Baryon masses in QCD+QED

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42nd International Symposium on Lattice Field Theory

06 November 2025

Summary

- Isospin breaking and C-periodic boundary conditions
- 2 kinds of contributions to baryonic 2-point functions
    1. **3-quark connected**: Ongoing measurements
    2. **1-quark connected**: Implementation and preliminary results
- Outlook

QCD+QED on the lattice

Isospin symmetry in **QCD+QED** is an **approximate symmetry** due to $m_u \neq m_d$ (**strong isospin breaking**) and $q_u \neq q_d$ (**electromagnetic isospin breaking**)

To perform lattice simulations we need to define **QED in finite volume**, which is challenging. One solution comes from **C-periodic boundary conditions** in space, consisting in the charge conjugation of quark and gauge fields when crossing the boundaries of the lattice, as we can see from **Gauss law**

$$Q = \int_0^L dx^3 \rho(x) = \int_0^L dx^3 \nabla \cdot E(x) = 0 \longrightarrow \neq 0 \qquad (1)$$

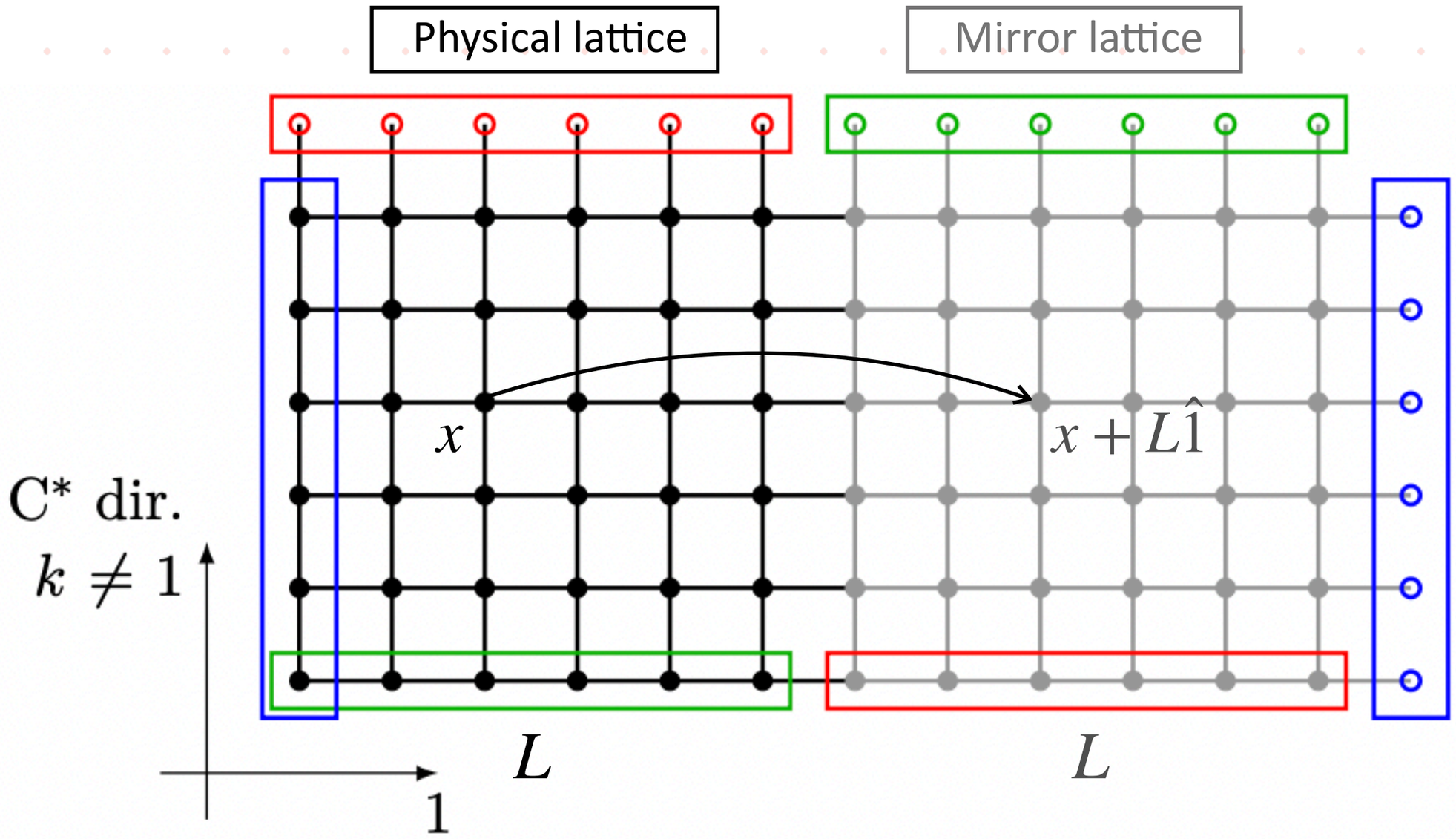
