Born-Oppenheimer Effective Theory for multiquark states

42nd International Symposium on Lattice Field Theory (Lattice 2025)
Tata institute of Fundamental Research
Nov 5, 2025

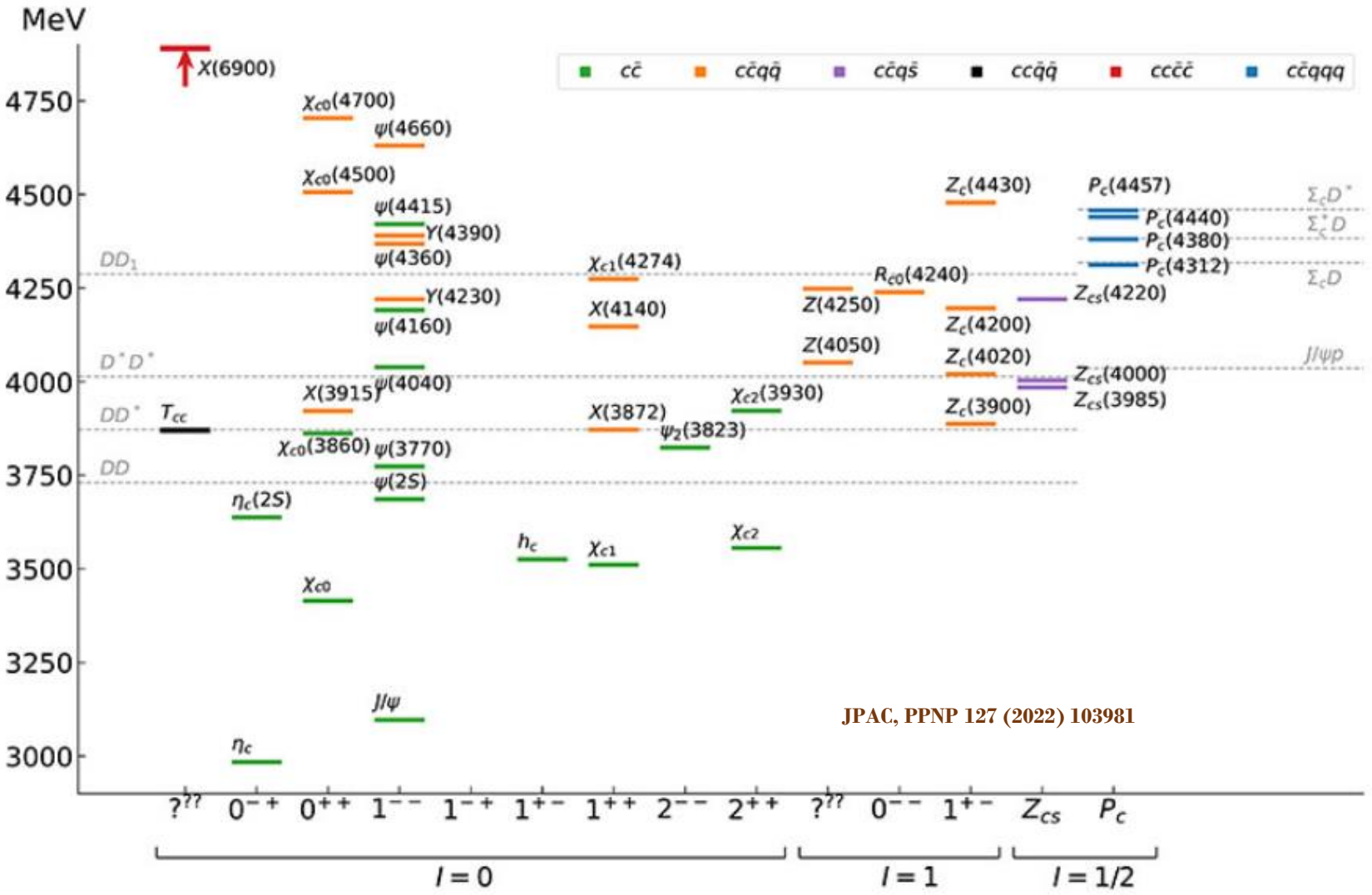
Abhishek Mohapatra (TU Munich)

Phys. Rev. D 110, 094040 (2024) (Editors Suggestion), Phys. Rev. Lett 135, 121902 (2025),
arXiv 2508.13050 (accepted in Physical Review D)

Technical University of Munich

DFG Deutsche Forschungsgemeinschaft

Exotic mesons



JPAC, PPNP 127 (2022) 103981

Recent count:

- 54 in $c\bar{c}$ sector
- 5 in $b\bar{b}$ sector
- 4 with all c and $\bar{c}$
- 1 with two charm quarks.

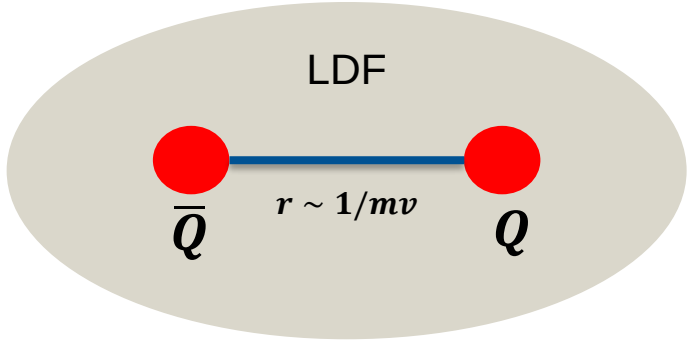
Lebed 2024, PDG 2025

XYZ hadrons: QCD bound states of **at least two heavy quarks** (charm or bottom).

Born-Oppenheimer EFT

Exotic Hadron

- Exotic hadron $(Q\bar{Q}X, QQX, \ldots\ldots)$, X: any combination of light quark and gluons (LDF) for color singlet.
- Multiple scales in Exotics:
  - ❖ Mass of heavy quark: $\boldsymbol{m}$
  - ❖ Energy scale for LDF: $\boldsymbol{\Lambda_{QCD}}$
  - ❖ Relative momentum between heavy quarks: $mv \sim 1/\mathrm{r}$
  - ❖ Heavy Quark kinetic energy scale: mv^2



$$m \gg mv \gtrsim \Lambda_{\mathrm{QCD}} \gg mv^2$$

- Time-scale for dynamics of $Q\bar{Q}$: $\sim \frac{1}{mv^2} \gg \frac{1}{\Lambda_{QCD}}$

Heavy quarks **static** with respect to light quarks or gluons

Born-Oppenheimer (BO) Approximation

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)

Juge, Kuti, Morningstar, Phys. Rev. Lett. 90, 161601 (2003)

- **Bound-state dynamics** at energy scale $\boldsymbol{mv^2}$! Integrate out all energy scales above $\boldsymbol{mv^2}$.

QCD → NRQCD → BOEFT

BOEFT: Quantum #'s

- **BO-quantum number** (**r ≠ 0**): heavy quarks static, Cylindrical symmetry group $D_{\infty h}$

Labelling LDF static energies:

✓ Absolute value of component of **LDF angular momentum** $\boldsymbol{K}$

$$|\boldsymbol{r}\cdot\boldsymbol{K}| \equiv \boldsymbol{\Lambda} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots\ldots\ldots (\mathbf{or}\ \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \ldots\ldots)$$

✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

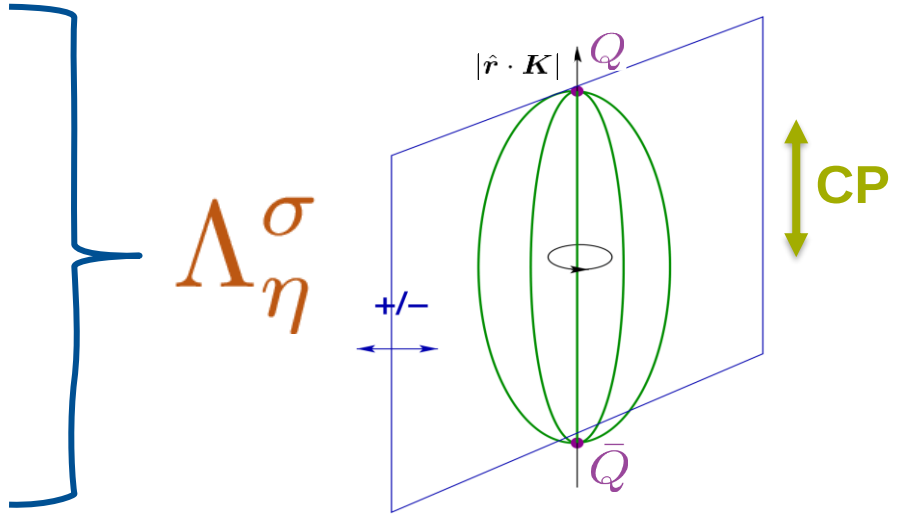
$$\boldsymbol{\eta} = +\mathbf{1}\ (\mathbf{g}),\ -\mathbf{1}\ (\boldsymbol{u})$$

✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

$$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

Born, Oppenheimer, Annalen der Physik 389 (1927)

Landau, Lifshitz & Pitaevskii, QM book



Examples:

K^{PC}	Λ_η^σ
0^{++}	Σ_g^+
0^{+-}	Σ_u^+
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$

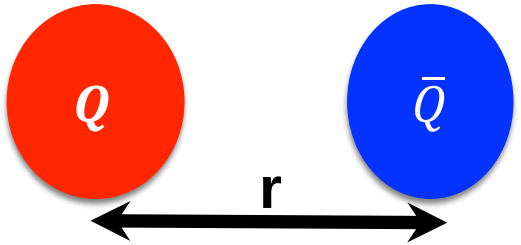
- **Spherical symmetry** restored in **r → 0** limit: Labelled by LDF quantum #'s:

$$\kappa = \{K^{PC}, f\}$$

Exotic Hadron

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$QQ\bar{Q}X$



color: $3 \otimes \bar{3} = 1 \oplus 8$

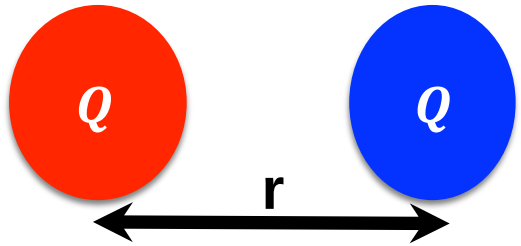
X = gluon → **Hybrid**

$X = (q\bar{q})_8$ → **Tetraquark**

$X = (qqq)_8$ → **Pentaquark**

and so on

QQX



color: $3 \otimes 3 = \bar{3} + 6$

$X = q_3$ → **Double heavy baryon**

$X = (\bar{q}\bar{q})_3$ / $(\bar{q}\bar{q})_{\bar{6}}$ → **Tetraquark**

$X = (qq\bar{q})_3$ / $(qq\bar{q})_6$ → **Pentaquark**

and so on

X_8 : **Adjoint hadrons** (gluelump, adjoint meson, adjoint baryon….)

$X_{3/6}$: **Triplet or sextet hadrons** (meson, baryon….)

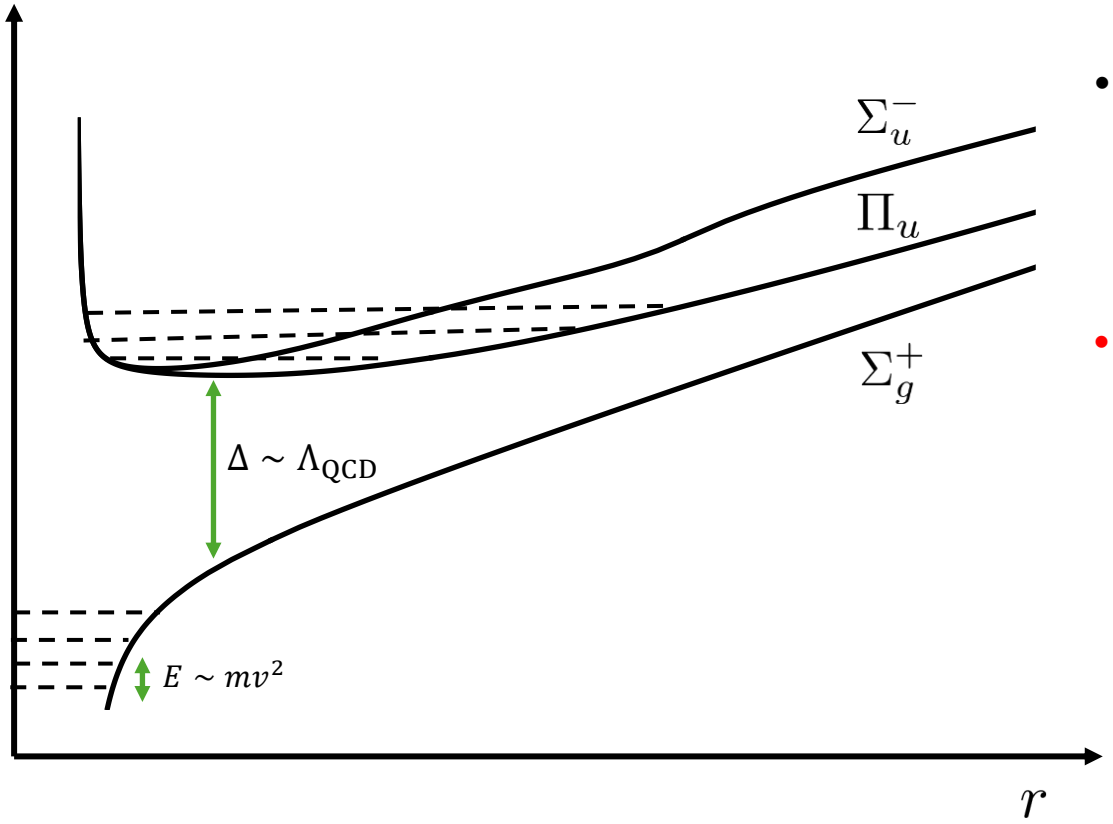
- BOEFT can address all these states with required inputs from Lattice QCD !!

BOEFT

- BOEFT Lagrangian: $L_{\mathrm{BOEFT}} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{Q\bar{Q}q\bar{q}} + L_{\mathrm{mixing}} + \cdots$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Soto & Castella Phys. Rev. D. 102, (2020), 014012



- Gap of order Λ_{QCD} allows us to focus individually on low-lying states corresponding to quarkonium, hybrid, tetraquark etc.

- L_{mixing}: Mixing between different states with similar masses and same quantum-numbers.

Ex: Hybrid-quarkonium mixing, tetraquark-hybrid & tetraquark-quarkonium mixing etc.

R. Oncala, J. Soto, Phys. Rev. D96 014004 (2017)

Σ_g^+, Σ_u^-, Π_u: represent different potentials between heavy quark-antiquark

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- **BO potentials:** **Potential between heavy quark pair** due to LDF (light quarks, gluons).

