Electromagnetic form factors and structure of the T_{bb} tetraquark

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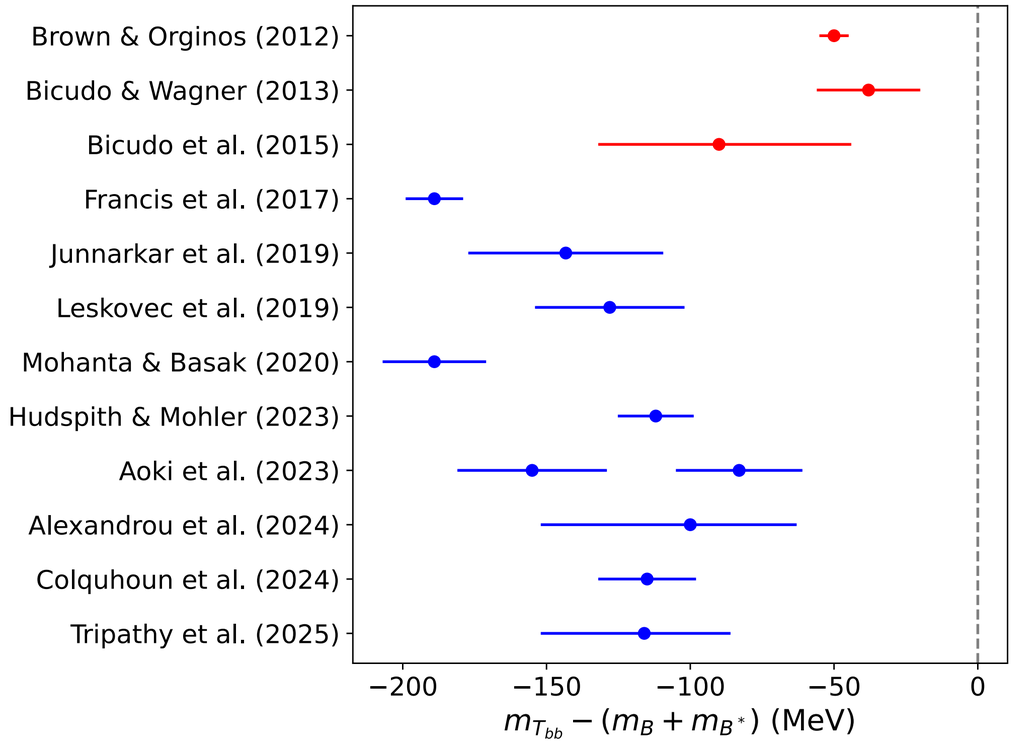
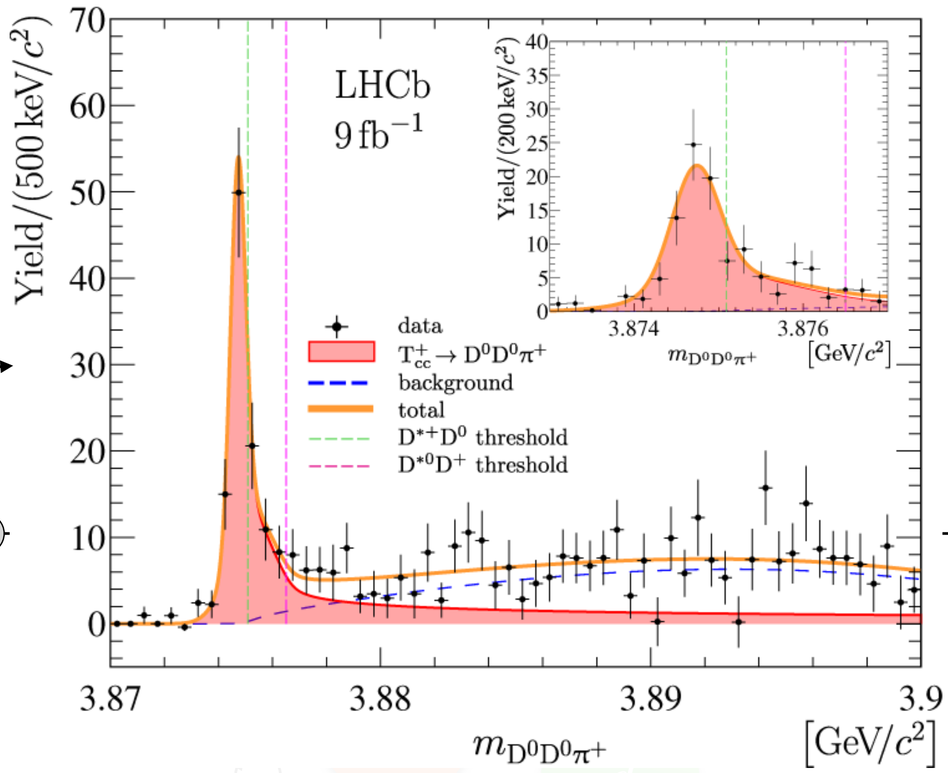
Introduction

- $T_{bb} = bb\bar{u}\bar{d},\ I(J^P) = 0(1^+)$
- yet-experimentally-undiscovered: one of the most promising candidates for an exotic QCD bound state
  - $T_{cc} = cc\bar{u}\bar{d}, I(J^P) = 0(?^?)$ found just below $DD*$ thr. (LHCb)
- quark models + **lattice QCD (spectroscopy):**



static $b-$quark

$b-$NRQCD



Introduction

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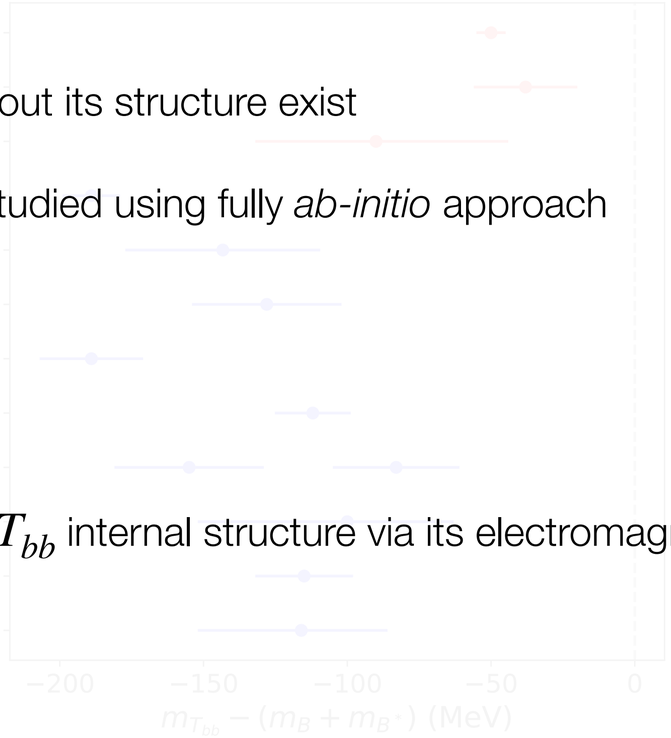
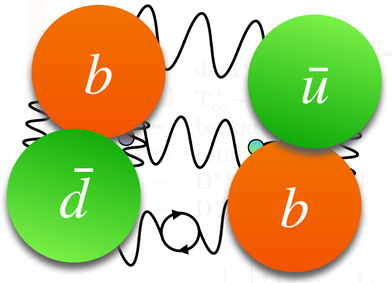
- quark models + **<u>lattice QCD (spectroscopy):</u>**