Periodic boundary conditions

C-periodic boundary conditions

This choice preserves locality, gauge and translational invariance

C-periodic boundary conditions

In the openQxD code these boundary conditions are implemented through the **orbifold construction**



From: Isabel Campos et al. “openQ*D code: a versatile tool for QCD+QED simulations”.In: The European Physical Journal C (Mar. 2020).

C-periodic boundary conditions

Due to C-periodic boundary conditions there are **additional contributions to quark propagators**:

$$\langle \overbrace{q_a^A(x)\ \overline{q}_b^B(y)}\rangle = D^{-1}(x,y)_{ab}^{AB} \qquad (1)$$

$$\langle \overbrace{q_a^A(x) q_b^{B,T}(y)}\rangle = -\, D^{-1}(x, y + L\hat{1})_{ad}^{AB} C_{db} \qquad (2)$$

$$\langle \overbrace{\overline{q}_a^{A,T}(x) \overline{q}_b^B(y)}\rangle = C_{ad} D^{-1}(x + L\hat{1}, y)_{db}^{AB} \qquad (3)$$

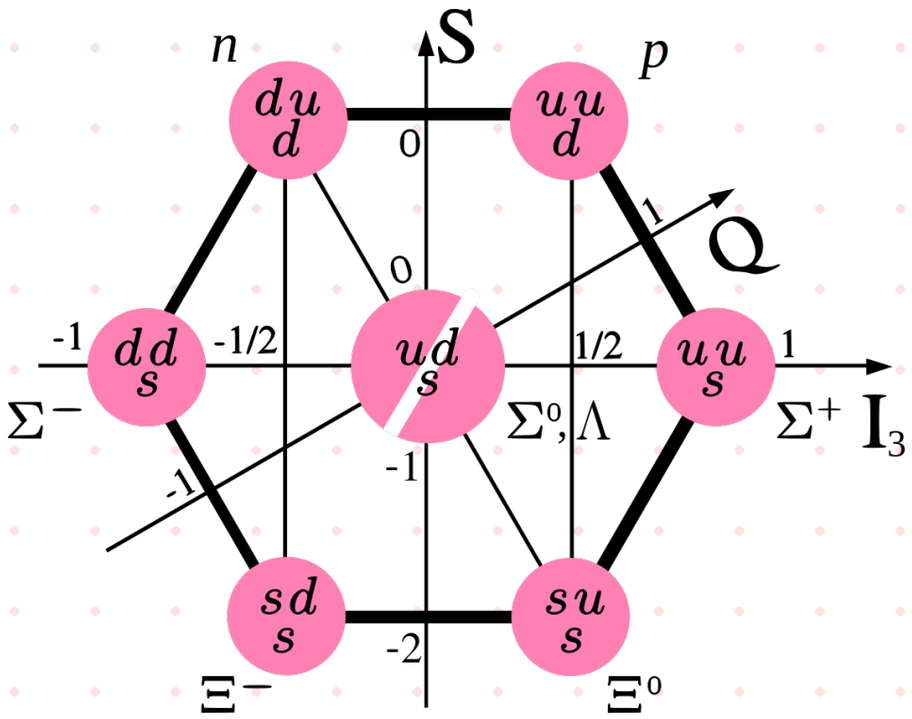
This is the reason for the presence of additional contributions in **baryonic two-point functions**

