The $\Lambda(1405)$ at the $SU(3)$ point in lattice QCD

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- The quark content (uds), however it does not fit within the quark model: It has a lower mass than predicted.
- Chiral $SU(3)$ dynamics finds a two pole structure in the baryon-meson interaction [Jido et al, 2003]

Theoretical background

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- $\mathbf{8}_B \otimes \mathbf{8}_M = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8}' \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$

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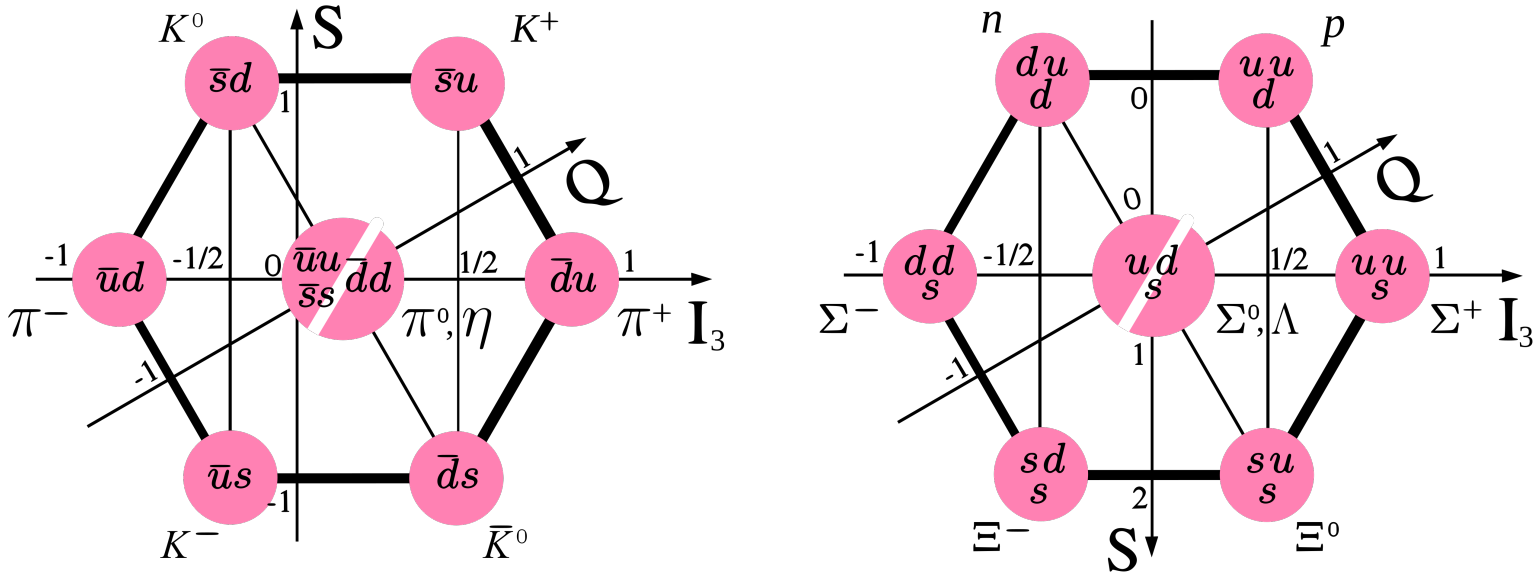
- Recently Lattice data away from $SU(3)$ point [Bulava *et al.*, 2024] and using a separable potential in HALQCD [Murakami *et al.* 2024]
- We aim to look directly at the $SU(3)$ point to learn about the structure of the levels

Interpolation operators

- Interpolation operators belonging to $SU(3)$



- Use gamma matrices for the correct spin and parity

$$\mathcal{O} = \sum_{\{q_1 \ldots \bar{q}_5\}} C_{\{q_1 \ldots \bar{q}_5\}} \left(\gamma_5 P_+ q_1 (q_2 C \gamma_5 q_3)(q_4 \gamma_5 \bar{q}_5)\right)$$

- We have checked that the correlations among irreps are zero and that any state in an irrep gives the same contraction

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- Measure two point functions:

$$C(t) = \langle \mathcal{O}(t)\bar{\mathcal{O}}(0)\rangle = \sum_n Z_n e^{-E_n t}$$

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- Determine the optimal combination with the GEVP [Blossier 2009]

$$C^{(t)}v^{(n)} = \lambda^{(n)} C(t_0) v^{(n)}(t), \lambda_n(t) \sim e^{-E_n(t)}(1+\mathcal{O}(e^{-\Delta E_n(t-t_0)}))$$

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- We evaluate the correlation functions using Distillation because of the all-to-all propagators appearing due to the meson operators.

