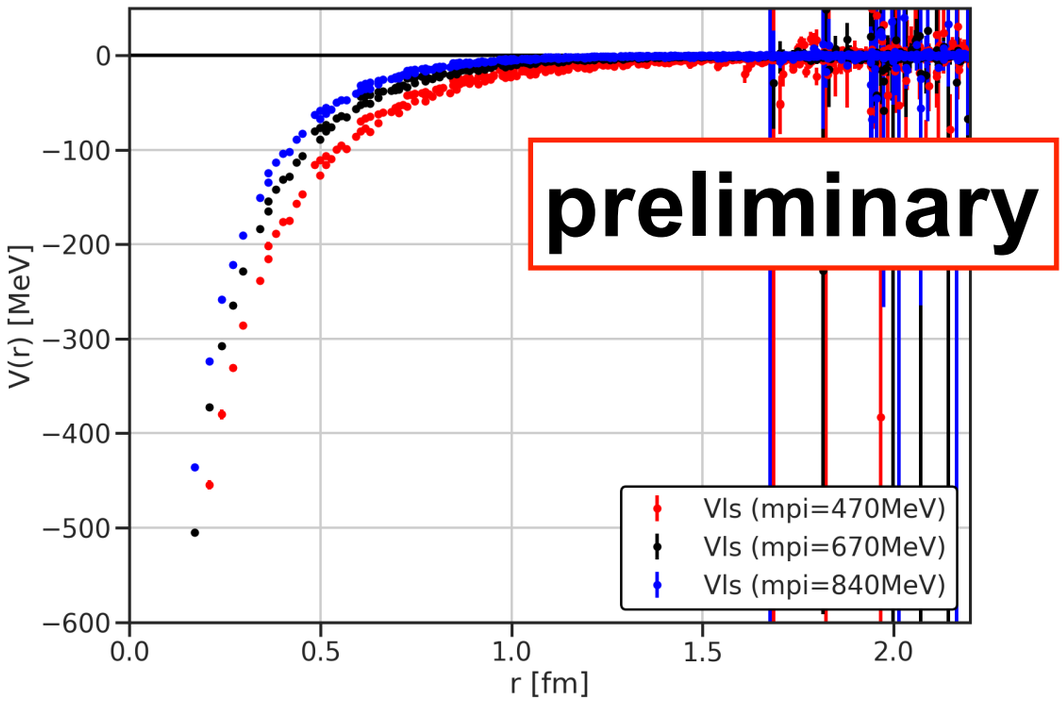
Central (mpi=470,670,840 MeV)



Tensor (mpi=470,670,840 MeV)



LS (mpi=470,670,840 MeV)



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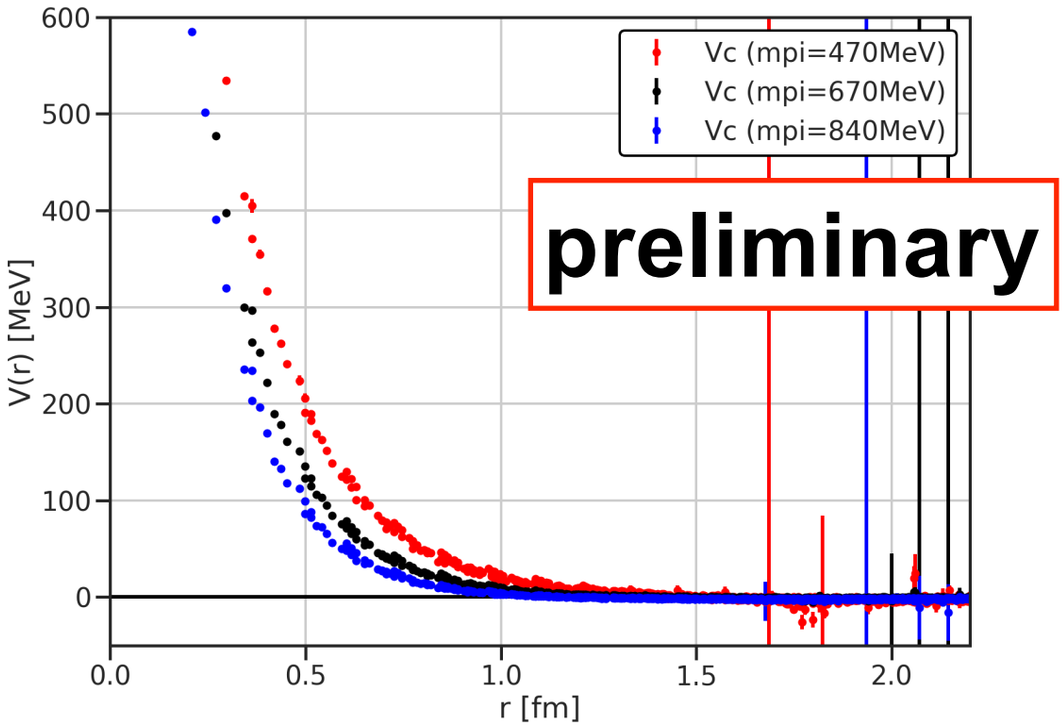