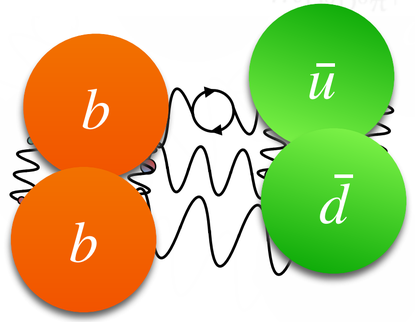


- well-motivated ideas about its structure exist
  - was never before studied using fully *ab-initio* approach
- in this talk:
  - first glimpse of the T_{bb} internal structure via its electromagnetic (EM) form factors

- <u>molecular binding</u>: $(b\bar{u})_1(b\bar{d})_1,\ (b\bar{u})_8(b\bar{d})_8$



- <u>diquark-antidiquark binding</u>: $[bb]_{\bar{3}}[\bar{u}\bar{d}]_3,\ [bb]_6[\bar{u}\bar{d}]_{\bar{6}}$



Elastic EM form factors

- matrix elements: $\mathscr{M}^{\mu}_{EM} \equiv \langle h(p_2,\lambda_2)\,|\,\hat{j}^{\mu}_{EM}\,|\,h(p_1,\lambda_1)\rangle = \sum_j K_j(p_{1,2},\lambda_{1,2})\cdot F_j(Q^2)$

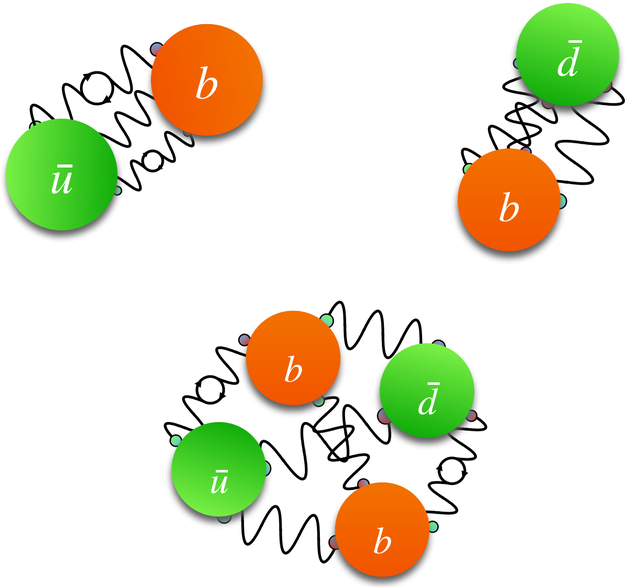
$$\hat{j}^{\mu}_{EM} = \sum_q e_q \bar{q}\gamma^{\mu} q = \sum_q \hat{j}^{\mu}_q \qquad Q^2 \equiv -(p_2-p_1)^2 > 0$$

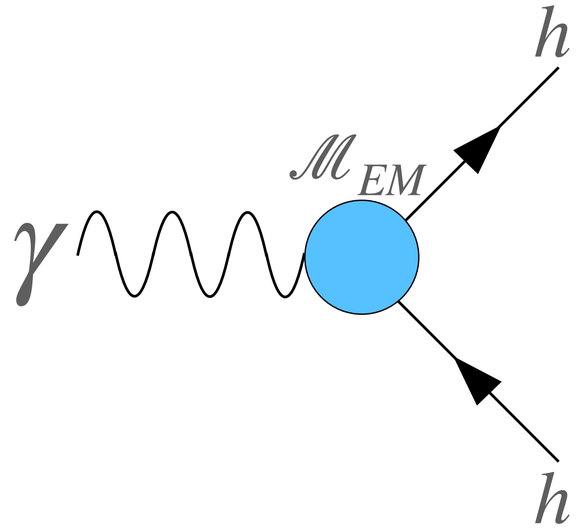
- additive current → flavor decomposition: $F_j = \sum_q F_j^q(Q^2)$
  - can probe distributions of individual quarks
- relevant states h:
  - T_{bb}
  - $B,\quad B^*$ ⟶ set the scale of the problem

$$J^P = 0^-: \quad \mathscr{M}^{\mu}_{EM} \sim F_C(Q^2)$$
$$J^P = 1^{\pm}: \quad \mathscr{M}^{\mu}_{EM} \sim F_C(Q^2), F_M(Q^2), F_{\mathcal{Q}}(Q^2)$$

- infinite-volume, continuum normalizations: $F_C(0) = -1 \qquad F_M(0) = 2m\cdot\mu$

