Adjoint meson and tetraquark static energies using novel BOEFT interpolators

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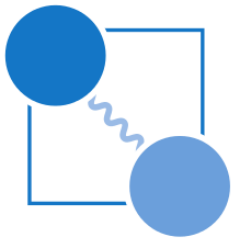
Juan Andrés Urrea-Niño, Nora Brambilla, Francesco Knechtli, Michael Peardon, Saša Prelovšek



T30f
Theoretische Teilchen-
und Kernphysik

Motivation

- Explore the $I = 1$ hidden-heavy Z_b $(\bar{b}b\bar{q}q)$ tetraquark channel with $J^{PC} = 1^{+[-]}$ to provide theoretical input for interpreting the Belle experiment's discovery of the exotic $Z_b(10610)$ and $Z_b^{'}(10650)$ (near degenerate) states close to the $B\bar{B}^*$ and $B^*\bar{B}^*$threshold.

  Belle, A. Bondar et al., [arXiv:1110.2251], Belle, A. Garmash et al., [arXiv:1403.0992]

- In particular, validate BOEFT (Born-Oppenheimer Effective field Theory) reasoning for this observed degeneracy.

  M. Berwein, N.Brambilla, A. Mohapatra, A. Vairo, arXiv:2408.04719v2 [hep-ph]

Background

- BOEFT exploits the energy hierarchy between heavy quarks and the light degrees of freedom (LDF) (light quarks and gluons) of QCD.
- It provides an elegant alternative to computationally challenging Lüsher's method by facilitating the extraction of the static energies for infinitely heavy quarks in the presence of LDF on the lattice.
- These static energies can be plugged into the coupled Schrödinger equations in the framework of BOEFT to extract the spectrum of XYZ states comprising of two heavy quarks or one heavy quark-antiquark pair.

See A.Mohapatra's talk on Wednesday 11:10 am

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- In particular, Z_b and Z_b' with $J^{PC} = 1^{+-}$ are superpositions of Z_1 and Z_2 with 1^{--} and 0^{-+} as LDF quantum numbers respectively:

$$|Z_b'\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle}_{Z_1} - \underbrace{|S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle}_{Z_2}\Big)$$

$$|Z_b\rangle = \frac{1}{\sqrt{2}}\Big(|S_{b\bar{b}} = 0\rangle|K^{PC} = 1^{--}\rangle + |S_{b\bar{b}} = 1\rangle|K^{PC} = 0^{-+}\rangle\Big)$$

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- Z_b lies slightly above $B\bar{B}^*$ threshold and Z_b' lies slightly above $B^*\bar{B}^*$. This near degeneracy of Z_b and Z_b', and the suppression of Z_b' decay to $B\bar{B}^*$, strongly points towards the degeneracy of adjoint mesons associated with the LDF configurations of Z_1 and Z_2.

Tetraquark and adjoint meson operators

- As $r \to \infty$, states with 3 and $\bar{3}$ sources should become pairs of a $3-$hadron and a $\bar{3}-$hadron.
- As $r \to 0$, the 3 and $\bar{3}$ sources reduce to a linear combination of $1-$source and $8-$source.

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- For $Q\bar{Q}$ (Σ_g^+), BO potentials should smoothly connect $1-$potential at small r to $(3+\bar{3})-$potential at large r.
- Rest $(3+\bar{3})-$potentials at large r should connect to $8-$potential at small r.

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  - Rest $(3+\bar{3})-$potentials at large r should connect to $8-$potential at small r.
- Following novel tetraquark operator is proposed to have a zero overlap with quarkonium and pions, and contains the heavy-light component:

$$O_{BO} = \bar{Q}(0)\Gamma_1\, U(0, r/2)\, T^a \underbrace{\bar{q}(r/2)\Gamma_2 T^a q(r/2)}_{\text{adjoint meson} \equiv O^a_{AM}} U(r/2, r) Q(r)$$

$$a \in [1, 8]$$

- For short distances, at the leading order, $\langle O_{BO} | \bar{Q}Q \rangle$ will result in a Tr $(T^a) = 0$.
- For $r \to 0$, $V_{O_{BO}} = V_8 + \Lambda_{AM}$

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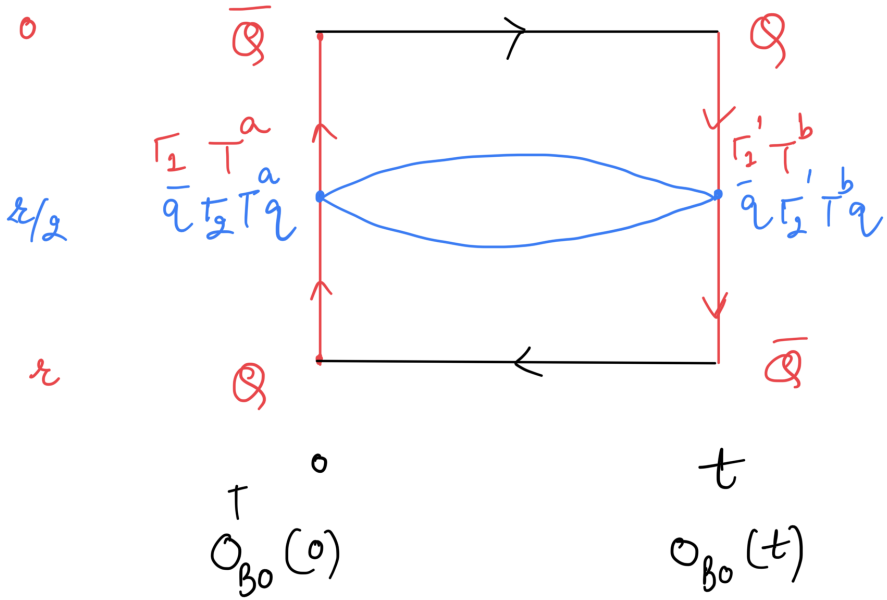

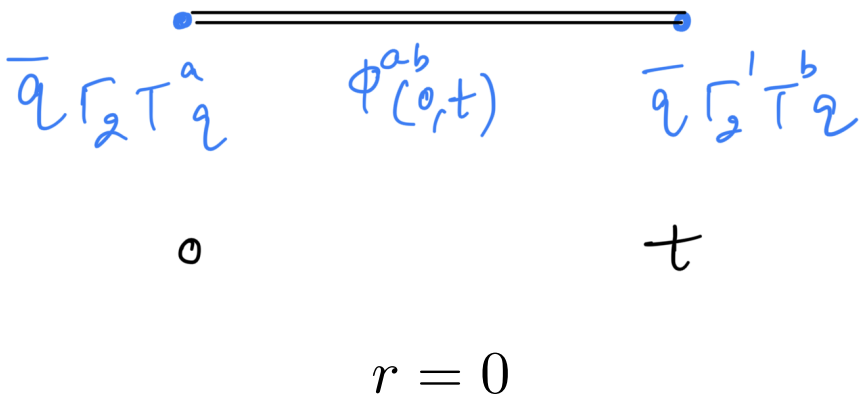

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- The adjoint meson correlator:

$$\langle O^a_{AM}(t)\, \phi^{ab}_{adj}(t, 0)\, O^{b\dagger}_{AM}(0)\rangle,$$

where

$$\phi^{ab}_{adj}(t, 0) = \mathrm{Tr}\left[T^a U(t, 0) T^b U(0, t)\right]$$

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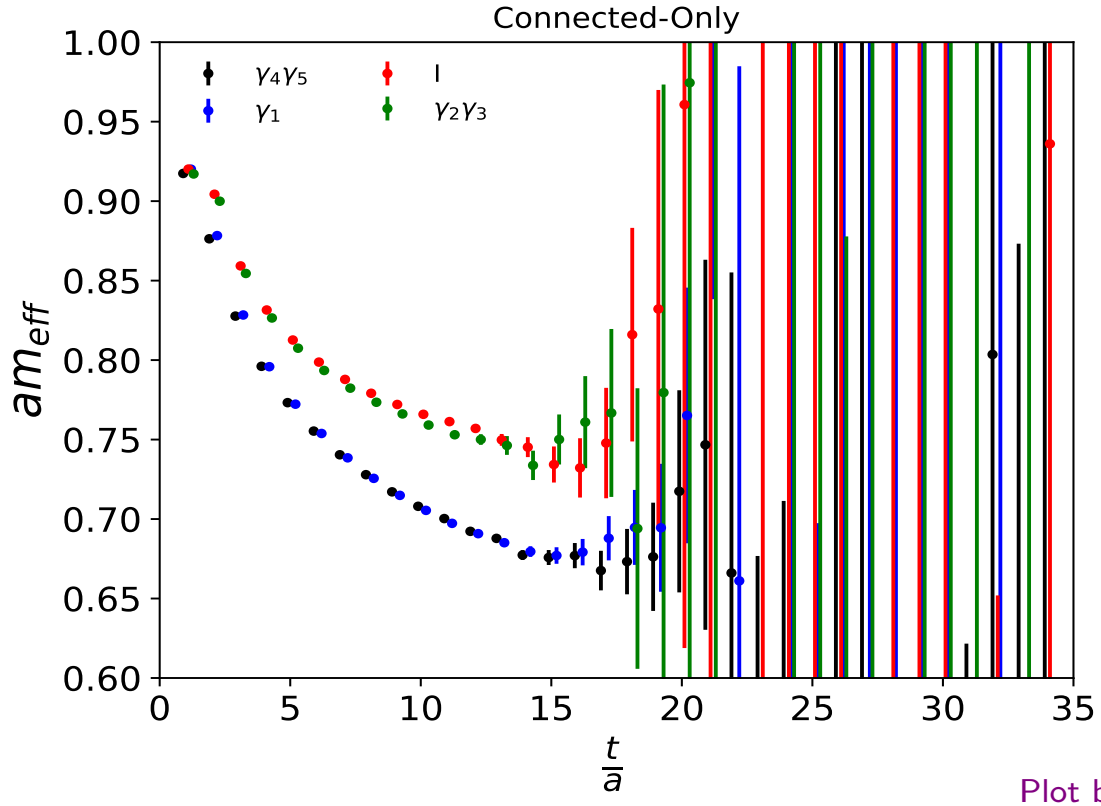
- $\Gamma_2 = \gamma_5 \Rightarrow 0^{-+}$ adjoint meson LDF associated with Z_2
  $\Gamma_2 = \gamma_i \Rightarrow 1^{--}$ adjoint meson LDF associated with Z_1 In the quenched case, vector and PS adjoint mesons were found to be -10(103) MeV and 34(161) MeV heavier than the lightest gluelump, M. Foster and C. Michael. Phys. Rev. D, 59:094509, 1999

Lattice setup for adjoint meson correlators

- $N_f = 3 + 1$ ensembles produced with the openQCD package, with $m_\pi = 406$ MeV, $32^3 \times 96$ lattices with open boundary conditions in time, and $a = 0.05359$ fm ALPHA Collaboration, Eur. Phys. J. C 80 (2020) 349 [2002.02866].
- For the light quark, we use 100 distillation vectors and standard distillation on 4000 gauge configurations. The eigenvectors were computed from links that have 20 steps of 3D APE smearing with $\alpha = 0.5$. The temporal links entering the temporal lines have HYP2 smearing with parameters $\alpha_1 = 1$, $\alpha_2 = 1$, $\alpha_3 = 0.5$.

Preliminary analysis of adjoint mesons

$$am^{i}_{eff}(t, t+1) = \log \frac{C_{ii}(t)}{C_{ii}(t+1)};\ C_{ij}(t) = \sum_{n=1}^{\infty} e^{-E_n t} \langle \hat{O}_i | n \rangle \langle n | \hat{O}_j \rangle$$



Plot by Juan Andrés Urrea-Niño

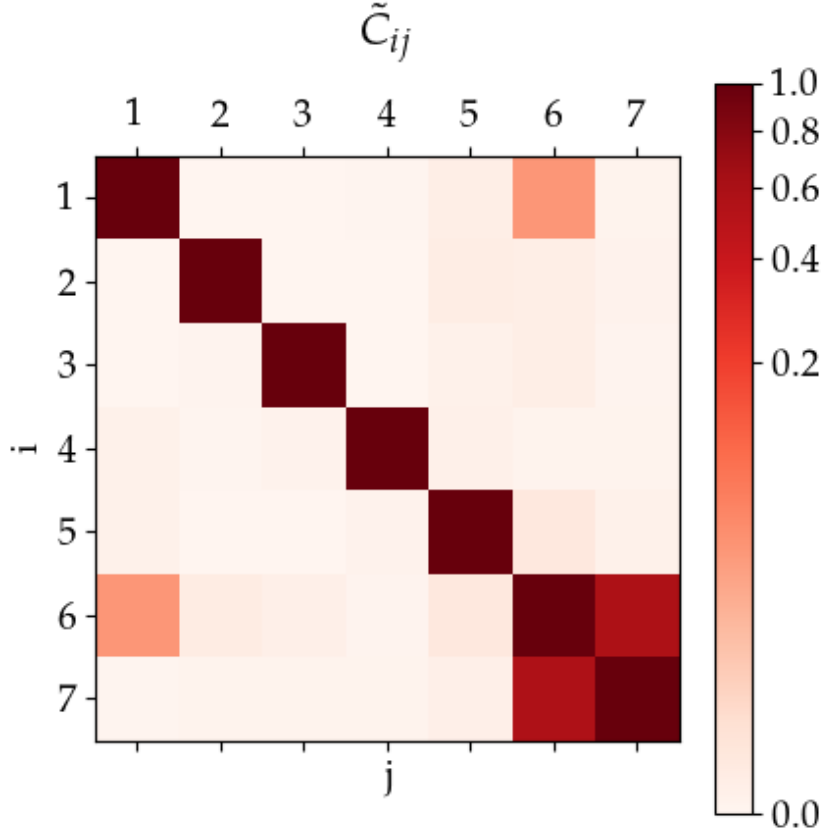
- 0^{-+} and 1^{--} agree validating the BOEFT prediction.
- Agreement of 0^{++} and 1^{+-} hints at a larger symmetry.

Lattice set up for extracting Z_b static energies

- Analysis was done on 134 gauge configurations part of a CLS ensemble with $a = 0.08636$ fm, $N_f = 2 + 1$, $m_\pi = 290$ MeV, $24^3 \times 128$ lattices with open boundary conditions in time.
- For the light quark, we used 60 distillation vectors and standard distillation. Spatial Gauge links are 3D APE smeared with 25 steps and $\alpha \approx 0.7$.
- We use five time sources per configuration situated at $t/a = 36, 40, 44, 48, 52$.

