Physical-point Lattice QCD study of baryon-baryon interactions in the $S = -1$ channel

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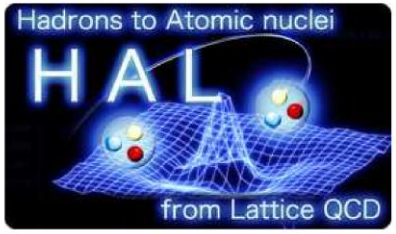
for HAL QCD Collaboration



Lattice 2025, 2025-11-07, TIFR, India

Background for S = -1 interaction

***S* = -1**: Systems of minimum number of strangeness

→ Most common/accessible exotic systems

- Neutron star (NS): EoS, M-R, hyperon puzzle (massive NS)
- Λ-hypernuclear spectroscopy: binding energy
- Scattering experiment: dσ/dΩ, phase shift, cusp, …
- Femtoscopy: correlation function
- PID flows in high-energy heavy-ion collisions through single-particle potential
- etc.

This study: two-body force *YN* (*S* = -1) with the HAL QCD method

HAL QCD method

Time-dependent Schroedinger eq. for *R* correlator with coupled channels

$$\left(\frac{1+3\delta_c^2}{8\mu_c}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_c}\right)R^c{}_d(\vec{r},t)=\sum_{c'}\int d^3r' U^c{}_{c'}(\vec{r},\vec{r}')\Delta^c{}_{c'}R^{c'}{}_d(\vec{r}',t),$$

Measure normalized 4pt function ***R(r,t)***

Assumption: $U(r,r') \sim \delta(r-r')$ ***V(r)*** → Solve $V(r)$

K-conf

- N_f = 2+1, *almost physical point* (m_π, m_K) = (146, 525) MeV at K-computer
- size 96^4, a = 0.085 fm, 400 configurations
- Existing analyses: H. Nemura, AIP Conf. Proc. 2130 (2019) 1, 040005 [HYP2018], H. Nemura, PoS LATTICE2021 (2022) 272, etc.

F-conf (HAL-conf-2023) Aoyama et al, PRD110 (2024) 9, 094502.

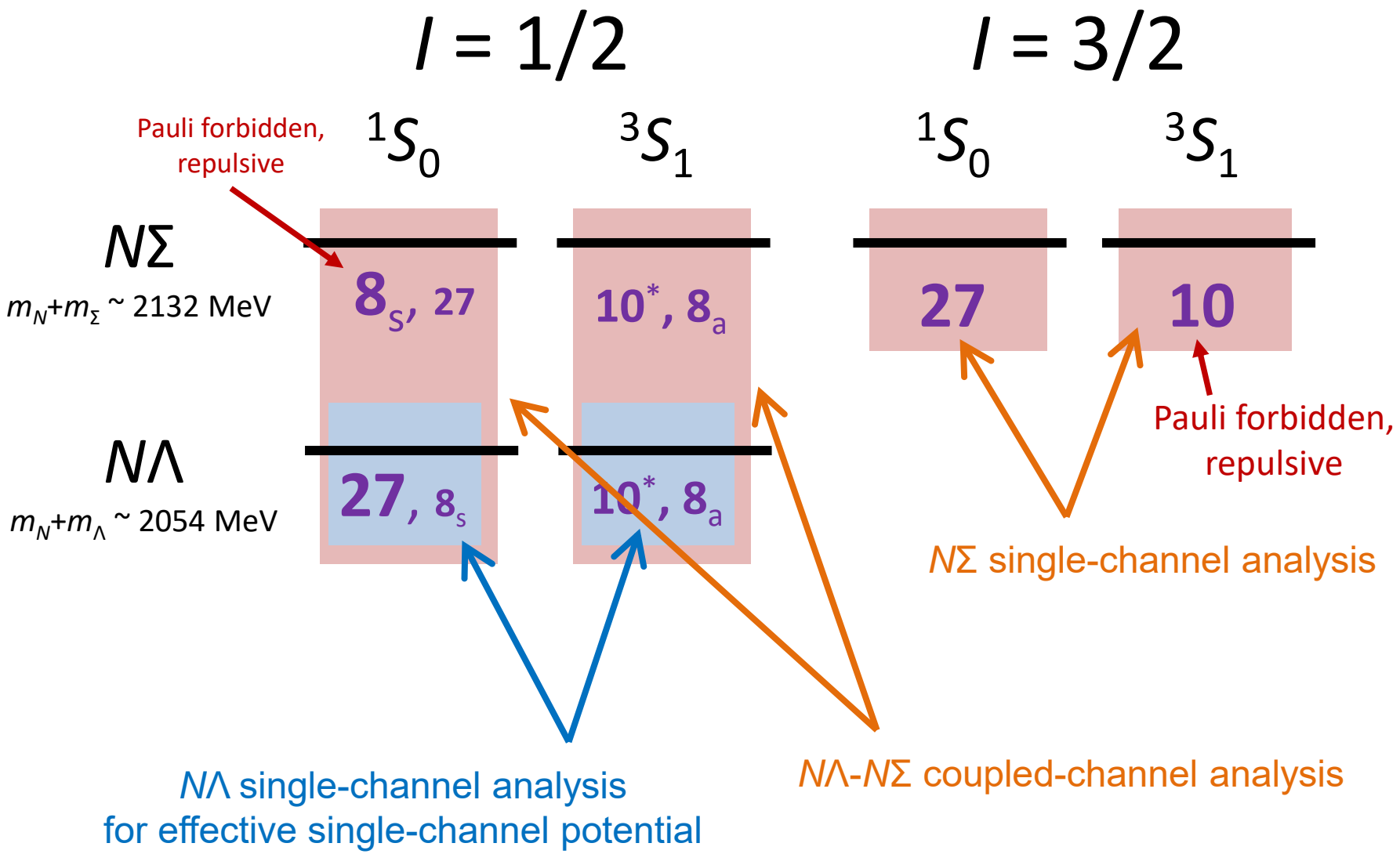
- N_f = 2+1, *physical point* (m_π, m_K) = (137, 502) MeV at Fugaku supercomputer
- size 96^4, a = 0.084372(54) fm, 1600 configurations

→ Use F-conf to analyze *S* = -1 potentials

S = -1 Baryon-Baryon (S-wave)

Note: $^3D_1$ & tensor forces are not considered **for now**

27, 10, 8$_s$ etc.: SU(3)$_f$ rep.



