# Negative parity hyperons from coupled channel scattering

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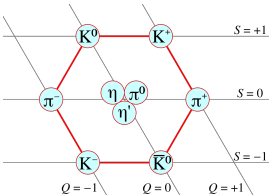


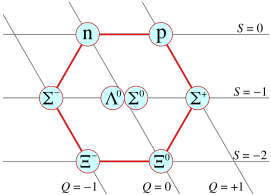


**Meson-baryon scattering**

- First way to study baryon excitations (since $\Delta(1232)$ in the '50s)
- Approachable from first principles on the lattice with Lüscher method

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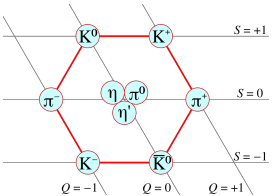


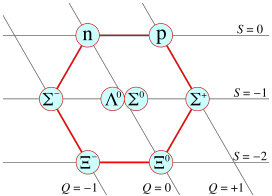


## Meson-baryon scattering

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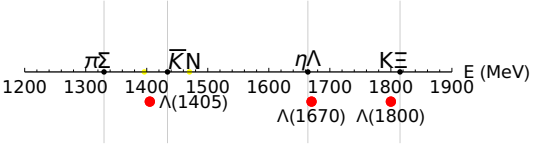
## Talk overview

1. Meson-baryon scattering in s-wave $\Rightarrow J^P = 1/2^-$:
   $N^*$, $\Lambda^*$, $\Sigma^*$, $\Xi^*$
2. $\Lambda(1405)$
3. $\Xi^*$ channel (preliminary)

# Negative parity hyperons from coupled channel scattering

## $\Lambda(1405)$ channel

- '60s-'80s bubble chamber exps. $K^- p$ [Alston et al PRL 6 (1961) 698]
- $J^P = 1/2^-$ confirmed in $\gamma p \to K^+ \Lambda(1405)$ polarisation by CLAS [Moriya et al., PRC 87 035206 (2013)]
- $\Lambda(1405)$ (****) & $\Lambda(1380)$ (**)?



# Negative parity hyperons from coupled channel scattering

Simplified illustration of $m_q$ dep. of $\boldsymbol{\Lambda(1405)}$

- SU(3): $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \bar{10} \oplus 27$

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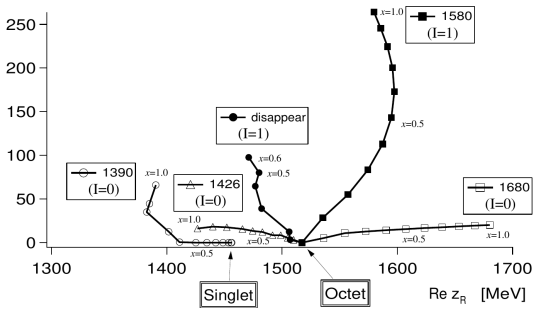
- If one observes one pole with $I = 0$, then one could expect three isosinglets
- $\sim$UChPT with SU(3) broken by $m_q$ predicts only two poles arround 1400 MeV

[Oller & Meissner PLB 500 (2001); Jido, Oller, Oset, Ramos, Meissner NPA 725 (2003)]

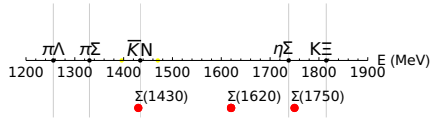



Flg. from [Jido, Oller, Oset, Ramos, Meissner NPA 725 (2003)]

## $\Sigma$ flavour channel

*lattice results in this sector are highly desired* [Mai et al 2507.14283]

- $\Sigma(1430)$
  - predicted by ∼UChPT, e.g. [Oller & Meissner PLB 500 (2001)]
  - experimental analyses are inconclusive
- $\Sigma(1620)$ (*) [Van Horn NPB ('75)]



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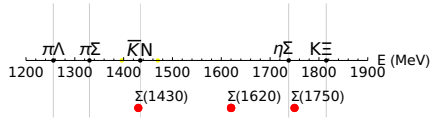
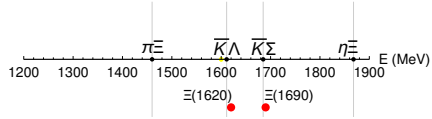