Baryon masses

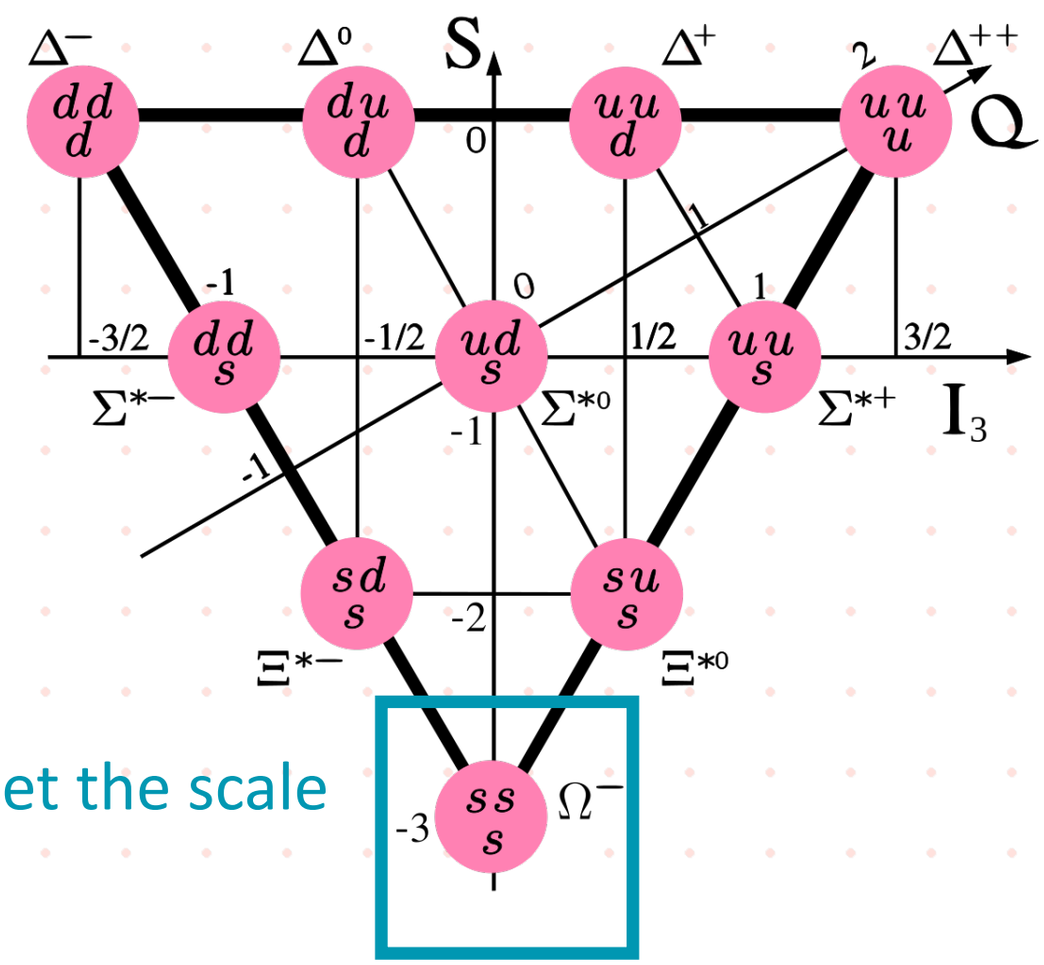
Definitions

Baryons are hadrons composed of **three valence quarks**

Baryon octet



Baryon decuplet



the Ω^- baryon has been used to set the scale of lattice simulations

Baryon masses

Definitions

To measure baryon masses on the lattice we have to define **interpolating operators**