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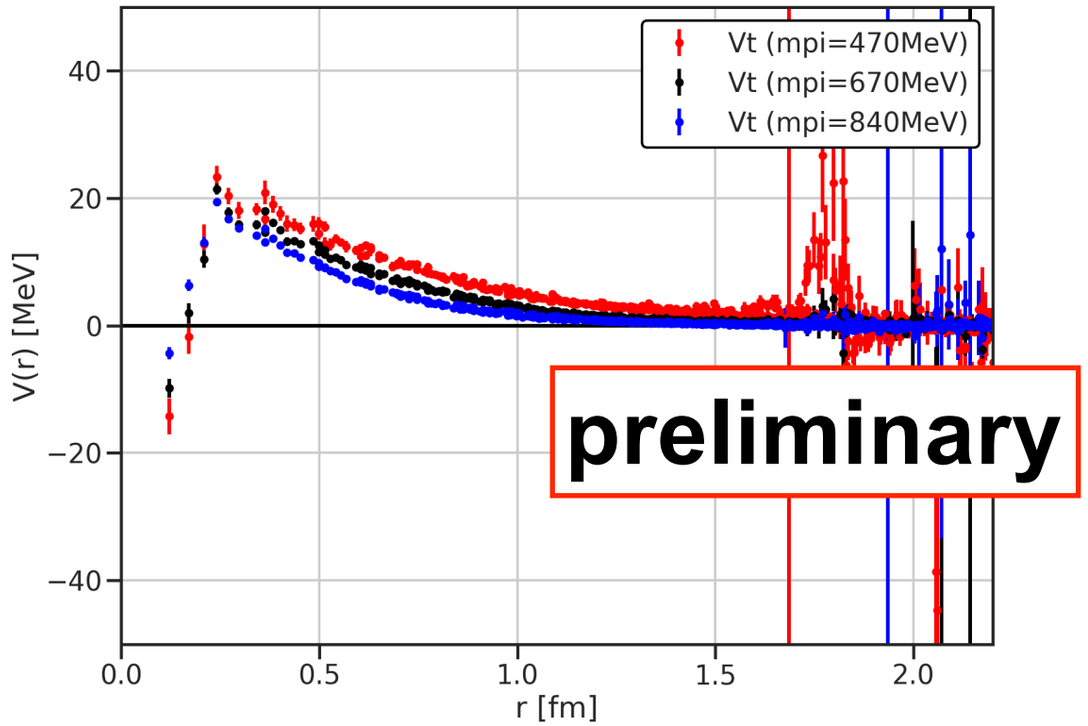