## $\Xi$ flavour channel

- $\Xi(1620)$ (**) Belle $\Xi_c \to \Xi\pi\pi$ [Sumihama et al PRL (2019)]
  - UChPT, e.g. [Ramos et al PRL 89 (2002)] & [García-Recio, Lutz & Nieves, PLB 582 (2004)]
  - NLO UChPT [Feijoo et al PLB 841 (2023)]
  - femtoscopy [Mantovani et al PRD 110 (2024)] argues that it is mostly $\eta\Xi$ in contrast with previous literature
- $\Xi(1690)$ (***), e.g. diffractive dissociation of $\Xi$ [Biagi et al ZPC (1987)]
  - UChPT: also predicted in García-Recio's and Feijoo's and many others



# Negative parity hyperons from coupled channel scattering

Lattice QCD approach

- Past works have studied the $\Lambda(1405)$ only with single-hadron operators

  [Nemoto PRD 68 (2003) 094505, Burch PRD 74 (2006) 014504, Takahashi PRD 81 (2010) 034505, Menadue PRL 108 (2012) 112001, Engel PRD 87(2013), 034502 & 034504, Hall PRL 114 (2015), Gubler PRD 94 (2016) 114518]

- Necessity of multi-hadron operators in $\phi\phi$, e.g. [Mohler et al PRD 87 (2013), Wilson et al PRD 92 (2015)]
- $\pi N$ operators, e.g.
  - $N(1535)$ [Lang & Verduci PRD 87 (2013)]
  - $N(1440)$ [Lang et al PRD 95(2017)]
  - $\Delta(1232)$ [Alexandrou et al PRD 96 (2017), Bulava et al NPB 987 (2023)]
- The $\Lambda(1405)$ study [Bulava et al PRL 132 (2024), Bulava et al PRD 109 (2024)]

## Lattice setup: a pathway through CLS ensembles

(joint effort of the **BaSc** collaboration)

∋Alharazin/FA/Baião Raposo/Bulava/Cid-Mora/Green/Hanlon/Hörz/Laudacina/Mohler/Morningstar/Moscoso/Nicholson/Romero-Lopez/Salg/Skinner/Toma/Walker-Loud

- $\Lambda(1405)$ already studied on D200 ($m_\pi = 200$ MeV) with CLS action [Bulava et al]

  (= tree-level improved Lüscher-Weisz gluons and nonperturbatively $\mathcal{O}(a)$ improved Wilson fermions)

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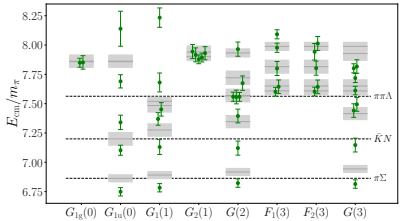
- $\Rightarrow$ Motivates current analysis on X451: $\neq m_q, a, L$ (and next N451)
  - $m_\mathrm{q}^\mathrm{phys}$ on E250 is on the horizon
- We also study $\Xi$ channel on X451 (N451 next)
- We have set up the $\Sigma$ channel run, but we are not producing it yet

| ensembles | boundary | $a$ | $T \times L^3$ | $m_\pi$ | $m_K$ | $m_\pi L$ | $N_\mathrm{cnfg}$ |
|---|---|---|---|---|---|---|---|
| D200 | open | 0.064 fm | $128 \times 64^3$ | 200 MeV | 480 MeV | 4.3 | 2000 |
| X451 | periodic (better) | 0.075 fm | " $\times\, 40^3$ | 287 MeV | 464 MeV | 4.4 | 2000 |
| N451 | " | " | " $\times\, 48^3$ | " | " | 5.3 | 1000 |
| E250 | " | 0.064 fm | $192 \times 96^3$ | 131 MeV | 493 MeV | 4.1 | 1000 |