$$F_{\mathcal{Q}}(0) = m^2\cdot\mathcal{Q}$$



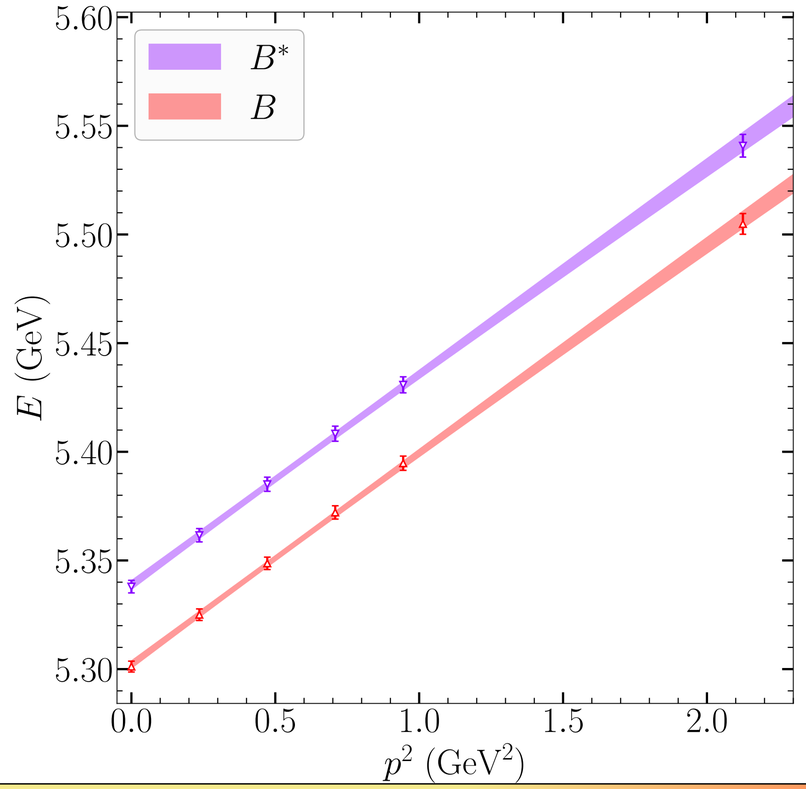


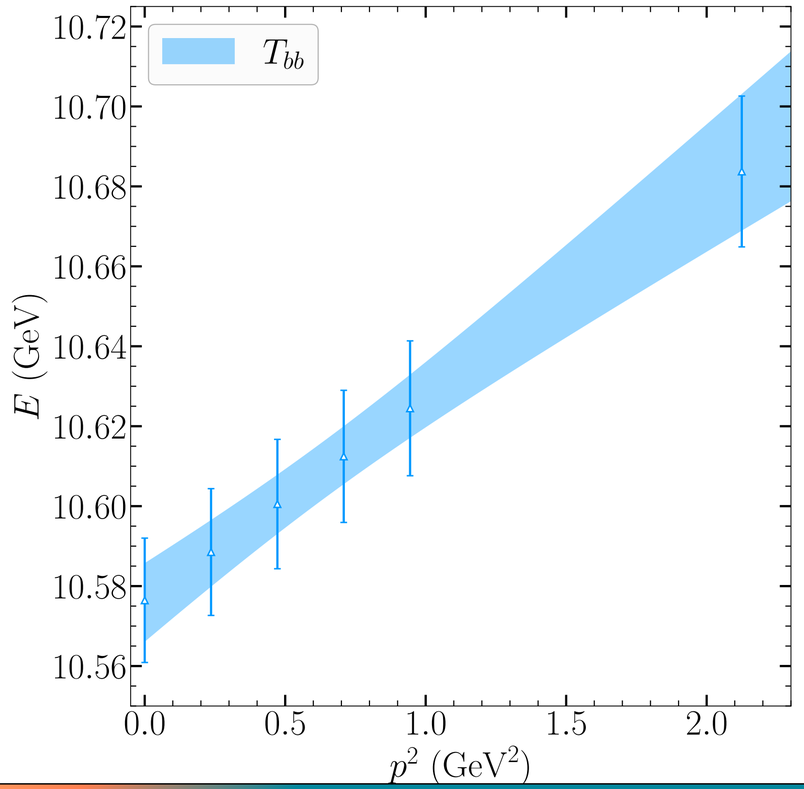
Lattice setup

- single $N_f = 2 + 1$ gauge ensemble, generated by CLS
- $V = N_L^3 \times N_T = 40^3 \times 128$
- $a \approx 0.06379(37)$ fm, $L \approx 2.56$ fm
- $Z_b^V = 3.97(11), \quad Z_{u/d}^V = 0.725(16)$
- u/d quarks: isospin symmetry; isotropic, $\mathrm{O}(a)$ improved Wilson action → pion mass $m_\pi \approx 290$ MeV
- b quarks: anisotropic relativistic heavy quark action (RHQ)

h	m_h (GeV)	c
B	5.3020(17)	1.020(12)
B^*	5.3387(20)	1.021(14)
T_{bb}	10.5765(98)	1.04(11)

B, B^* lattice masses well within 1% of PDG values





$$m_{T_{bb}} - (m_B + m_{B^*}) = -64(10) \text{ MeV}$$

$$E = \sqrt{m^2 + (c\vec{p})^2}$$

Lattice observables

- what is sought: $\langle h(\vec{p}_2)\,|\,\hat{j}^{\mu}_{q}\,|\,h(\vec{p}_1)\rangle,\ h = T_{bb}, B, B^*, \pi$

$$q = u, d, b$$

  - three-pt. correlators:

$$C_3(\vec{p}_2,\vec{q},T,t) = \langle\Omega\,|\,\mathcal{O}_h(\vec{p}_2,T)\hat{j}_{EM}(\vec{q},t)\mathcal{O}_h^{\dagger}(0)\,|\,\Omega\rangle =$$

$$= \sum_{n,m}\frac{\mathcal{Z}_n^{f*}\mathcal{Z}_m^{i}}{(2E_n^f)(2E_m^i)}\cdot\mathcal{M}_{nm}\cdot e^{-E_n^f(T-t)}e^{-E_m^i t}$$

  - **three-point ratio R_3:** $\vec{q} = \vec{p}_2 - \vec{p}_1$

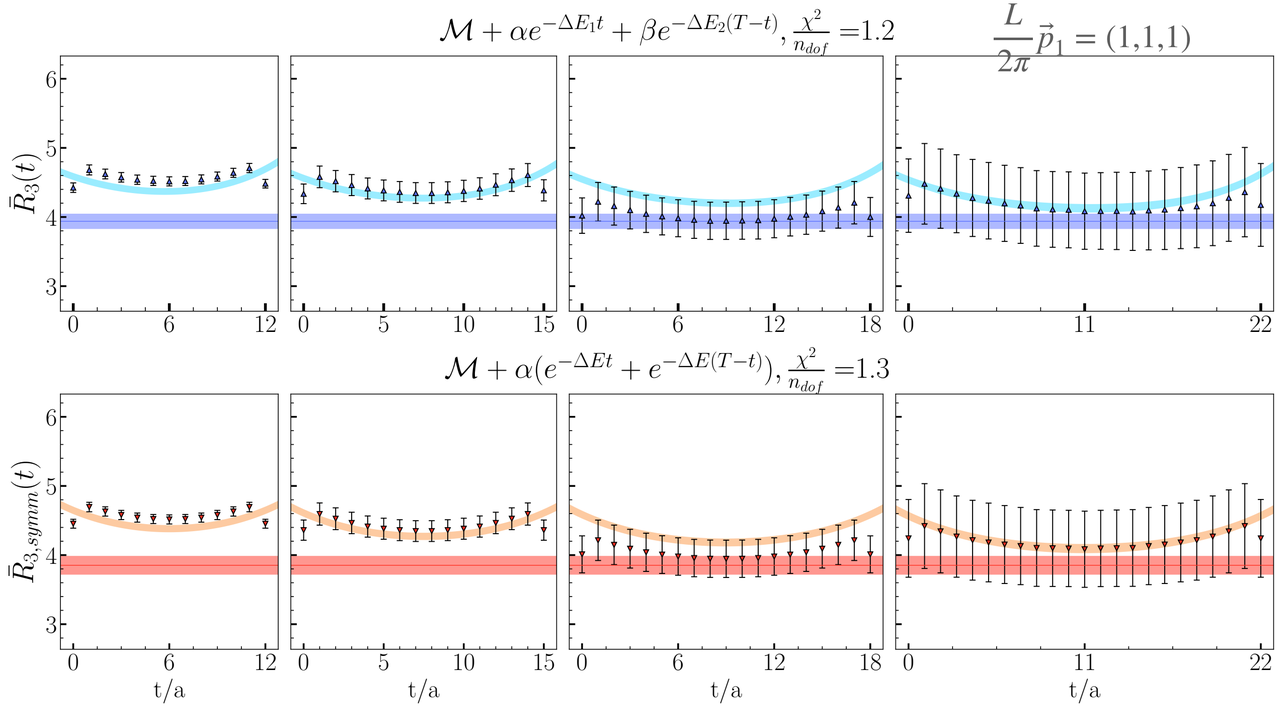
$$R_3 \equiv \frac{(2E_0^f)(2E_0^i)}{\mathcal{Z}_0^{f*}\mathcal{Z}_0^{i}}\cdot C_3\cdot e^{E_0^f(T-t)}e^{E_0^i t} = \boxed{\mathcal{M}_{00}} + \text{excited st. cont.}$$

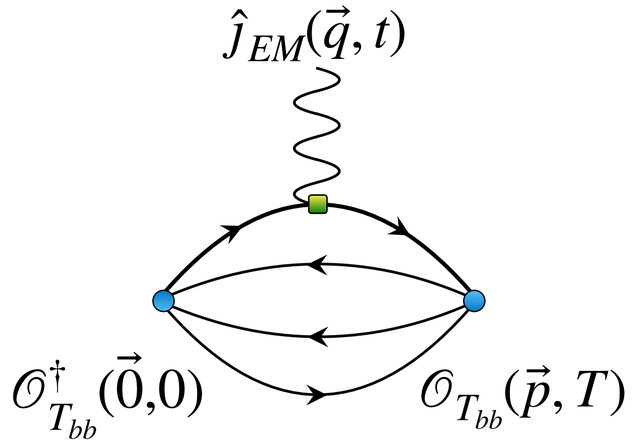
    - $\vec{p}_2 = \vec{0}$ in all computations
    - average over different directions $\hat{p}_1$ that yield same Q^2