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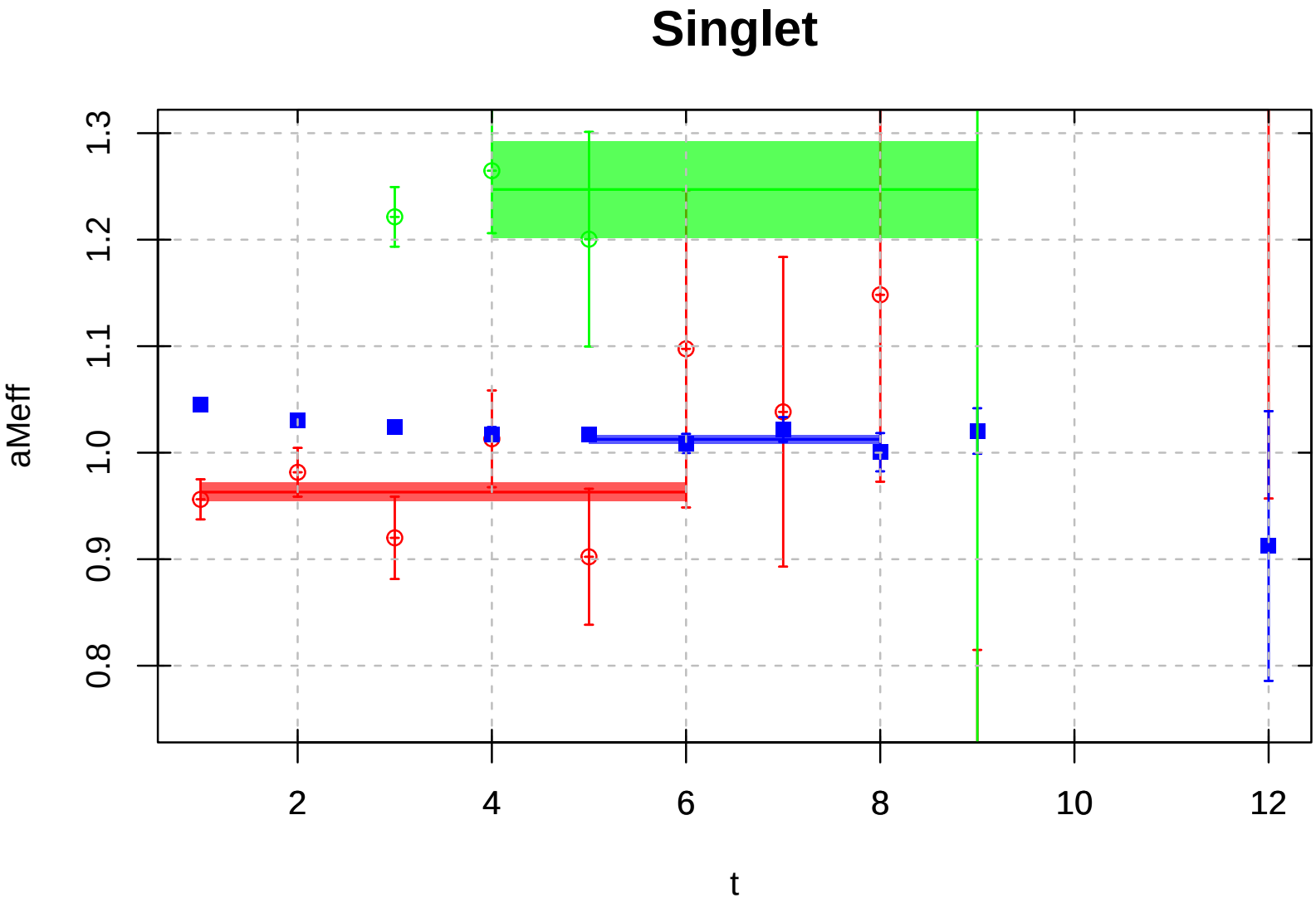
- Clover fermions $N_f = 3$ at the $SU(3)$ point. [Hörz *et al* 2020]