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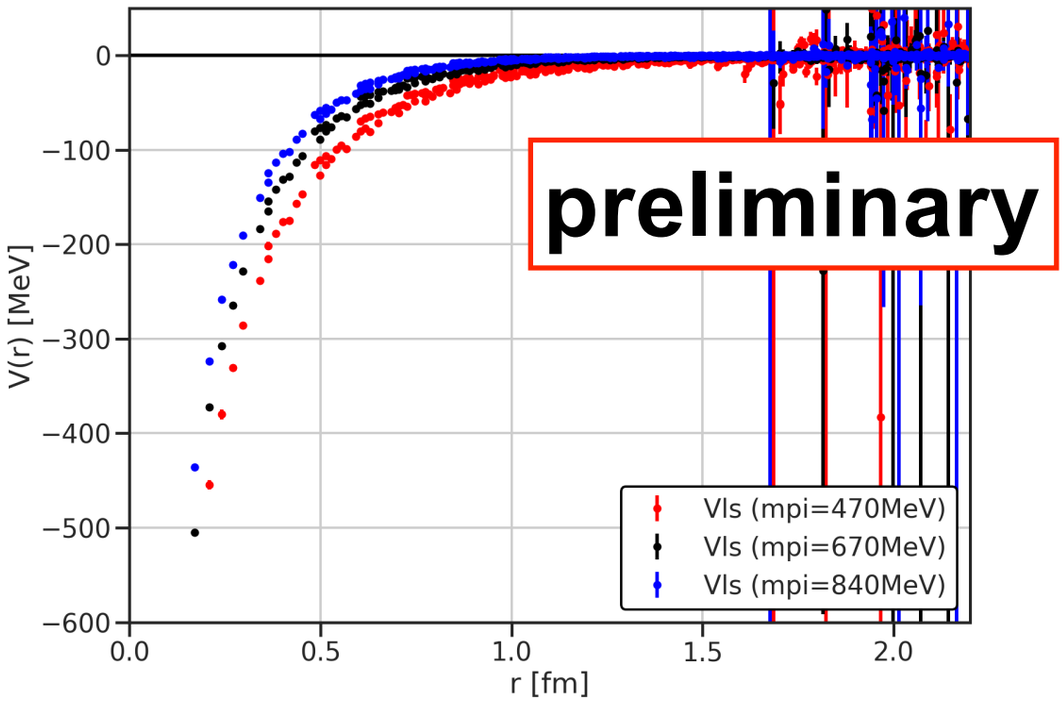
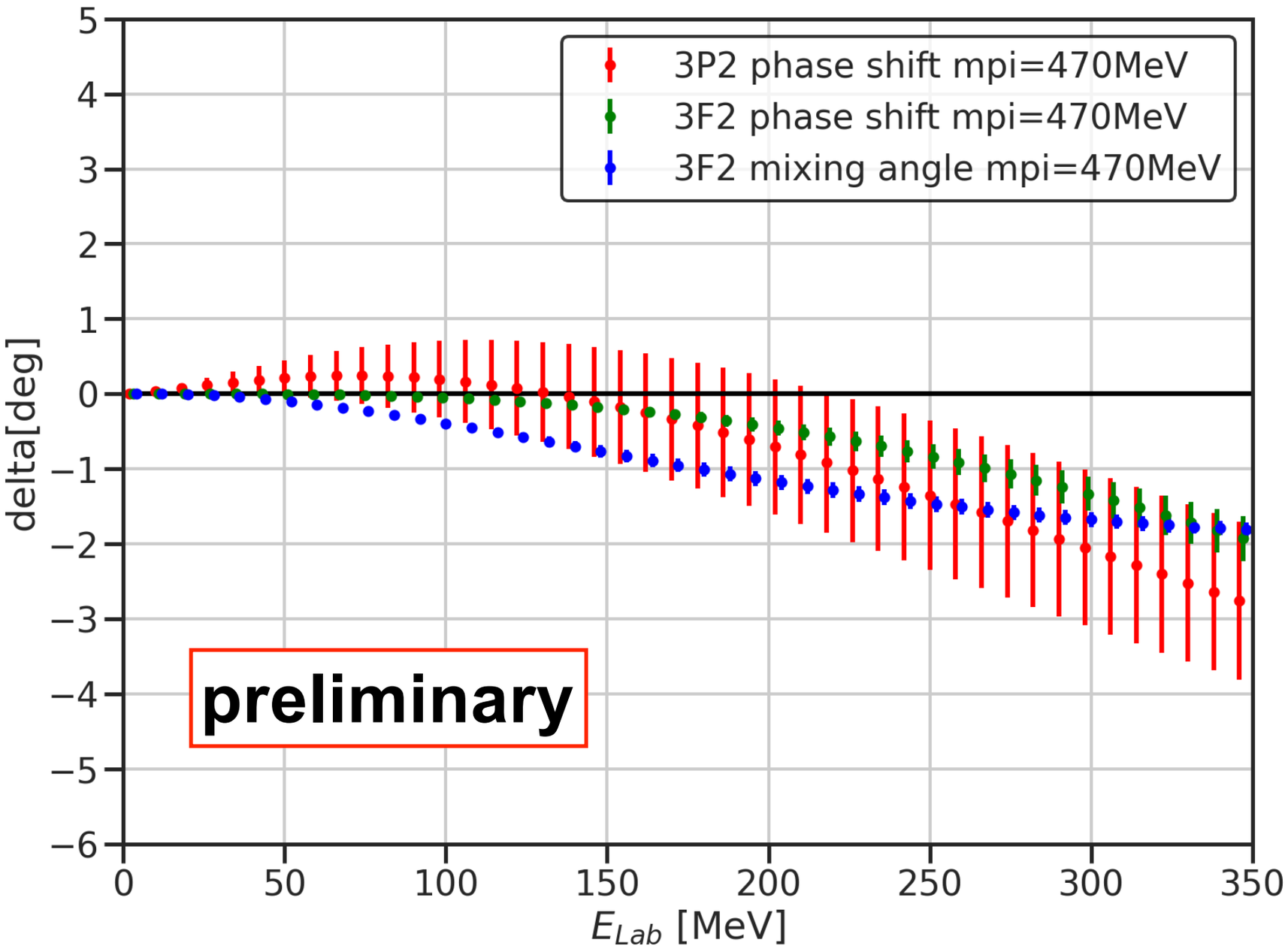