- fit models:

  constant $\mathcal{M}$

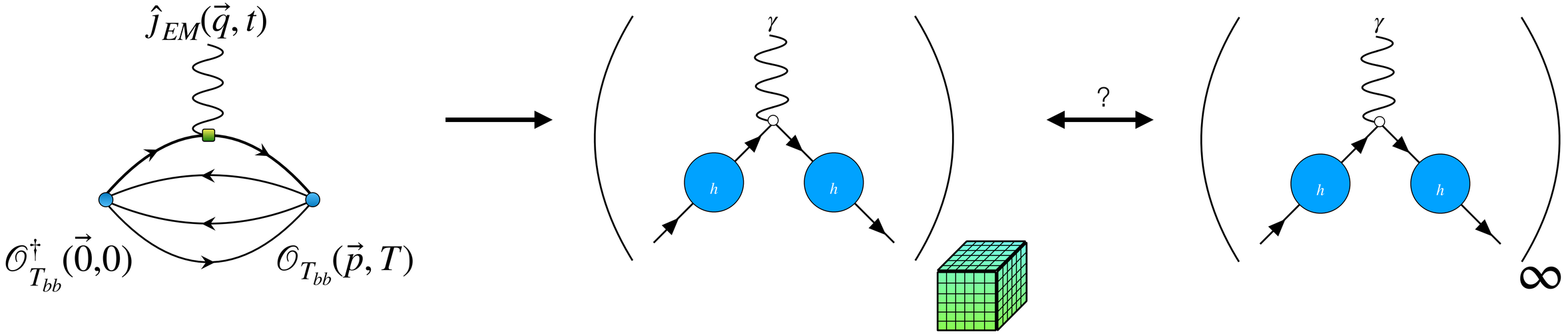
  first excited state model $\mathcal{M} + \alpha e^{-\Delta E_1 (T-t)} + \beta e^{-\Delta E_2 t}$



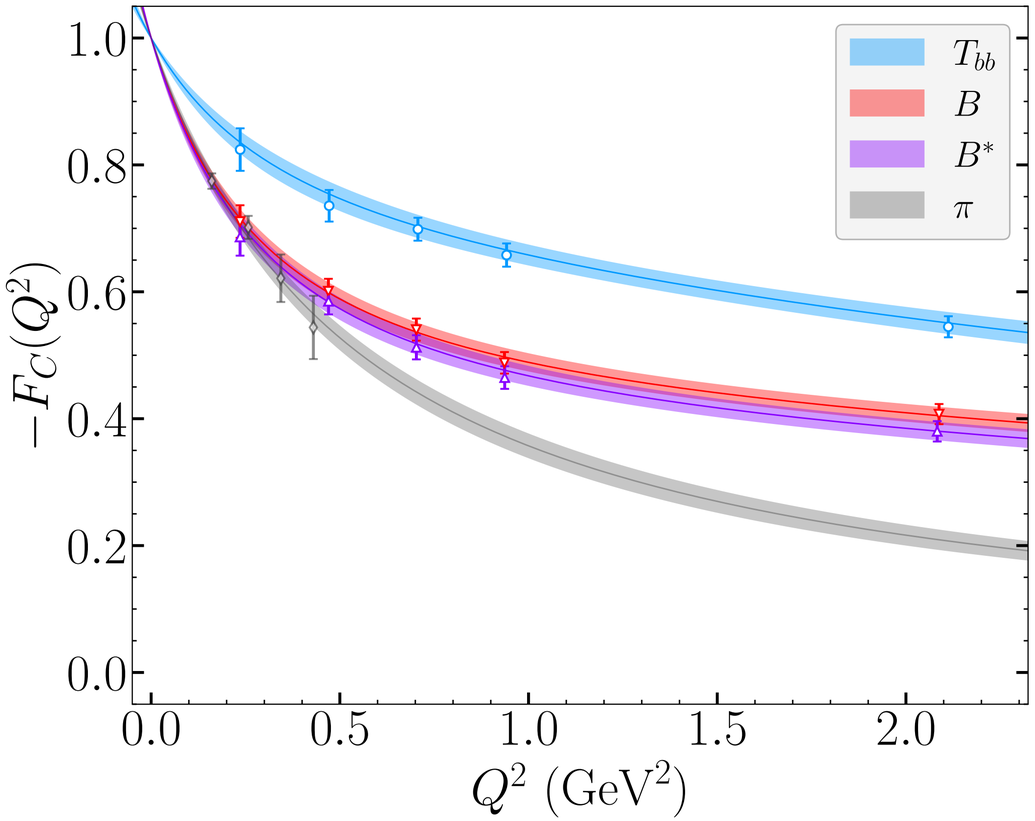


Finite-volume effects

- $h = T_{bb}, B, B^*, \pi \rightarrow$ all stable against the strong decay on the lattice
  - $\langle h | \hat{j} | h \rangle$ accessible directly from the lattice
  - exponentially suppressed finite-volume corrections
    - $\langle h | \hat{j} | h \rangle_L - \langle h | \hat{j} | h \rangle_\infty \sim \mathcal{O}(e^{-\kappa L})$



Results: charge distributions of $T_{bb}, B^{(*)}, \pi$



- general parametrisation: $z-$expansion
  - captures the effect of the closest branch cut t_+ and resonance pole m_r

$$F(Q^2) = \frac{1}{1+\frac{Q^2}{m_r^2}} \sum_{n=0}^{\infty} a_n z^n(Q^2, t_+, t_0)$$

$$z(Q^2, t_+, t_0) = \frac{\sqrt{t_+ + Q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ + Q^2} + \sqrt{t_+ - t_0}}$$

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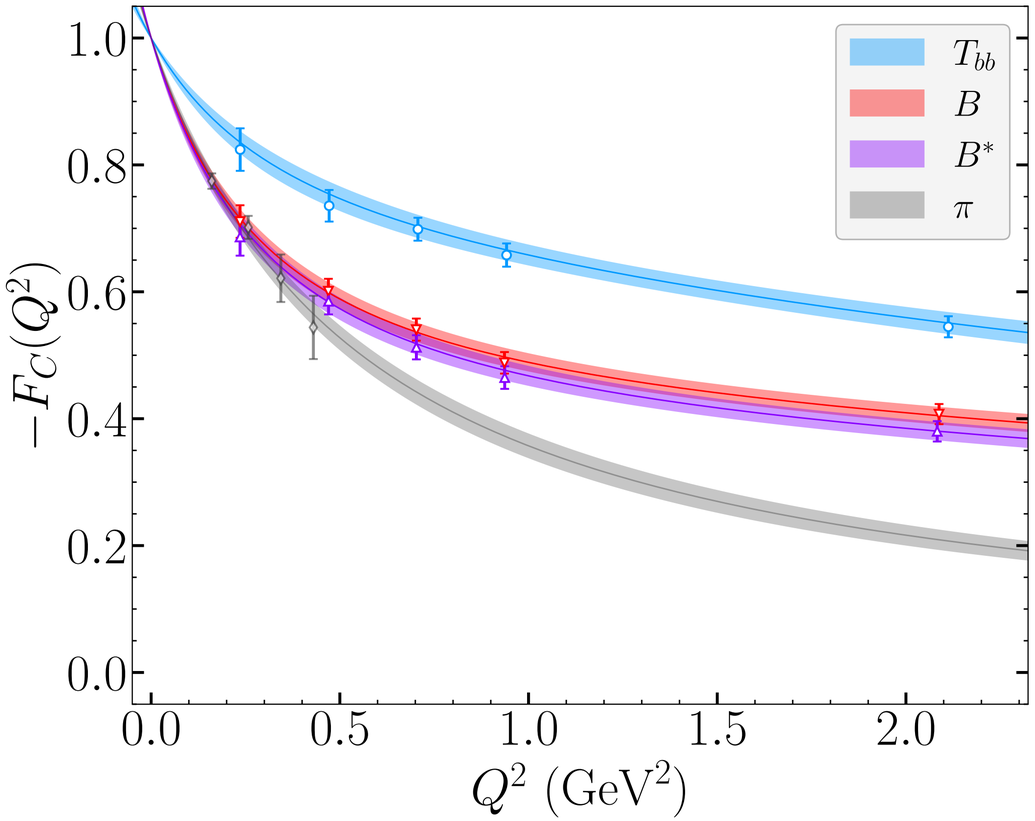
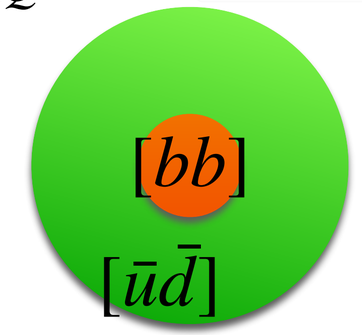