Ensemble	Vol	M_π	$M_\pi L$	N_e
C103	$48^3 \times 96$	714 MeV	14.5	100

Results

Effective mass singlet



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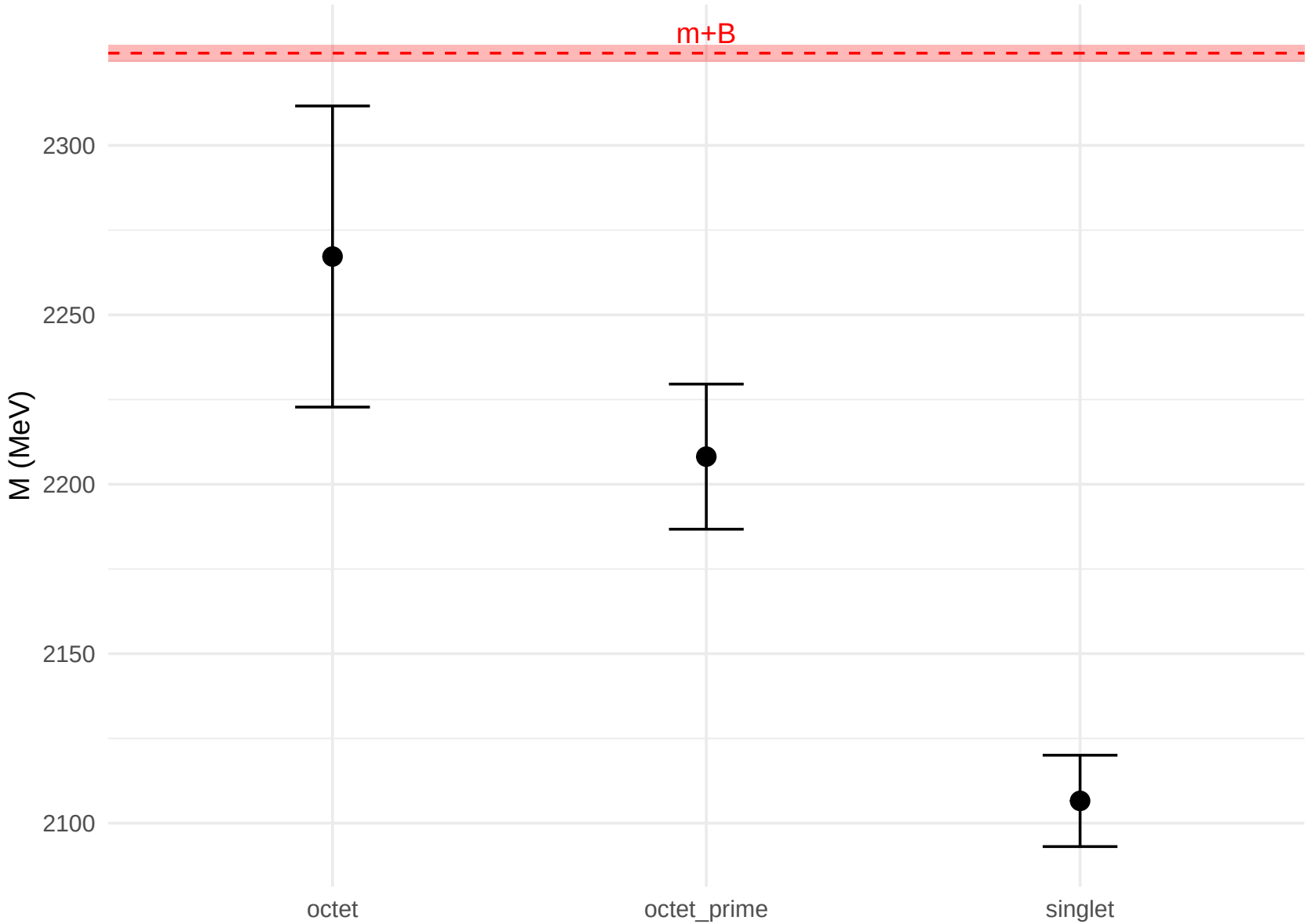
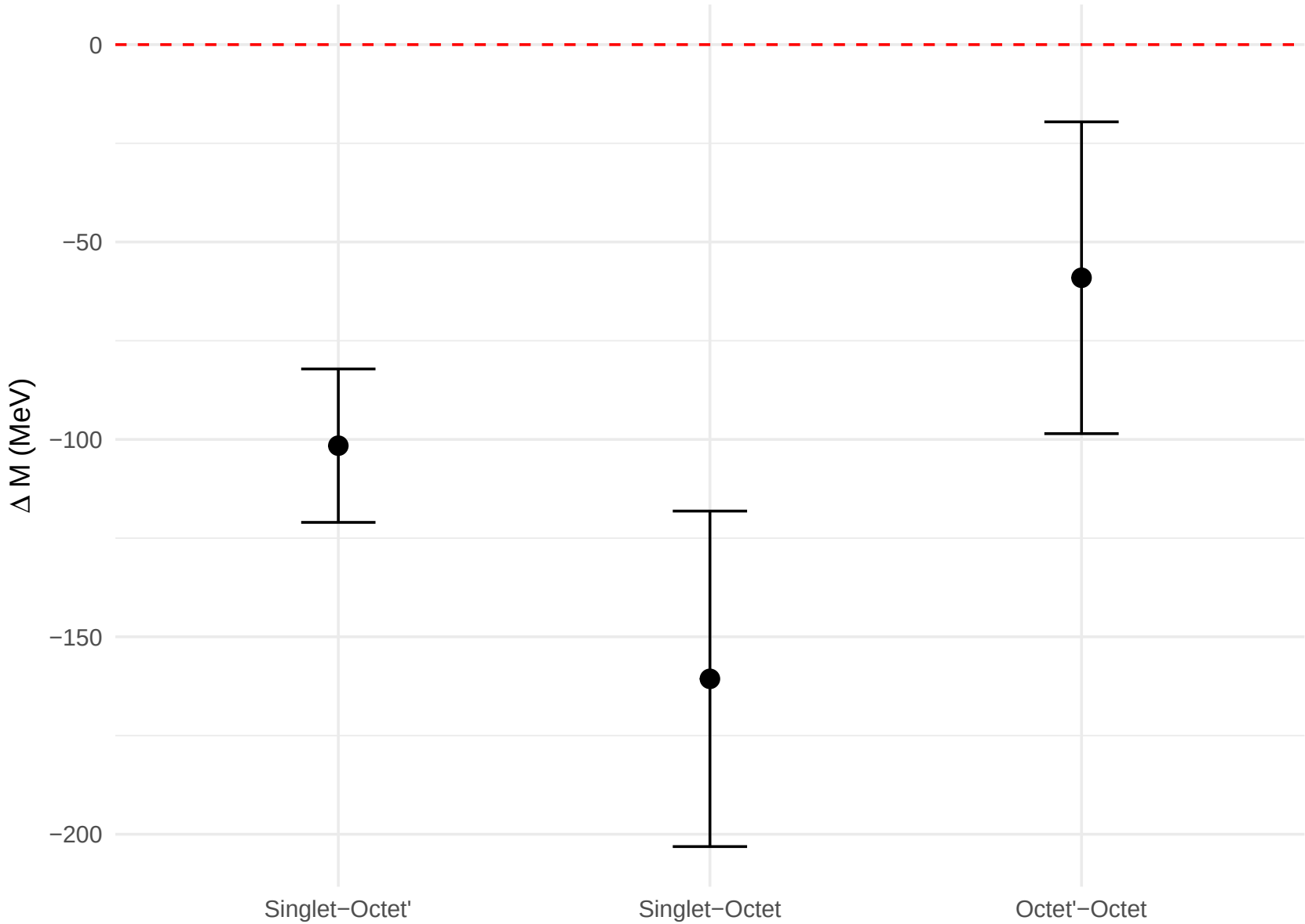
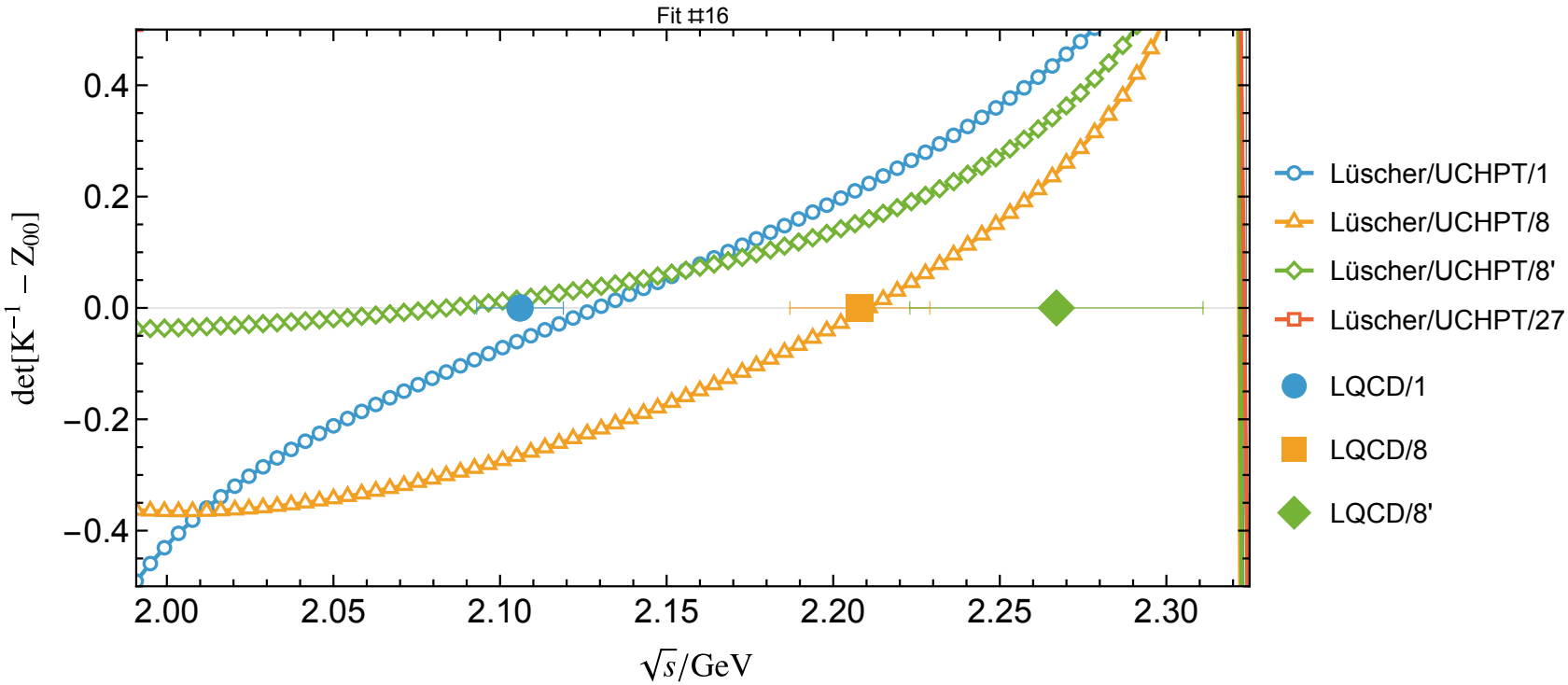