With smaller m_π,

- Vc gets more positive
- Vls gets more negative
- Vt remains small

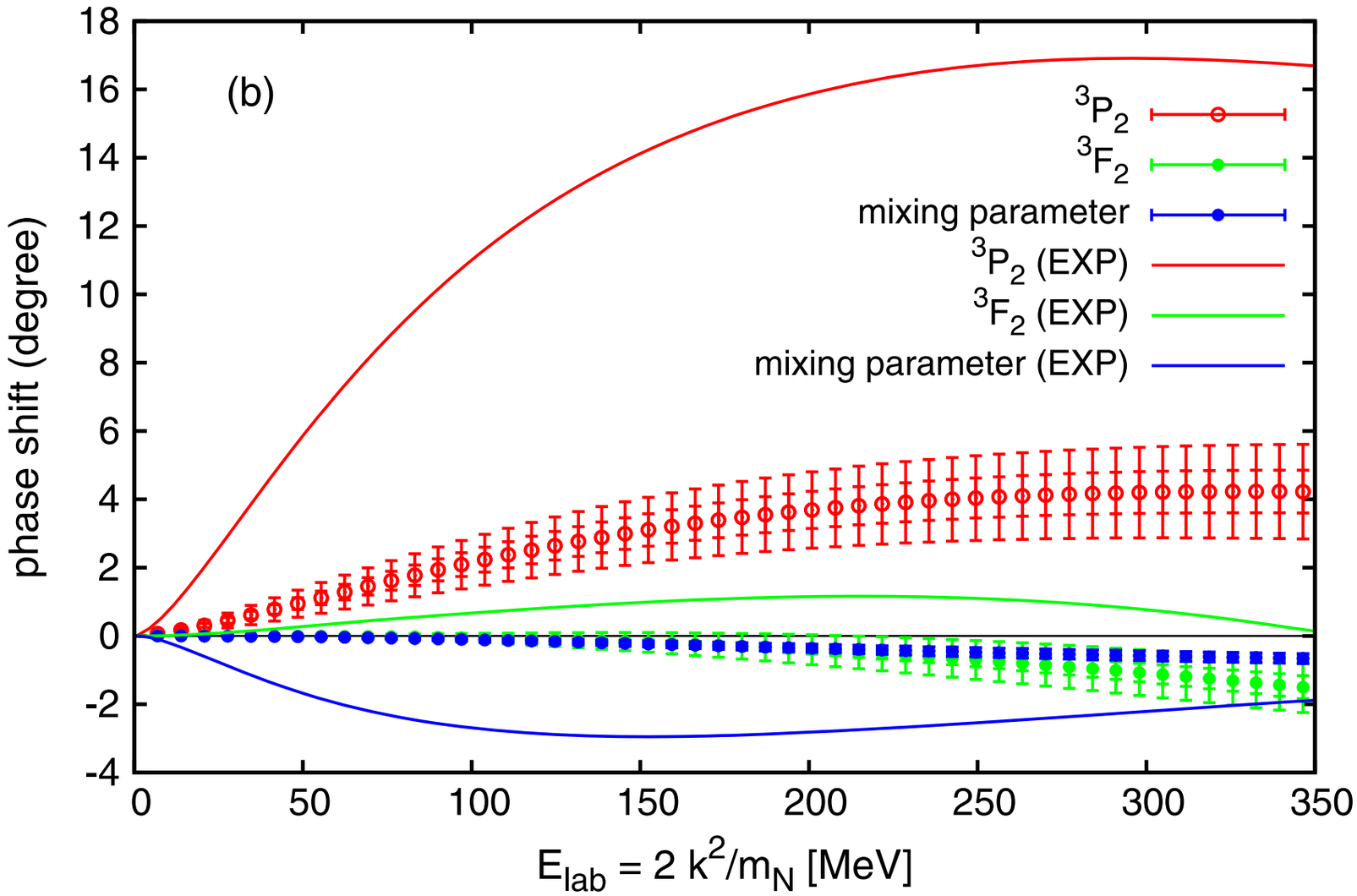
Phase Shift (3P2+3F2 channel)

red: $^3P_2$ phase shift green: $^3F_2$ phase shift blue: mixing angle



Comparison w/ previous results

Murano and Ishii [HAL QCD], Phys. Lett. B **735** (2012) 19



- Small lattice volume: $16^3 \times 32$
- Heavy quark masses: $m_\pi = 1133$ MeV
- nucleon source = wall source×2q + 1q with finite momentum

Yet 3P2 attraction was stronger than ours : Non-monotonic m_q dependence??