# $\pi\Sigma - \bar{K}N$ and the $\Lambda(1405)$ resonance

Single- & two-hadron ops. are necessary to extract levels:

$$\Lambda[G_{1u}(0)]_0$$
$$\Lambda[G_{1u}(0)]_1$$
$$\Lambda[G_{1u}(0)]_2$$
$$\Lambda[G_{1u}(0)]_3$$
$$\bar{K}[A_{1u}(0)]_0 \; N[G_{1g}(0)]_0$$
$$\pi[A_{1u}^{-}(0)]_0 \; \Sigma[G_{1g}(0)]_0$$
$$\bar{K}[A_2(1)]_1 \; N[G_1(1)]_0$$
$$\pi[A_2^{-}(1)]_1 \; \Sigma[G_1(1)]_0$$

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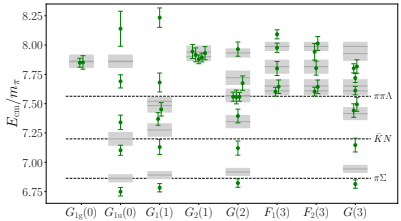
$$\pi[A_{1u}^-(0)]_0\ \Sigma[G_{1g}(0)]_0$$
$$\bar{K}[A_2(1)]_1\ N[G_1(1)]_0$$
$$\pi[A_2^-(1)]_1\ \Sigma[G_1(1)]_0$$




D200 extracted $E$ levels

## Rest Frame Irreps

| $J^P$ | [000] | [00n] | [0nn] | [nnn] | |
|---|---|---|---|---|---|
| $1/2^+$ | $G_{1g}$ | $G_1$ | $G$ | $G$ | $\Lambda$, $\Lambda(1600)$ |
| $1/2^-$ | $G_{1u}$ | $G_1$ | $G$ | $G$ | $\Lambda(1405)$, $\Lambda(1670)$ |
| $3/2^+$ | $H_g$ | $G_1$, $G_2$ | $2G$ | $F_1$, $F_2$, $G$ | - |
| $3/2^-$ | $H_u$ | $G_1$, $G_2$ | $2G$ | $F_1$, $F_2$, $G$ | $\Lambda(1520)$, $\Lambda(1690)$ |

- $G_{1u} \simeq 1/2^-$ (only contaminated by $\ell = 4$)

Single- & two-hadron ops. are necessary to extract levels:

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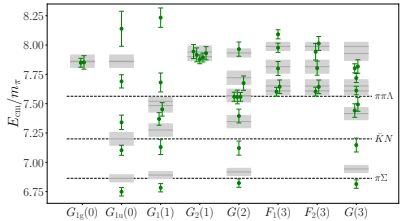


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| $3/2^+$ | $H_g$ | $G_1$, $G_2$ | $2G$ | $F_1$, $F_2$, $G$ | - |
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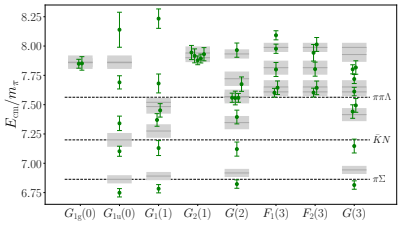
- $G_{1u} \simeq 1/2^-$ (only contaminated by $\ell = 4$)
- moving frames: mixing of quantum numbers (more operators)
  - $J^P = 1/2^+, 3/2^+$ contamination seems negligible close to lowest threshold, but more data will improve analysis

- Correlation matrix
  - $C_{ij}(t) = \langle \mathcal{O}_i(t) \bar{\mathcal{O}}_j(0) \rangle = \sum_k A_k^{(ij)} e^{-tE_k}$
  - Stochastic Laplacian Heaviside distillation [Peardon et al PRD 80 (2009)], [Morningstar et al PRD 83 (2011)]
    - source-sink factorization allows to use optimised tensor contractions to evaluate correlation matrices containing large sets of multi-hadron ops
    - only low modes of the Laplacian are kept (smearing) (quark perambulators can be stored)

- GEVP (increases $\Delta E_{\text{eff}}$) and ratio fits to extract lowest lying $E_k$ from earlier plateaus
  - $C(t)\,\vec{v} = \lambda(t)\,C(t_0)\,\vec{v}$