Results

Correlated mass differences



CHPT prediction



- Preliminary hint from the next steps. . .
- We compare the poles predicted from the Lattice data away from the $SU(3)$ point

Conclusion and outlook

Conclussions

- We have measured the energy levels of the $SU(3)$ states
- We find splitting between the singlet and octets with the singlet coming from lower energy as predicted from ChPT
- The octets are non-degenerate, increase statistics to ensure from this splitting we could constrain Lec's in ChPT

Outlook

- Simulate at a different lower pion mass $M_\pi \sim 450 Mev$
- Inclusion of local operators to maximize overlap with the states of the Lambda

Thank you for your attention!
Any questions?

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Backup slides

Lattice setup

Ensemble	Vol	M_π	$M_\pi L$	a
C103	$48^3 \times 96$	714 Mev	14.5	0.086fm

β	c_0	κ	c_{sw}
3.6	1.66	0.136497	1.986246

- Tree-level Luscher-Weisz gauge action
- Wilson-Clover fermions

Distillation

- Apply a spatial smearing to the fermion fields

$$\tilde{\psi}_{a\alpha}(x) = S_{ab}(x,y)\psi_{b\alpha}(y)$$

- Use eigenvectors of the Laplacian to construct the smear matrix

$$J_{\sigma,n_\sigma} = \left(1 + \frac{\sigma\nabla^2(t)}{n_\sigma}\right)^{n_\sigma \to \infty} \exp \sigma\nabla^2(t)$$

- The highest modes are exponentially suppressed

$$S = V_\Delta \Theta(\sigma_s^2 + \Lambda_\Delta) V_\Delta^\dagger \approx V_s V_s^\dagger$$

- We are replacing position space by a truncated Lap. eigenstate space.

$$\mathcal{Q} = V^i(t') \underbrace{V^{i\dagger}(t')\Omega^{-1}_{\alpha\beta}(t',t)V^j(t)}_{\tau^{i,j}_{\alpha\beta}(t',t)} V^{j\dagger}(t)$$