Summary and Outlook

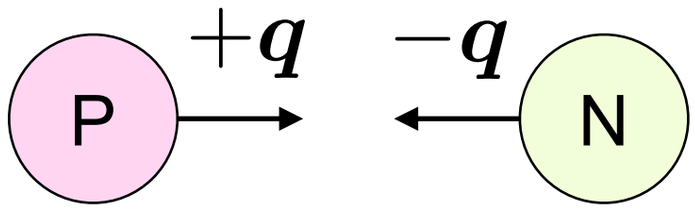
- fLapH: free Laplacian Heaviside, is an efficient way of computing all-to-all propagators
- P-wave potentials (central, tensor, LS) are determined with the HAL QCD method
- Experimentally expected attraction in the 3P2 channel is not evident in our study

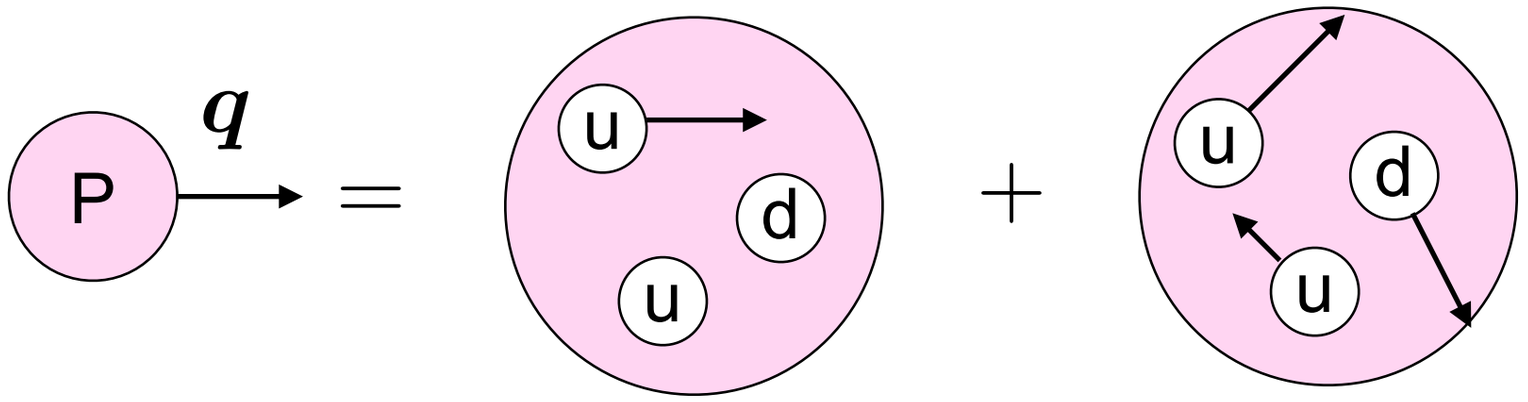
- P-wave Hyperon forces (YN and YY): flavor SU(3) symmetric point will be useful
- Lighter quarks, hopefully at the physical point
- For larger volumes, we might need a better method to compute all-to-all propagators

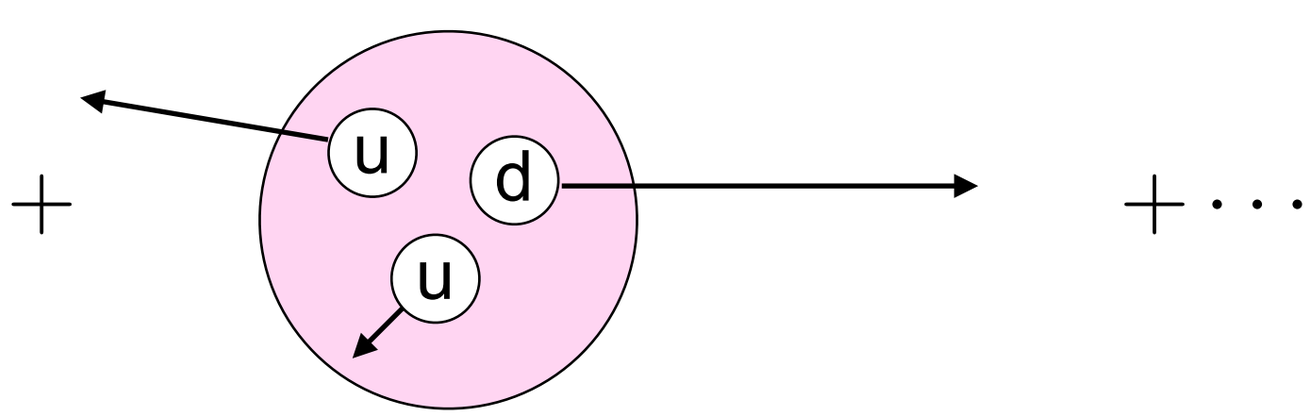
Backup Slides

fLapH法 (2/2)