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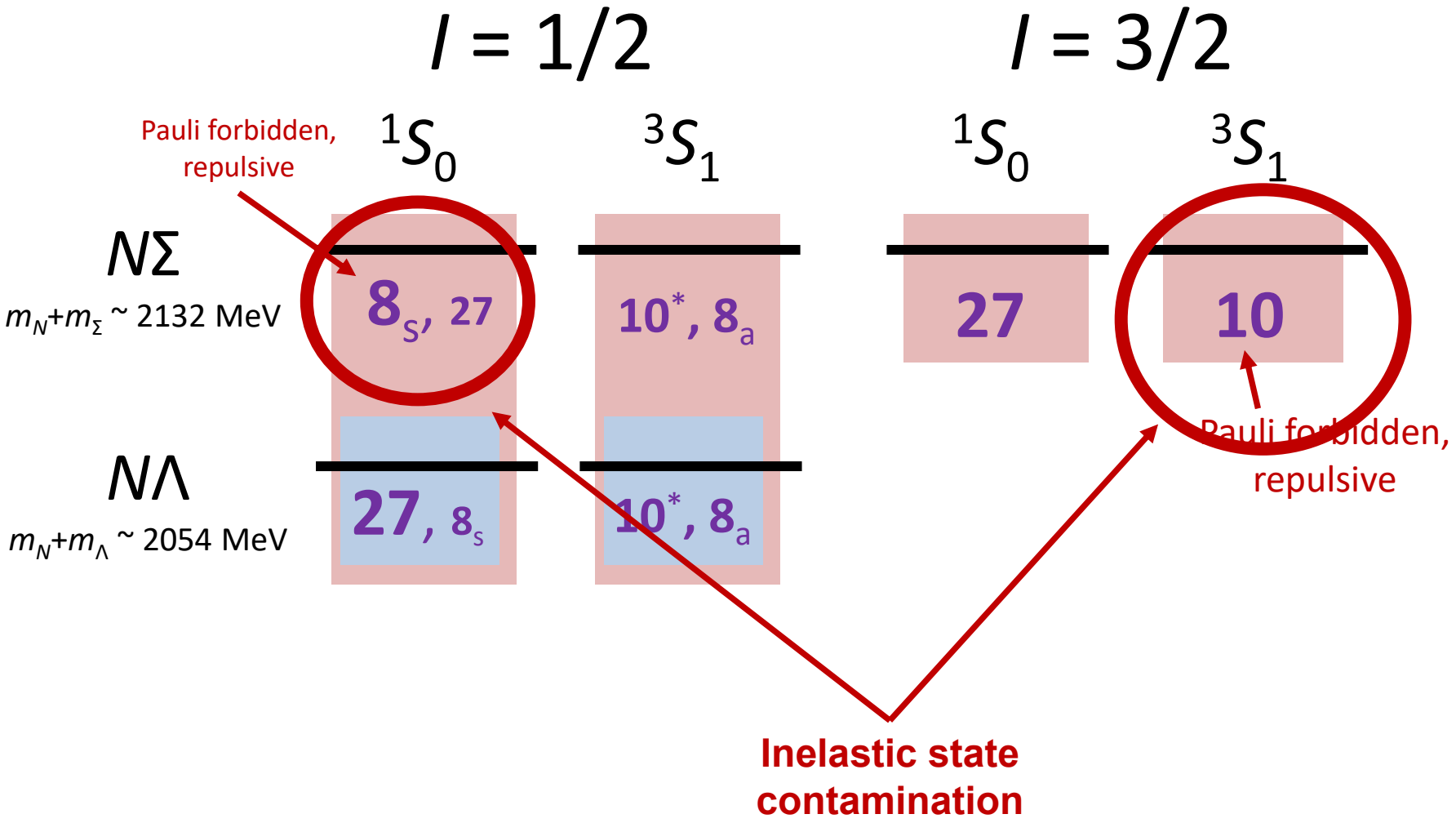
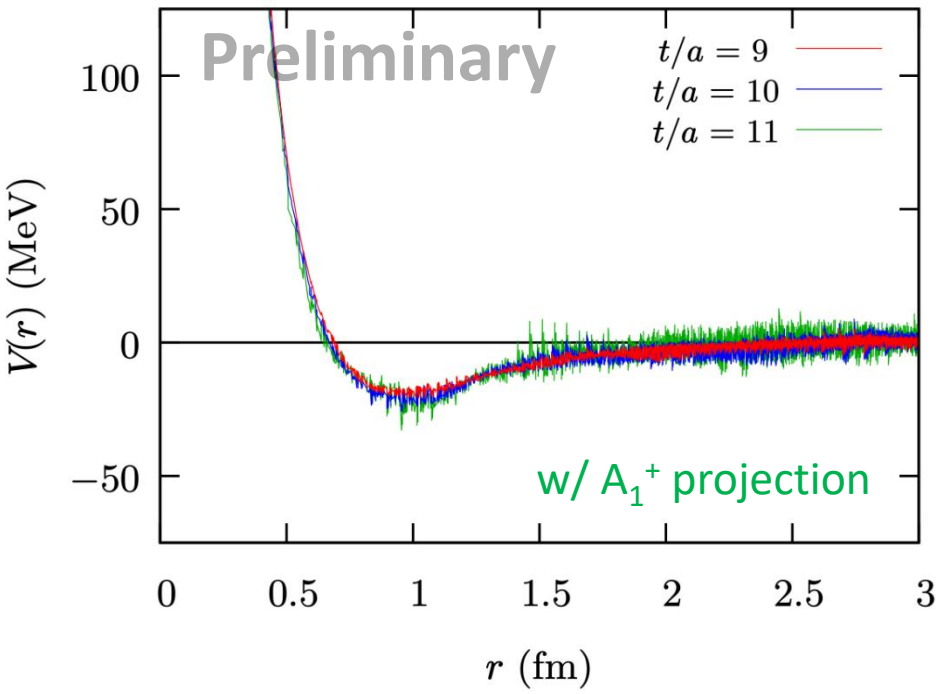


$N\Sigma$(I=3/2) potentials

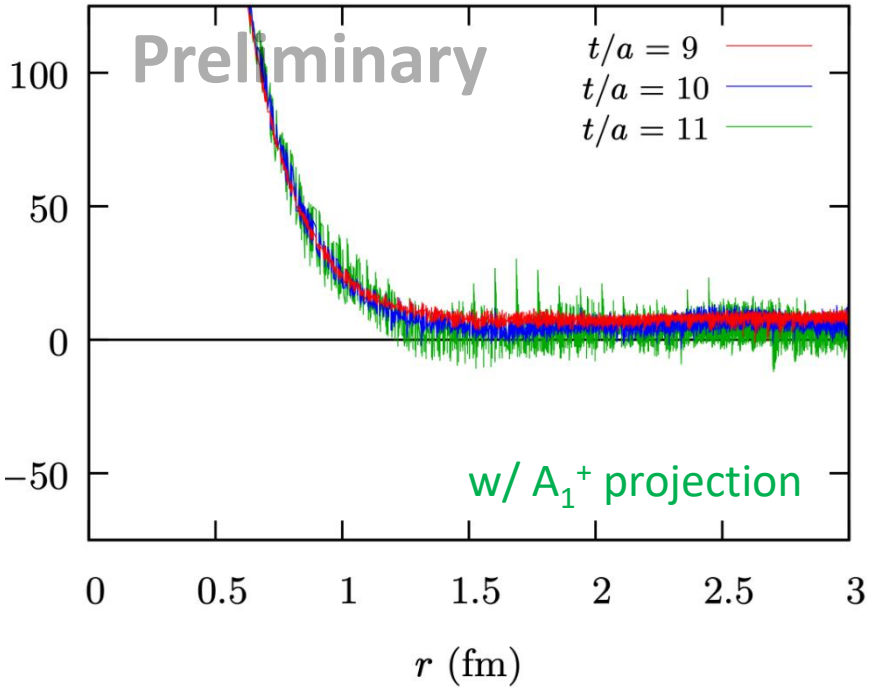
$^1S_0$



~ SU(3) **27**

~ *NN*(I=1)