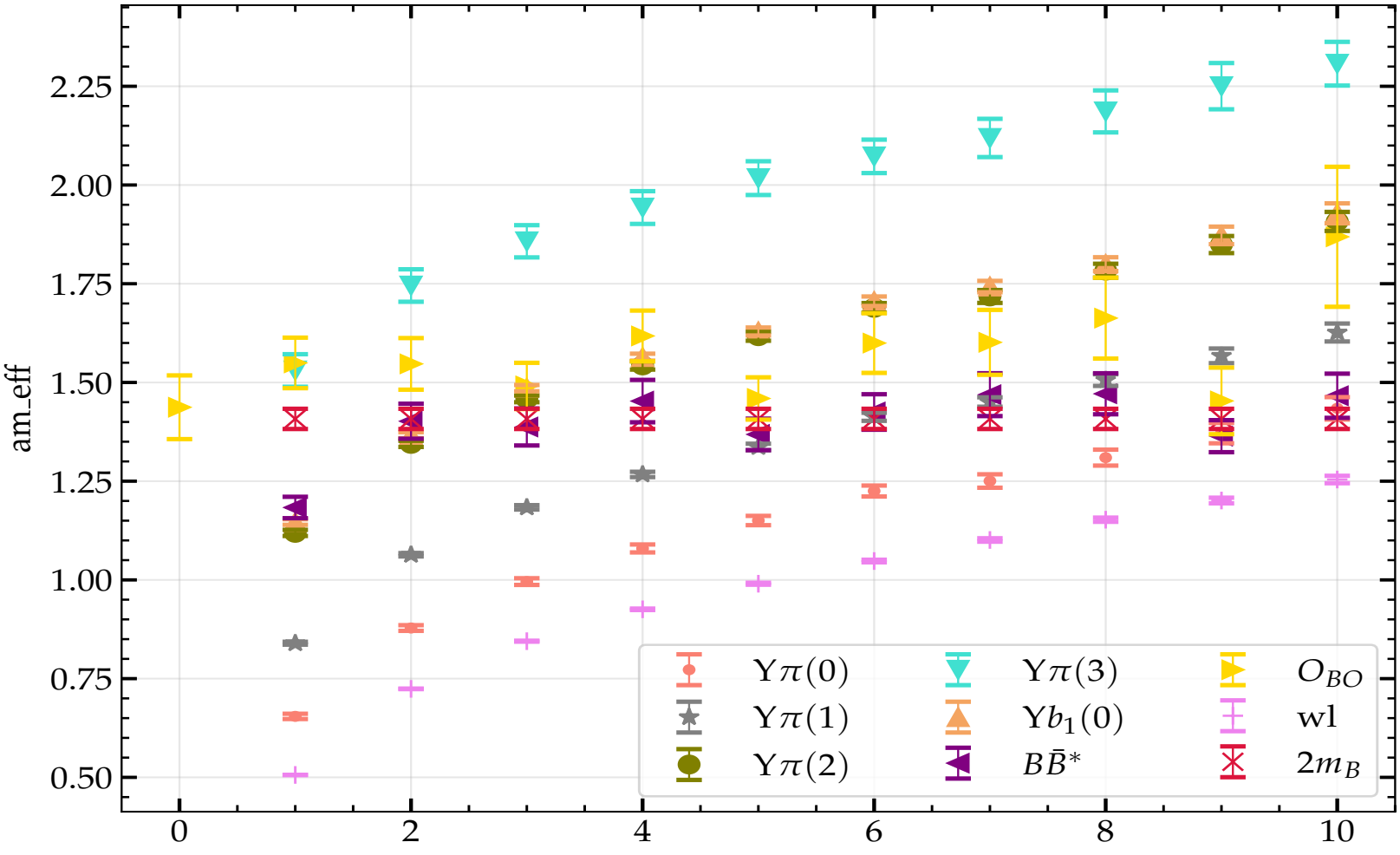
Preliminary steps towards extracting Z_b static energies

- We constructed a 7×7 correlation matrix, with
  - Op1: $\Upsilon\pi(0)$
  - Op2,3,4: $\Upsilon\pi(1,2,3)$
  - Op5: $\Upsilon b_1(0)$
  - Op6: $B\bar{B}^*$
  - Op7: O_{BO} with 0^{-+} LDF configuration (Σ_u^-)
- $\tilde{C}_{ij}(r) = \dfrac{|\sum_{t=1}^{n_t} C_{ij}(r,t)|}{\sqrt{|\sum_{t=1}^{n_t} C_{ii}(r,t)||\sum_{t=1}^{n_t} C_{jj}(r,t)|}}$
- Op6 has a visible overlap with Op1 but Op7 does not have a visible overlap with Op1.



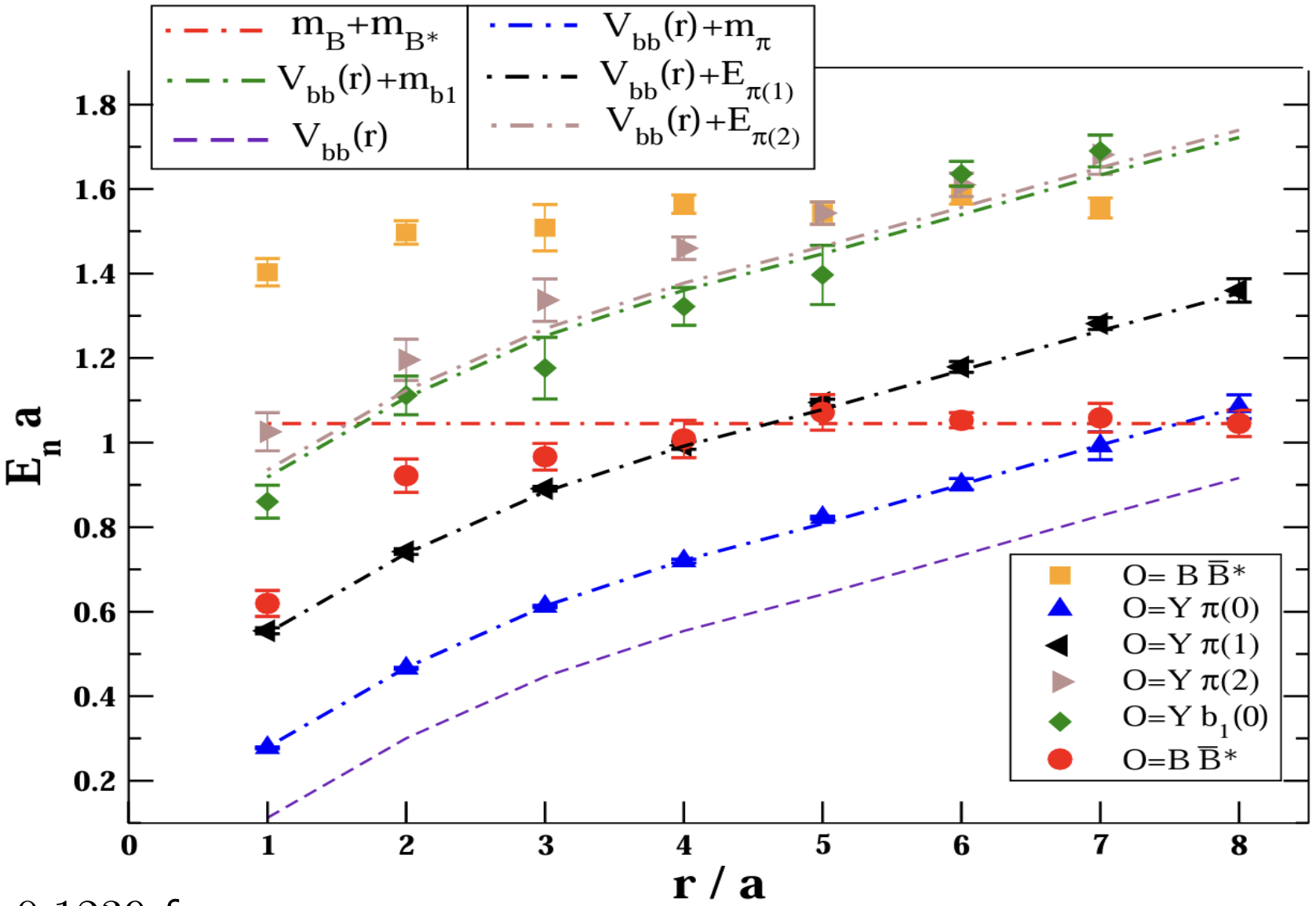
$r/a = 1$

Preliminary steps towards extracting Z_b static energies



- Plateaus extracted from am_{eff} of various diagonal correlators plotted a function of r. No GEVP.