D200 extracted $E$ levels

Amplitude analysis

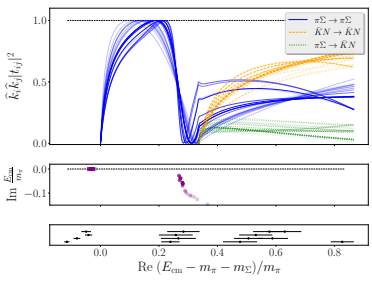
- five $\tilde{K}_{ij}$ parametrisations, $t^{-1} = \tilde{K}^{-1} - i\hat{k}$

Two poles extracted

(at $a \simeq 0.064$ fm, $m_\pi \simeq 200$ MeV, $m_K \simeq 480$ MeV)

1. virtual state:
   $E_1 = 1392(9)_{\rm st}(2)_{\rm md}(16)_{\rm sc}$ MeV
2. resonance pole:
   $E_2 = [1455(13)_{\rm st}(2)_{\rm md}(17)_{\rm sc} - i\ 11.5(4.4)_{\rm st}(4)_{\rm md}(0.1)_{\rm sc}]$ MeV

- These data has been re-analysed in [Pittler et al 2507.14283] and [Salg, Romero-López & Jay, 2506.16161]



- **Ongoing work: Moving the analysis to other $m_q$, $a$ and $L$ in CLS**

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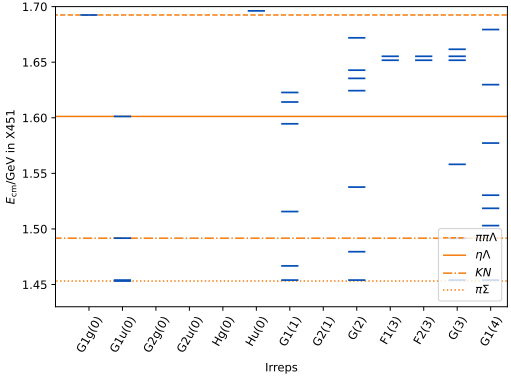
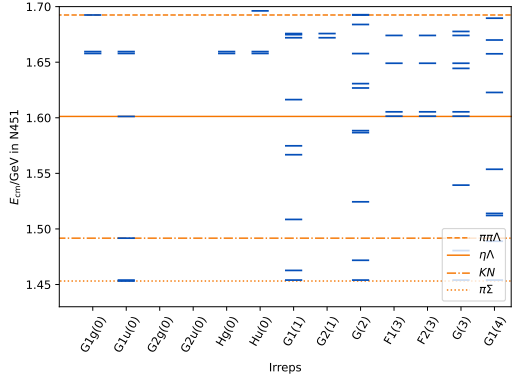
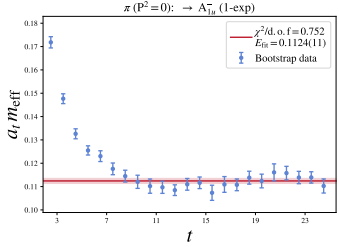
| N451 | " | " | " $\times 48^3$ | " | " | 5.3 | 1000 |

- Figs: free $E_n$ in X451 and N451
- They only differ in volume $\Rightarrow$ constrain more $E_{\text{cm}}$

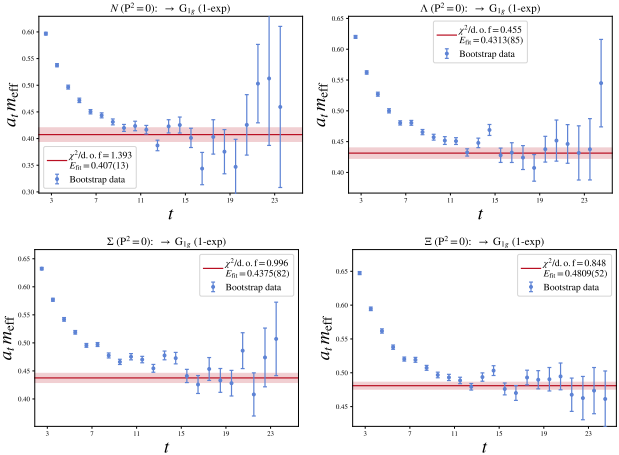





Production on X451:

- in Figs.: single hadron $m_{\rm eff}$ on on 135 configs, one source and direction ($\sim 1/100$ of target statistics)

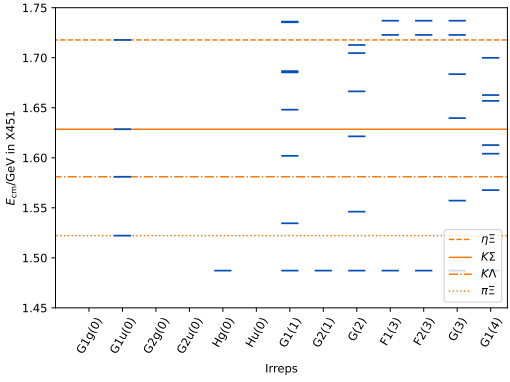


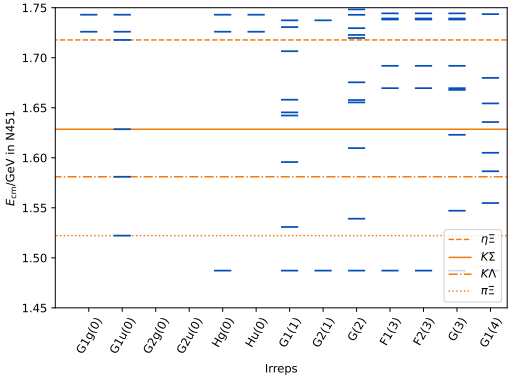

Production on X451:

- in Figs.: single hadron $m_{\rm eff}$ on on 135 configs, one source and direction ($\sim 1/100$ of target statistics)
- periodic boundary conditions and source time varied randomly: reduce baryon noise
- consistency with expectations

## $\pi\Xi - \bar{K}\Lambda - \bar{K}\Sigma$ channels and the $\Xi(1620)$ and $\Xi(1690)$ resonances

- $\Xi$ channel expected free levels in X451 and N451
- double strangeness should improve signal with respect to $\Lambda$ channel
- absence of nucleons should reduce the baryon noise

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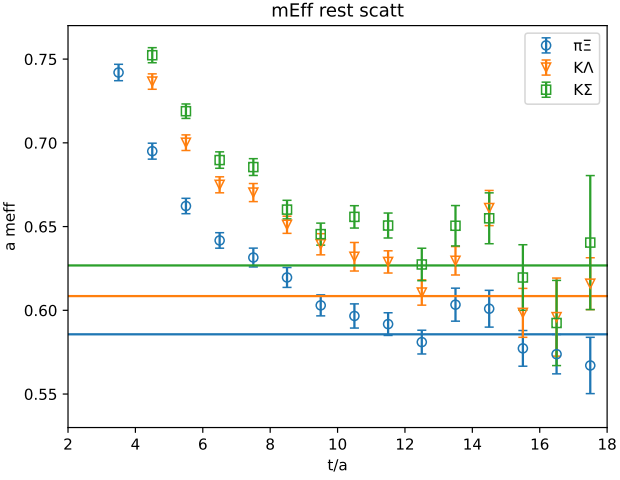
| $3/2^-$ | $H_u$ | $G_1$, $G_2$ | $2G$ | $F_1$, $F_2$, $G$ | $\Xi(1820)$ |

# $\pi\Xi - \bar{K}\Lambda - \bar{K}\Sigma$ and the $\Xi(1620)$ and $\Xi(1690)$ resonances

Production on X451:

- three coupled channels
  $\mathcal{O}(50\mathrm{MeV})$ separation
- 135 cnfgs on one source and direction ($\sim$1% data)
  - consistent with noninteracting thresholds