$^3S_1$

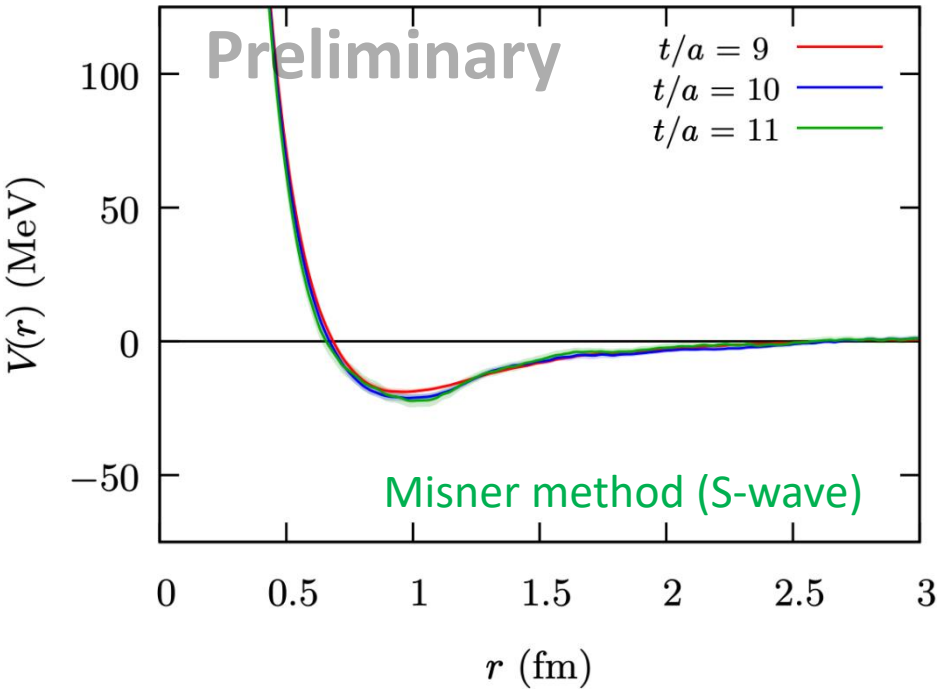


~ SU(3) **10**

~ Pauli forbidden, repulsive

$N\Sigma$(I=3/2) potentials

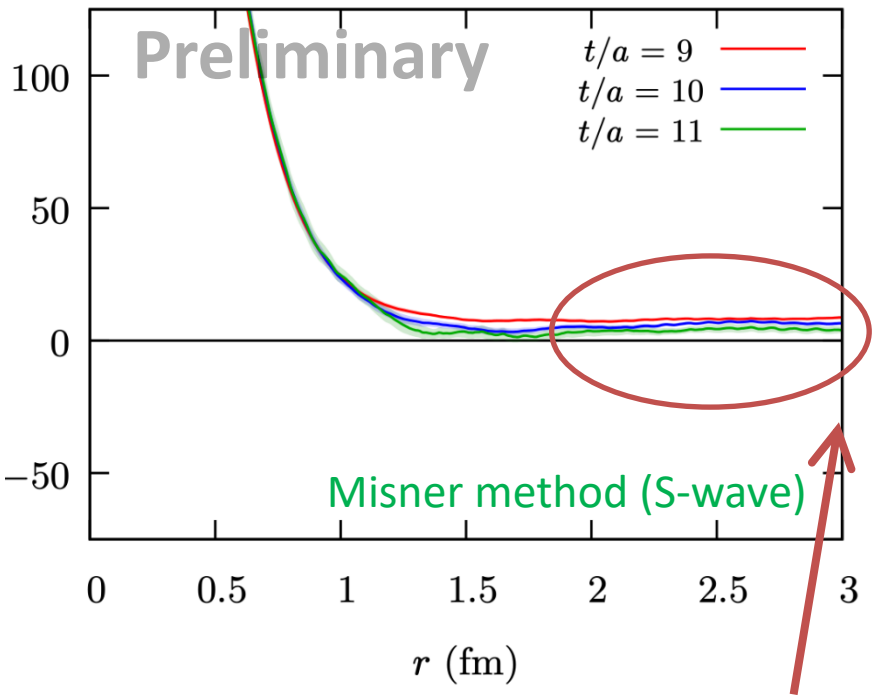
$^1S_0$



~ SU(3) **27**

~ *NN*(*I*=1)

$^3S_1$



~ SU(3) **10**

~ Pauli forbidden, repulsive

Constant-offset coming from inelastic contamination

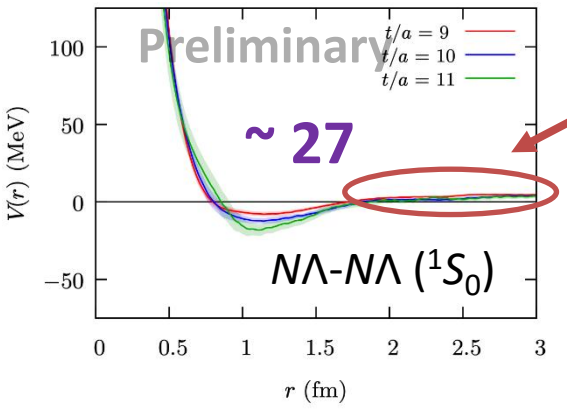
Misner method: Extract partial wave and average in [*r*-*a*, *r*+*a*]

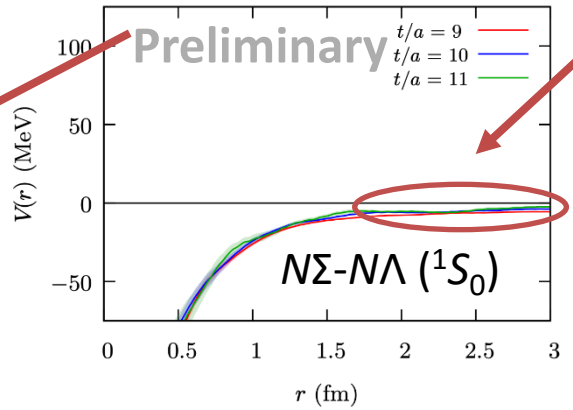
$N\Lambda$-$N\Sigma$ coupled-channel (I=1/2)

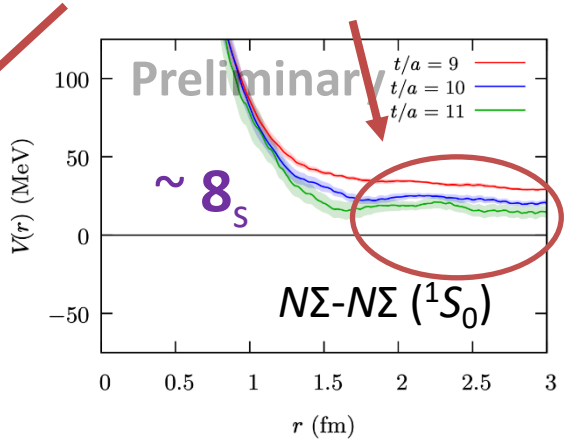
Analysis ongoing…

Other channels also suffer from **constant offsets**

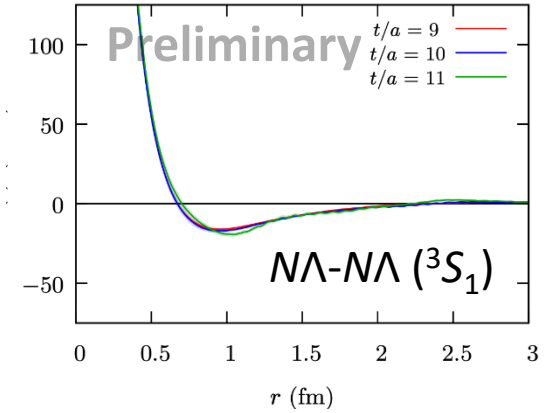
2x2 $N\Lambda$-$N\Sigma$ coupled-channel potential ($^1S_0$)