Distillation

- The eigenvector matrices are collected in the elmentals:

$$\Phi^{ij}(t) = \sum_x V^{i*}(x,t) V^j(x,t)$$

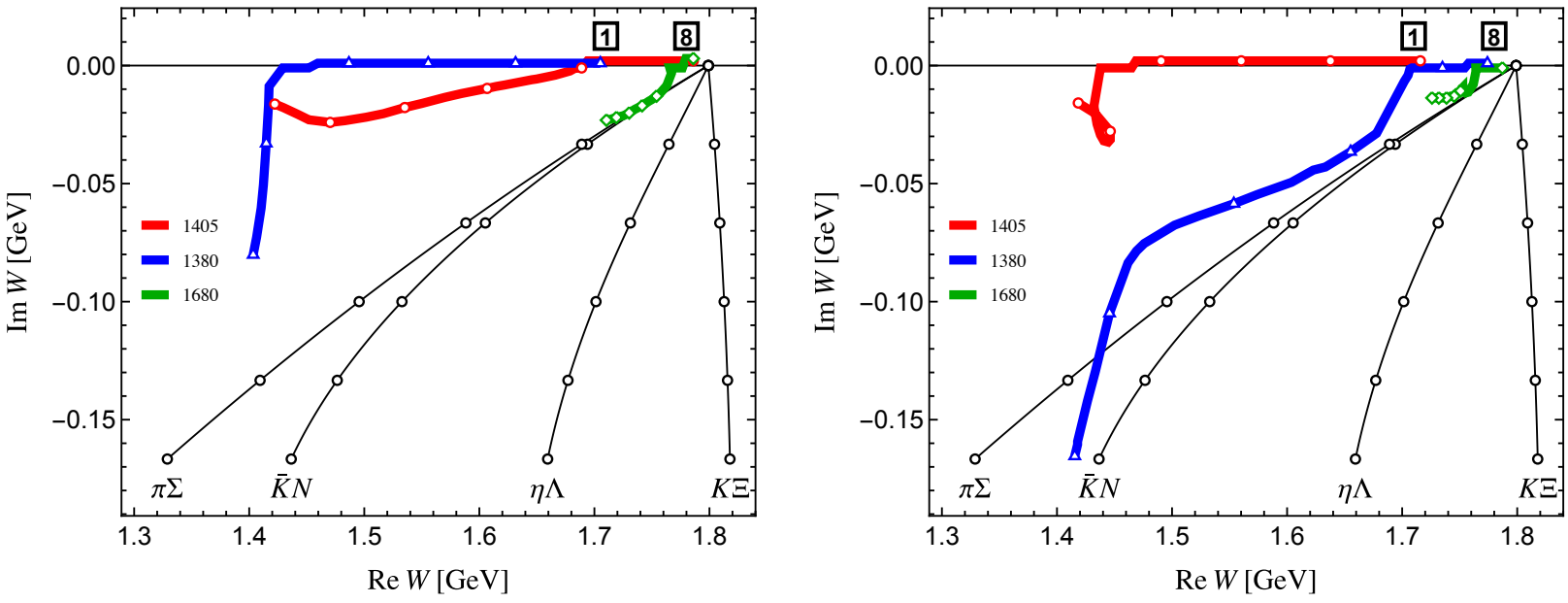
- The elemental for the baryon is

$$\Phi^{ijk}(t) = \sum_x \epsilon^{abc} V^{i,a}(x,t) V^{j,b}(x,t) V^{k,c}(x,t)$$

- The local elemental for a baryon-meson is

$$\Phi^{ijklm}(t) = \sum_x \epsilon^{abc} V^{i,a}(x,t) V^{j,b}(x,t) V^{k,c}(x,t) V^{l*}(x,t) V^m(x,t)$$