$$v_{\Omega^-}^{m;d}(x) = \sum_{\substack{abc \\ ABC}} W_{abc}^{d;m} \epsilon^{ABC} \psi_c^C(x) \psi_a^A(x) \psi_b^B(x) \tag{1}$$

$$\overline{v}_{\Omega^-}^{m;d}(x) = \sum_{\substack{abc \\ ABC}} \overline{W}_{abc}^{d;m} \epsilon^{ABC} \overline{\psi}_b^B(x) \overline{\psi}_a^A(x) \overline{\psi}_c^C(x)$$

Where

$$W_{abc}^{d;m} = P_{dc}^{ml} \Gamma_{ab}^{l} + P_{db}^{ml} \Gamma_{ac}^{l} + P_{da}^{ml} \Gamma_{cb}^{l} \tag{2}$$

$$\overline{W}_{abc}^{d;m} = P_{cd}^{ml} \Gamma_{ab}^{l} + P_{bd}^{ml} \Gamma_{ac}^{l} + P_{ad}^{ml} \Gamma_{cb}^{l}$$

$$P^{ml} = [\delta^{ml} Id_{4\times4} - \frac{1}{3} \gamma^m \gamma^l]$$

$$\Gamma^l = C\gamma^l \tag{3}$$

Baryon masses

Definitions

With these interpolating operators we can construct the **two-point correlation function at zero momentum** projected to positive parity

$$C_{\Omega^-}(x_0) = \sum_{m} \sum_{dd'} P^{+}_{dd'} \; v^{m;d}_{\Omega^-}(x) \overline{v}^{m;d'}_{\Omega^-}(0) \qquad (1)$$

Where:

$$P^{+} = \frac{1+\gamma^0}{2} \qquad (2)$$

Due to C-periodic boundary conditions the two point function has contributions from two kind of diagrams: **3-quark connected** and **1-quark connected**

3-q connected contributions

Two types of contributions to the 2-point function

The two space-time points are connected by **three quark propagators**, we obtain these contributions also in the standard case of periodic boundary conditions in space

$$C_{\Omega^-}(x_0) = \sum_{\substack{d'd \\ m}} \sum_{\substack{\alpha bc \\ \alpha' b' c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC \\ A'B'C'}} \Big[\overline{W}^{d';m}_{a'b'c'} P^{+}_{dd'} W^{d;m}_{abc} \epsilon^{ABC} \epsilon^{A'B'C'} \overbrace{\psi^{A}_{a}(x) \overline{\psi}^{A'}_{a'}}(0) \overbrace{\psi^{B}_{b}(x) \overline{\psi}^{B'}_{b'}}(0) \overbrace{\psi^{C}_{c}(x) \overline{\psi}^{C'}_{c'}}(0) \Big] \tag{1}$$

In our code the inversions are performed using **point sources**

Smearing

Can be applied to source and sink to better the superposition of the interpolating operators with the desired state.

<u>Wilson flow gauge smearing</u> in the three space directions and **Gaussian fermion smearing**

$$\Psi_{\text{smeared}}(x) = \left(\frac{1 + \kappa H_t(x,y)}{1 + 6\kappa} \right)^N \Psi(y) \qquad (1)$$

$$H_t(x,y) = \sum_{j=1}^{3} \left[V_t(x,j)\,\delta(x+\hat{j},y) + V_t(x-\hat{j},j)^\dagger\,\delta(x-\hat{j},y) \right] \qquad (2)$$

H_t is the spatial hopping operator, where V_t are the gauge-smeared links. The strength κ, smearing levels (allow GEVP) and smearing steps are set in input file

1-q connected contributions

Two types of contributions to the 2-point function

They are finite-volume effect due to C-periodic boundary conditions, obtained when we consider quark-quark and antiquark-antiquark propagators. We want to compare the size of the two at finite volume.