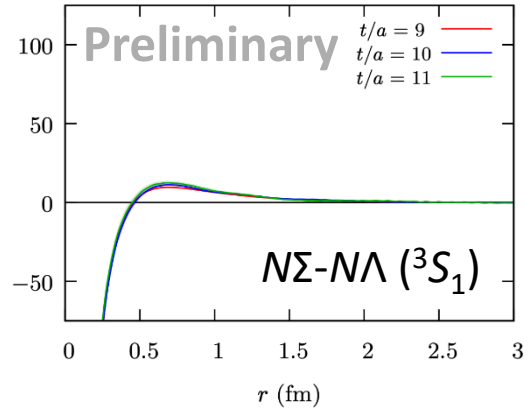


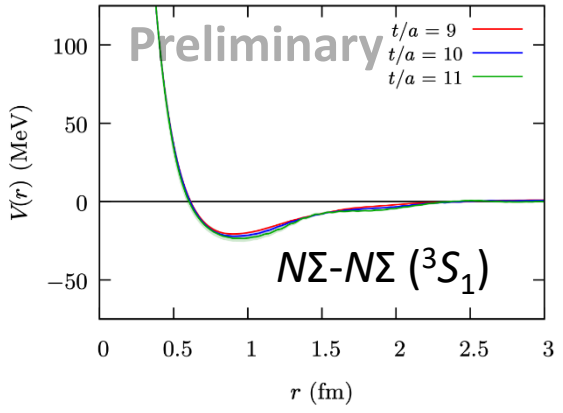




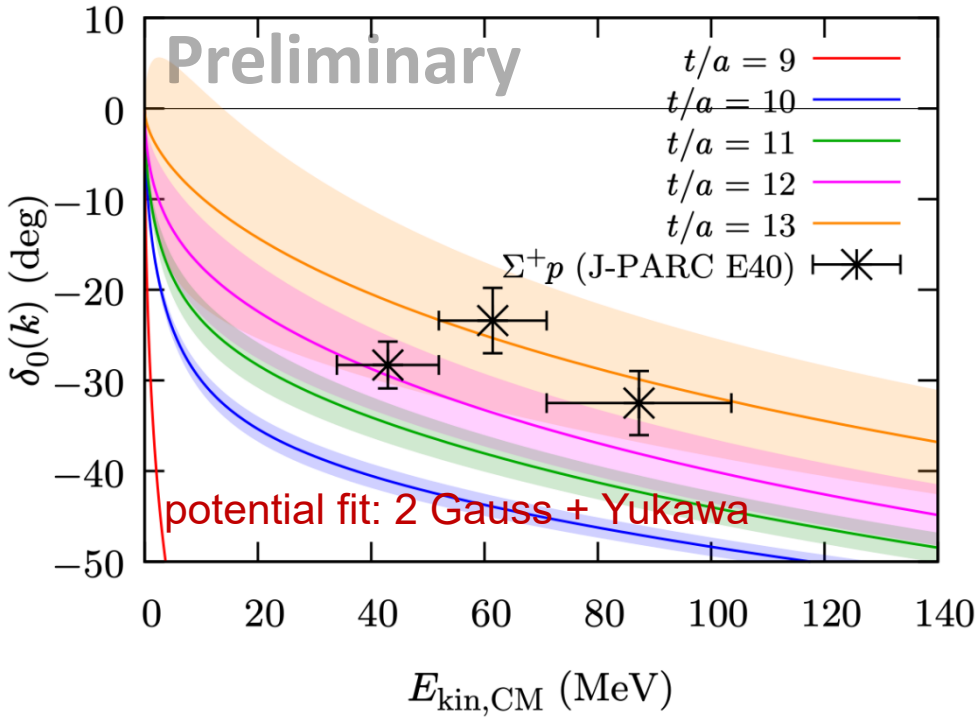
2x2 $N\Lambda$-$N\Sigma$ coupled-channel potential ($^3S_1$)





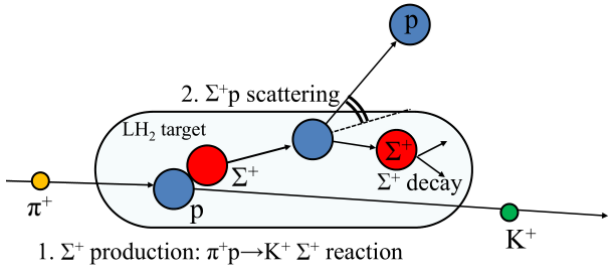


$N\Sigma$(I=3/2) $^3S_1$ phase shift



Experimental data:
Σ^+p elastic scattering [J-PARC E40]
to determine the size of repulsion

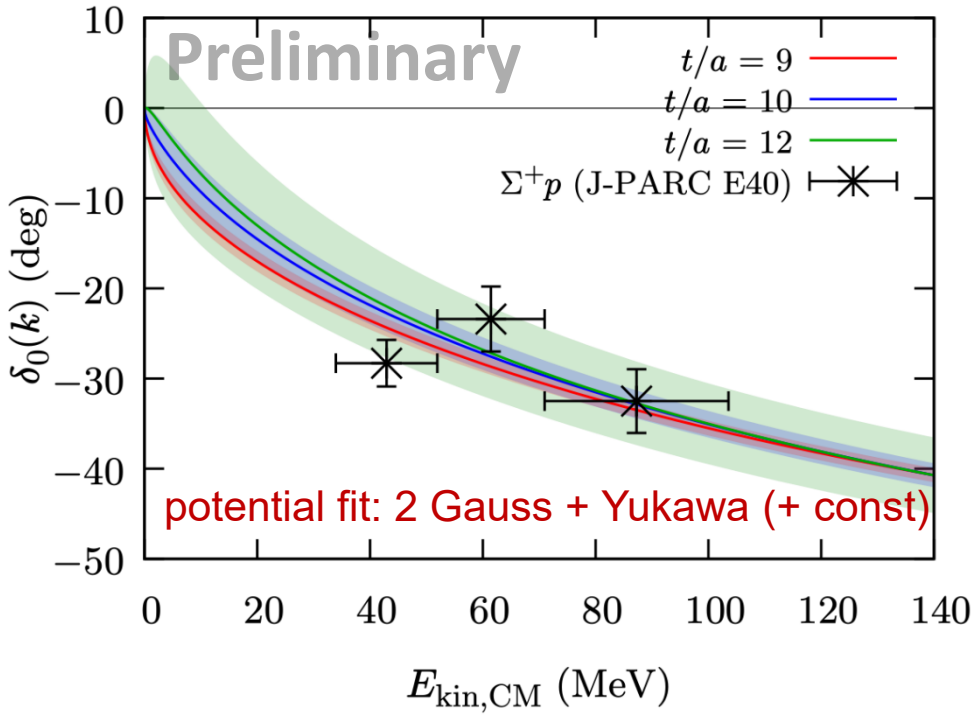
Nanamura et al. [J-PARC E40] PTEP 2022 (2022) 9, 093D01