Past work



$a = 0.1239$ fm

S. Prelovsek, H. Bahtiyar and J. Petkovic, arxiv:1912.02656 [hep-lat]

Conclusions and Outlook

- Preliminary results for the adjoint mesons associated with the Z_b tetraquark channel validate the predictions of BOEFT, and hint at a larger symmetry.
- Calculation of the Z_b static energies with an operator basis containing the new BOEFT interpolator is in progress.

$Q\bar{Q}q\bar{q}$: Operator Overlap

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

NRQCD operator (gauge invariant) for exotic hadron: $Q\bar{Q}$ pair in **octet** color

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^\dagger(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,0)\,P^{\alpha\dagger}_{\kappa,\lambda}\,H^\alpha_\kappa(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

$$\mathbf{H_K}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\tilde{\mathbf{\Gamma}}T^a q(t,\boldsymbol{x})\right]T^a$$

$\tilde{\mathbf{\Gamma}}$: Dirac matrices based on quantum #'s

Quarkonium + Pions

Quarkonium state:

$$|\mathcal{Q}\rangle = \mathcal{N}\int d^3\boldsymbol{r}\,\Psi^{(n)}(\boldsymbol{r})\;\psi_b^\dagger(t,-\boldsymbol{r}/2)\,\phi_{bc}(t;-\boldsymbol{r}/2,\boldsymbol{r}/2)\,\chi_c(t,\boldsymbol{r}/2)\,|\Omega\rangle$$

Overlap of adjoint operator on quarkonium + pion:

$$\langle\mathcal{Q}|\mathcal{O}_K^{Q\bar{Q}}(t,\boldsymbol{r})\,|\mathcal{Q}\rangle = 0$$

(at leading order in multipole expansion)

Meson-antimeson

Meson-antimeson state:

$$|M\bar{M}\rangle = \Big[\mathcal{N}\int d^3\boldsymbol{x}\,\Psi_J(\boldsymbol{x}) \times \int d^3\boldsymbol{y}\,\varphi_{J_1}(\boldsymbol{y}+\boldsymbol{x}/2)\;\psi_c^\dagger(t,-\boldsymbol{x}/2)\,\phi_{cd}(t;-\boldsymbol{x}/2,\boldsymbol{y}+\boldsymbol{x}/2)\,[P_+\Gamma_1\,q_d(t,\boldsymbol{y}+\boldsymbol{x}/2)] \times \int d^3\boldsymbol{z}\,\varphi_{J_2}(\boldsymbol{z}-\boldsymbol{x}/2)\;[\bar{q}_b(t,\boldsymbol{z}-\boldsymbol{x}/2)\,\Gamma_2\,P_-]\,\phi_{be}(t;\boldsymbol{z}-\boldsymbol{x}/2,\boldsymbol{x}/2)\,\chi_e(t,\boldsymbol{x}/2)\Big]|\text{vac}\rangle$$

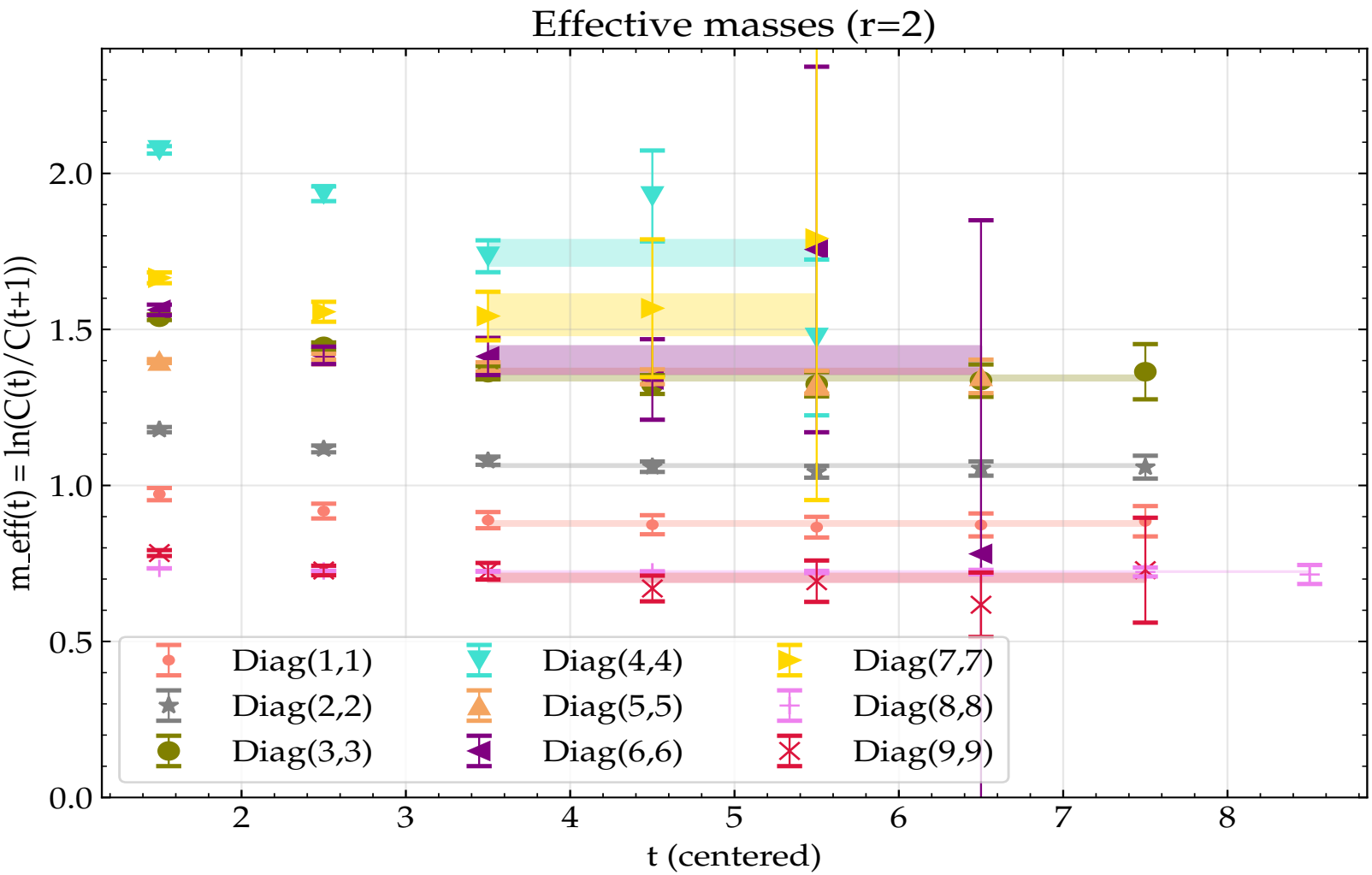
Overlap of our operator on meson-antimeson:

$$\langle M\bar{M}|\mathcal{O}_K^{Q\bar{Q}}(t,\boldsymbol{r})\,|M\bar{M}\rangle \neq 0$$

Adjoint operators could be good operators for lattice computation for $Q\bar{Q}q\bar{q}$ potentials