Born-Oppenheimer (BO) potential:

$$V_{\kappa\lambda\lambda'}(r) = E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'} + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \dots ,$$

Static Energy

Spin-dependent potentials

LDF-quantum #:

$$\kappa = \{K^{PC}, f\}$$

BO-quantum #:

$$\Lambda^{\sigma}_{\eta} \quad \lambda = \pm\Lambda$$

Brambilla, Lai, Segovia, Castellà, Phys. Rev. D. 101, (2020)

Castellà , Soto Phys. Rev. D. 102, (2020)

Soto, Valls, Phys. Rev. D 108 (2023)

- Radial Schrödinger equation:

$$\sum_{\lambda}\left[-\frac{1}{m_Q r^2}\partial_r\, r^2 \partial_r + \frac{1}{m_Q r^2} M_{\lambda'\lambda} + E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'}\right]\psi^{(N)}_{\kappa\lambda}(r) = \mathcal{E}_N\,\psi^{(N)}_{\kappa\lambda'}(r)$$

Mixing term $M_{\lambda'\lambda}$: Mixing static potentials with different BO-quantum numbers Λ^{σ}_{η} corresponding to same LDF-quantum # κ

- BO potentials are inputs into Schroedinger equations.

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

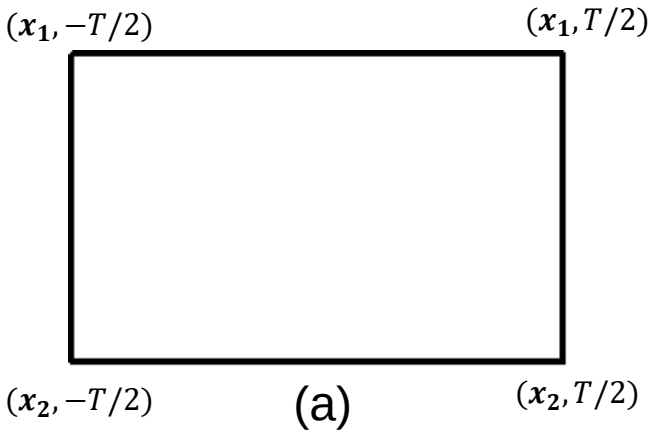
NRQCD operator (gauge invariant) for exotic hadron $Q\bar{Q}X$ or QQX :

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^{\dagger}(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\mathbf{0})\,P^{\alpha\dagger}_{\kappa,\lambda}H^{\alpha}_{\kappa}(t,\mathbf{0})\,\phi(t;\mathbf{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

H^{α}_{κ} : LDF (gluon or light-quarks) operator characterizing X based on quantum # κ (isospin, color etc..)

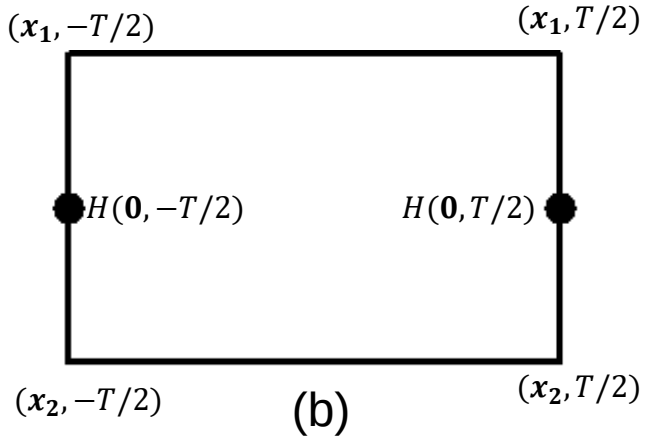
$P^{\alpha}_{\kappa,\lambda}$: Projection vectors for projecting onto cyclindrical symmetry $D_{\infty h}$ representations.

$$E^{(0)}_{\kappa,|\lambda|}(r) = \lim_{T\to\infty}\frac{i}{T}\log\left[\langle \mathrm{vac}|\mathcal{O}_{\kappa,\lambda}(T/2,\,\boldsymbol{r},\,\boldsymbol{R})\,\mathcal{O}^{\dagger}_{\kappa,\lambda}(-T/2,\,\boldsymbol{r},\,\boldsymbol{R})|\mathrm{vac}\rangle\right]$$



(a)

Quarkonium



(b)

Wilson loop for exotics

Examples of H^{α}_{κ}

Quarkonium tetraquarks $\mathbf{Q\bar{Q}q\bar{q}}$ **(I=0)**

Λ^{σ}_{η}	k^{PC}	Representation	Operator Examples $H^{\alpha,a}_{8,\kappa}\,(I=0)$	Projectors $P^{\alpha}_{\kappa\lambda}$
Σ_g^+	0^{++}	scalar	$\bar{q}T^a q$	1
Σ_u^-	0^{-+}	pseudoscalar	$\bar{q}\gamma^5 T^a q$	1
$\{\Sigma_g^+, \Pi_g\}$	1^{--}	vector	$\bar{q}\,\boldsymbol{\gamma}^i\, T^a q$	$\{\hat{r}^i, \hat{r}^i_{\pm}\}$
$\{\Sigma_g^-, \Pi_g\}$	1^{++}	pseudovector	$\bar{q}\,\boldsymbol{\gamma}^i\,\gamma^5\, T^a q$	$\{\hat{r}^i, \hat{r}^i_{\pm}\}$
$\{\Sigma_u^-, \Pi_u\}$	1^{+-}	pseudovector	$\bar{q}\,(\boldsymbol{\gamma}\times\boldsymbol{\gamma})^i\,\gamma^5\, T^a q$	$\{\hat{r}^i, \hat{r}^i_{\pm}\}$

I=1 operator: Insert $e_{I_3}\cdot\tau$ between light quarks

Doubly heavy tetraquarks $\mathbf{QQ\bar{q}\bar{q}}$ **(I=0)**

Λ^{σ}_{η}	k^{P}	Representation	Operator Examples $H^{\alpha,\ell}_{3,\kappa}\,(I=0)$	Operator Examples $H^{\alpha,\sigma}_{\bar{6},\kappa}\,(I=0)$	Projectors $P^{\alpha}_{\kappa\lambda}$
Σ_g^+	0^+	scalar	$\bar{q}\gamma^5\gamma^2\tau^2\,\underline{T}^a\,q^*$	–	1
Σ_u^-	0^-	pseudoscalar	–	$\bar{q}\gamma^2\tau^2\,\underline{\Sigma}^a\,q^*$	1
$\{\Sigma_u^+, \Pi_u\}$	1^-	vector	$\bar{q}\boldsymbol{\gamma}^i\,\gamma^5\gamma^2\tau^2\,\underline{T}^a\,q^*$	$\bar{q}\boldsymbol{\gamma}^i\,\gamma^5\gamma^2\tau^2\,\underline{\Sigma}^a\,q^*$	$\{\hat{r}^i, \hat{r}^i_{\pm}\}$
$\{\Sigma_g^-, \Pi_g\}$	1^+	pseudovector	–	$\bar{q}\boldsymbol{\gamma}^i\,\gamma^2\tau^2\,\underline{\Sigma}^a\,q^*$	$\{\hat{r}^i, \hat{r}^i_{\pm}\}$

Good diquark

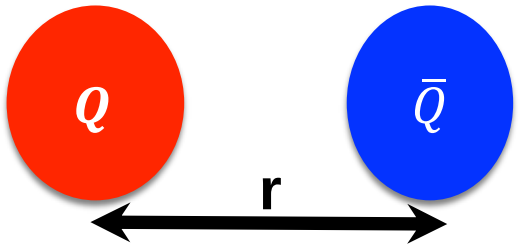
Also have operators for pentaquarks $\mathbf{Q\bar{Q}qqq}$, $\mathbf{QQqq\bar{q}}$

BOEFT: Potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- Exotic hadron ($\boldsymbol{Q\bar{Q}X}, \boldsymbol{QQX}, \ldots\ldots$), $\boldsymbol{X}$ is light d.o.f.

:



spin: $1/2 \otimes 1/2 = 0 \oplus 1$

color: $3 \otimes \bar{3} = 1 \oplus 8$

- EFT matching for operators characterizing $\boldsymbol{Q\bar{Q}X}$:

$$\mathcal{O}_{\kappa,\lambda}(t,\,\mathbf{r},\,\mathbf{R}) \longrightarrow \mathbf{Z}_{\mathbf{H}_\kappa}(\mathbf{r})\mathbf{O}^{\mathbf{a}}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})\mathbf{P}^{\alpha\dagger}_{\kappa,\lambda}\mathbf{H}^{\mathbf{a},\alpha}_{\kappa}(\mathbf{t},\,\mathbf{R}) \longrightarrow \mathbf{Z}_{\mathbf{\Psi}_\kappa}(\mathbf{r},\,\mathbf{\Lambda}_{\mathrm{QCD}})\mathbf{\Psi}_{\kappa,\lambda}(\mathbf{t},\,\mathbf{r},\,\mathbf{R})$$

NRQCD — Multipole expansion — BOEFT

$\mathbf{O^a}$**: Octet field**

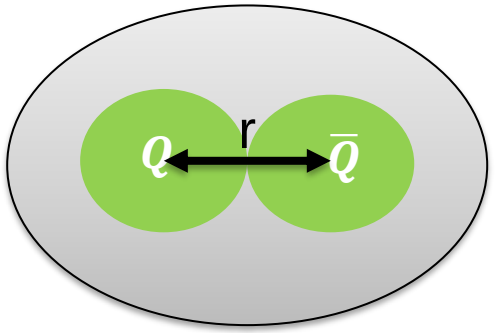
Only short-distance matching $(r \to 0)$

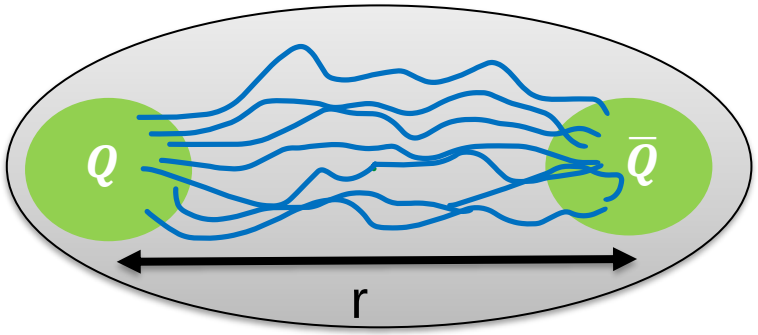
BOEFT: Potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

LDF-quantum #: $\kappa = \{K^{PC}, f\}$

BO-quantum #: Λ_η^σ





Short-distance $(r \to 0)$

Large-distance $(r \to \infty)$

$Q\bar{Q}$: $E^{(0)}_{\Sigma_g^+}(r) = V_s(r) + b_{\Sigma_g^+} r^2 + \cdots$

$Q\bar{Q}X$: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_o(r) + \Lambda_{H_\kappa} + b_{\Lambda_\eta^\sigma} r^2 + \cdots$

QQX: $E^{(0)}_{\Lambda_\eta^\sigma}(r) = V_l(r) + \Lambda_{H_\kappa,l} + b_{\kappa\lambda,l}\, r^2 + \cdots \; (l = T, \Sigma)$

$$V_s(r) = -\frac{4\alpha_s}{3r},\; V_o(r) = \frac{\alpha_s}{6r}$$
$$V_T(r) = -\frac{2\alpha_s}{3r},\; V_\Sigma(r) = \frac{\alpha_s}{3r}$$

- String behavior (**pure SU(3) gauge**)

$$E_N(r) = \sqrt{\sigma^2 r^2 + 2\pi\sigma\,(N - 1/12)}$$

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

- Mixing with pair of heavy-light states based on BO-quantum number Λ_η^σ representations

BOEFT: Potentials

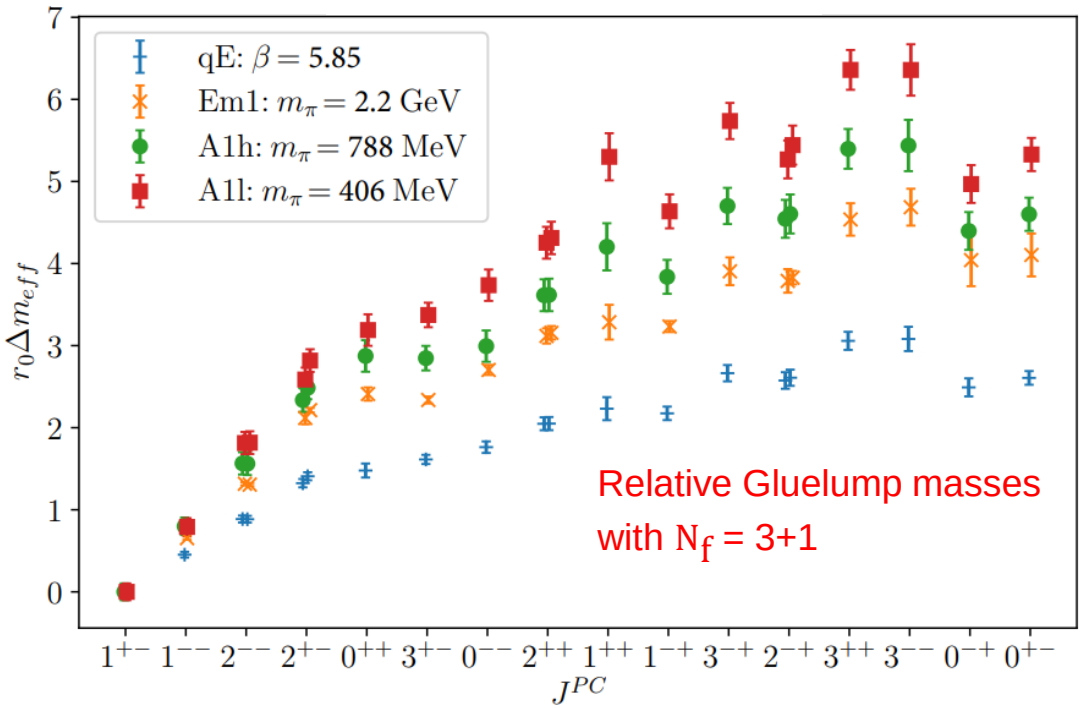
Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$$\Lambda_{H_\kappa} = \lim_{T\to\infty} \frac{i}{T} \langle \mathrm{vac}| H^a_\kappa(T/2,\, \boldsymbol{R})\, \phi^{ab}\,(T/2, -T/2)\, H^{a\dagger}_\kappa(-T/2,\, \boldsymbol{R})|\mathrm{vac}\rangle$$

Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

Campbell, Jorysz, Michael Phys. Lett. B 167 (1986)

- Gluelump / adjoint meson or baryon mass for $Q\bar{Q}X$ states
- Triplet meson or baryon / Sextet meson or baryon mass for QQX states



Höllwieser, Knechtli, Korzec, Peardon, Struckmeier, Andres Urea-Nino
PoS LATTICE2024 (2025) 102