$t/a \leqq 11$: large contamination from inelastic excited states

$t/a \geqq 12$: larger noise

NΣ(I=3/2) $^3S_1$ phase shift



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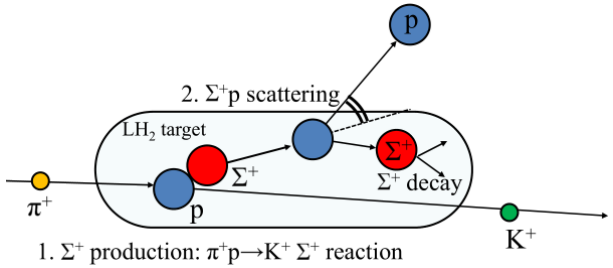


With the constant offset of the potential subtracted in fitting, the result becomes **stable** for a larger *t/a* range.

→ Consistent with experimental estimation of phase shifts (with $\delta < 0$ assumed)

Extrapolation of $V(r)$ to $t/a \to \infty$

Naively, **inelastic state contamination** enter
the *R* correlator in a sum of exponential forms exp(-bt):

$$R(r,t) = \sum_{k:\text{elastic}} \psi_k(r) e^{-\Delta E_k t} + \psi_{\text{inel}}(r) e^{-\Delta E_{\text{inel}} t} + \cdots .$$

Elastic part

Inelastic contamination...

Assumption for extracted potential: V = (1/R) (— H_0 — ∂/∂t) R

$$V(r,t) \simeq V_{\text{el}}(r) + V_{\text{inel}}(r) e^{-bt}$$

we want this

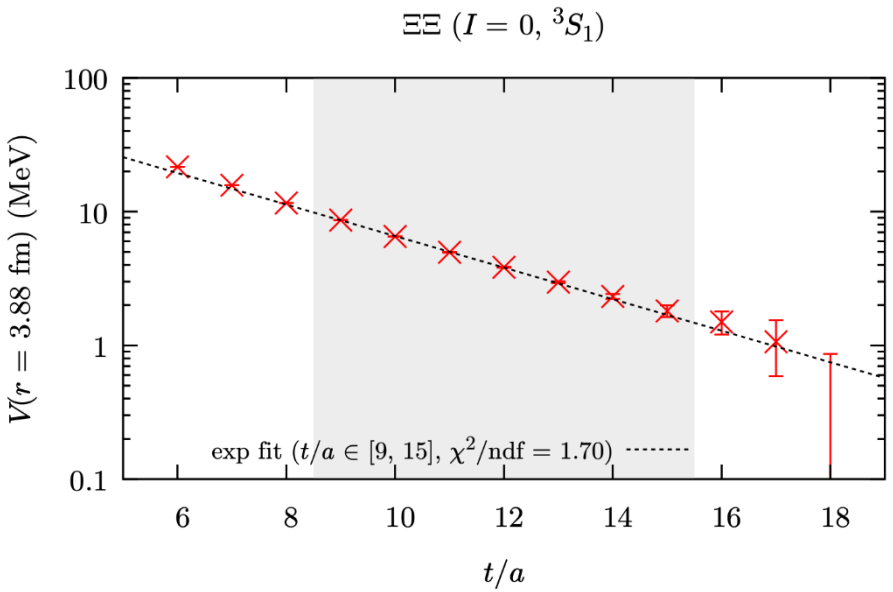
contamination (leading)

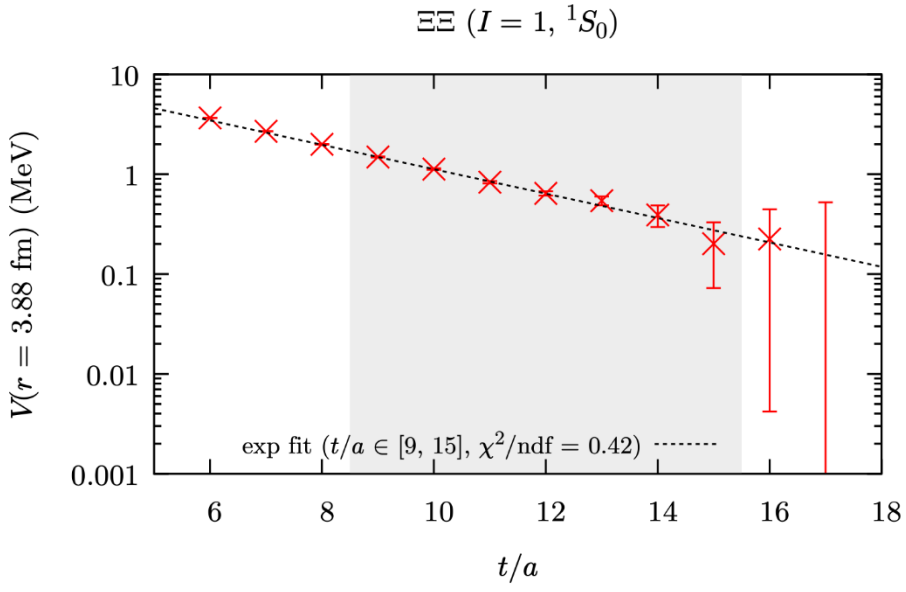
Steps to get *V*(*r*)

1. Determine ***b*** (> 0) at a **large *r*** of $V(r, t)$
2. Fit V_{el} and V_{inel} at each ***r***. → Pick only V_{el} (Extrapolation for $t/a \to \infty$)