1

Slide by A. Mohapatra


Effective masses (r=2)
m_eff(t) = ln(C(t)/C(t+1))
t (centered)
0.0
0.5
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1.5
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Diag(1,1)
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Diag(3,3)
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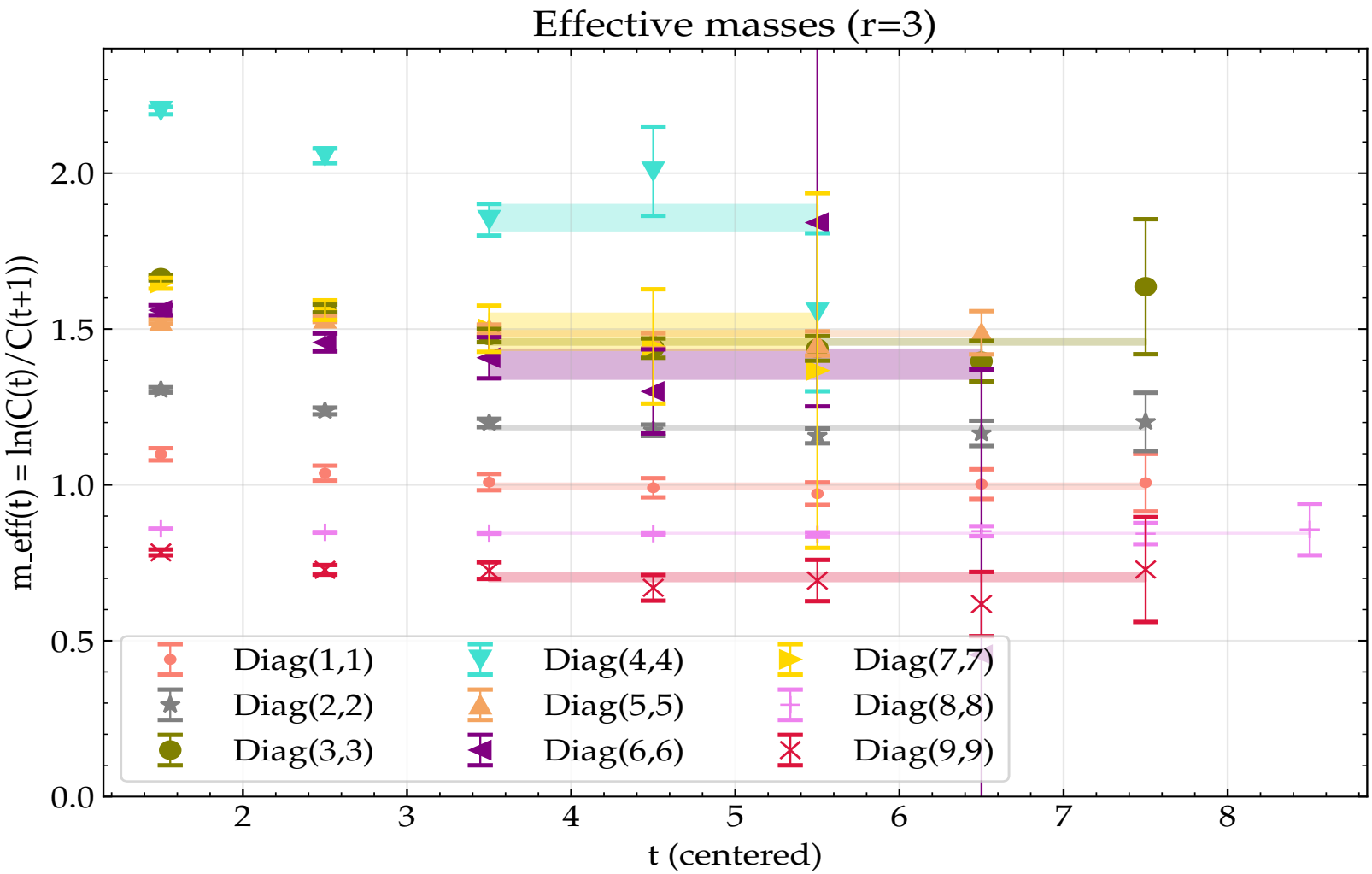
Effective masses (r=3)
m_eff(t) = ln(C(t)/C(t+1))