- Single hadron operators in the $J^P = 1/2^-$ channel

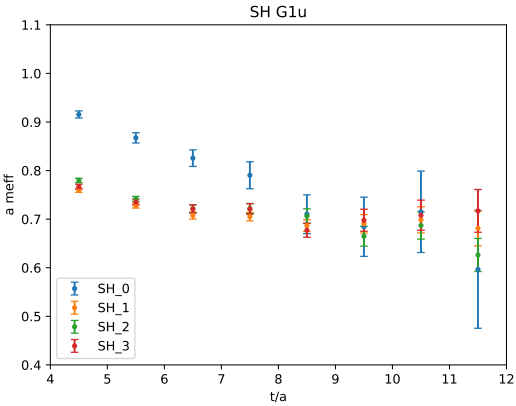



**Figure 1:** before GEVP

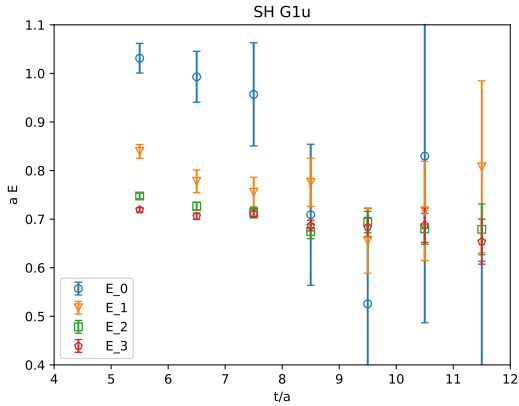



**Figure 2:** after GEVP

## Displaced Operators

# Displaced Operators

- We are studying displaced operators in order to improve the description of hadrons in general

SS: $\epsilon_{abc}\tilde{\psi}^{A}_{a\alpha}\tilde{\psi}^{B}_{b\beta}\tilde{\psi}^{C}_{c\gamma}$

SD: $\epsilon_{abc}\tilde{\psi}^{A}_{a\alpha}\tilde{\psi}^{B}_{b\beta}\left(\tilde{D}^{(p)}_{j}\tilde{\psi}\right)^{C}_{c\gamma}$

  - We have started studying $\Lambda$ ops. on X451
    - displaced ops. should help to disentangle states
    - drawback: noisier than local ones
  - theoretical input is beneficial: $\Lambda(1520)$ case

- **Displaced operators: $\Lambda(1520)$**, a $J^P = \frac{3}{2}^-$ state
  - ∼UChPT meson-decuplet scatt. [Kolomeitsev & Lutz PLB 585 (2004)]
  - quark model: $L = 1$, $S = 1/2$, flav. singlet [Gromes ZPC 18 (1983)]

$$(O_{\Lambda^*})_{j\gamma} = \epsilon^{abc}(C\gamma_5)_{\alpha\beta}\left(\frac{1+\gamma_0}{2}\right)_{\gamma\delta}[\tilde{s}^a_\alpha \tilde{d}^b_\beta(\tilde{\nabla}_j\tilde{u})^c_\delta$$
$$- \tilde{s}^a_\alpha \tilde{u}^b_\beta(\tilde{\nabla}_j\tilde{d})^c_\delta + \tilde{u}^a_\alpha(\tilde{\nabla}_j\tilde{d})^b_\beta \tilde{s}^c_\delta - \tilde{d}^a_\alpha(\tilde{\nabla}_j\tilde{u})^b_\beta \tilde{s}^c_\delta]$$

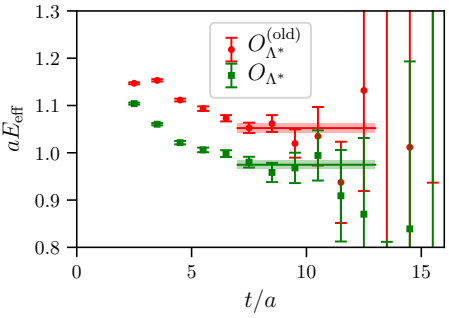



**Figure 3:** $\Lambda(1520)$ from [Meinel et al. PRD 103 (2021) 074505]

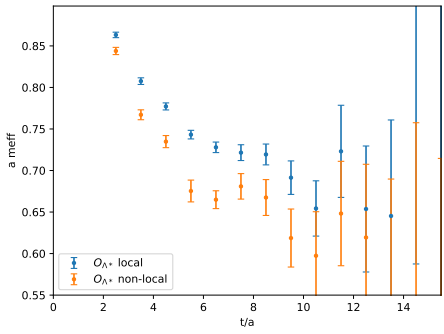



**Figure 4:** $\Lambda(1520)$ on X451

# Conclusions and Outlook

- A comprehensive set of single- and multi-hadron operators is indispensable to study scattering
- $\Lambda(1405)$ analysis:
  - Investigation of $m_q$ dependence by computing at $m_\pi \simeq 287$ MeV, $m_K = 464$ MeV (X451 box)
  - This and other ensembles will allow us to obtain a very robust amplitude parametrization and extrapolate to the physical world