Inelastic contamination in ΞΞ

Small noise in ΞΞ → We can check the contamination behavior for a wide t/a range



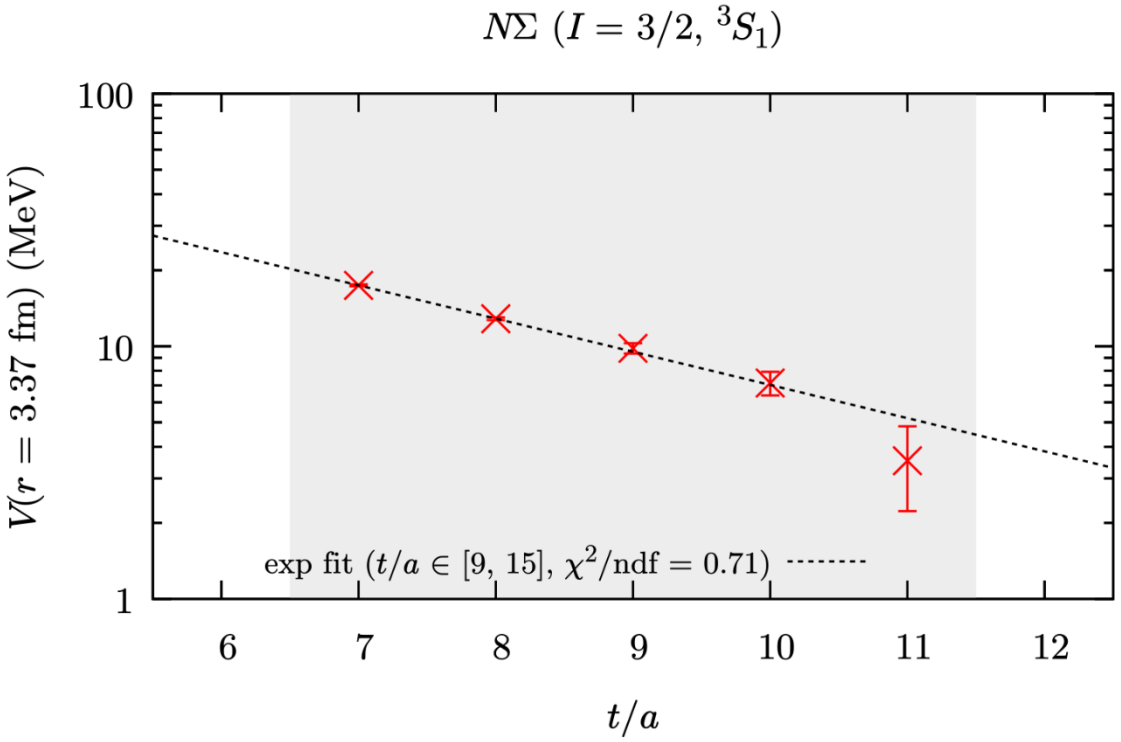


points: calculated potential values
line: exponential fit by $a\exp(-bt)$
shaded region: fitting range

Contamination is well described by a single exponential form
at large r (i.e., constant offsets)
in ΞΞ (where the contamination is small)

Inelastic contamination in $N\Sigma$ (I=3/2, $^3S_1$)

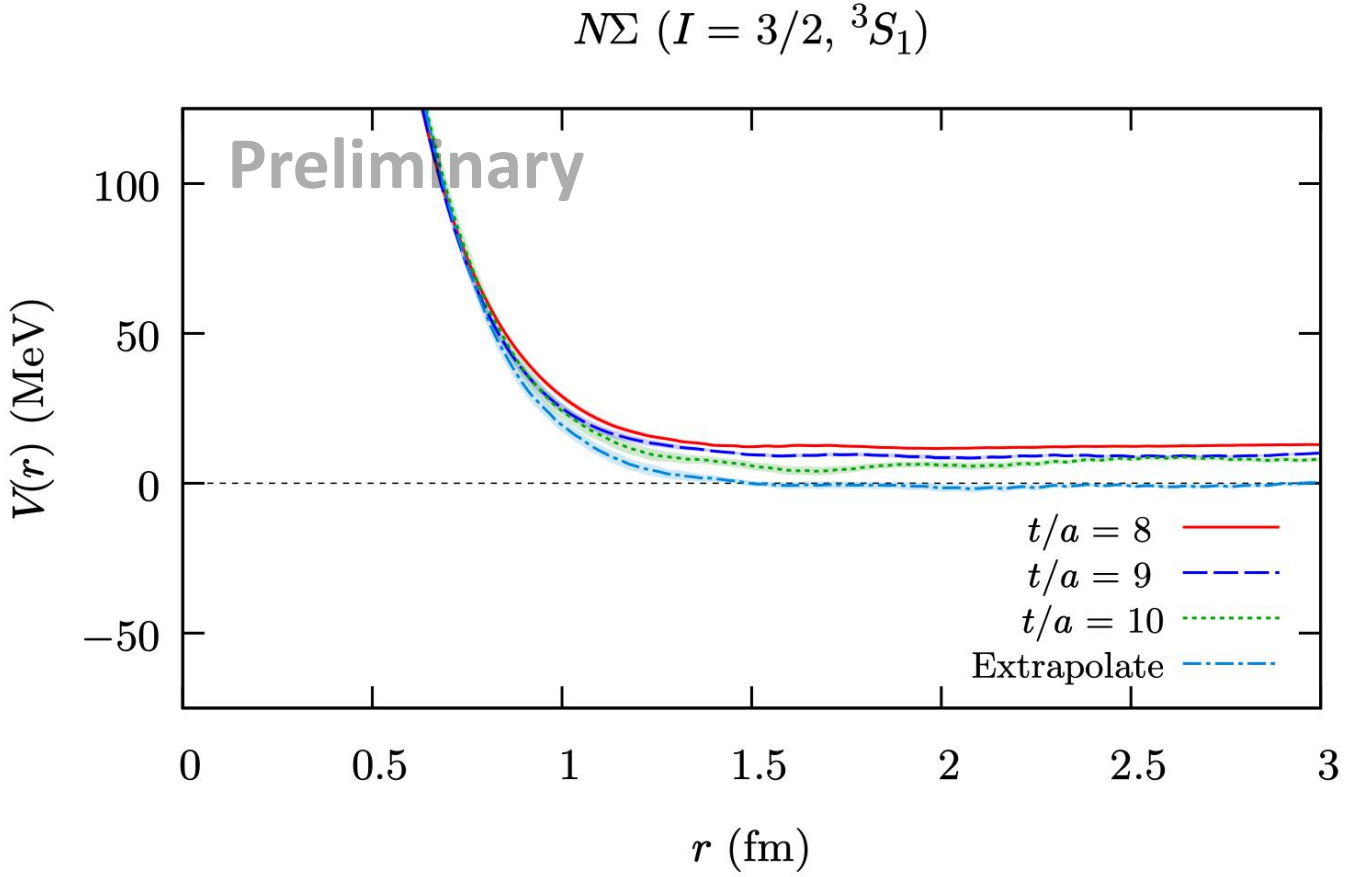
$N\Sigma$ is noisier … We need to fit to fewer data points



The constant offset of $N\Sigma$ also behaves in a single exponential decay

Extrapolated potential $V_{N\Sigma}(r)$ ($I = 3/2$, $^3S_1$)