The two space-time points are connected by 1 quark propagator only

$$C_{\Omega^-}(x_0) = -\sum_{\substack{d'd \\ m}} \sum_{\substack{\alpha bc \\ \alpha' b' c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC \\ A'B'C'}} \Big[\overline{W}^{d';m}_{a'b'c'} P^{+}_{dd'} W^{d;m}_{abc} \epsilon^{A'B'C'} \epsilon^{ABC} \overline{\psi}^{A'}_{a'}(0) \overline{\psi}^{B'}_{b'}(0) \tag{1}$$

$$\overline{\psi^{C}_{c}(x) \overline{\psi}^{C'}_{c'}(0)}\ \overline{\psi^{B}_{b}(x) \psi^{A}_{a}(x)} \Big]$$

1-q connected contributions

Computation strategy

$$C_{\Omega\text{-}}(x_0) = -\sum_{\substack{d'd \\ m}} \sum_{\substack{\alpha abc \\ \alpha' a' b' c'}} \sum_{\mathbf{x}} \sum_{\substack{ABC \\ A'B'C'}} \Big[\overline{W}^{d';m}_{a'b'c'} P^{+}_{dd'} W^{d;m}_{abc} \epsilon^{ABC} \epsilon^{A'B'C'} \tag{1}$$

$$C_{a'\alpha'} \boxed{D^{-1}(L\hat{1},0)^{A'B'}_{\alpha'b'} D^{-1}(x,0)^{CC'}_{cc'}} \boxed{D^{-1}(x,x+L\hat{1})^{BA}_{b\alpha}} C_{\alpha a} \Big]$$

The result is obtained with a combination of point sources and stochastic sources inversions

3-q connected contributions

Ongoing measurements

3-q connected contributions

New measurements: O(100) point sources

Two already generated QCD ensembles, at the **isospin symmetric point** where we only have two non degenerate masses, with C-periodic boundary conditions and O(a) improved Wilson fermions.