$$\bar{J}(t) =$$







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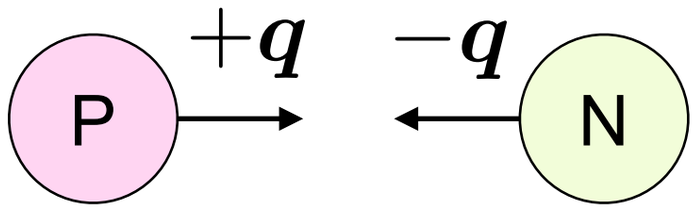
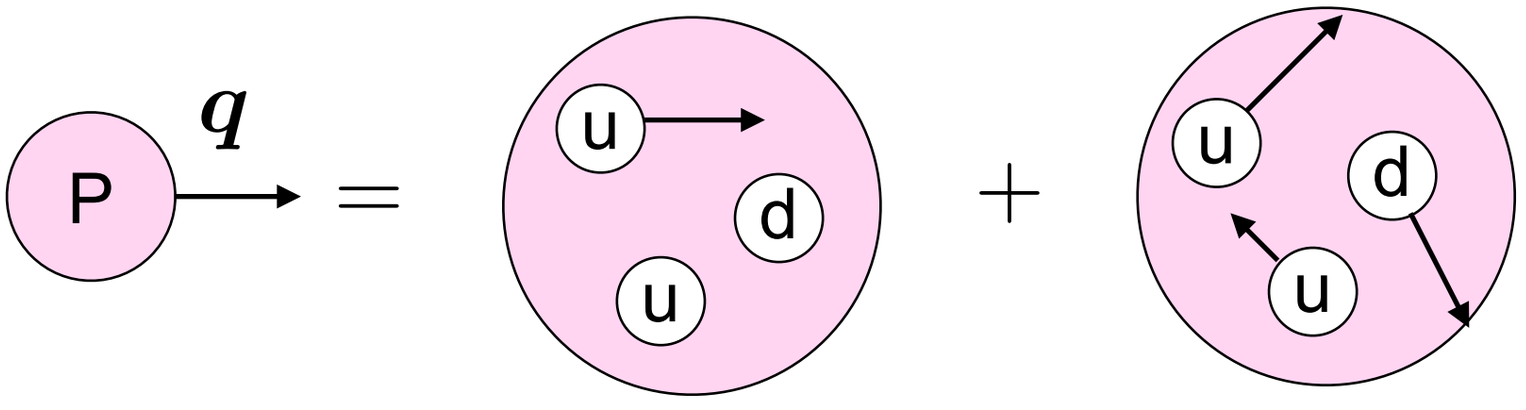
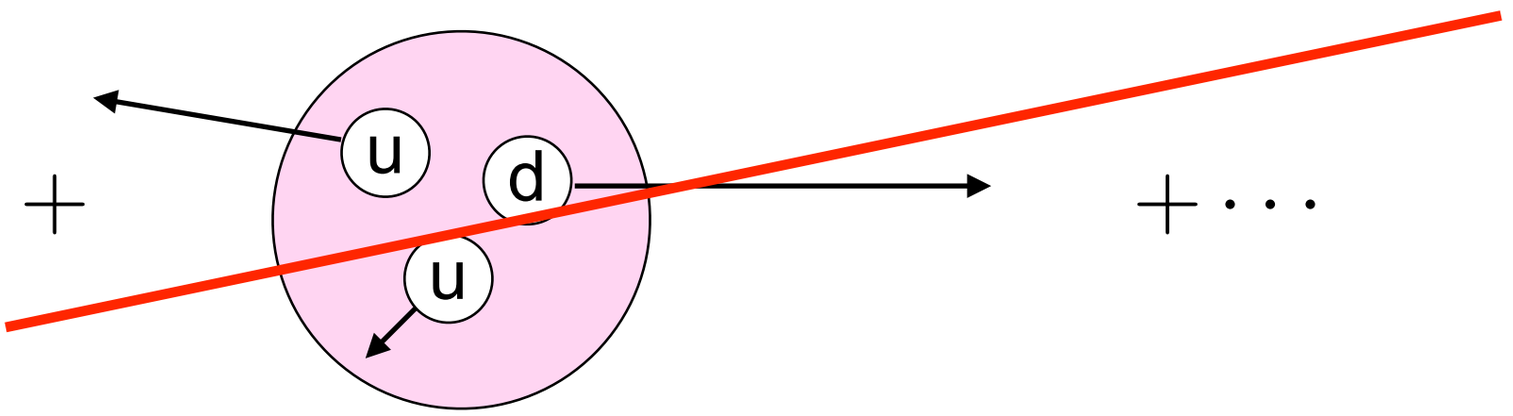