Pole evolution



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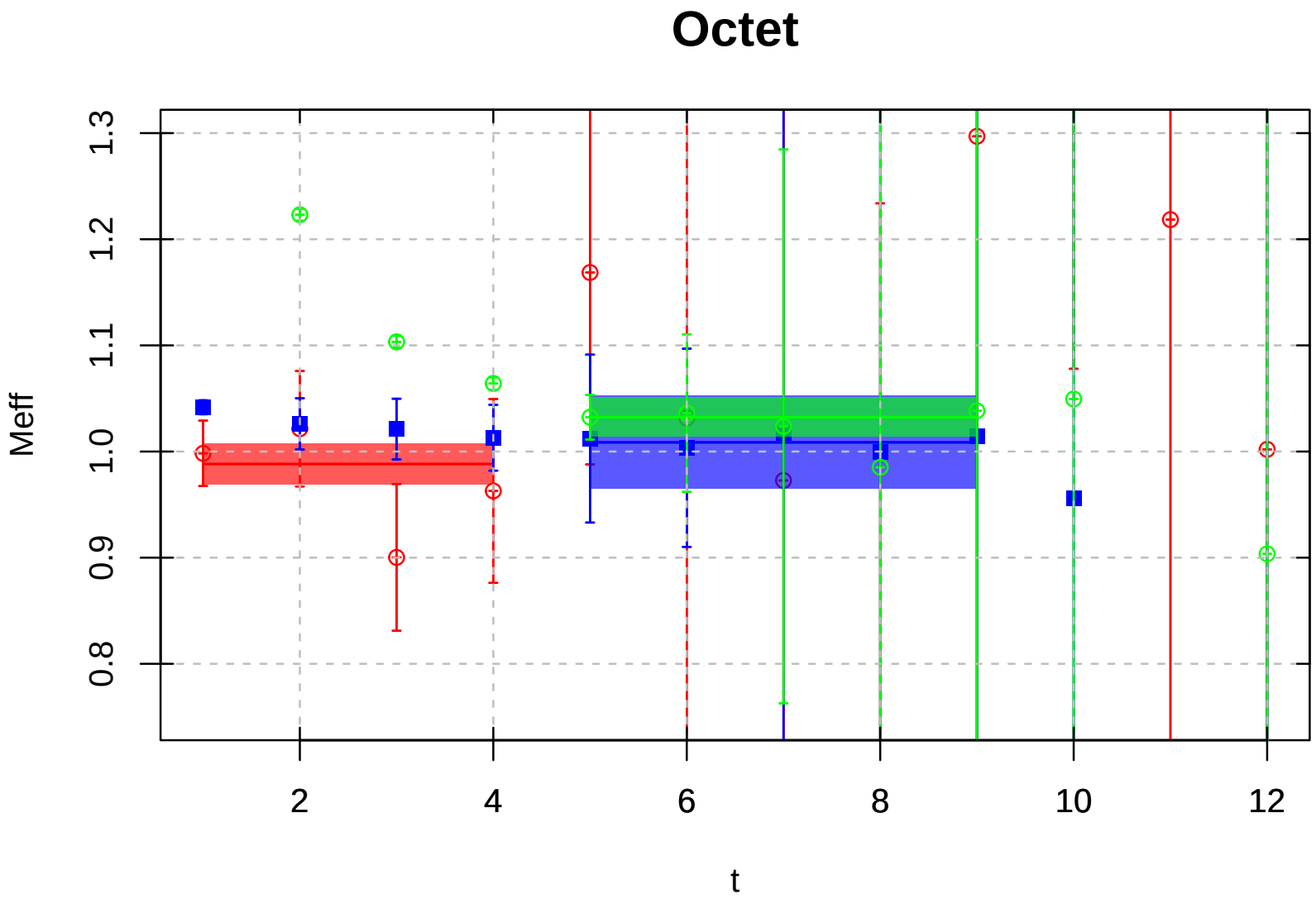
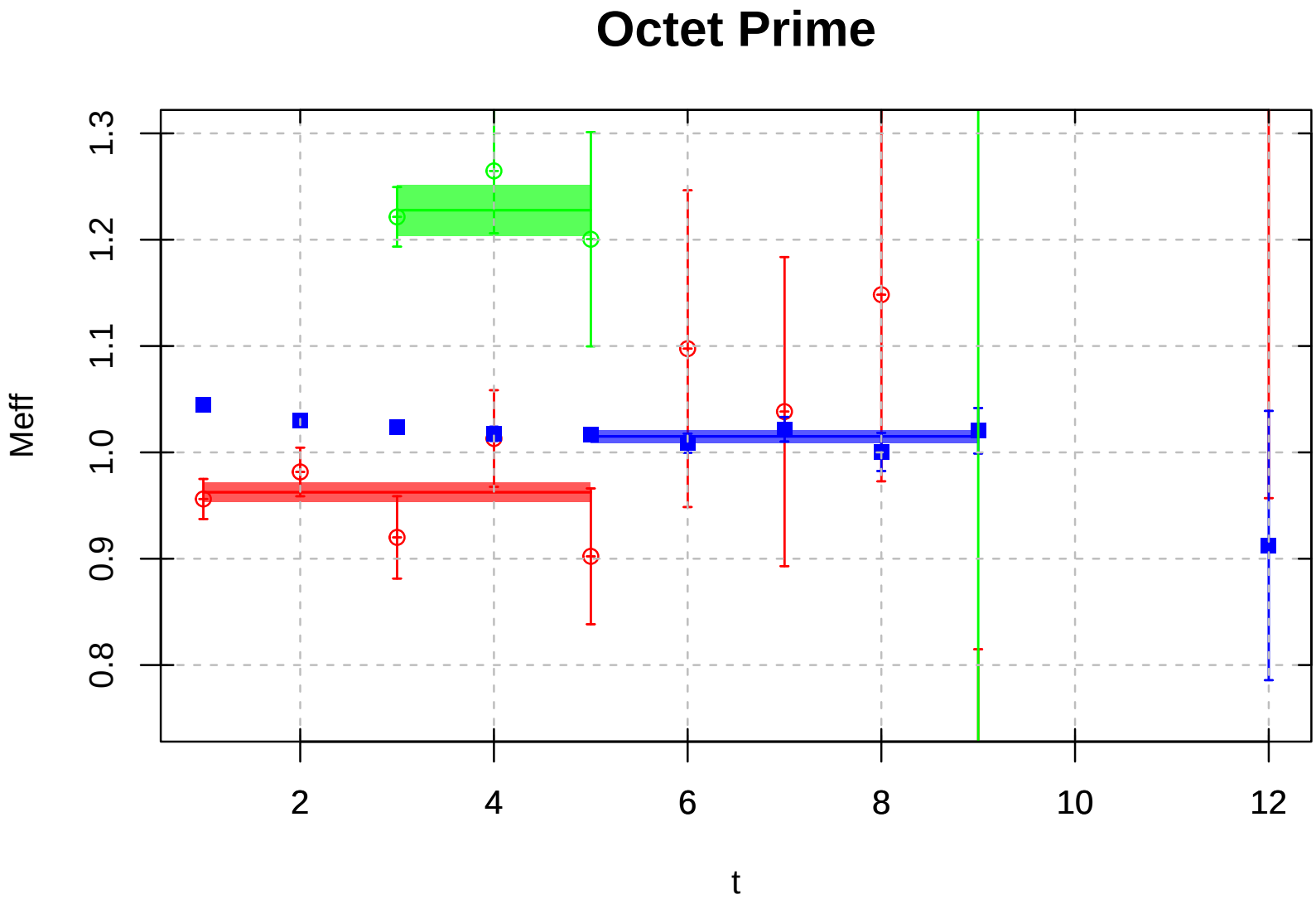
Results