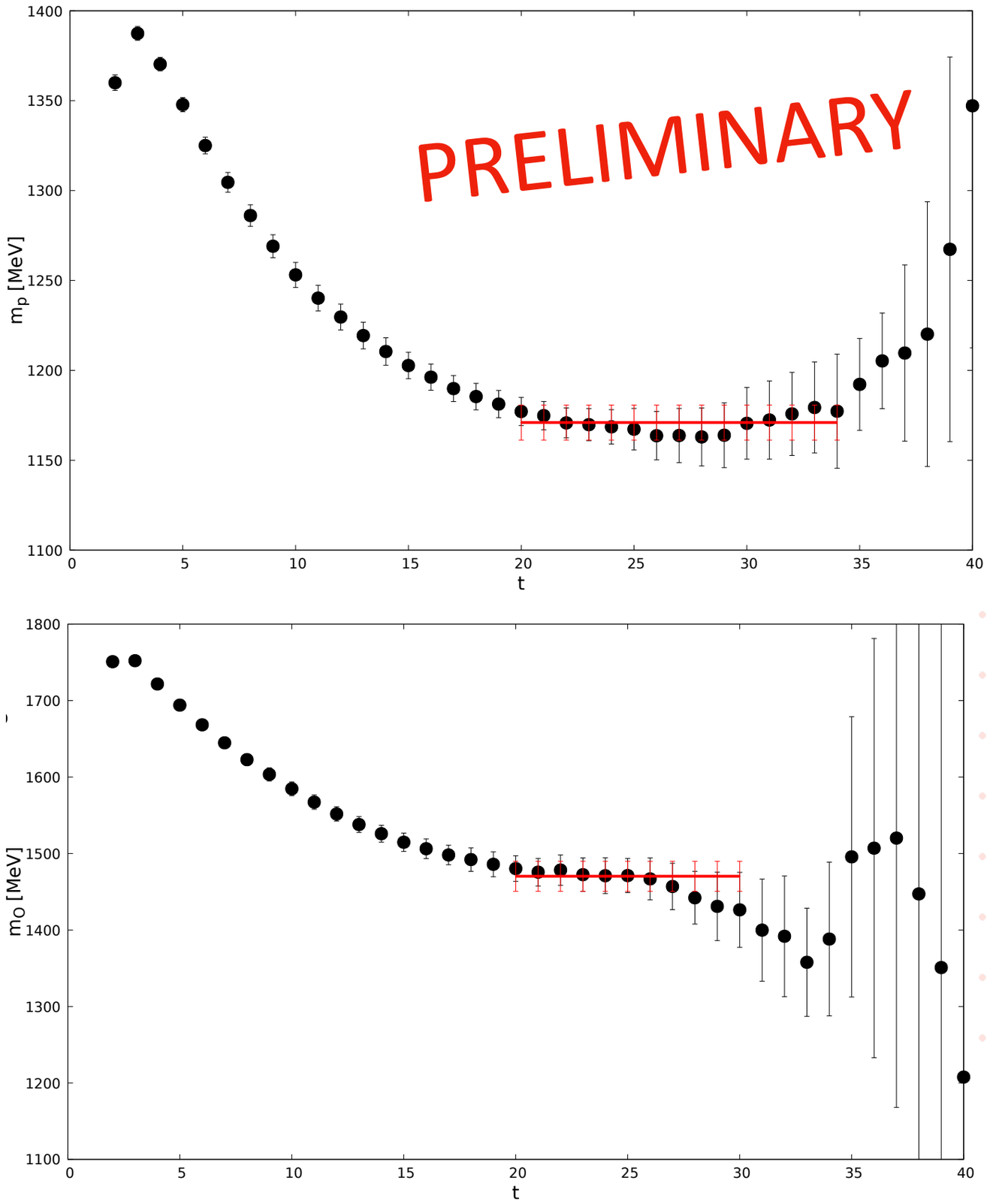
1. **B400b324a00** volume $80x48^3$, $a \simeq 0.05$ fm, 50 sources, 1000 configurations, $M_\pi \simeq 400$ MeV

2. **A400b324a00** volume $64x32^3$, $a \simeq 0.05$ fm, 40 sources, 2000 configurations, $M_\pi \simeq 400$ MeV

Ensemble	m_p[MeV]	$m_{\Omega-}$ [MeV]
B400b324a00	1170(10)	1470(20)
A400b324a00	1190(10)	1450(20)