t (centered)
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Diag(1,1)
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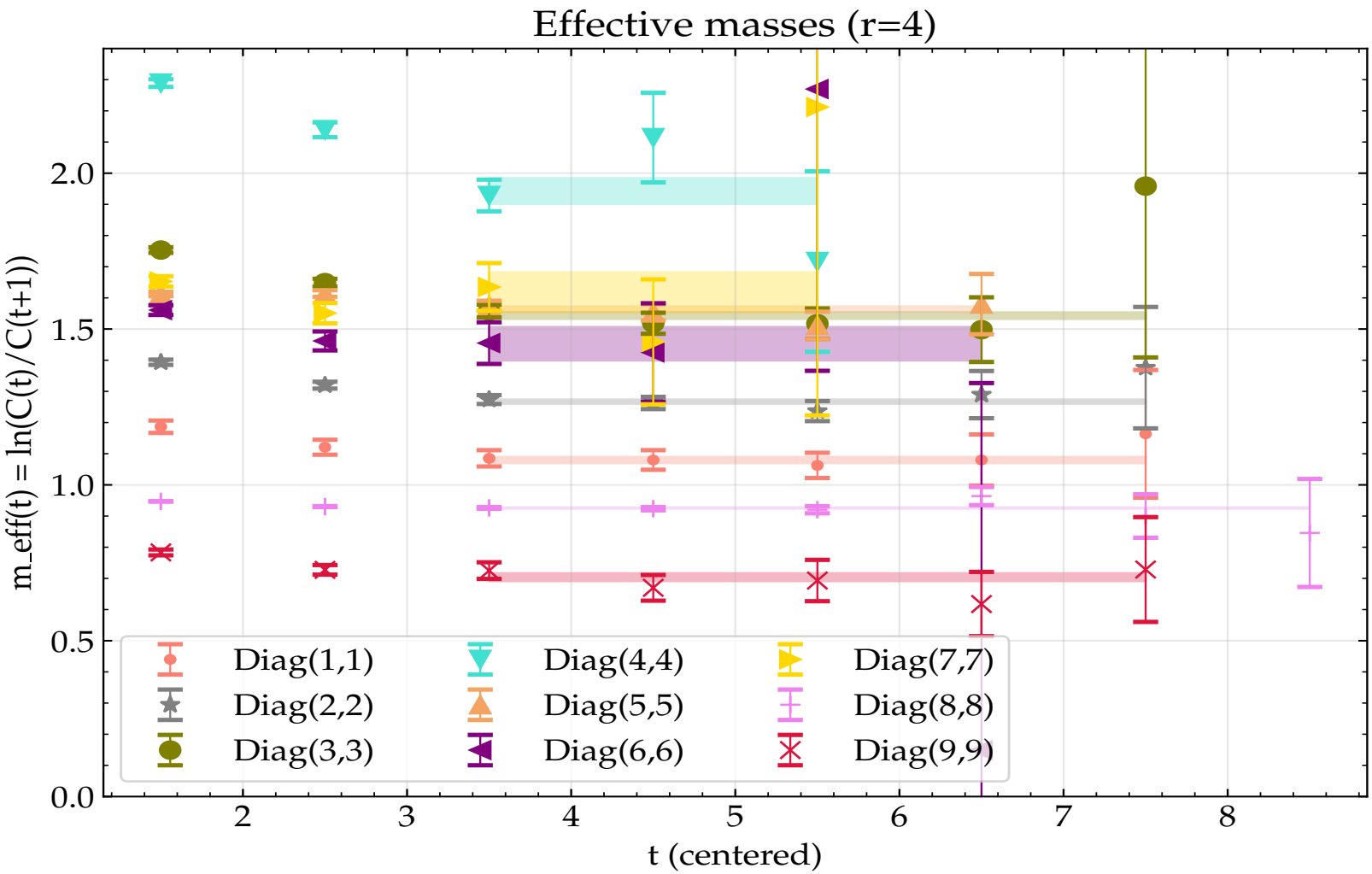
Effective masses (r=4)
m_eff(t) = ln(C(t)/C(t+1))
t (centered)
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Diag(1,1)
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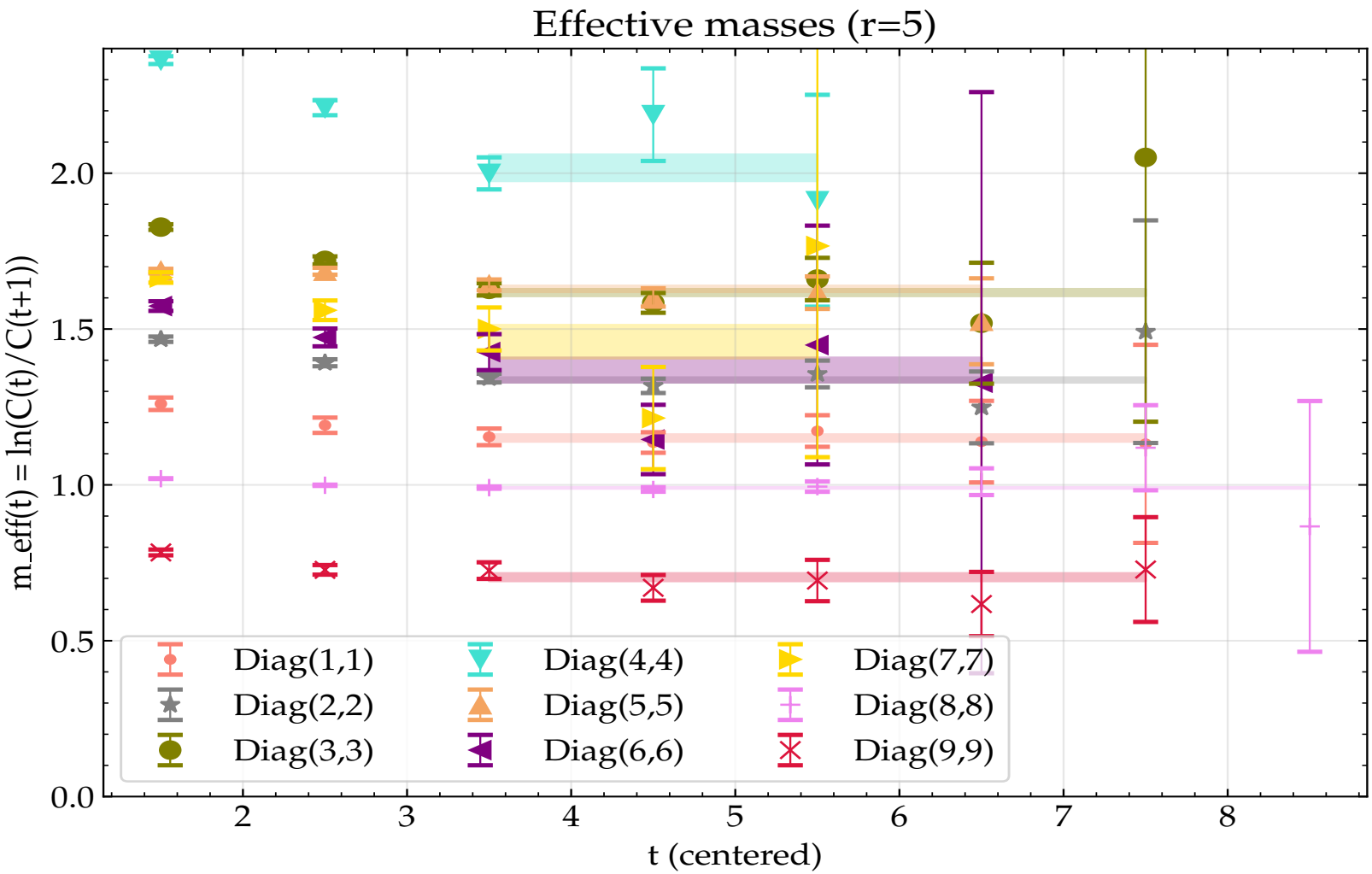
Effective masses (r=5)
m_eff(t) = ln(C(t)/C(t+1))