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- Displaced operators and the $\Lambda(1520)$:
  - synergy with phenomenology
  - significant reduction of excited state contamination

# Appendix I

Using the above definition, it is straightforward to show that the unitarity of S implies the Hermiticity of K. Time reversal invariance of the S-matrix implies that the K-matrix must be symmetric. Given its Hermiticity, this means that K is real and symmetric. Rotational invariance implies that the K-matrix must have the form

- $t^{-1} = \tilde{K}^{-1} - i\hat{k}$, with $m_\pi \hat{k} = \text{diag}\ (k_{\pi\Sigma}, k_{\bar{K}N})$ and $\tilde{K}$ hermitic due to unitarity
- $\det[\widetilde{K}^{-1}(E_{\rm cm}) - B^P(E_{\rm cm})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$
- $\langle J' m'_J \ell' S' a' | K | J m_J \ell S a \rangle = \delta_{J'J} \delta_{m'_J m_J} K^{(J)}_{\ell' S' a'; \ell S a}(E_{\rm cm})$
- all models with $\mathrm{AIC} - \mathrm{AIC}_{\rm min} < 1$, and taking half the difference in the maximal spread of values as the model uncertainty. In future work, when a full extrapolation to the physical point is performed, a more thorough determination of the model-average and systematic uncertainty will be performed.
- $\mathrm{AIC}_{\rm min}$ for ERE (and $\chi^2/\mathrm{dof} = 10.52/11$), values
  - ERE: $\tilde{K}_{ij} = \frac{m_\pi}{E_{cm}}(A_{ij} + B_{ij}\Delta(E_{cm}))$, with $\Delta(E_{cm}) = (E_c m^2 - (m_\pi + m_\Sigma)^2)/(m_\pi + m_\Sigma)^2$

# Appendix II

- link variables are stout smeared, $\rho = 0.1$, $n_\rho = 36$
- An advantage feature of LapH is source-sink factorization of the correlators which greatly facilitates the evaluation of large Hermitian correlation matrices via optimized tensor contractions
- Low-lying eigenvalues of the Dirac matrix of either the light quark doublet or the strange quark can cause instabilities in the HMC an issue which is sufficiently ameliorated by light- and strange-quark reweighting.
- relative quark lines only at the sink

D200

- $N_{\mathrm{ev}} = 448$
- $N_{\mathrm{bin}} = 10$, jackknife and bootstrap ($N_B = 800$)
- single-pivot with $t_0 = 4a$ and $t_{\mathrm{diag}} = 16a$
  - $t^{-1} = \tilde{K}^{-1} - i\hat{k}$
  - $\det[\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B^P(E_{\mathrm{cm}})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$
  - several $\tilde{K}_{ij}$ parametrisations (ERE, ERE of $\tilde{K}_{ij}^{-1}$, Blatt-Biederharn, W-T inspired, lin. around $\pi\Sigma_{\mathrm{thr}}$)
- sixteen energies are used, which include the fifteen energies from the s-wave fits and one G1g(0) level, and we incorporate a $1/2+$ wave in the K-matrix parametrization. In the second additional fit, seventeen energies are used, which include the fifteen energies from the s-wave fits and one level each in the F1(2), F2(3) irreps, and we add a $3/2+$ wave in the K-matrix parametrization.
- coupling from residues of poles 1 and 2:
  $\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\mathsf{st}}(6)_{\mathsf{md}}$, $\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)_{\mathsf{st}}(10)_{\mathsf{md}}$

# Appendix III

X451

- $N_{\mathrm{ev}} = 180$