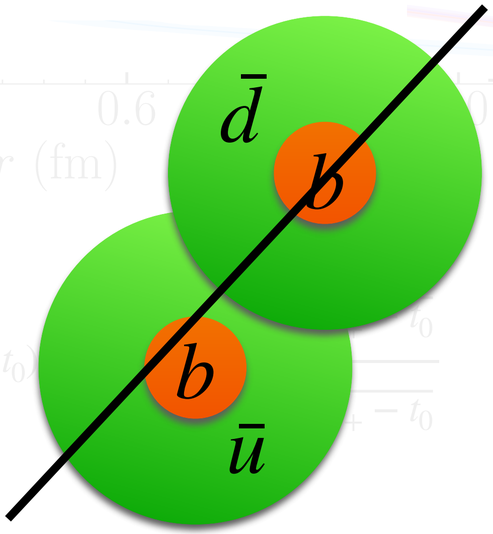
- charge form factors in momentum-space
- primary observable: $r_C \equiv \sqrt{\langle r_C^2 \rangle} = 6\frac{\mathrm{d}F_C}{\mathrm{d}Q^2}(0)$

h	$\sqrt{\langle r_C^2 \rangle}$ (fm)
T_{bb}	0.499(31)
B	0.692(21)
B^*	0.698(23)
π	0.652(20)



$$r_C^{T_{bb}} \ll r_C^B + r_C^{B^*}$$

- disfavors $B - B^*$ structure



Results: charge distributions of $T_{bb}, B^{(*)}, \pi$

- position-space charge densities
  - non-relativistic approximation
  - justified for hadrons with heavy quarks

$$\rho(r) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} F_C(|\vec{q}|^2)$$

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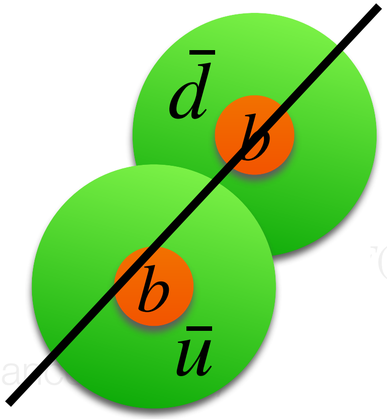
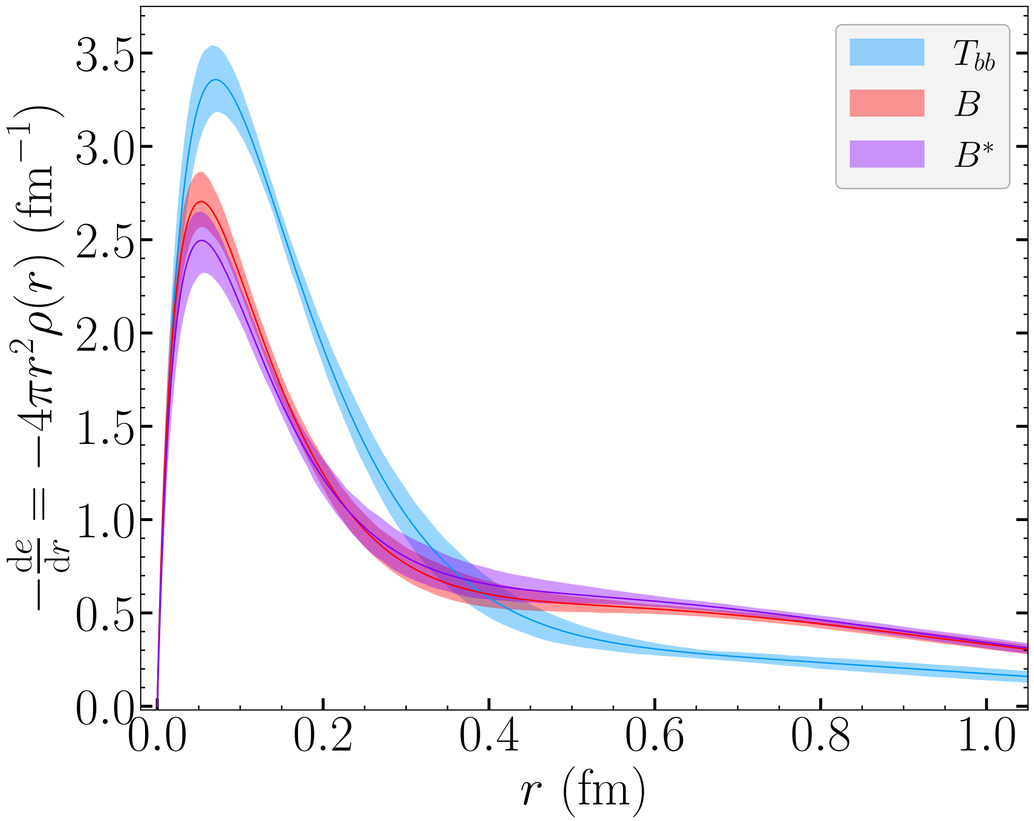
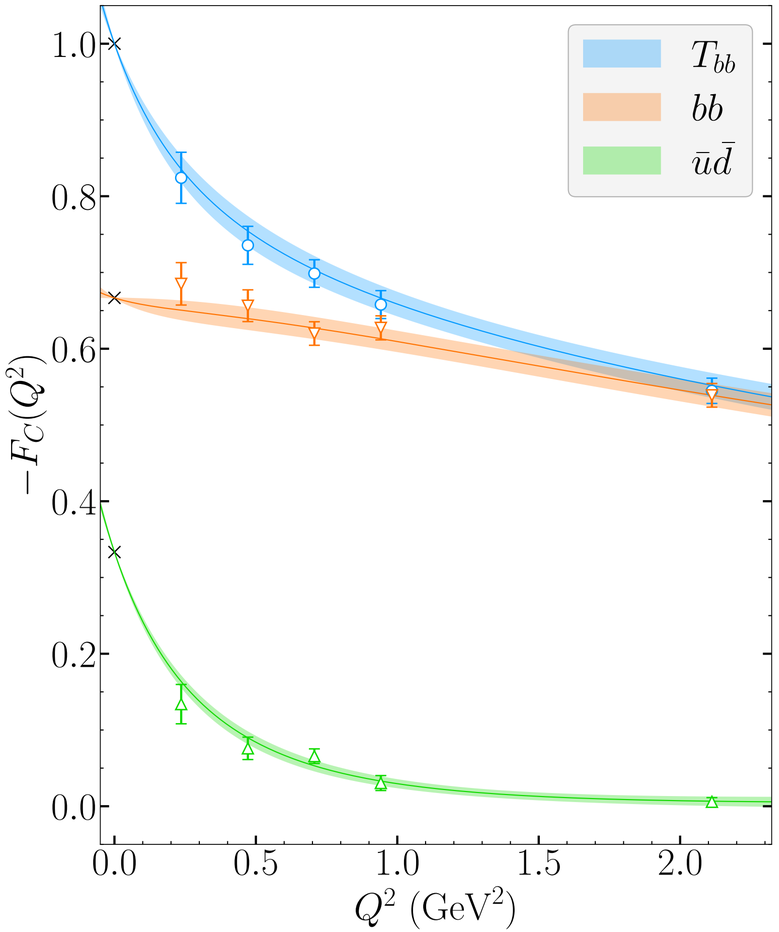
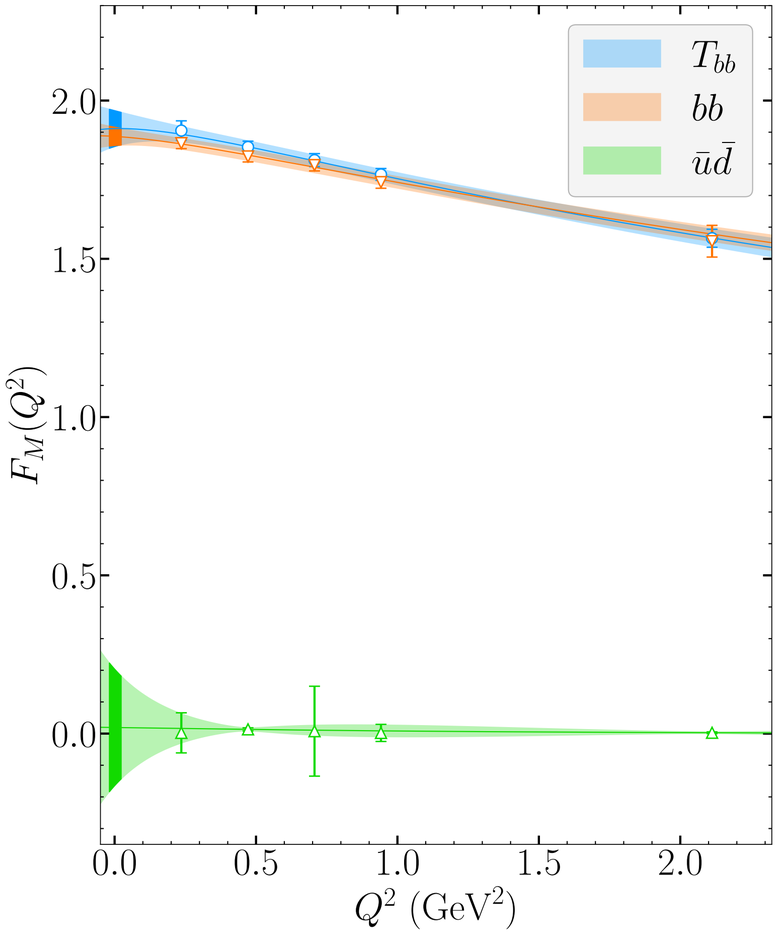
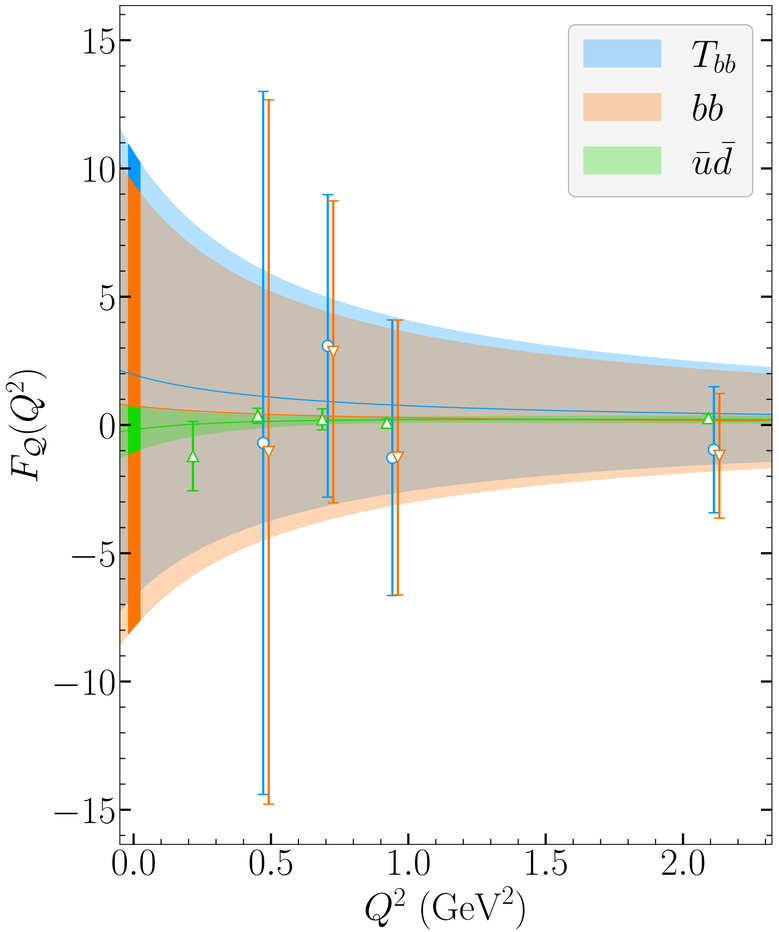