- Adjoint meson (1^{--} & 0^{-+}) : quenched with valence quarks

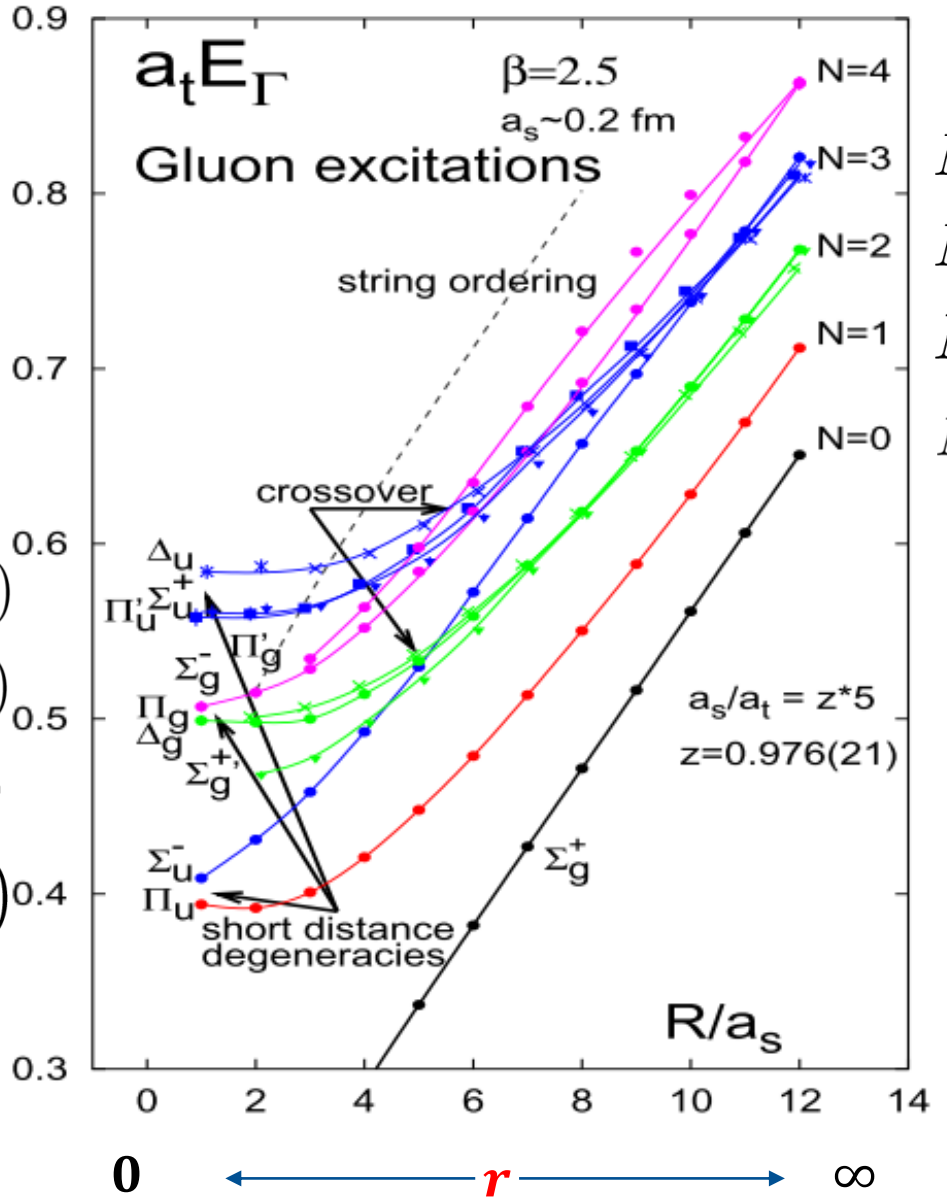
Foster, Michael (UKQCD) Phys. Rev. D 59 (1999)

$$m_A\left(1^{--}\right) - m_G\left(1^{+-}\right) = -10(103)\ \mathrm{MeV}$$
$$m_A\left(0^{-+}\right) - m_G\left(1^{+-}\right) = 34(161)\ \mathrm{MeV}$$

New results on **isospin=1** adjoint mesons**: See Sipaz Sharma talk on Wed at 11:30 !**

- In context of **QQX states**, emergence of **diquark**: BOEFT operator for triplet meson coincides with good diquark but directly gauge-invariant !

BO potential: Quenched



$$N = 3\ (\Sigma_u^-, \Sigma_u^+, \Pi_u', \Delta_u, \cdots)$$
$$N = 2\ (\Sigma_g^{+\prime}, \Pi_g, \Delta_g)$$
$$N = 1\ (\Pi_u)$$
$$N = 0\ (\Sigma_g^+)$$

Λ_η^σ corresponding to **gluelump** quantum # K^{PC}

$$2^{+-}\ (\Sigma_u^+, \Pi_u', \Delta_u)$$
$$2^{--}\ (\Delta_g, \Sigma_g^-, \Pi_g')$$
$$1^{--}\ (\Sigma_g^{+\prime}, \Pi_g)$$
$$1^{+-}\ (\Pi_u, \Sigma_u^-)$$

Observation:

BO-quantum # Λ_η^σ **conserved** at all values of r

K. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. 90 (2003)

0 ⟷ r ⟷ ∞

BO potential: Unquenched

String breaking: quarkonium & meson-antimeson

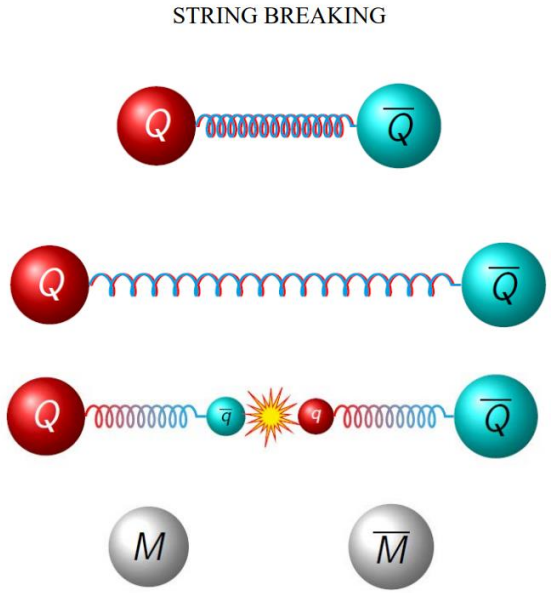
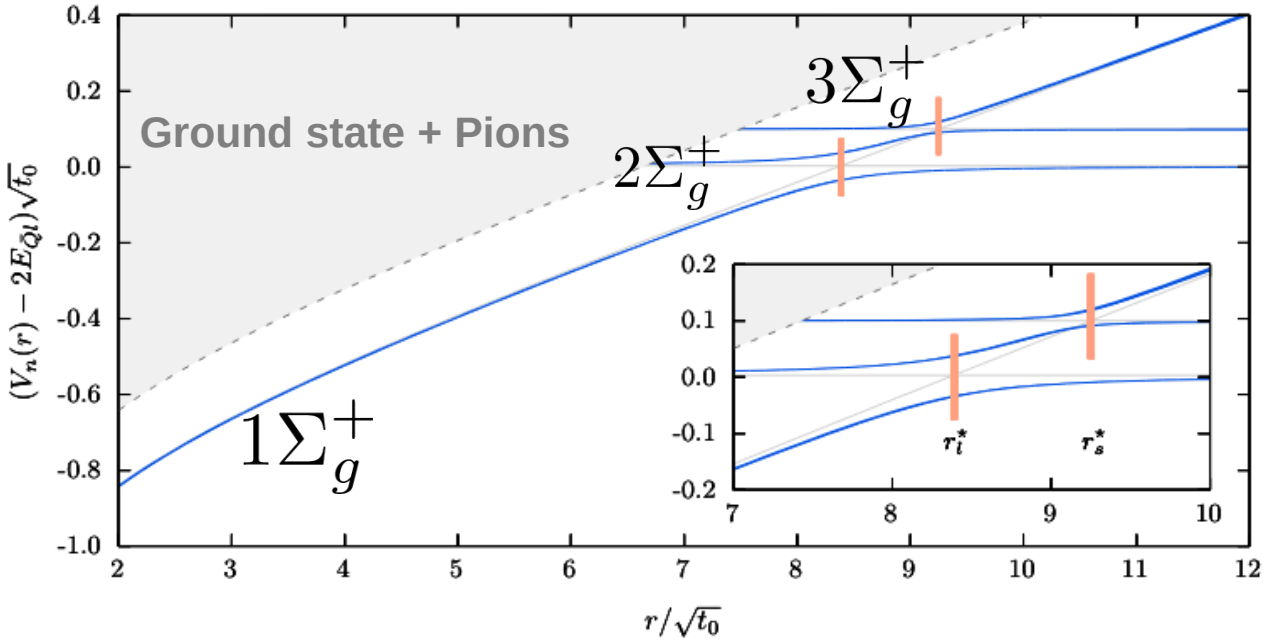


Figure from R. Bruschini talk

Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

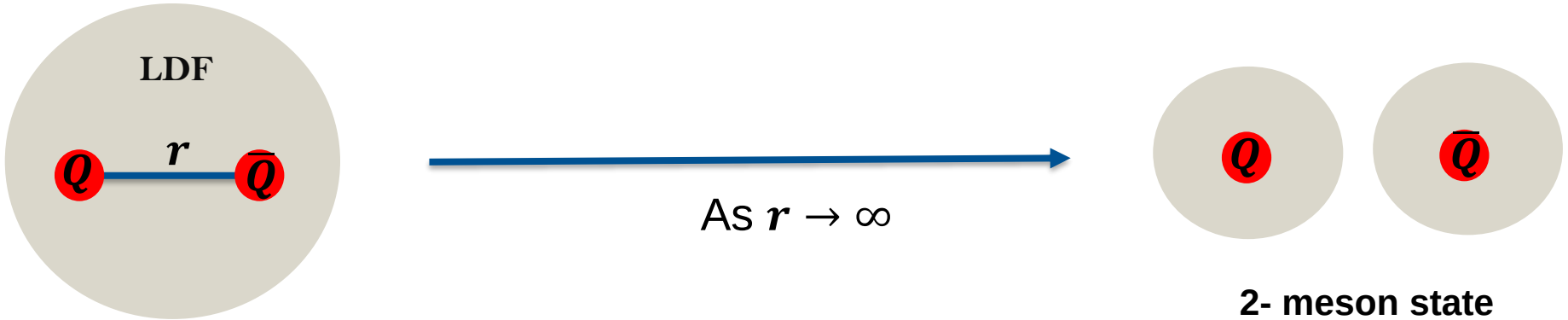
$m_\pi \approx 200 - 340$ MeV $\quad m_K \approx 440 - 480$ MeV



String breaking radius ≈ 1.22 fm

Avoided crossing between **degenerate** static potentials with **same** BO-quantum # Σ_g^+

BO potentials: Tetraquarks



2- meson state

Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ (color)	Light Spin K^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

BO-quantum # Λ_η^σ for meson-antimeson as $r \to \infty$

$K_q^P \otimes K_{\bar{q}}^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

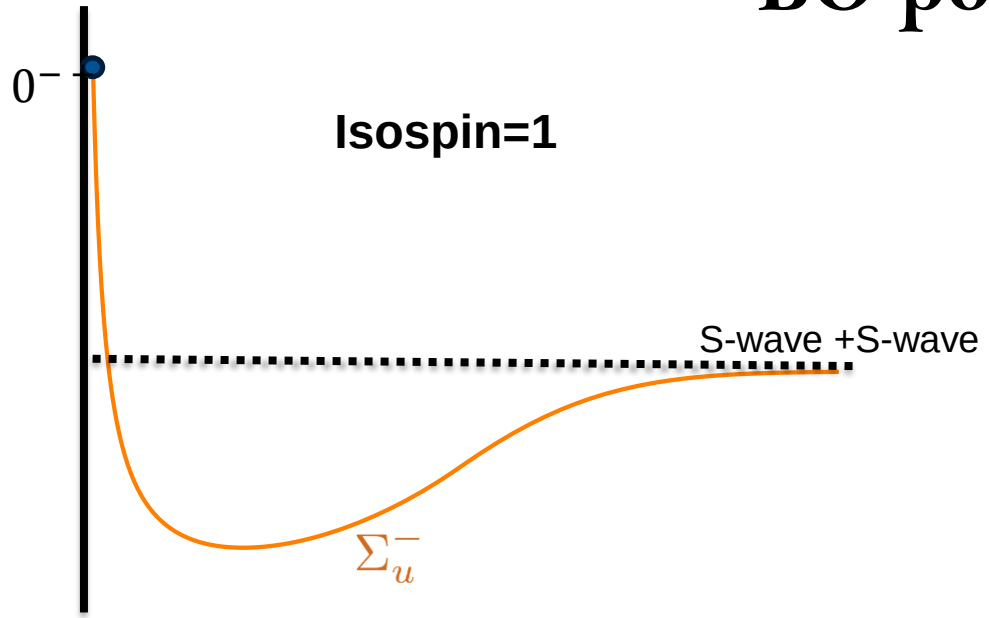
s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

Meson-antimeson threshold have same BO-quantum # Λ_η^σ !!!