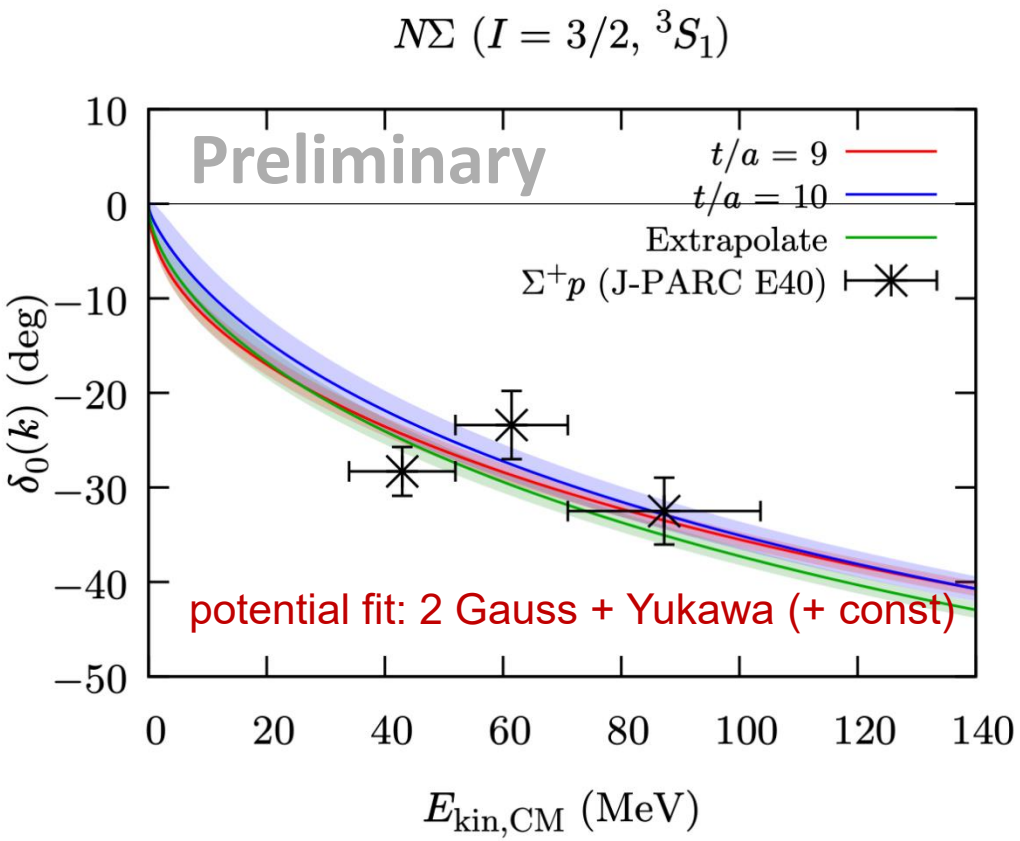
Result of the extrapolation to $t/a \to \infty$



Modification by the extrapolation is small at $r < 1$

This is not actually constant subtraction.

Compare const sub and extrapolation with δ_0



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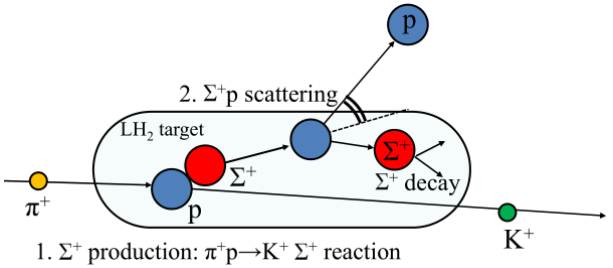


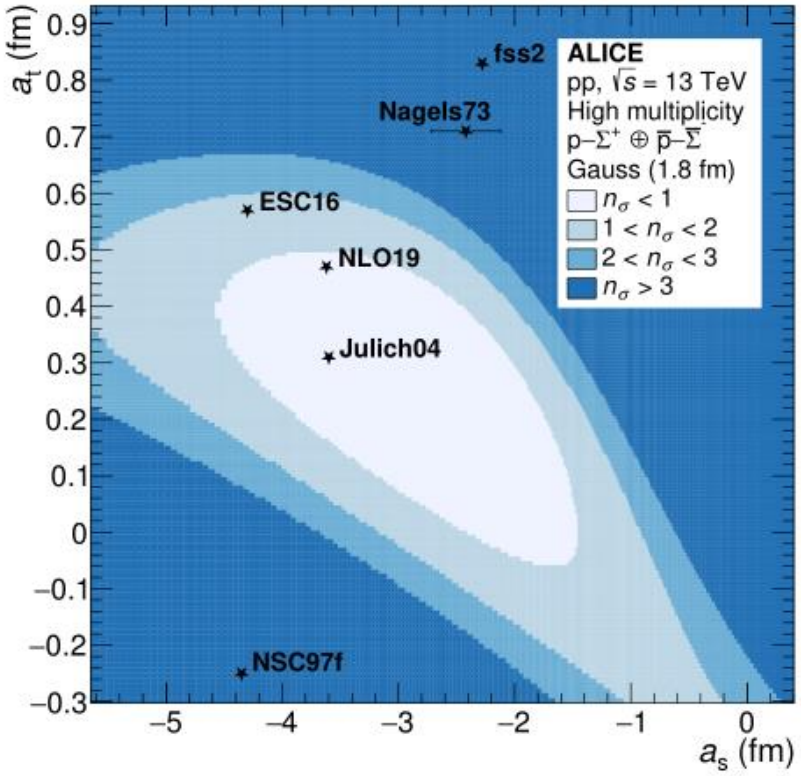
The phase shift with the constant subtraction (red/blue) is consistent with the result with the extrapolation method (green).

← the potential at short range (r < 1) does not affect δ_0 much.

Femtoscopy of $p\Sigma^+$ (ALICE)

Legend (fitting function):
G = Gauss, **Y** = Yukawa OPE,
const = const subtraction

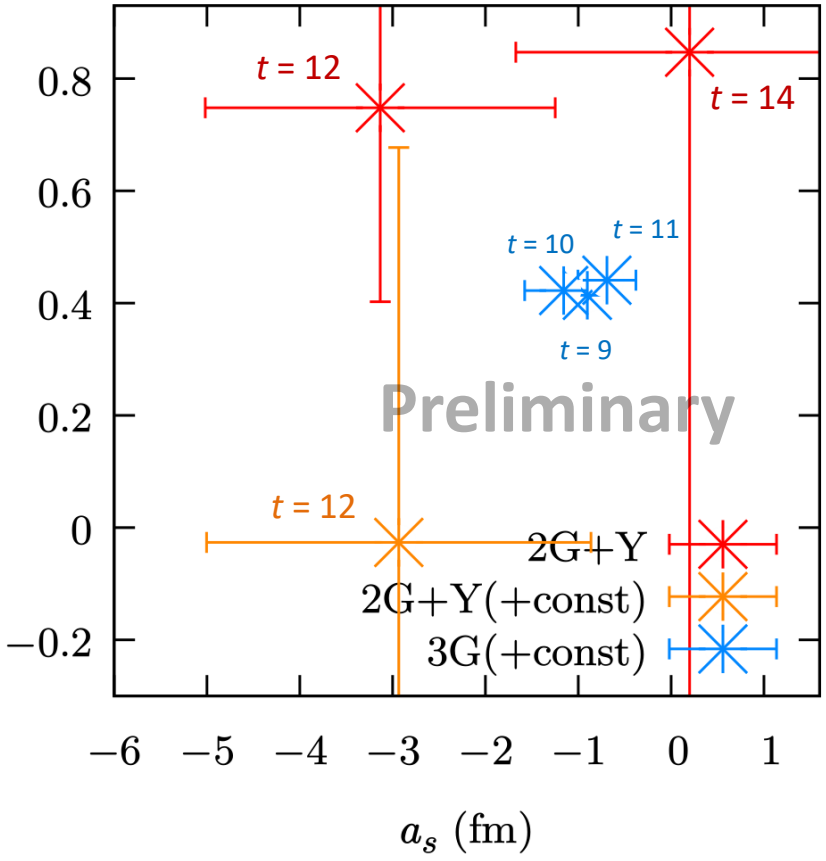
ALICE, arXiv:2510.14448v1 [nucl-ex] (2025)
(a_s, a_t) for Gaussian potential (1.8 fm)



$^3S_1$ scattering length →

$^1S_0$ scattering length →

(Coulomb **Un**modified) Scattering length
from HAL-conf-2023
in the same plotting range



Fitting function dependence &
time dependence are large.