Results: T_{bb} form factors



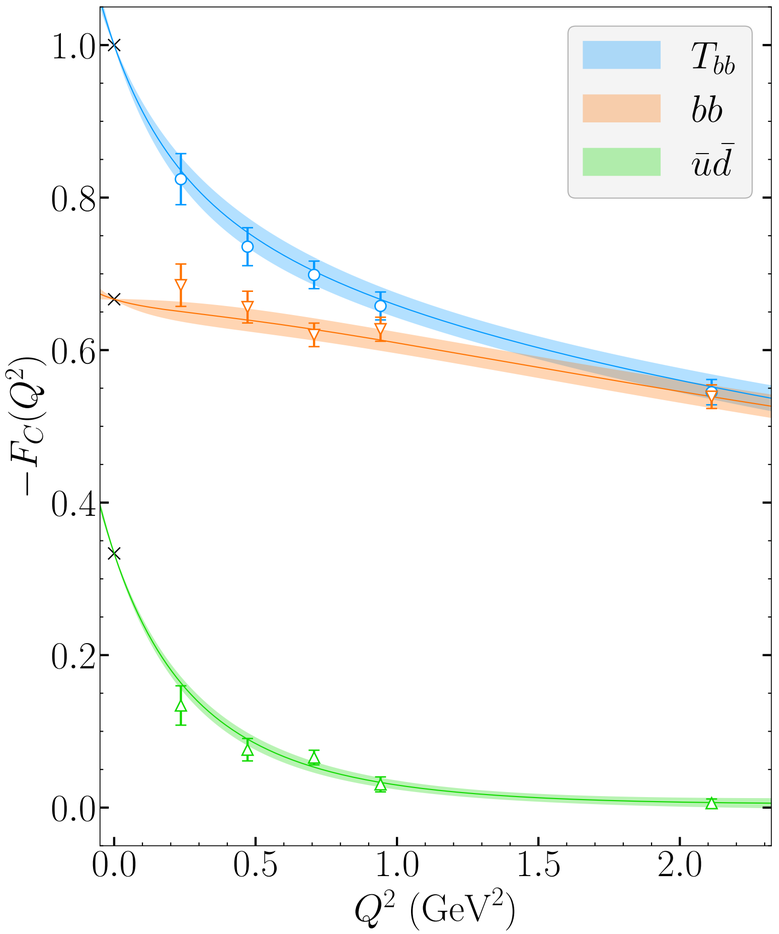




$$T_{bb} = bb\bar{u}\bar{d} \qquad \hat{j}^{\mu}_{u/d} = Z^V_{u/d} \cdot \left(\frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d\right) \qquad \hat{j}^{\mu}_{b} = Z^V_{b} \cdot \left(-\frac{1}{3}\bar{b}\gamma^{\mu}b\right)$$