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

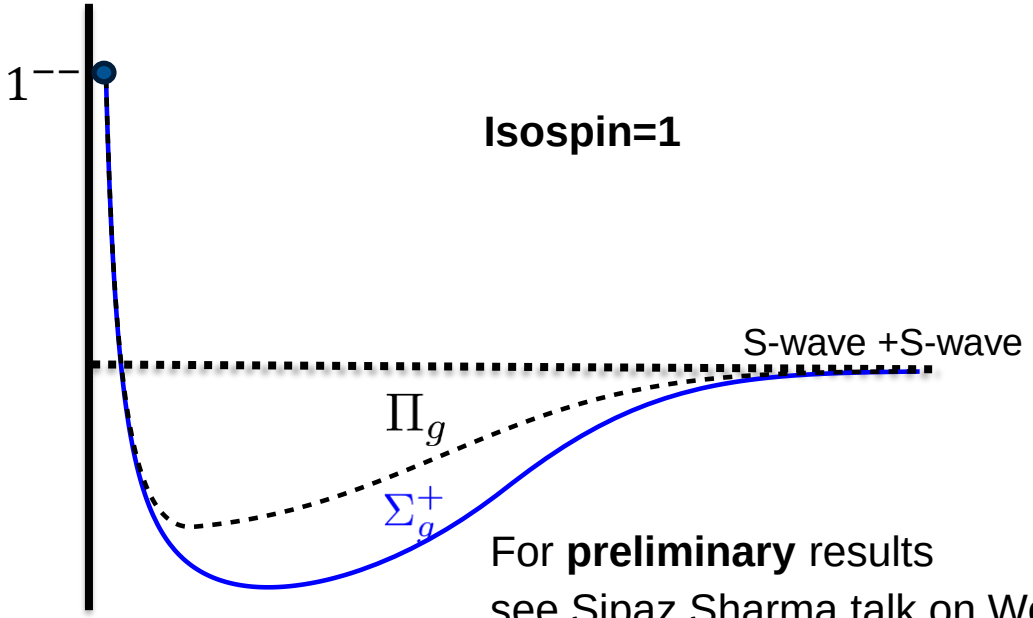
Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

BO potentials: Tetraquarks

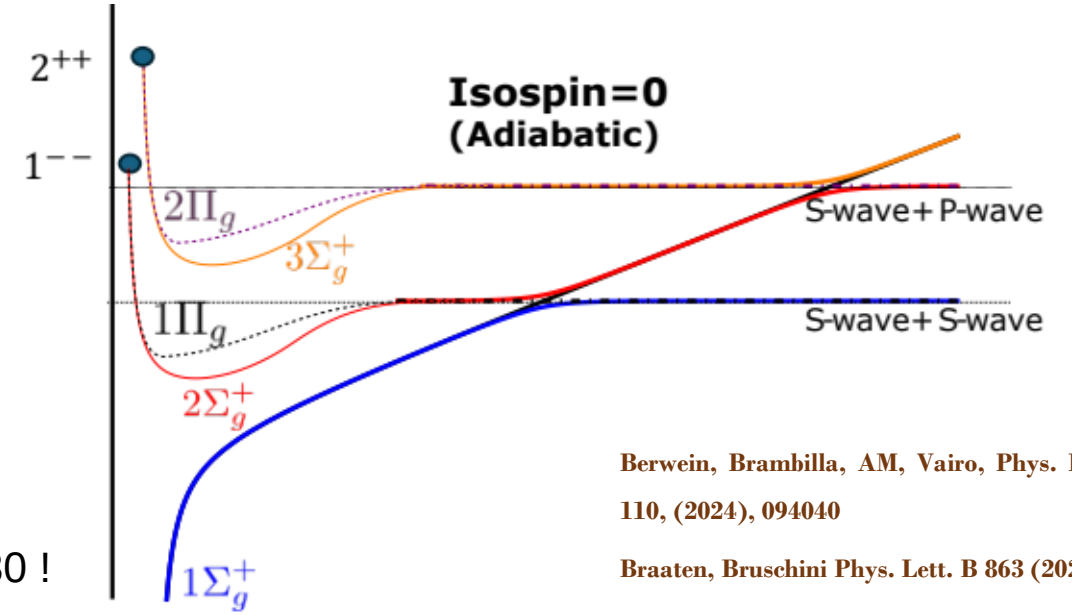


Key Takeaways: Tetraquark static potential behavior

- ❑ Repulsive behavior at small r $(r \to 0)$
- ❑ Heavy meson pair threshold at large r $(r \to \infty)$
- ❑ Avoided crossing with quarkonium static energy (Isopsin=0)



For **preliminary** results
see Sipaz Sharma talk on Wed 11:30 !



Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

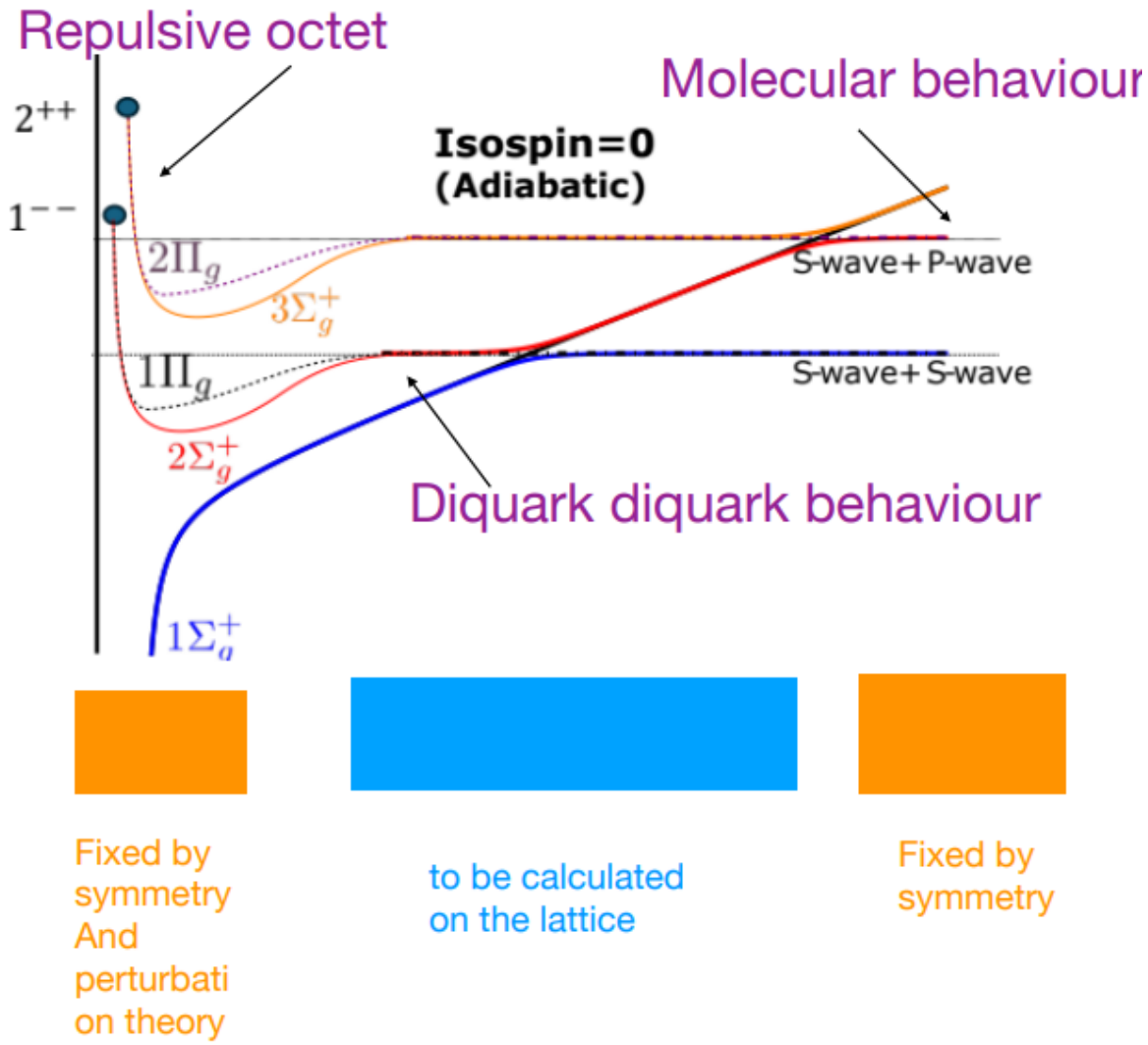
BO potentials

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

The BOEFT

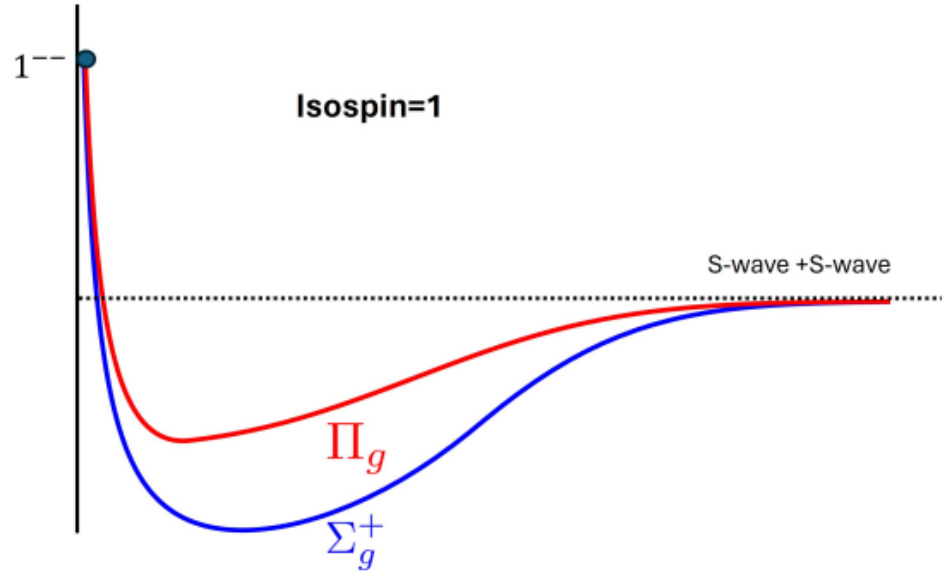
Behavior of tetraquark static energy:

- Adjoint meson behavior at small r $(r \to 0)$
- Heavy meson pair threshold at large r $(r \to \infty)$

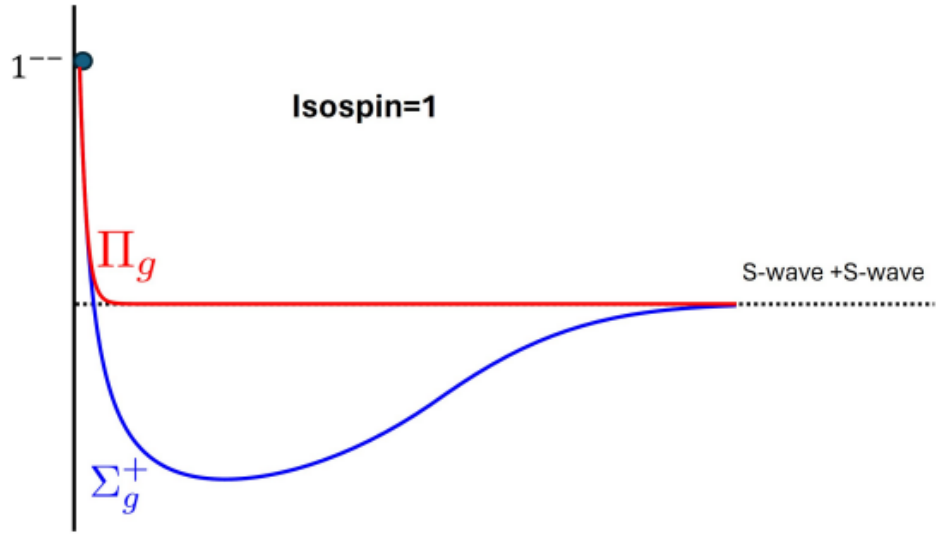


BOEFT Could subdue molecular and compact tetraquark pictures

Need lattice evaluation of static potentials:



Pig supports bound states



Pig does not support bound states

Intermediate region dictates if bound states are supported.

$Q\bar{Q}q\bar{q}$: Operator Overlap

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

NRQCD operator (gauge invariant) for exotic hadron: $\boldsymbol{Q\bar{Q}}$ pair in **octet** color

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^\dagger(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\boldsymbol{0})\,P^{\alpha\dagger}_{\kappa,\lambda}\,H^\alpha_\kappa(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

$$\mathbf{H}_{\boldsymbol{K}}(t,\boldsymbol{x}) = \left[\bar{q}(t,\boldsymbol{x})\tilde{\boldsymbol{\Gamma}}T^a q(t,\boldsymbol{x})\right]T^a$$

$\tilde{\Gamma}$: Dirac matrices based on quantum #'s

Quarkonium + Pions

Quarkonium state:

$$|\mathcal{Q}\rangle = \mathcal{N}\int d^3\boldsymbol{r}\,\Psi^{(n)}(\boldsymbol{r})\;\psi^\dagger_b(t,-\boldsymbol{r}/2)\,\phi_{bc}(t;-\boldsymbol{r}/2,\boldsymbol{r}/2)\,\chi_c(t,\boldsymbol{r}/2)\,|\Omega\rangle$$

Overlap of adjoint operator on quarkonium + pion:

$$\langle\mathcal{Q}|\mathcal{O}^{Q\bar{Q}}_K(t,\boldsymbol{r})\,|\mathcal{Q}\rangle = 0$$

(at leading order in multipole expansion)

Meson-antimeson

Meson-antimeson state:

$$\begin{aligned}|M\bar{M}\rangle = \Big[&\mathcal{N}\int d^3\boldsymbol{x}\,\Psi_J(\boldsymbol{x})\\ &\times\int d^3\boldsymbol{y}\,\varphi_{J_1}(\boldsymbol{y}+\boldsymbol{x}/2)\;\psi^\dagger_c(t,-\boldsymbol{x}/2)\,\phi_{cd}(t;-\boldsymbol{x}/2,\boldsymbol{y}+\boldsymbol{x}/2)\,[P_+\,\Gamma_1\,q_d(t,\boldsymbol{y}+\boldsymbol{x}/2)]\\ &\times\int d^3\boldsymbol{z}\,\varphi_{J_2}(\boldsymbol{z}-\boldsymbol{x}/2)\;[\bar{q}_b(t,\boldsymbol{z}-\boldsymbol{x}/2)\,\Gamma_2\,P_-]\,\phi_{be}(t;\boldsymbol{z}-\boldsymbol{x}/2,\boldsymbol{x}/2)\,\chi_e(t,\boldsymbol{x}/2)\Big]|\text{vac}\rangle\end{aligned}$$

Overlap of our operator on meson-antimeson:

$$\langle M\bar{M}|\mathcal{O}^{Q\bar{Q}}_K(t,\boldsymbol{r})\,|M\bar{M}\rangle \neq 0$$

Adjoint operators could be good operators for lattice computation for $Q\bar{Q}q\bar{q}$ potentials

$\chi_{c1}(3872)$ & $T_{cc}^{+}(3875)$

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

$Q\bar{Q}$ color state	$q\bar{q}$ spin K^{PC}	Static energies	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

Isospin-1 channel:
$Z_c(3900), Z_c(4200), Z_b(10610), Z_b(10650)$

Mixing b/w adjoint mesons 0^{-+} and 1^{--}

Voloshin, Phys. Rev. D. 93, 074011 (2016)

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

See Sipaz Sharma talk on Wed at 11:30 !

Isospin-0 channel:
$\chi_{c1}(3872), X_b$

Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902

BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,

Phys. Rev. D. 110, (2024), 094040

Isospin-0

$q\bar{q}$ spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S_Q=0, S_Q=1\}$
1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$
	$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$
	Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$
	$\Sigma_g^{+\prime}, \Pi_g$	2	$\{2^{-+}, (1,2,3)^{--}\}$

Lowest multiplet with $\chi_{c1}(3872)$ **$J^{PC}=1^{++}$**

1^{--} Adjoint meson mass unknown.
Fix it to reproduce $\chi_{c1}(3872)$.