Summary

- Analysis *ongoing* for ***NΛ & NΣ potentials*** on physical pt. using HAL QCD method for **F-conf (HAL-conf-2023)**
- Repulsive channels, $N\Sigma(I=3/2)\ ^1S_0$ & $N\Sigma(I=3/2)\ ^3S_1$, receive constant offsets in the potential from inelastic contamination: (Large offset at **small** t/a) vs (Large noise at **large** t/a)
  - *Manual subtraction* of the offset stabilizes t/a deps.
  - *Extrapolation* from smaller t/a is consistent (assuming $\Delta V \sim e^{-bt}$)
- Comparison
  - I=3/2 $\delta(^3S_1)$ consistent with J-PARC E40 Σ^+p scattering?
  - I=3/2 $a(^1S_0)$ and $a(^3S_1)$ [~ ALICE femtoscopy] large uncertainty

Outlook

- Systematic validation of *extrapolation* method, better extrapolation
- fitting form dependence, $^3D_1$ and tensor forces, …

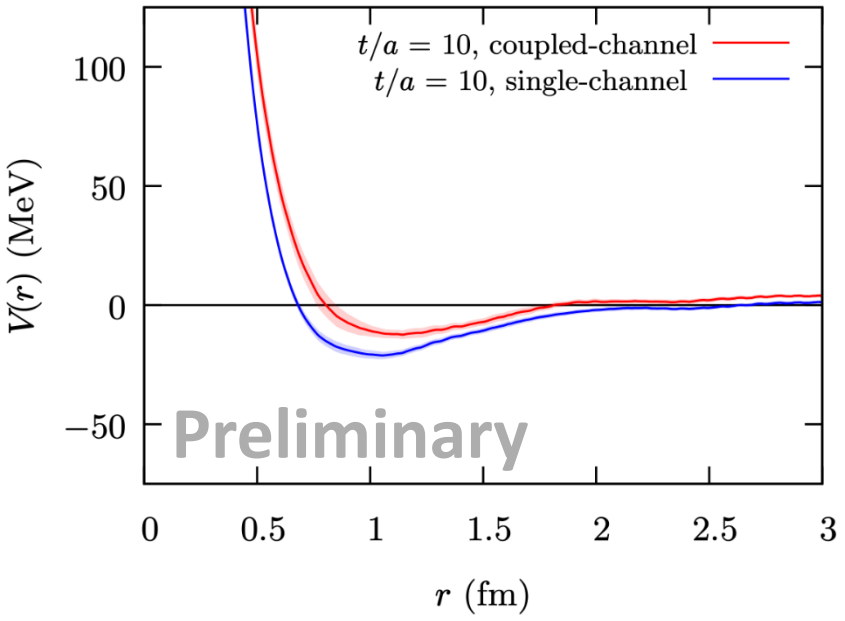
BACKUP

$N\Lambda$(I=1/2) Effective single-channel pot.

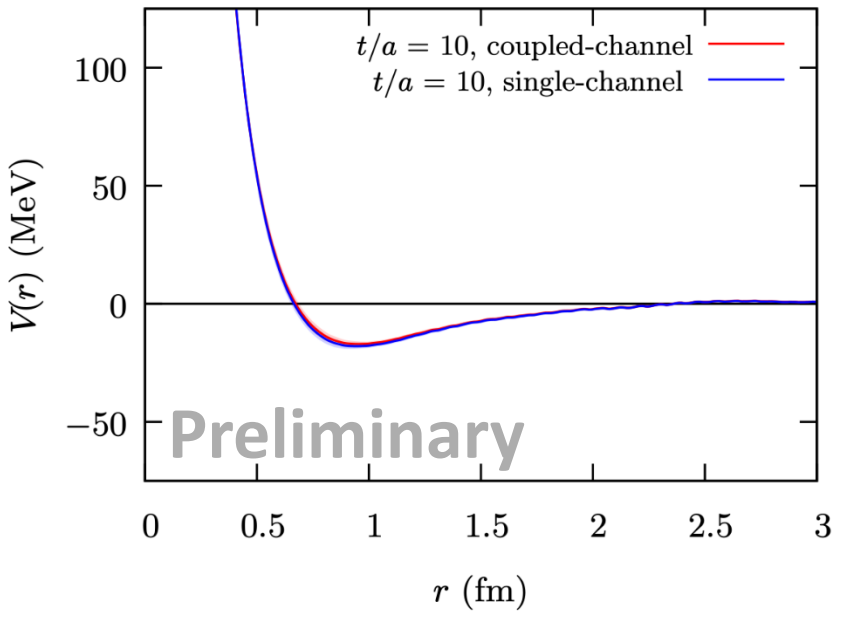
Analysis ongoing…

2x2 $N\Lambda$-$N\Sigma$ coupled-channel potential
vs **$N\Lambda$ effective single-channel potential**

$^1S_0$

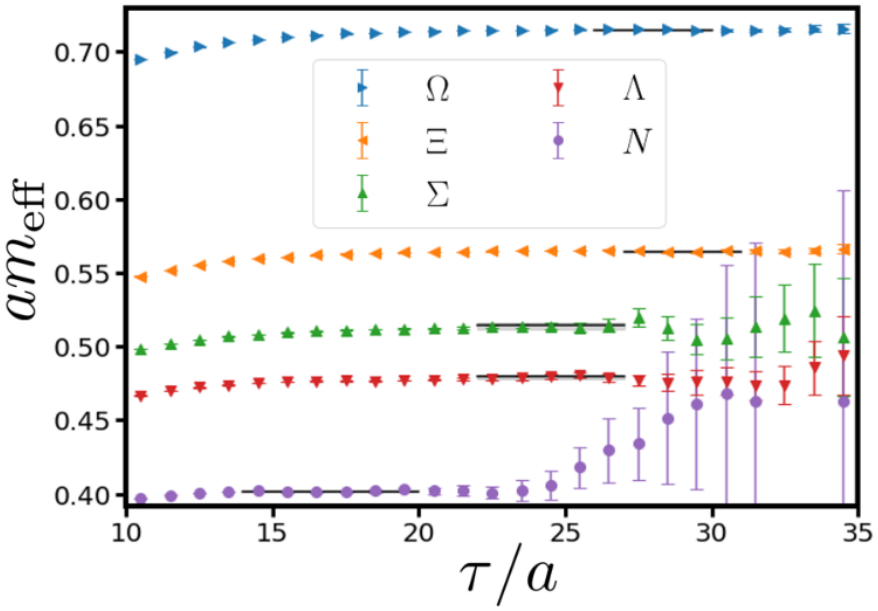


$^3S_1$



Baryon masses

Aoyama et al [HAL QCD], PRD110 (2024)



hadron	experiment [MeV]	lattice [MeV]
π	138.04	$137.1(0.3)(^{+0.0}_{-0.2})$
K	495.64	$501.8(0.3)(^{+0.0}_{-0.7})$
N	938.92	$939.7(1.8)(^{+0.2}_{-1.7})$
Λ	1115.68	$1121.4(3.6)(^{+0.5}_{-3.7})$
Σ	1193.15	$1202.5(5.6)(^{+0.3}_{-4.0})$
Ξ	1318.29	$1320.7(2.1)(^{+1.5}_{-1.7})$
Ω	1672.45	input