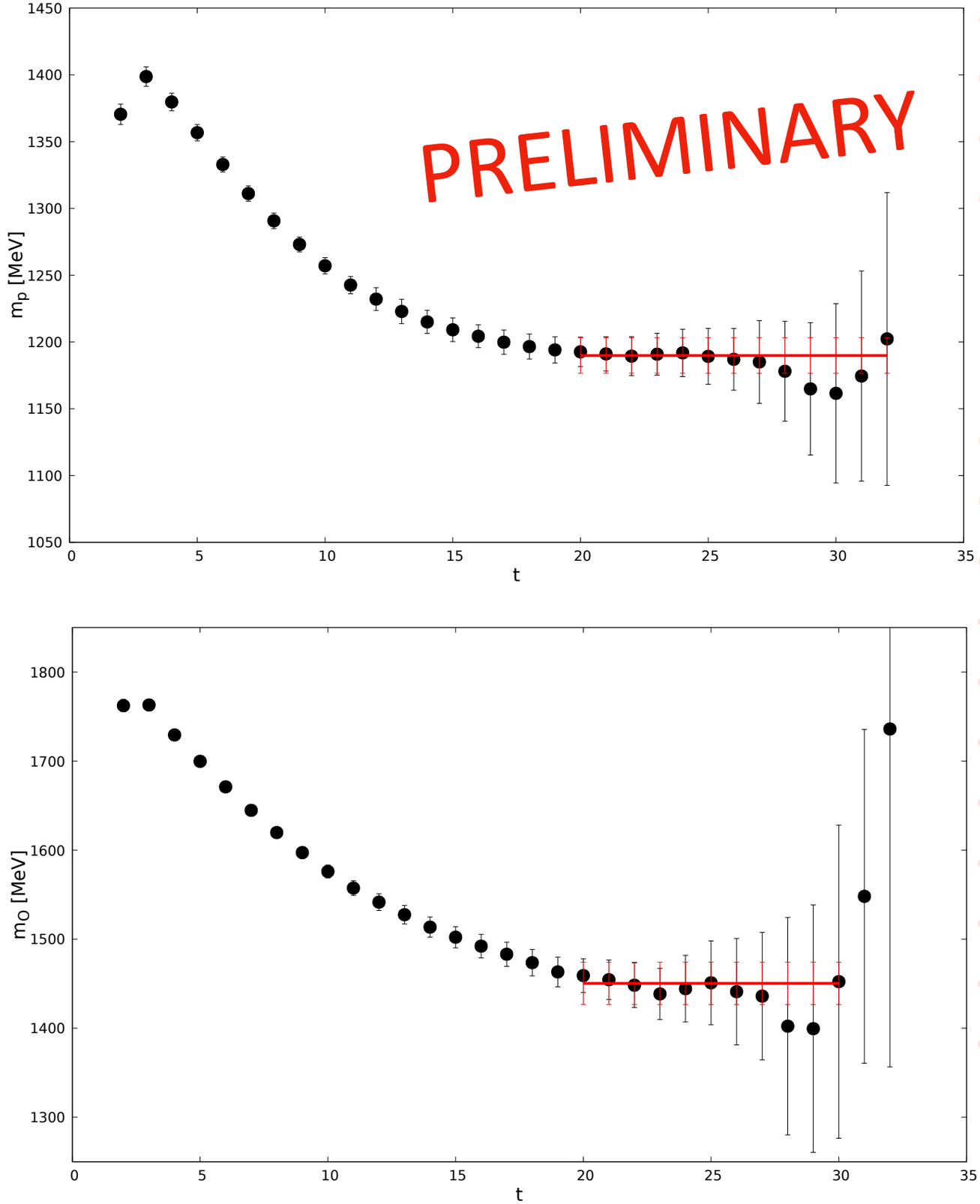
3-q connected contributions

Results for the proton and Ω^- effective mass and selected plateau on **B400a00b324** for 50 point sources, maximum smearing at the source, no smearing at the sink



3-q connected contributions

Results for the proton and Ω^- effective mass and selected plateau on **A400a00b324** for 40 point sources, maximum smearing at the source, no smearing at the sink