$$\hat{j}^{\mu}_{EM} = \hat{j}^{\mu}_{u/d} + \hat{j}^{\mu}_{b}$$

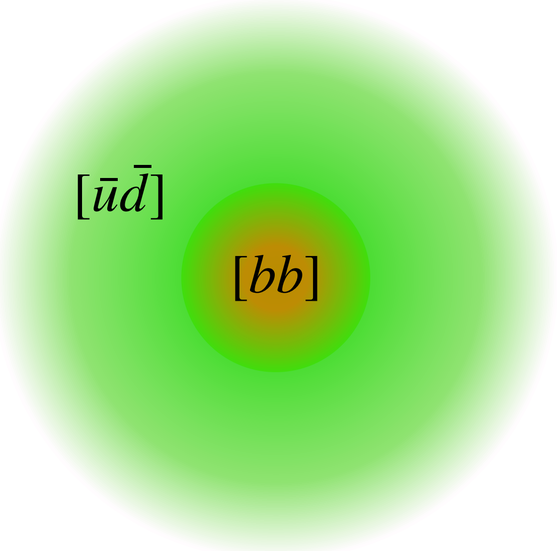
Results: T_{bb} form factors



$$r_C \equiv \sqrt{\langle r_C^2 \rangle} = 6\frac{\mathrm{d}F_C}{\mathrm{d}Q^2}(0)$$

- $r_C^{bb} \ll r_C^{\bar{u}\bar{d}}$
- strongly favors diquark-antidiquark structure

	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$\sqrt{\langle r_C^2 \rangle}$ (fm)	0.499(31)	0.174(59)	0.511(14)
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)
$m^2_{T_{bb}} \cdot Q$	1.9(8.7)	0.6(8.8)	−0.18(86)



- $\{[bb]^{l_{bb},s_{bb}}_{\bar{c}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}},s_{\bar{u}\bar{d}}}_{c}\}^{J^P=1^+}_{l_{12}}$

Results: T_{bb} form factors

- all three components components consistent with zero at $Q^2 = 0$
- can only happen with lowest config. of orbital angular momenta: $l_{bb} = l_{\bar{u}\bar{d}} = l_{12} = 0$

	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$\sqrt{\langle r_{\mathcal{O}}^2 \rangle}$ (fm)	0.499(31)	0.174(59)	0.511(14)
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)
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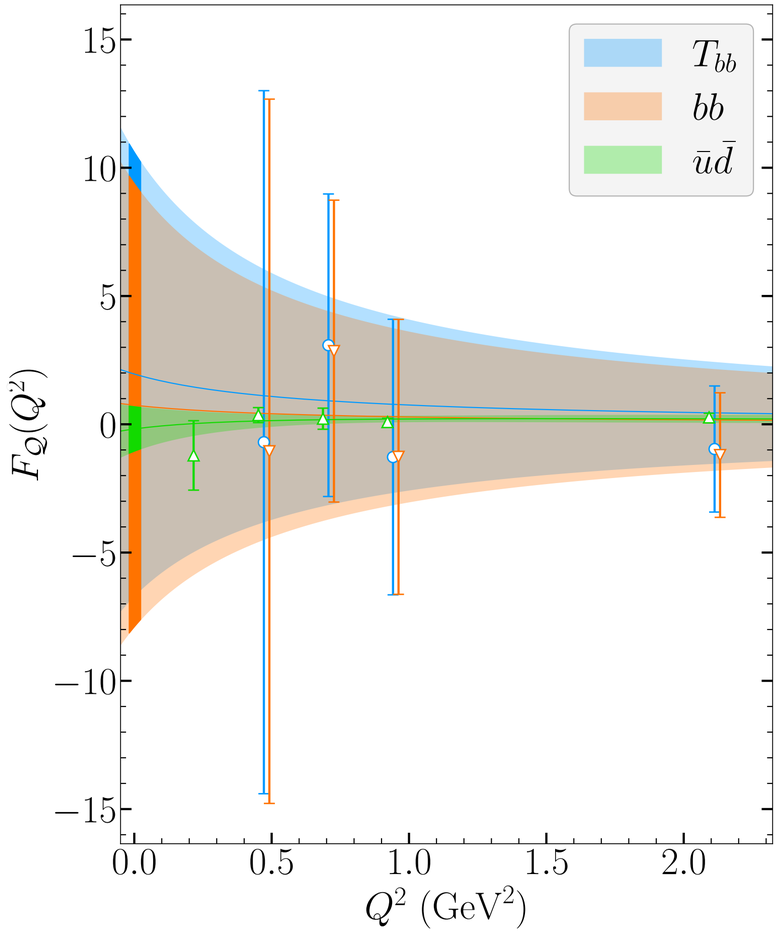
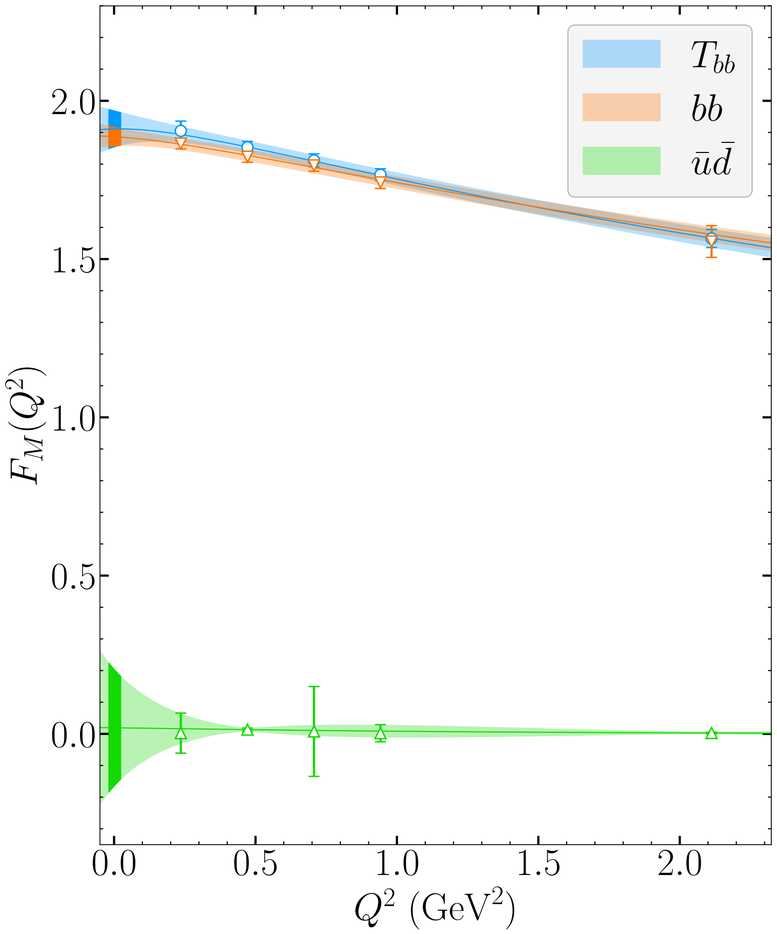


- $\{[bb]^{l_{bb}=0,s_{bb}}_{\bar{c}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}}_{c}\}^{J^P=1^+}_{l_{12}=0}$

Results: T_{bb} form factors

- heavy- and light-quark pair spin correlation
- total magnetic dipole moment saturated by $b-$quarks