cut high momentum modes

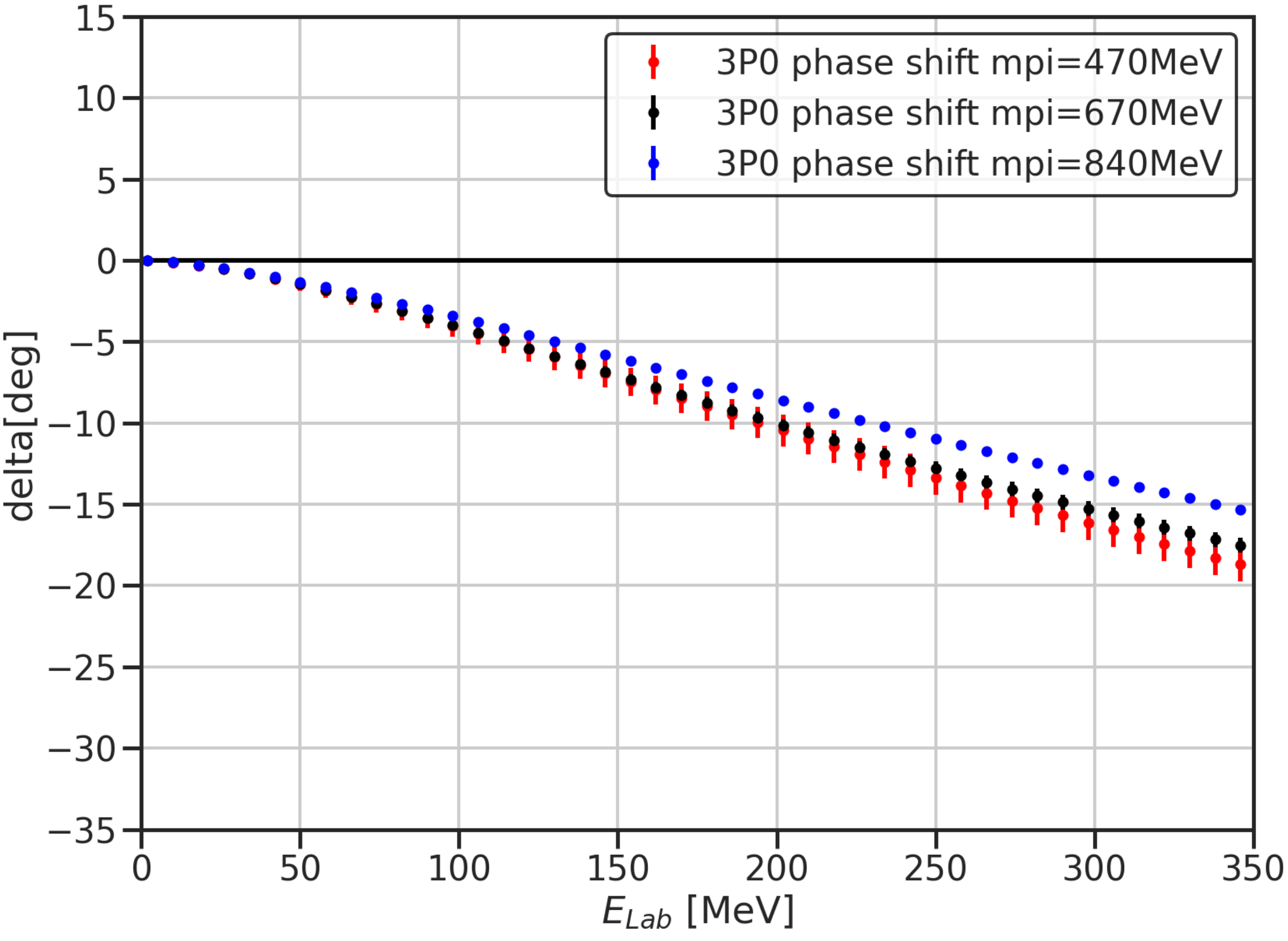


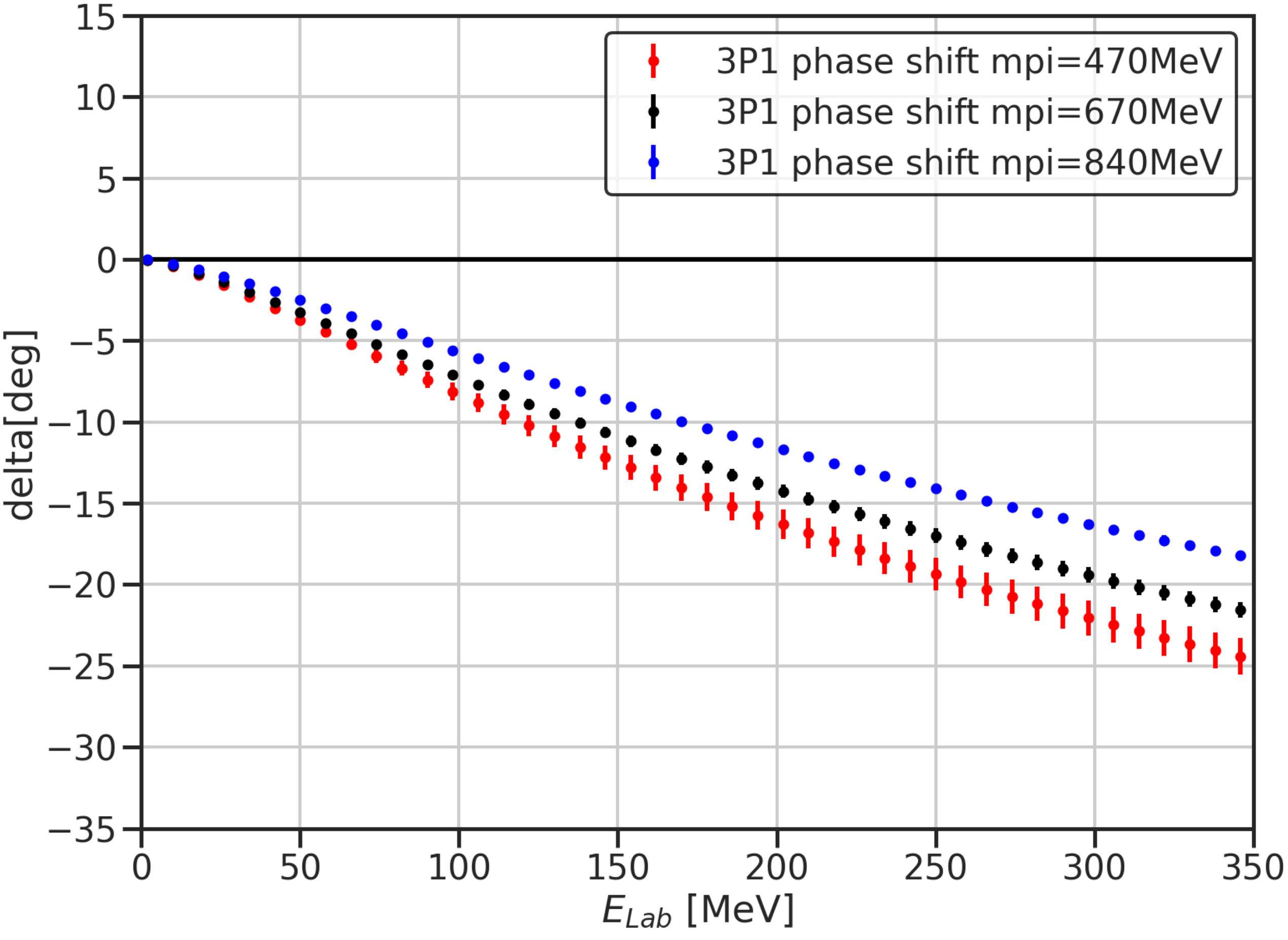
q = (0,0,0), $\rightarrow N_l = 1 \quad (|q|^2 = 0)$ ... wall source

(1,0,0), (0,1,0), (0,0,1), (-1,0,0), (0,-1,0), (0,0,-1), $\rightarrow N_l = 7 \quad (|q|^2 \leq 1)$

(1,1,0), (0,1,1), (1,0,1), (1,-1,0), (0,1,-1), (-1,0,1), (-1,1,0), (0,-1,1), (1,0,-1),

(-1,-1,0), (0,-1,-1), (-1,0,-1), $\rightarrow N_l = 19 \quad (|q|^2 \leq 2)$


3P0 phase shift mpi=470MeV
3P0 phase shift mpi=670MeV
3P0 phase shift mpi=840MeV
delta[deg]
E_{Lab} [MeV]
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
0
50
100
150
200
250
300
350


3P1 phase shift mpi=470MeV
3P1 phase shift mpi=670MeV
3P1 phase shift mpi=840MeV
delta[deg]
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
0
50
100
150
200
250
300
350
E_{Lab} [MeV]