Effective mass octet



Results

Effective mass octet prime



States

$$|\mathbf{1}\rangle = -\tfrac{1}{3\sqrt{2}}usdu\bar{u} - \tfrac{1}{3\sqrt{2}}dsdu\bar{d} - \tfrac{1}{3\sqrt{2}}ssdu\bar{s} + \tfrac{1}{3\sqrt{2}}udsu\bar{u} + \tfrac{1}{3\sqrt{2}}ddsu\bar{d}$$
$$+ \tfrac{1}{3\sqrt{2}}sdsu\bar{s} + \tfrac{1}{3\sqrt{2}}usud\bar{u} + \tfrac{1}{3\sqrt{2}}dsud\bar{d} + \tfrac{1}{3\sqrt{2}}ssud\bar{s} - \tfrac{1}{3\sqrt{2}}uusd\bar{u}$$
$$- \tfrac{1}{3\sqrt{2}}dusd\bar{d} - \tfrac{1}{3\sqrt{2}}susd\bar{s} - \tfrac{1}{3\sqrt{2}}udus\bar{u} - \tfrac{1}{3\sqrt{2}}ddus\bar{d} - \tfrac{1}{3\sqrt{2}}sdus\bar{s}$$
$$+ \tfrac{1}{3\sqrt{2}}uuds\bar{u} + \tfrac{1}{3\sqrt{2}}duds\bar{d} + \tfrac{1}{3\sqrt{2}}suds\bar{s}$$

$$|\mathbf{8}\rangle = \tfrac{1}{3\sqrt{2}}\Big(- usdu\bar{u} - dsdu\bar{d} + udsu\bar{u} + ddsu\bar{d} + usud\bar{u}$$
$$+ dsud\bar{d} - uusd\bar{u} - dusd\bar{d} - udus\bar{u} - ddus\bar{d} + uuds\bar{u} + duds\bar{d}\Big)$$

$$|\mathbf{8}'\rangle = \tfrac{1}{3\sqrt{2}}\Big(- usdu\bar{u} - dsdu\bar{d} - ssdu\bar{s}$$
$$+ udsu\bar{u} + ddsu\bar{d} + sdsu\bar{s} - udus\bar{u} - ddus\bar{d} - sdus\bar{s}$$
$$+ uuds\bar{u} + duds\bar{d} + suds\bar{s}\Big)$$