t (centered)
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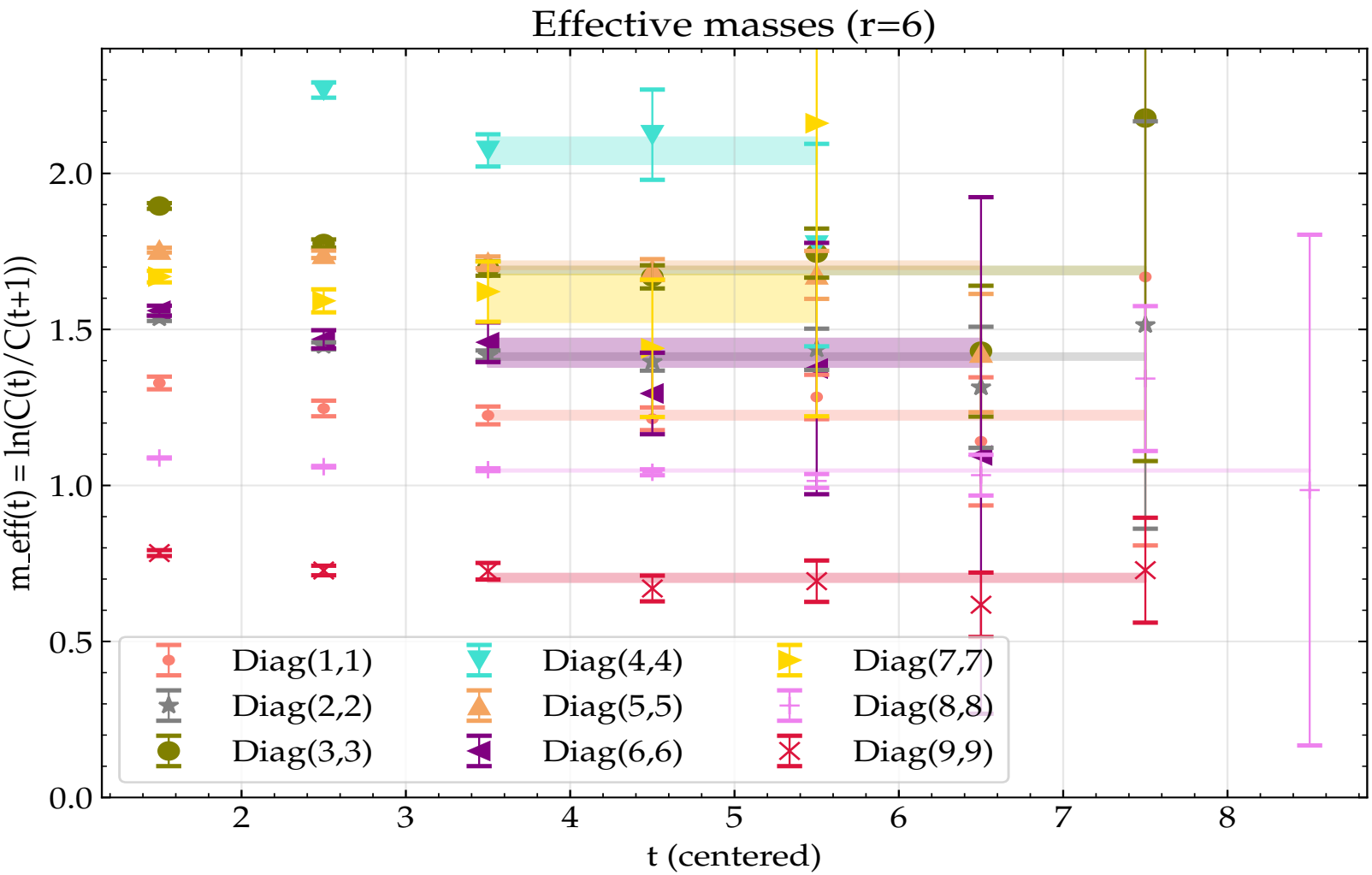
Effective masses (r=6)
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t (centered)
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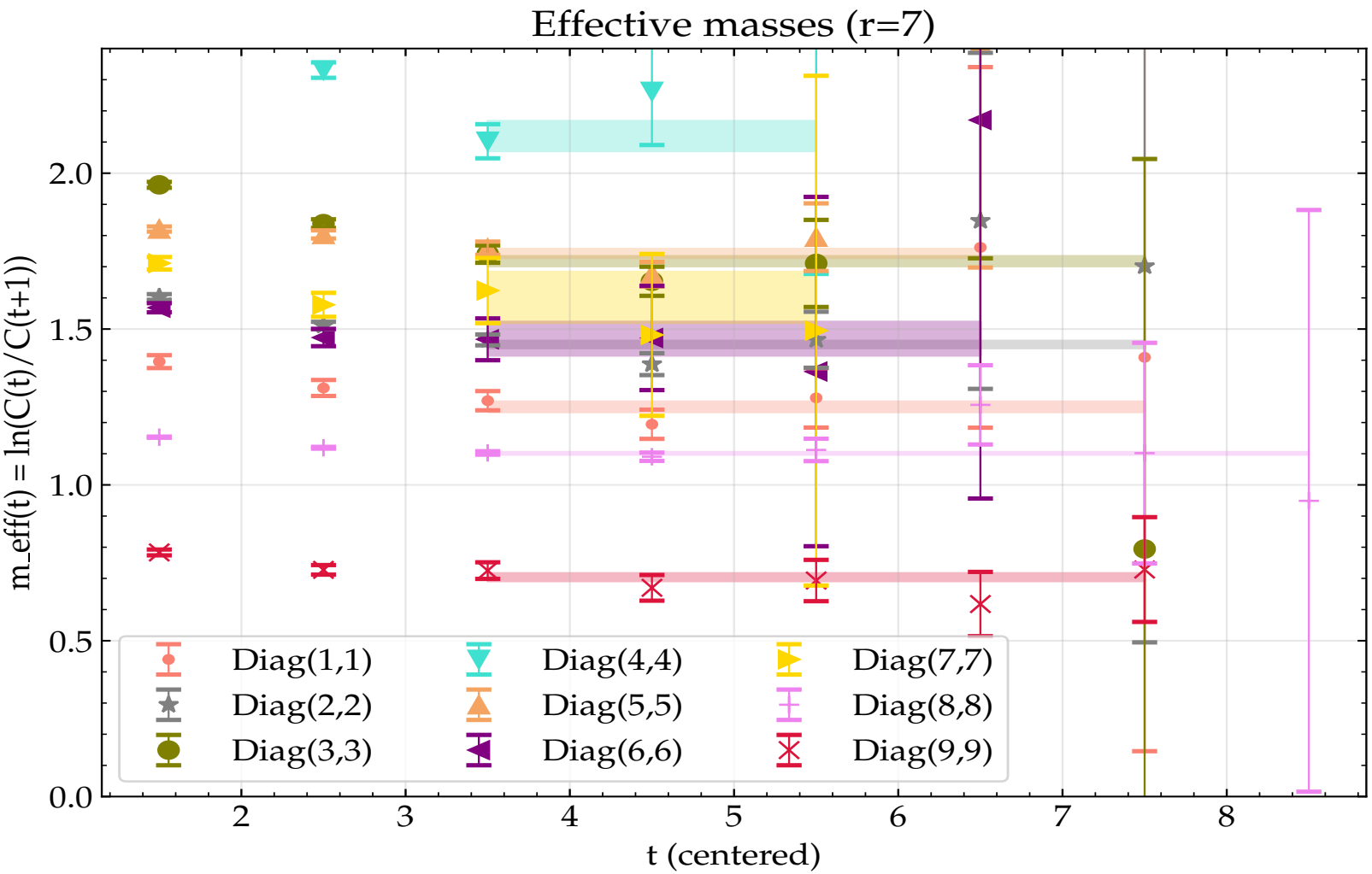
Effective masses (r=7)
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t (centered)