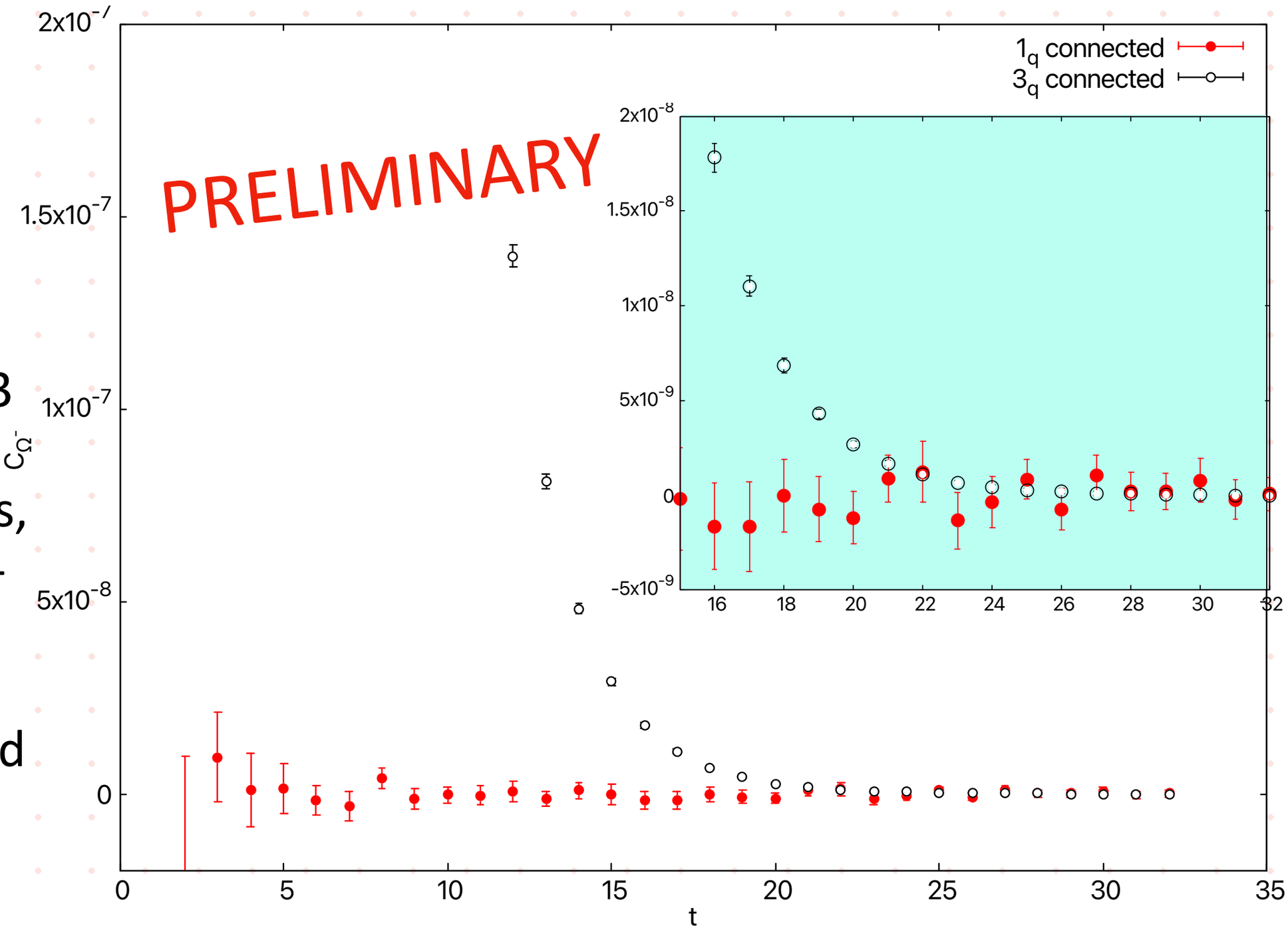
1-q connected contributions

Preliminary results

1-q connected contributions

First results for **A400a00b324** 40 decorrelated configurations (spaced 50) for 13 point and 13 stochastic sources, compared with 3-quark results for the same configurations and 10 point sources

No smearing in both cases



Conclusions and outlook

1. **3-q connected contributions**

   - New measurements of M_p and M_{Ω^-} with a **high number of point sources**

   - Increasing the number of point source up to 100 for **B400a00b324** and 150 for **A400a00b324,** add a QCD+QED ensemble and perform a **GEVP analysis**

2. **1-q connected contributions**

   - We implemented a code to measure these contributions for the first time and obtained the expected behaviour but significant noise at high time separations

   - Add **smearing**

   - Exploring **different choices for the stochastic sources**, sampling the position with point sources, differently distributed stochastic sources, …

Wilson flow gauge smearing

$$\dot{V}_t(x,k) = -g^2 \left\{ \delta_{x,k} S_W^{spatial}(V) \right\} V_t(x,k), \quad V_0(x,k) = V(x,k),$$

$V(x,\mu)$ are the $SU(3)$ gauge fields

$V_t(x,k)$ for $k = 1,2,3$ are the smeared gauge links.

$S_W^{spatial}(V)$ denotes the spatial part of the Wilson gauge action meaning the spatial plaquette sum in the absence of a prefactor