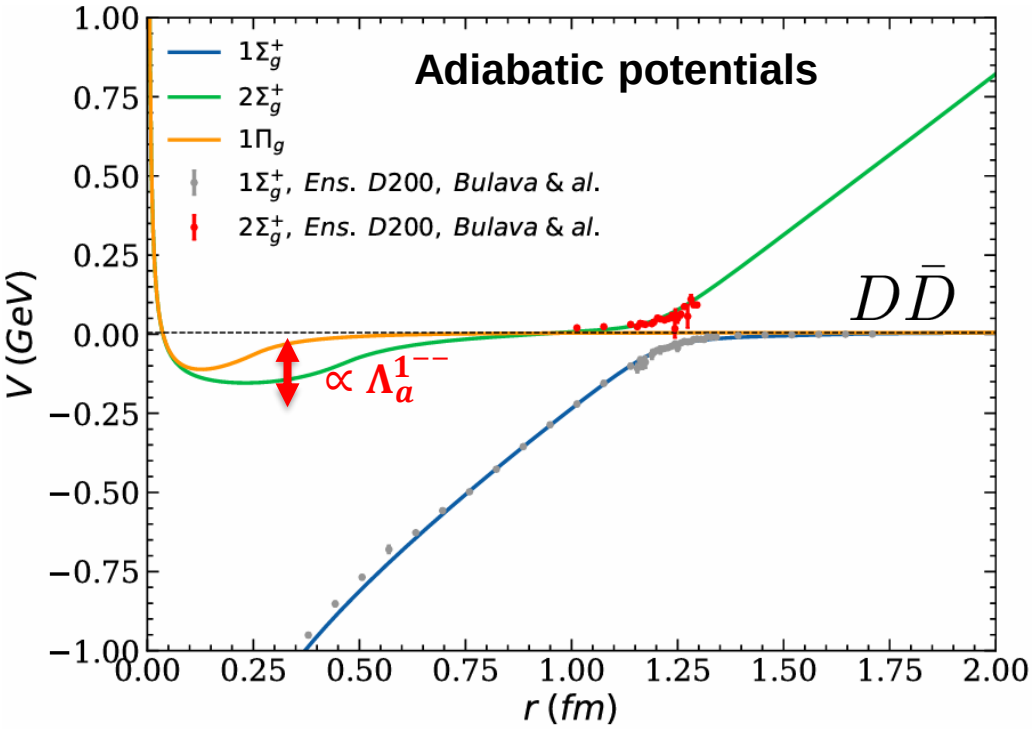
Coupled-channel Equations:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix}$$

Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902

$\chi_{c1}(3872)$



Bulava et al Phys. Lett. B. 854 (2024)

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

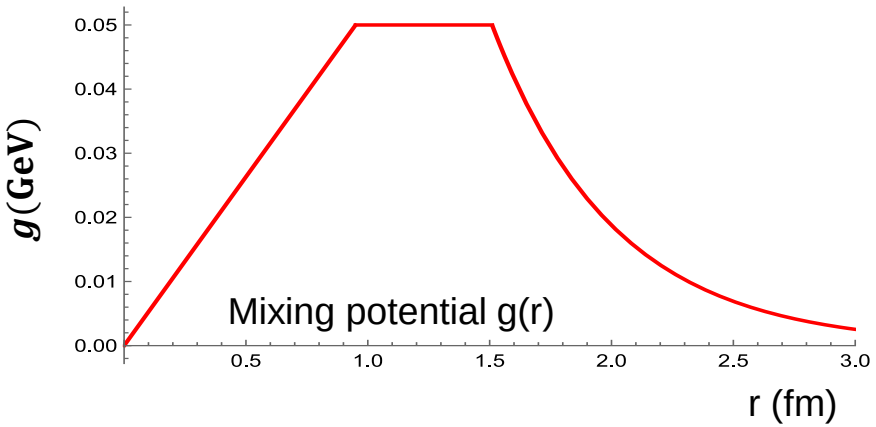
Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902

Coupled-channel Equations:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix} = \mathcal{E}\begin{pmatrix} \psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi \end{pmatrix}$$

$l = 1$

Adjoint $(q\bar{q})_8$: $\Lambda_a^{1^{--}} \approx 914$ MeV (pole mass scheme)



Results:

1) 2P Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius ~ 15 fm.
4) Deeper state in bottom sector: 15 MeV below spin-isospin averaged $B\bar{B}$ threshold.

Adjoint $(q\bar{q})_8$ energy $\Lambda_a^{1^{--}}$ can be confirmed by lattice QCD !!

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo,
Phys. Rev. D. 110, (2024), 094040

light antiquarks

q q'

{qq'}, 1^+ [qq'], 0^+

"Bad diquark" "Good diquark"

Defines the Born-Oppenheimer static potentials $\boldsymbol{\Sigma_g^+}, \{\boldsymbol{\Sigma_g^-}, \boldsymbol{\Pi_g}\}$

Bad diquark – Good diquark
$\approx$ 200 MeV

QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^{PC}	
					$S_Q = 0$	$S_Q = 1$
anti-triplet $\bar{\mathbf{3}}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$

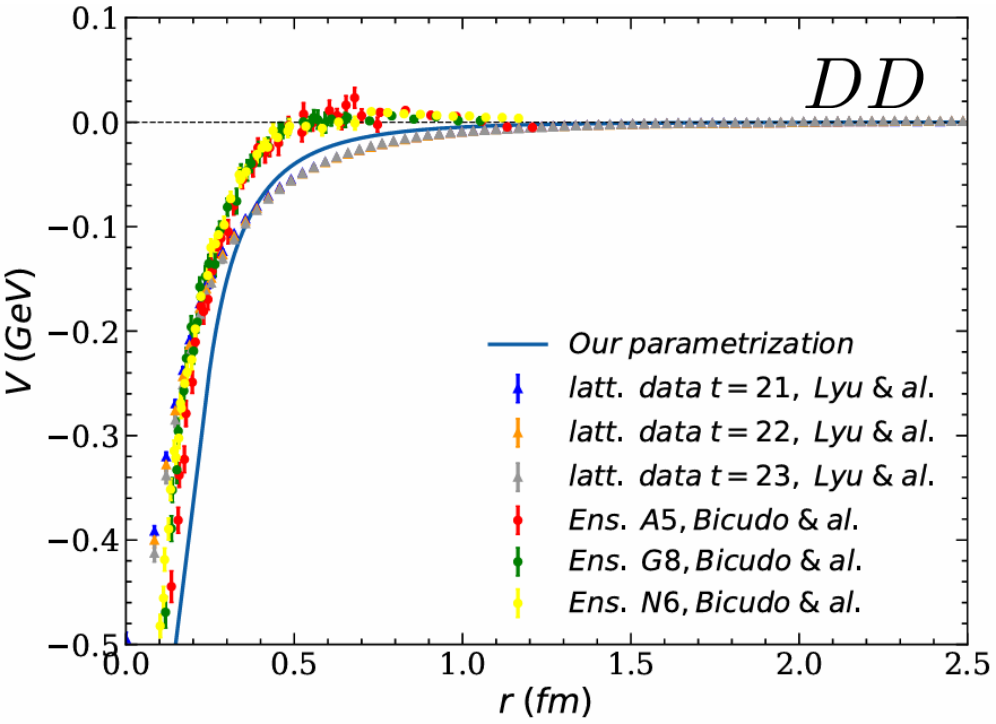
Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo Phys. Rev. Lett. 135 (2025), 131902

Schrödinger Equation:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\,\psi_{\Sigma_g^+}\,.$$

$l = 0$



Good diquark : $\Lambda_t^{0^+} \approx 664$ MeV (pole mass scheme)

Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng, Phys. Rev. Lett. 131, 161901 (2023)

Bicudo, Marinkovic, Mueller, Wagner, arXiv 2409.10786

Results:

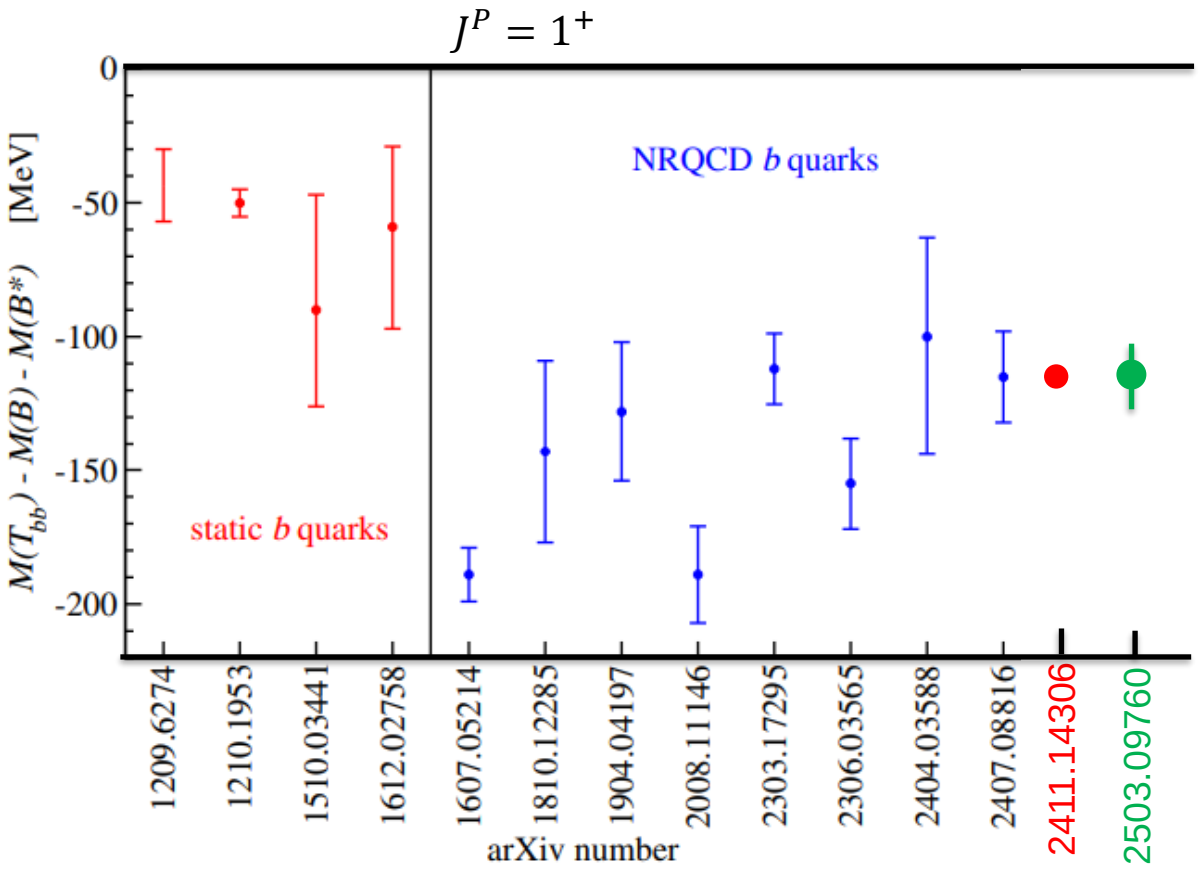
1) T_{cc} state : 320 keV below DD threshold
2) Radius ~ 8 fm or larger.
3) Deeper bound state in bb sector: T_{bb} 116 MeV below BB threshold.
4) Deeper bound state in bc sector: T_{bc} 25 MeV below DB threshold.

Good diquark energy $\Lambda_t^{0^+}$ can be confirmed by lattice QCD !!

T_{bb} & T_{bc}

Brambilla, AM, Scirpa, Vairo 2411.14306

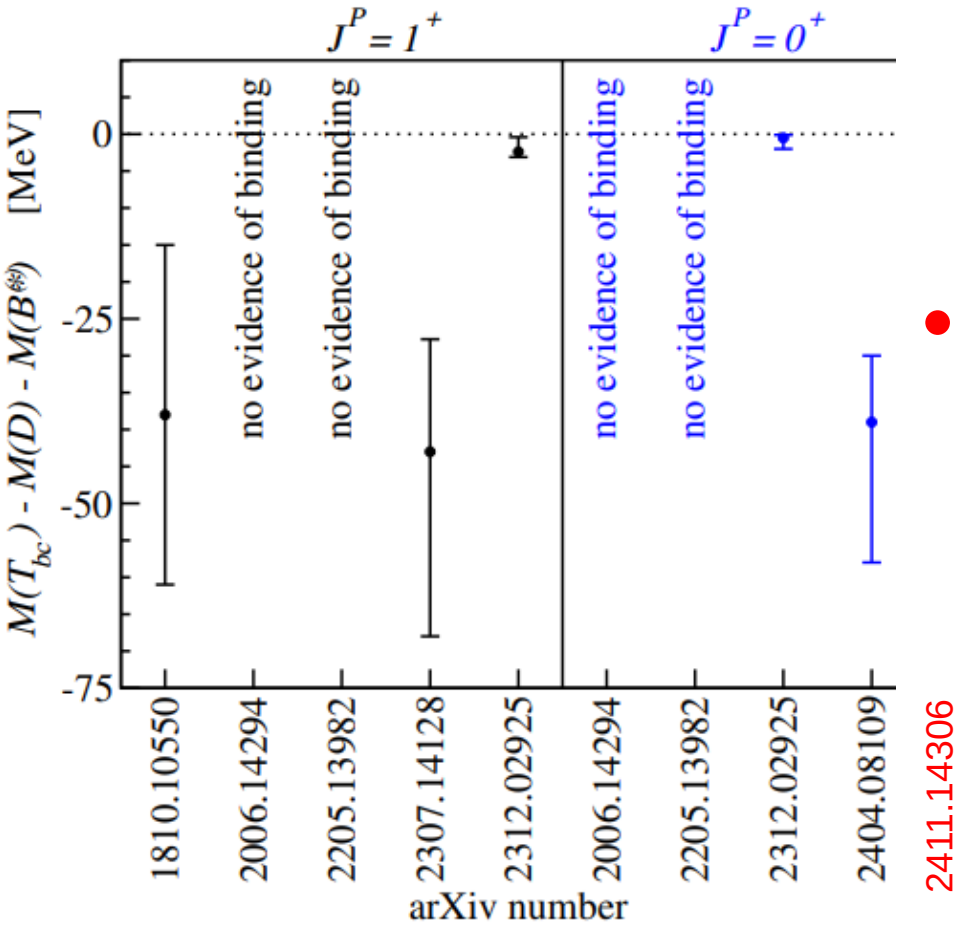
T_{bb} binding energy comparison:



EFT and heavy-quark-diquark symmetry prediction for T_{bb}: 133 ± 25 MeV

Braaten, He, AM, Phys. Rev. D. 103, 016001 (2021)

T_{bc} binding energy comparison:

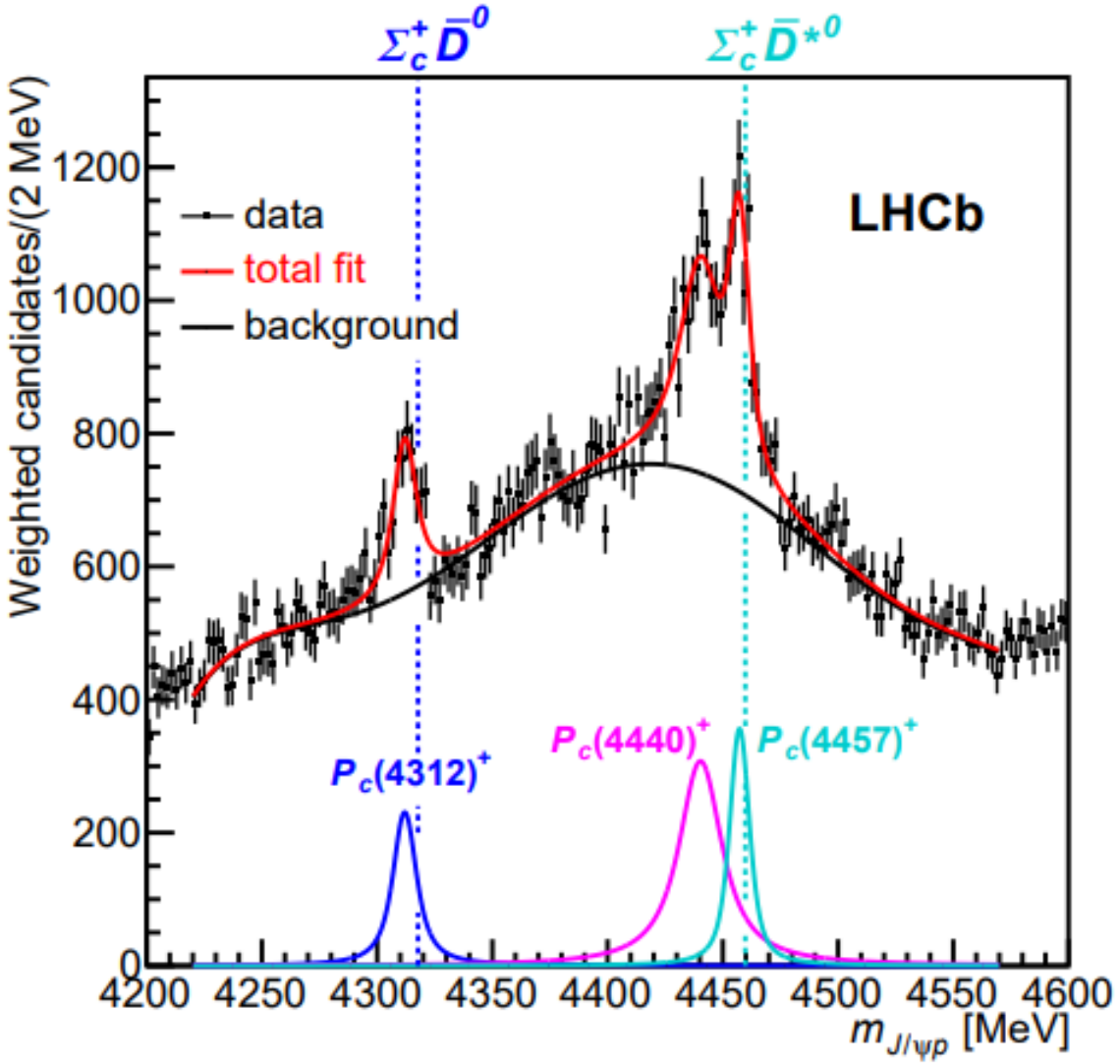


Our result 25 MeV for both $J^P = \{0^+, 1^+\}$

Figures from Randy Lewis talk: LepageFest 2024

Pentaquarks

Pentaquark



LHCb, Phys. Rev. Lett. 122, (2019), 222001

Observed states

- **4** states with isospin $I = 1/2$:

$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$

J^P quantum numbers not established

All pentaquark states seen by LHCb experiment

PDG 2025

Pentaquark

Brambilla, AM, Vairo, arXiv 2508.13050

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Lowest pentaquark multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q=0, S_Q=1\}$
Octet	$(1/2)^+$	$(1/2)_g$	1/2	$\{1/2^-, (1/2,3/2)^-\}$
8	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	3/2	$\{3/2^-, (1/2,3/2,5/2)^-\}$

No lattice QCD results on adjoint baryon mass: $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$

Treat $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$ as **free parameter** to fix on $P_{c\bar{c}}$ spectrum.

Coupled-channel Equations $k^P = (1/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + V_{(1/2)_g}\right]\psi^{(N)}_{(1/2)^+} = \mathcal{E}_{1/2}\,\psi^{(N)}_{(1/2)^+}$$

$l = 1/2$

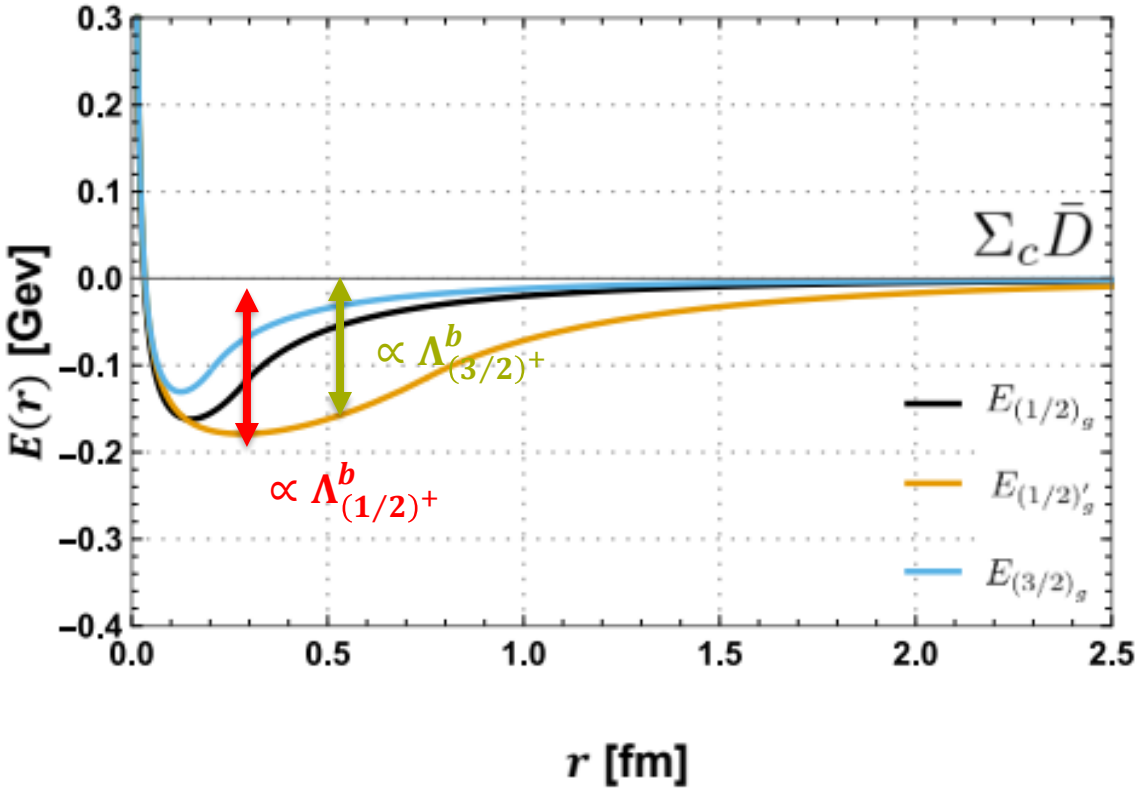
Coupled-channel Equations $k^P = (3/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)'_g} & 0 \\ 0 & V_{(3/2)_g}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2}\\ \psi^{(N)}_{3/2}\end{pmatrix} = \mathcal{E}_{3/2}\begin{pmatrix}\psi^{(N)}_{1/2}\\ \psi^{(N)}_{3/2}\end{pmatrix}$$

$l = 3/2$

Brambilla, AM, Vairo, arXiv 2508.13050

Pentaquark: $P_{c\bar{c}}$



Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx 1125$ MeV, $\Lambda^b_{(3/2)^+} \approx 1152$ MeV

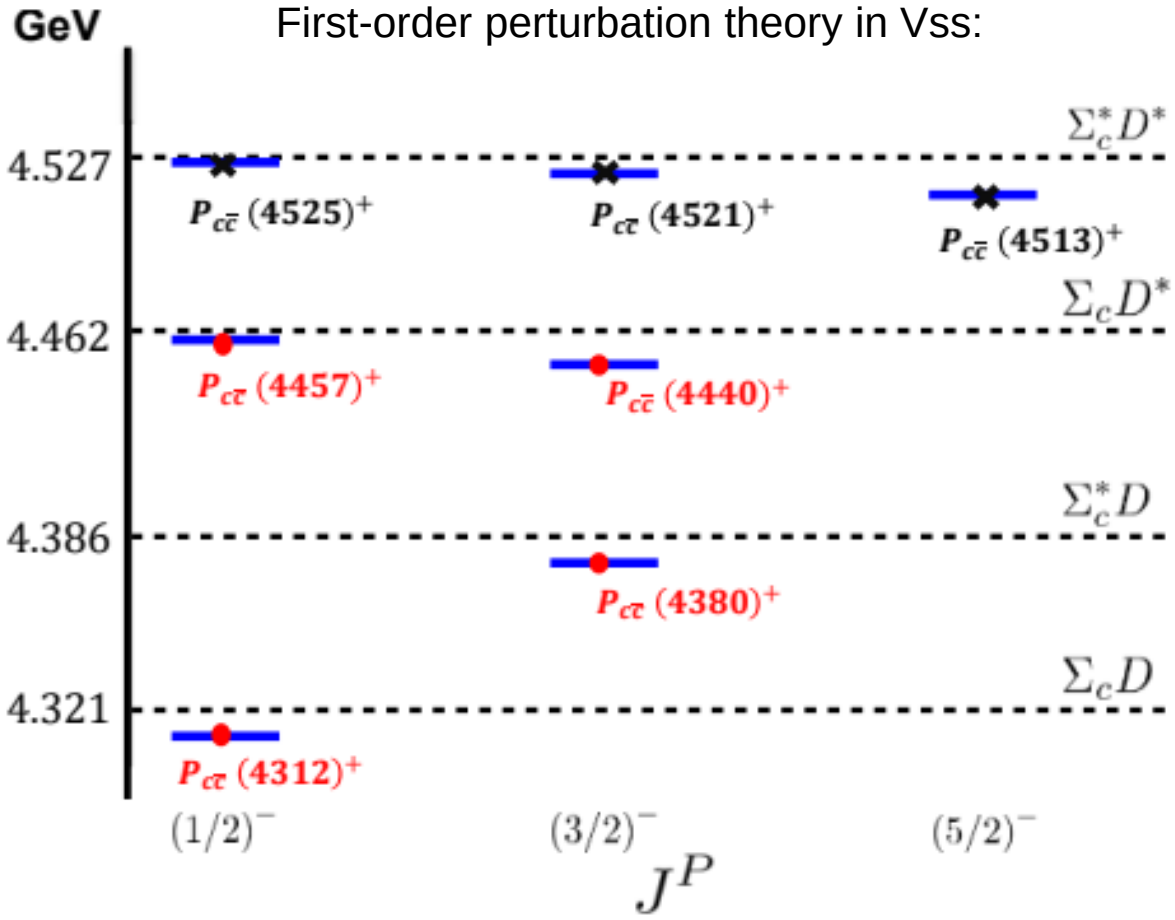
Compact pentaquark model: 10 states (some with parity = +)

Molecular model: 7 states

Spin-splitting in meson - baryon :

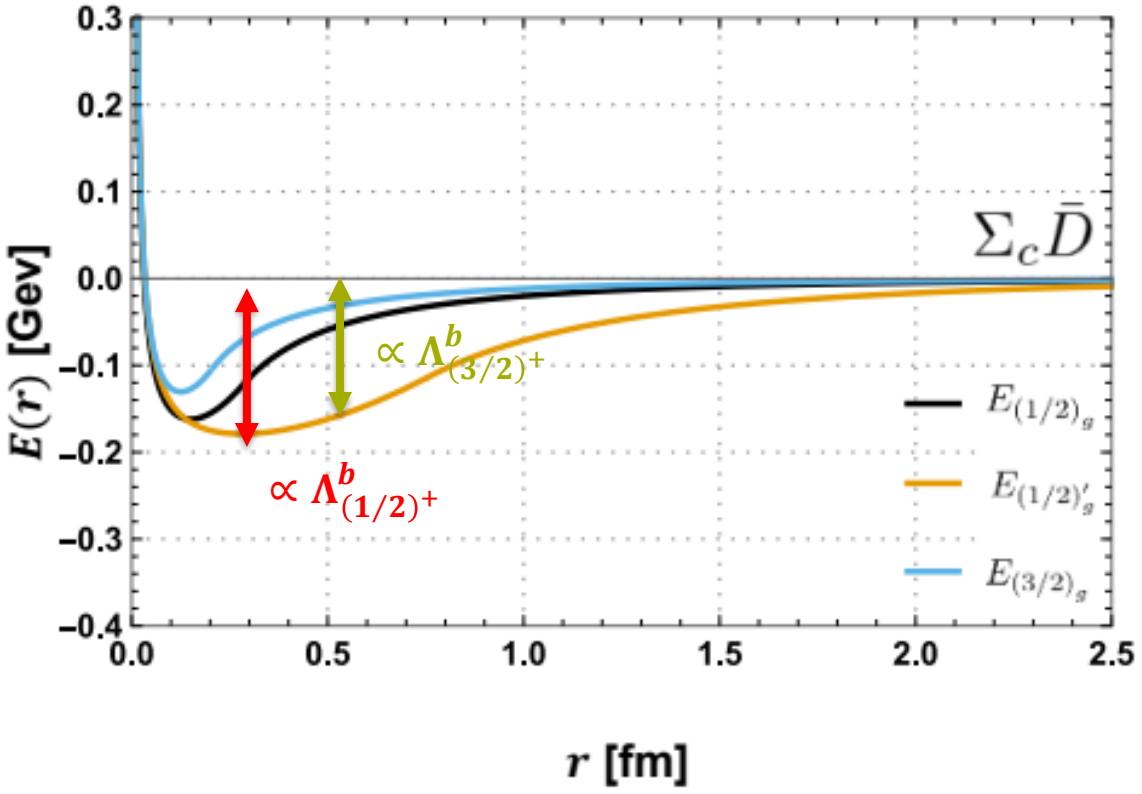
$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:



Brambilla, AM, Vairo, arXiv 2508.13050

Pentaquark: $P_{b\bar{b}}$



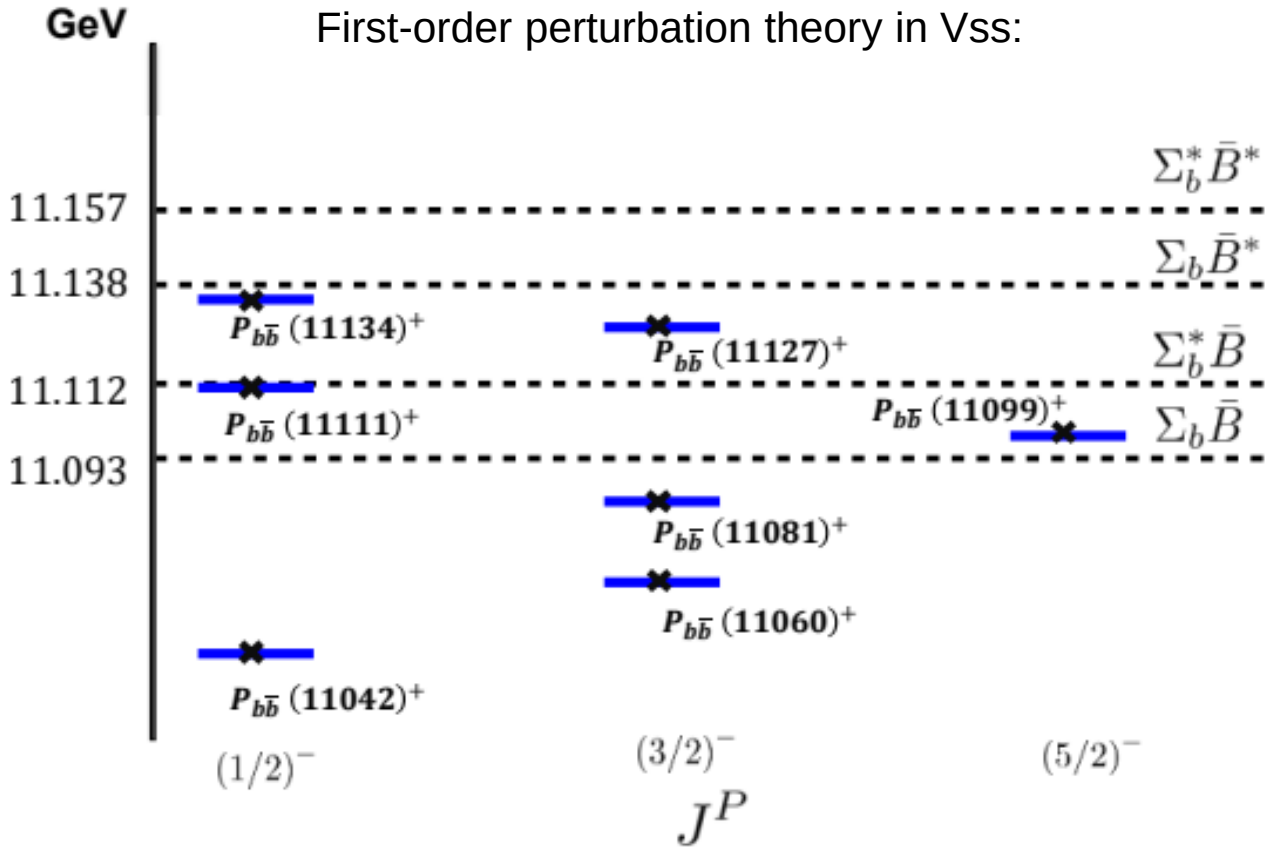
Adjoint baryon $(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx 1125$ MeV, $\Lambda^b_{(3/2)^+} \approx 1152$ MeV

Spin-splitting in meson - baryon :

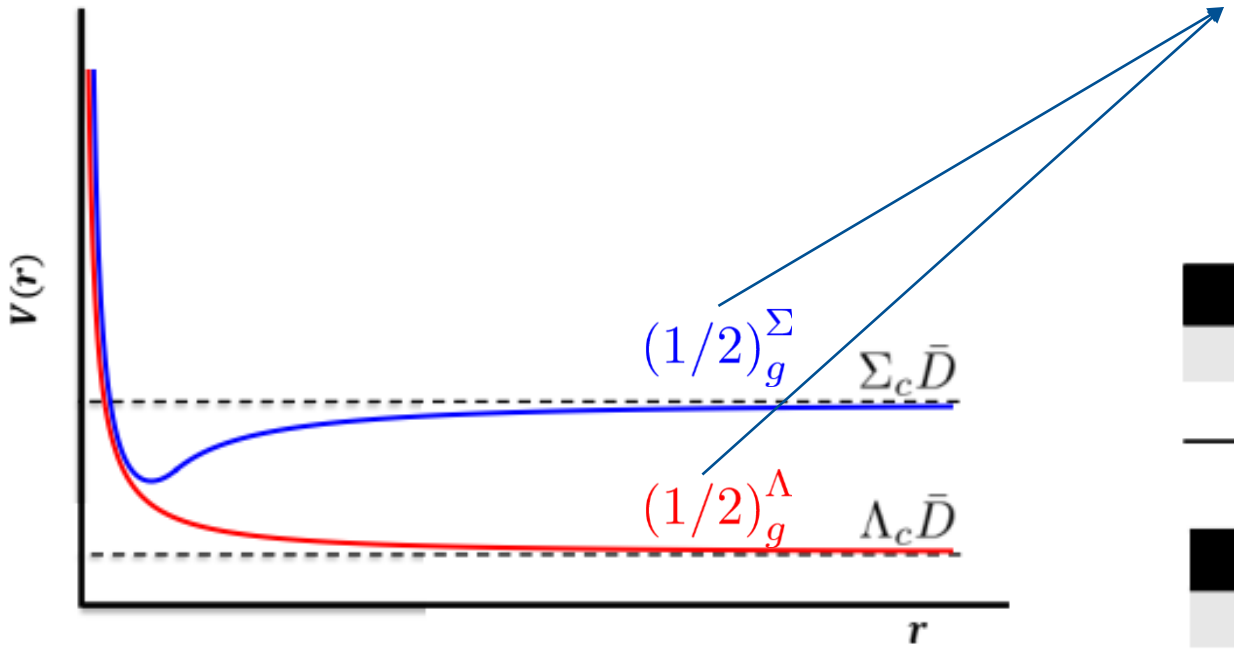
$$V_{SS} = \frac{2\Delta_1^Q}{3} \boldsymbol{S}_1 \cdot \boldsymbol{K}_1 + \Delta_2^Q \boldsymbol{S}_2 \cdot \boldsymbol{K}_2$$

First-order perturbation theory in Vss:



Pentaquark: Decays

❑ **Ratio of decays** to $\Lambda_c - \bar{D}$ and $\Lambda_c - \bar{D}^*$: $\frac{\Gamma(P_{c\bar{c}} \to \Lambda_c - \bar{D})}{\Gamma(P_{c\bar{c}} \to \Lambda_c - \bar{D}^*)}$



➢ Decay happens through **transition amplitude** $g_{(1/2)}^{\Sigma-\Lambda}$

➢ $g_{(1/2)}^{\Sigma-\Lambda}$ currently unknown can be **determined by lattice QCD**

$J^P = (1/2)^-$

	$P_{c\bar{c}}(4312)^+$	$P_{c\bar{c}}(4440)^+$	$P_{c\bar{c}}(4507)^+$
$\Lambda_c \bar{D}$	0	3.52	5.48
$\Lambda_c \bar{D}^*$	3.59	3.24	2.17

$J^P = (3/2)^-$

	$P_{c\bar{c}}(4380)^+$	$P_{c\bar{c}}(4457)^+$	$P_{c\bar{c}}(4515)^+$
$\Lambda_c \bar{D}$	0	0	0
$\Lambda_c \bar{D}^*$	3.74	1.17	4.09

$J^P = (5/2)^-$

	$P_{c\bar{c}}(4526)^+$
$\Lambda_c \bar{D}$	0
$\Lambda_c \bar{D}^*$	0

➢ $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ decays to $\Lambda_c - \bar{D}$ and $\Lambda_c - \bar{D}^*$ in d-wave !!!

Summary

- Born-Oppenheimer EFT (BOEFT):
  - ❑ Theoretical framework based on scale separation, symmetries with some inputs from lattice QCD.
  - ❑ Quarkonium and Exotic XYZ mesons can be described in the same framework.
- Tetraquark / pentaquark static potentials features based on BOEFT:
  - ❑ $Q\bar{Q}$ **systems: Repulsive** at small r **($r \to 0$).** Avoided crossing quarkonium static energy (isospin=0).
  - ❑ QQ **systems: Attractive (**Triplet) or repulsive (sextet) at small r **($r \to 0$).**
  - ❑ Heavy meson pair or heavy meson baryon threshold at large r **($r \to \infty$).**
  - ❑ $Q\bar{Q}$ systems: Avoided crossing between tetraquark and quarkonium static energy (Isospin=0).
- Inputs required from lattice QCD: adjoint meson or baryon spectrum, triplet & sextet meson or baryon spectrum, tetraquark & pentaquark static potentials.
- Exotic dynamics governed by Schrödinger equations. Universal non-perturbative parameters same for both charm and bottom.
- **BOEFT + lattice QCD**, enable quantitative predictions of spectra, decays, and even production rates of exotics, offering key insights into their underlying nature.

What is an XYZ Meson ???

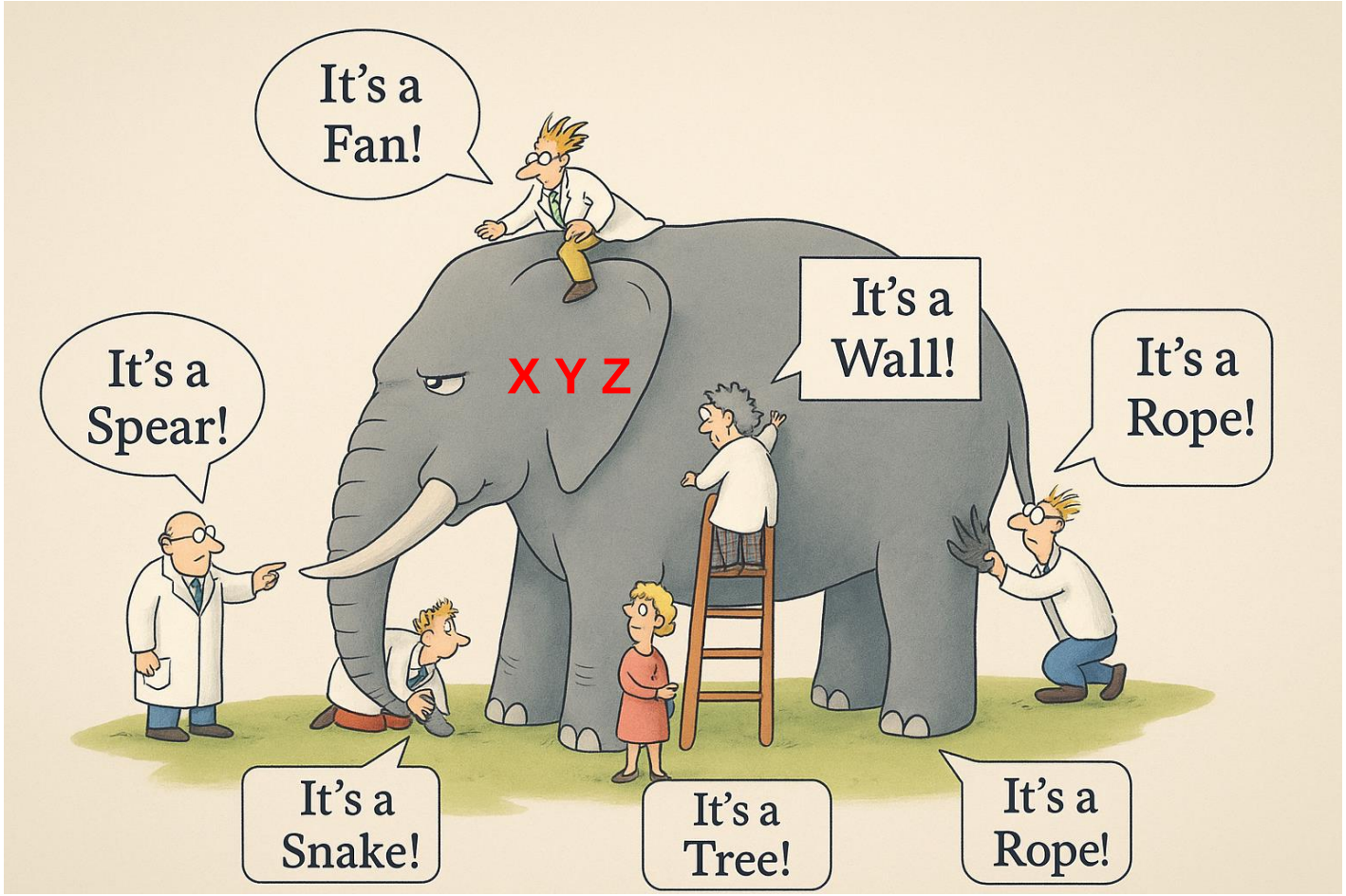


Figure from Eric Braaten talk: Charm 2020 conference and chatgpt

Thank you!!

Perhaps Born-Oppenheimer EFT + Lattice QCD is the key to unlock !!!

Backup Slides

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Coupled Equations for lowest Hybrids ($Q\bar{Q}g$) and Tetraquarks ($QQ\bar{q}\bar{q}$ or $Q\bar{Q}q\bar{q}$):

LDF quantum # K=1

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} \\ -2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma & 0 \\ 0 & E_\Pi\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_p} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + E_\Pi\right]\psi^{(N)}_{\Pi,-\sigma_P} = \mathcal{E}_N\,\psi^{(N)}_{\Pi,-\sigma_P}$$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92 (2015)

LDF quantum # K=2

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+6 & -2\sqrt{3l(l+1)} & 0 \\ -2\sqrt{3l(l+1)} & l(l+1)+4 & -2\sqrt{(l-1)(l+2)} \\ 0 & -2\sqrt{(l-1)(l+2)} & (l-1)(l+2)\end{pmatrix} + \begin{pmatrix} E_\Sigma & 0 & 0 \\ 0 & E_\Pi & 0 \\ 0 & 0 & E_\Delta\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_p} \\ \psi^{(N)}_{\Pi,\sigma_P} \\ \psi^{(N)}_{\Delta,\sigma_p}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P} \\ \psi^{(N)}_{\Delta,\sigma_p}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+4 & -2\sqrt{(l-1)(l+2)} \\ -2\sqrt{(l-1)(l+2)} & (l-1)(l+2)\end{pmatrix} + \begin{pmatrix} E_\Pi & 0 \\ 0 & E_\Delta\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Pi,-\sigma_P} \\ \psi^{(N)}_{\Delta,-\sigma_p}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Pi,-\sigma_P} \\ \psi^{(N)}_{\Delta,-\sigma_p}\end{pmatrix}$$

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Coupled Equations (**spin-averaged**) for Doubly Heavy Baryons (QQq) & Pentaquarks (QQq̄qq or QQ̄qqq):

LDF quantum # K=1/2

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + E_{K_\eta}\right]\psi^{(N)}_{K_\eta,\sigma_P} = \mathcal{E}_N\,\psi^{(N)}_{K_\eta,\sigma_P}$$

LDF quantum # K=3/2

parity σ_P:

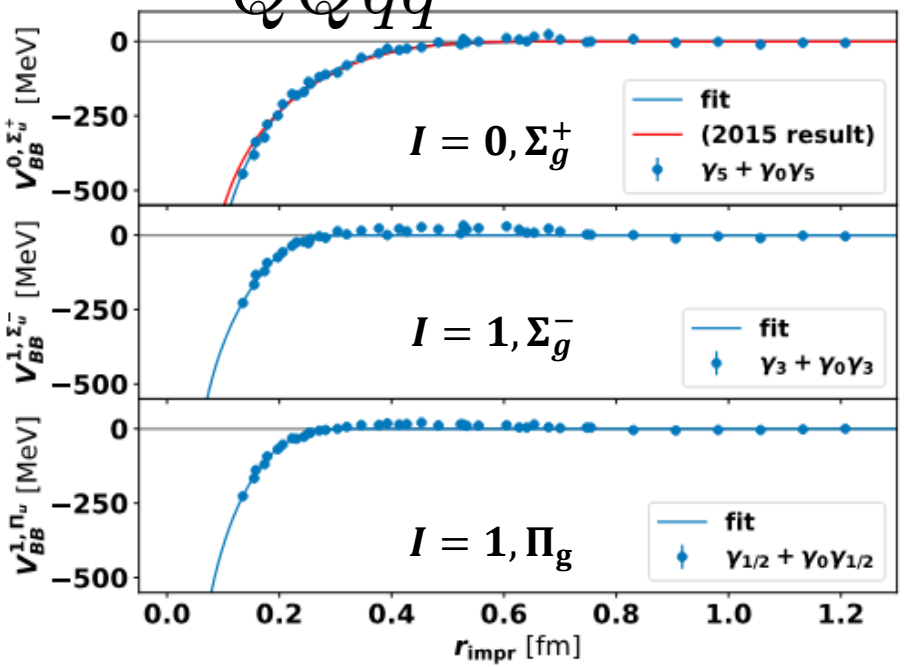
$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} E_{(1/2)_u} & 0 \\ 0 & E_{(3/2)_u}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P}\\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix} = \mathcal{E}_n\begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P}\\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix}$$

Opposite parity $-\sigma_P$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+3)+\frac{17}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} E_{(1/2)_u} & 0 \\ 0 & E_{(3/2)_u}(r)\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P}\\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix} = \mathcal{E}_n\begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P}\\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix}$$

BO potentials

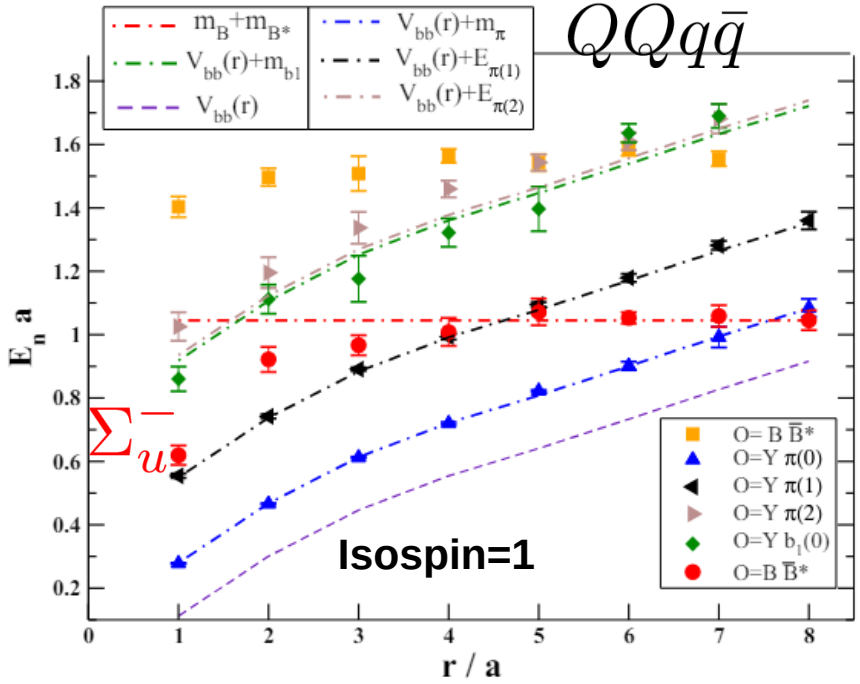
$QQ\bar{q}\bar{q}$



Mueller et al, PoS LATTICE2023, 64 (2024)

Bicudo, Cichy, Peters, Wagner, Phys. Rev. D. 93, (2016)

$Q\bar{Q}q\bar{q}$



Prelovsek, Bahtiyar, Petkovic, Phys. Lett. B. 805, (2020)

Tetraquark / pentaquark BO potentials:

Quark configurations can be rearranged to have two hadron state.

Similarity with molecular physics. Molecules going to constituent atoms when internuclear separation very large.

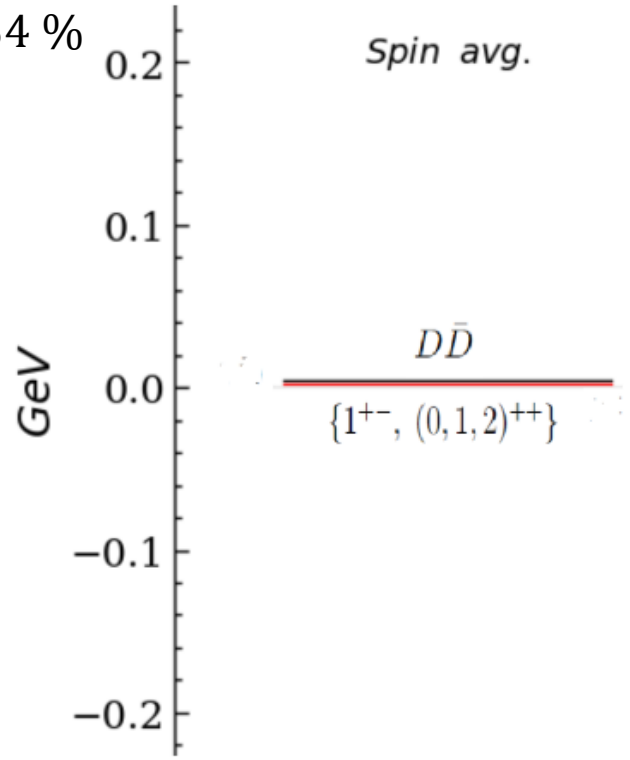
$\chi_{c1}(3872)$

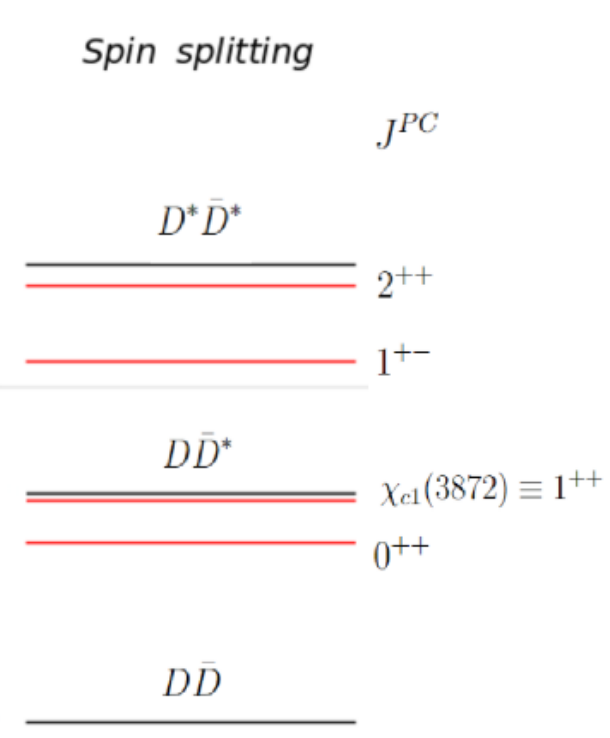
Brambilla, AM, Scirpa, Vairo
Phys. Rev. Lett. 135 (2025), 131902

Results:

Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius $\sim$ 15 fm.
4) **Adjoint** $(q\bar{q})_8$ energy: $\Lambda_a^{1^{--}} \approx -228$ MeV:
   No bound states in higher multiplets T_2^1, T_3^1, T_4^1 ….





Multiplet T_1^1: $\{1^{+-}, (0,1,2)^{++}\}$

1^{++} state: Identified with $\chi_{c1}(3872)$

1^{+-} state: Mass around 3.957 (11) GeV. Identified with X(3940) ?

2^{++} state: Mass around 4.004 (14) GeV.

0^{++} state: Mass around 3.846 (11) GeV. Also indicated in the lattice calculations: Prelovsek et al JHEP 06 (2021) 035.

X_b

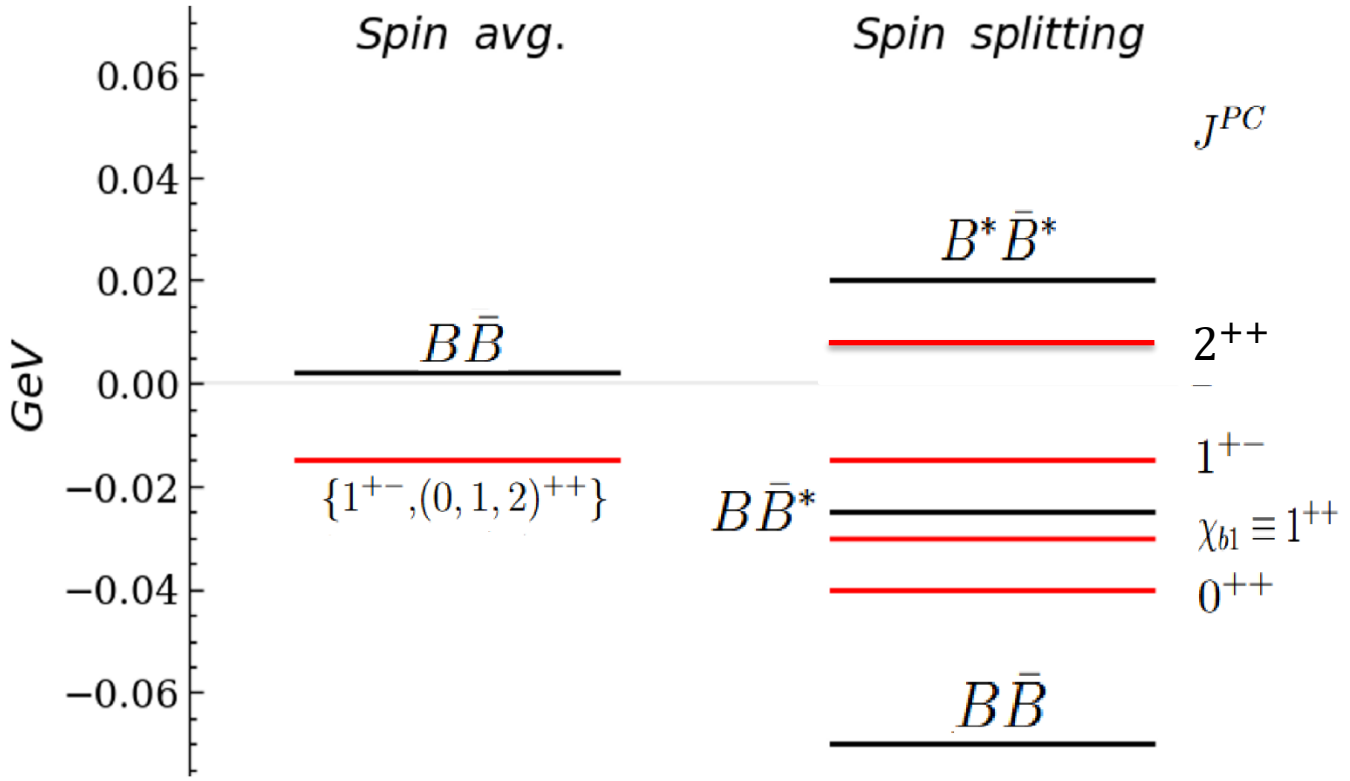
Brambilla, AM, Scirpa, Vairo

Phys. Rev. Lett. 135 (2025), 131902

Results with

adjoint meson energy ≈ -228 MeV

Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)



Multiplet T_1^1: $\{1^{+-},(0,1,2)^{++}\}$

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 1.5\ \%$
2) Tetraquark percentage:

$$|\psi_{\Sigma'}|^2 \approx 45.4\ \%,\ |\psi_\Pi|^2 \approx 53.1\ \%$$

1^{++}: identified with X_b: Mass around 10.595 GeV

1^{+-} state: Mass around 10.612 GeV.

2^{++} state: Mass around 10.635 GeV.

0^{++} state: Mass around 10.576 GeV.

$\chi_{c1}(3872)$

Brambilla, AM, Scirpa, Vairo 2411.14306

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8-13\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius $>$ 15 fm.

We **naturally** get $8-13\,\%$ quarkonium component in $\chi_{c1}(3872)$ due to **avoided level crossing**

Radiative decays:

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2s)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J\psi}},$$

Our estimate: $\boldsymbol{R_{\gamma\psi} = 2.99 \pm 2.36}$ (assuming only through $\boldsymbol{\chi_{c1}(2P)}$ **component**)

LHCb: $\boldsymbol{R_{\gamma\psi} = 1.67 \pm 0.25}$ Aaij et al arXiv: 2406.17006

Compositeness:

BES III: $\boldsymbol{Z = 0.18^{+0.20}_{-0.23}}$

Ablikim et al. Phys. Rev. Lett 132, 151903 (2024)

EMPPR: $0.052 < Z < 0.14$

Esposito, Maiani, Pilloni, Polosa, Riquer Phys. Rev. D 105, L031503 (2022)

Agreement with our $8-13\,\%$ quarkonium (compact) component

Lattice QCD:

$c\bar{c}$ operator along with $D\bar{D}^*$ relevant for $\chi_{c1}(3872)$ signal

Padmanath, Lang, Prelovsek Phys. Rev. D 92, 034501 (2015)

Prelovsek and Leskovec Phys. Rev. Lett 111, 192001 (2013)

Brambilla, AM, Vairo, arXiv 2508.13050

Pentaquark: Decays

- Semi-inclusive decays:

$$P_{c\bar{c}} \to Q_n + Y$$

Q_n: low-lying quarkonium **Y**: light hadrons

- ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{P_{c\bar{c}}} - E_n \gtrsim 1$ GeV.
- ✓ Constituent LDF of the exotic is a spectator.
- ✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$

Perturbative computation

Reported in PDG 2024:

$P_{c\bar{c}}$	Γ (total width)
$P_{c\bar{c}}(4312)$	10 ± 5 MeV
$P_{c\bar{c}}(4380)$	210 ± 90 MeV
$P_{c\bar{c}}(4440)$	21^{+10}_{-11} MeV
$P_{c\bar{c}}(4457)$	$6.4^{+6}_{-2.8}$ MeV

Semi-inclusive decays:

$P_{c\bar{c}}$	J^P	$\Gamma\,(P_{c\bar{c}} \to J/\psi + \cdots)$
$P_{c\bar{c}}(4312)$	$(1/2)^-$	$2^{+1\,+1}_{-1\,-1}$ MeV
$P_{c\bar{c}}(4457)$	$(1/2)^-$	$1^{+0.5\,+0.4}_{-0.3\,-0.3}$ MeV
$P_{c\bar{c}}(4380)$	$(3/2)^-$	$14^{+6\,+5}_{-3\,-4}$ MeV
$P_{c\bar{c}}(4440)$	$(3/2)^-$	$20^{+9\,+8}_{-5\,-6}$ MeV

➢ $P_{c\bar{c}}\left[J^P = (5/2)^-\right]$ only decays to η_c !!!