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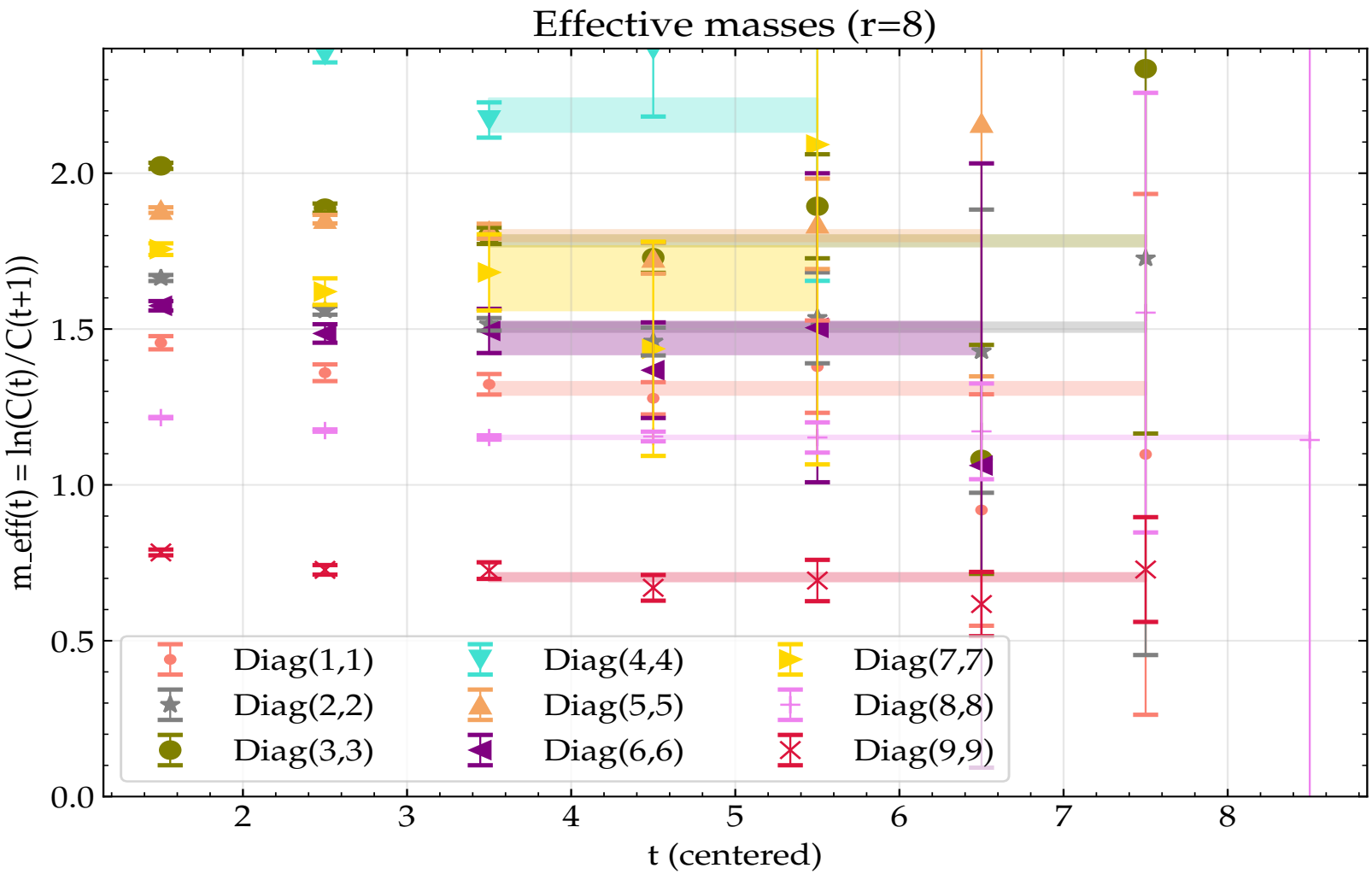
Effective masses (r=8)
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t (centered)
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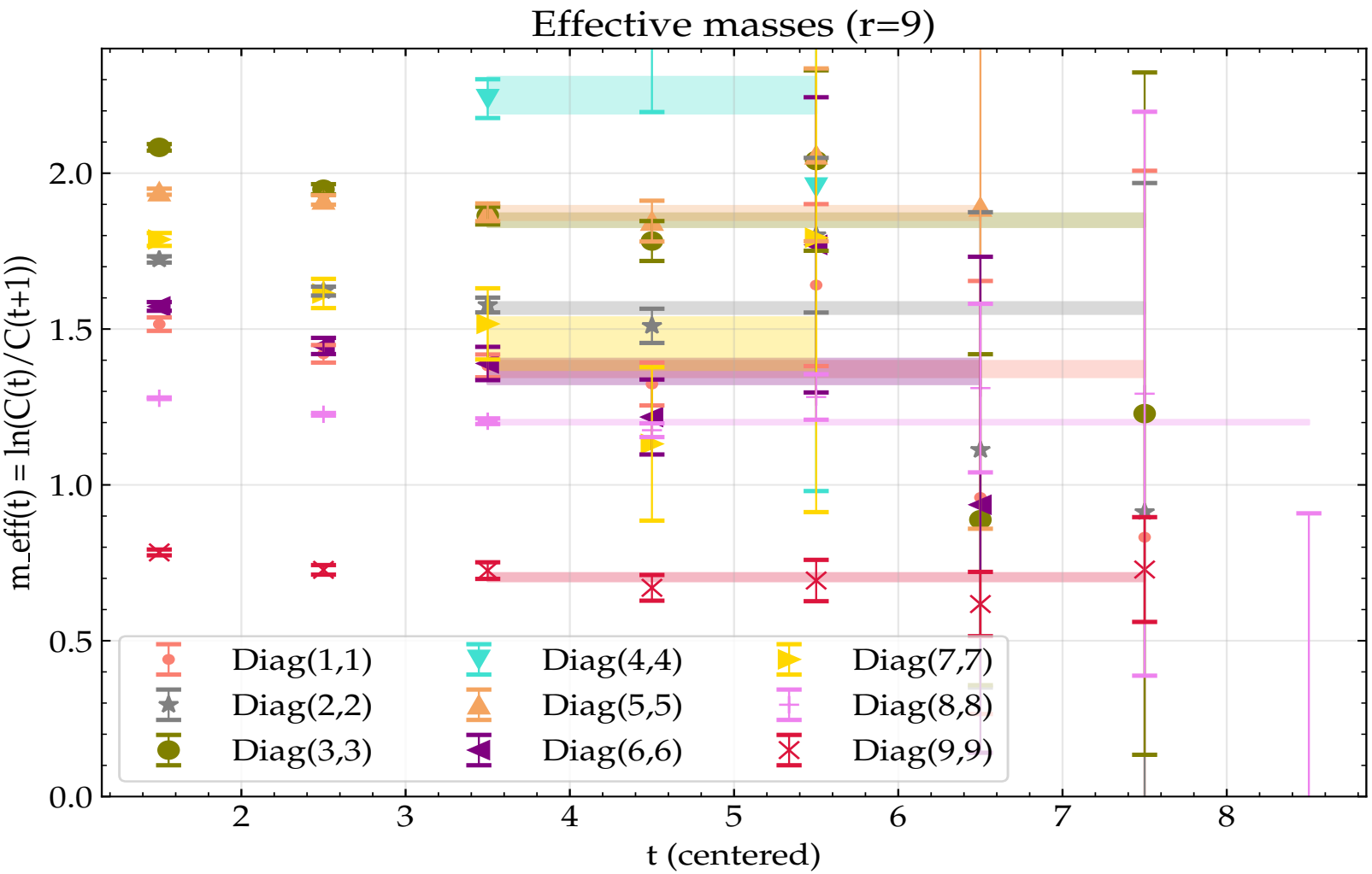
Effective masses (r=9)
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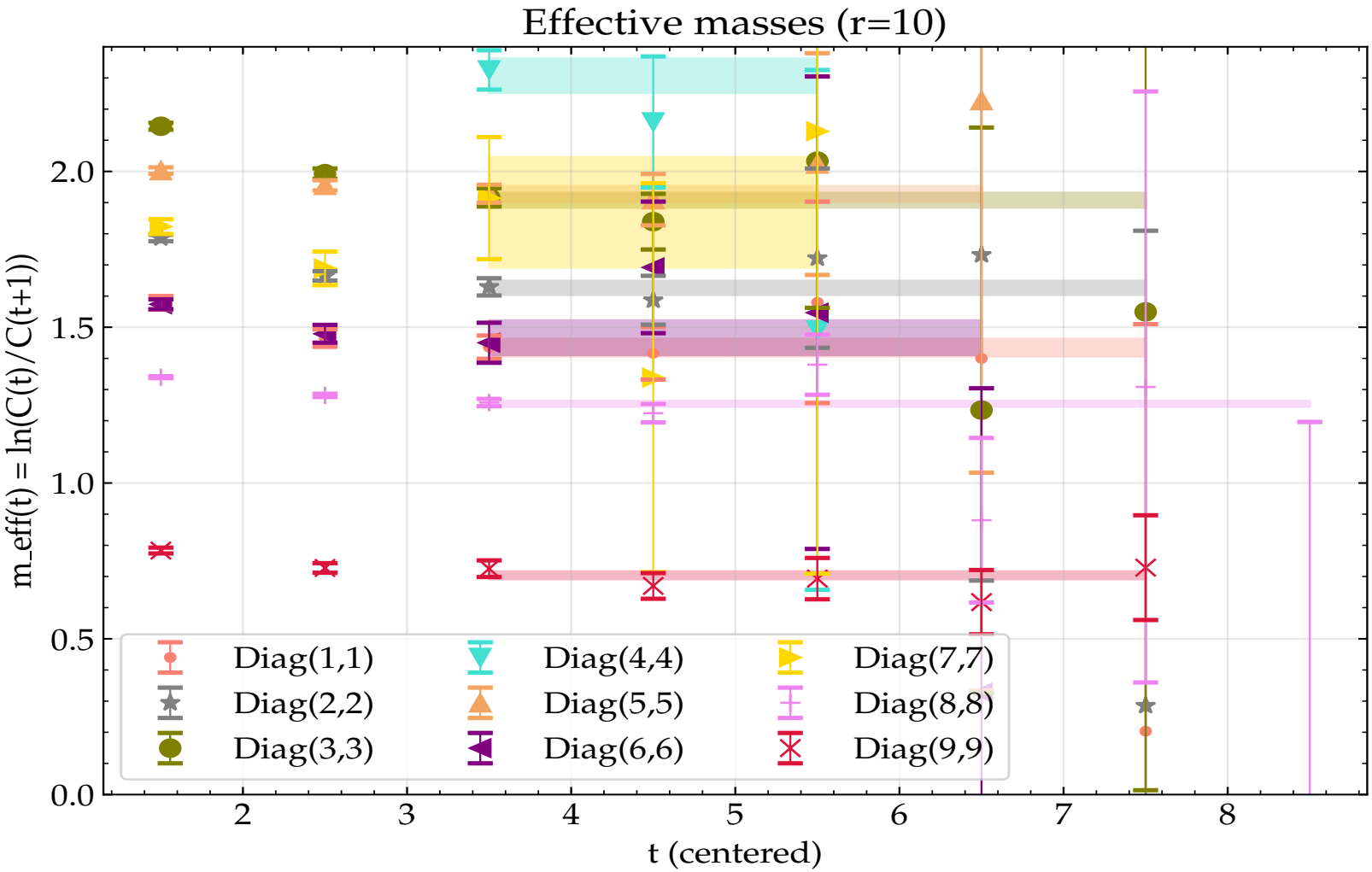
Effective masses (r=10)
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