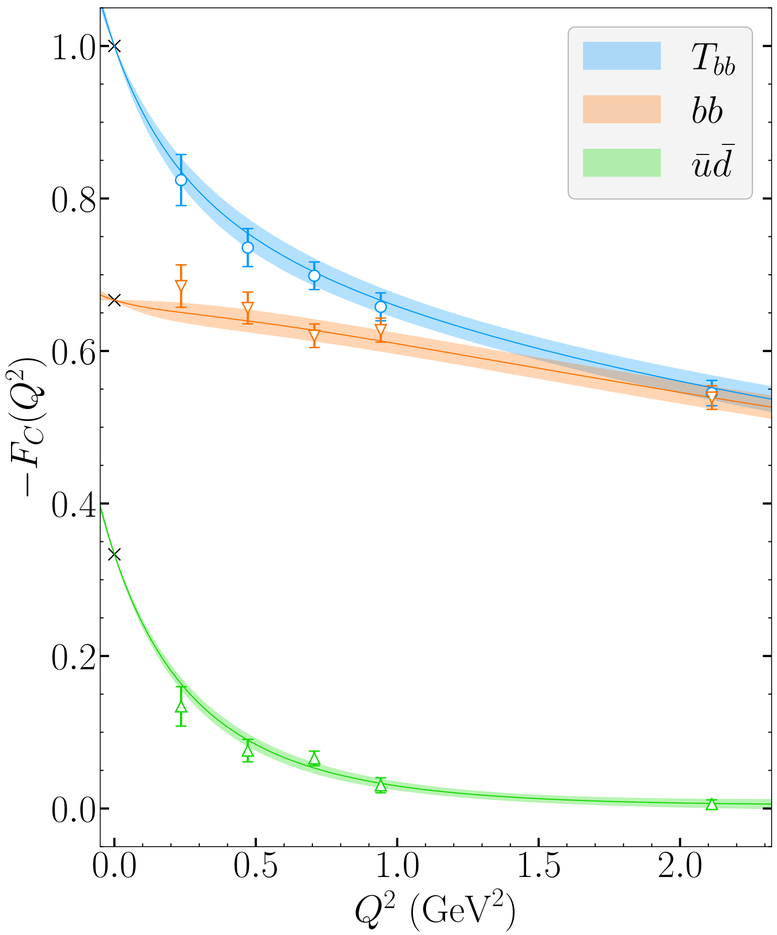
	T_{bb}	$[bb]$	$[\bar{u}\bar{d}]$
$2m_{T_{bb}} \cdot \mu$	1.912(57)	1.887(29)	0.02(18)

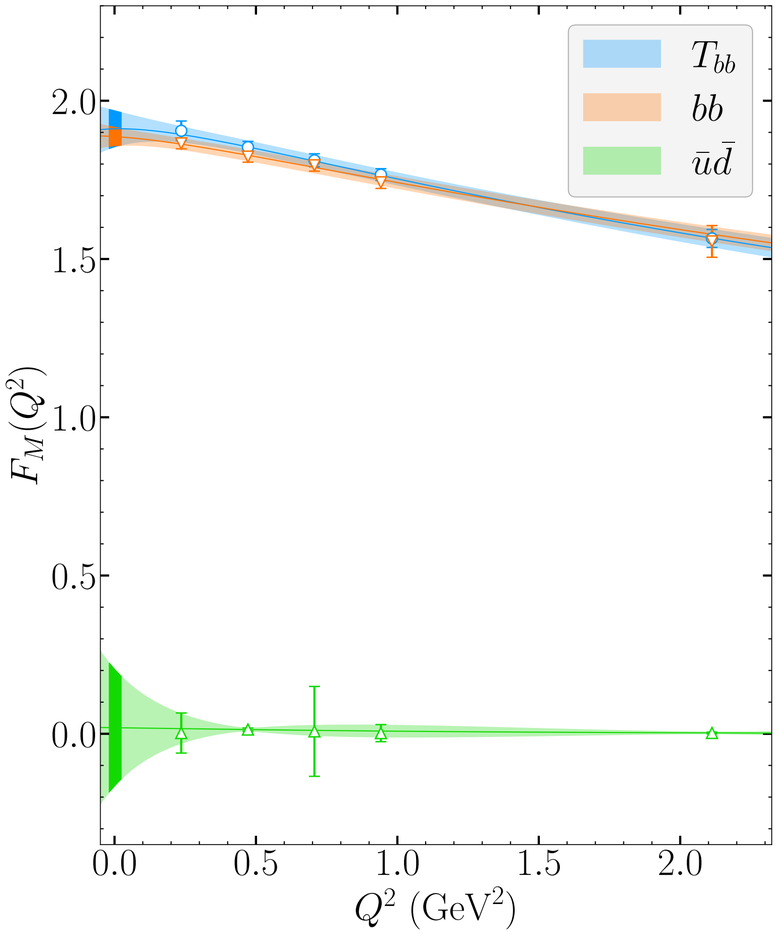


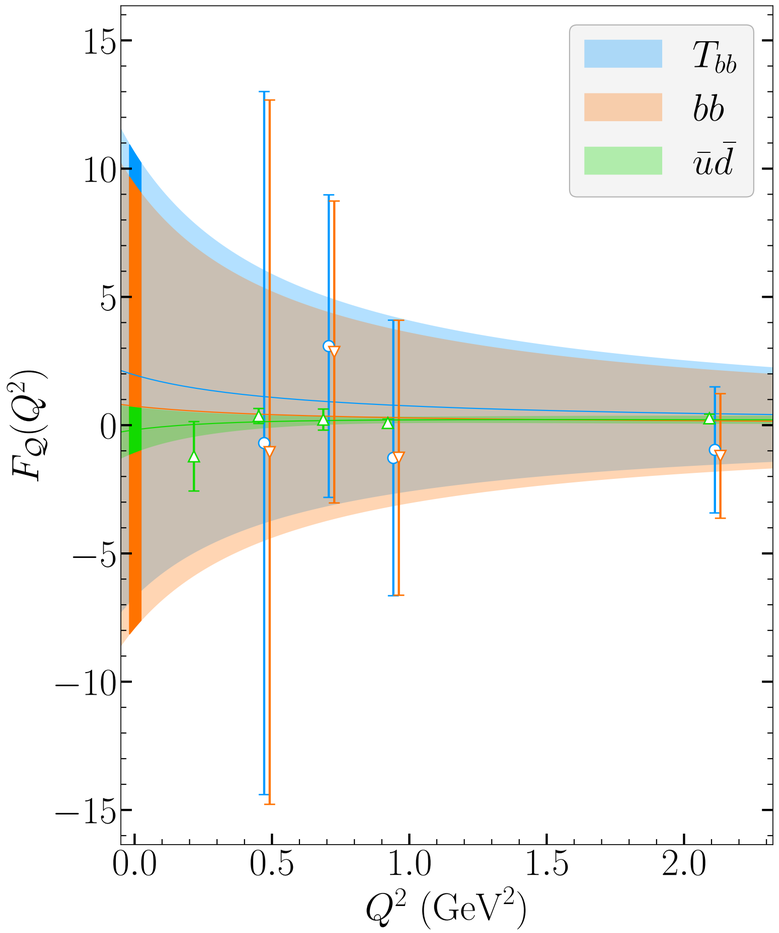
- $\mu_{T_{bb}} = \langle T_{bb} | \sum_{q=u,d,b} \frac{e_q}{2m_q} (\hat{l}_q + g_q \hat{s}_q) | T_{bb} \rangle = \frac{F_M(0)}{2m_{T_{bb}}}$
- $s_{bb} = 1, \quad s_{\bar{u}\bar{d}} = 0$

- $\{ [bb]^{l_{bb}=0, s_{bb}=1}_{\bar{c}} [\bar{u}\bar{d}]^{I=0, l_{\bar{u}\bar{d}}=0, s_{\bar{u}\bar{d}}=0}_{c} \}^{J^P=1^+}_{l_{12}=0}$

Results: T_{bb} form factors



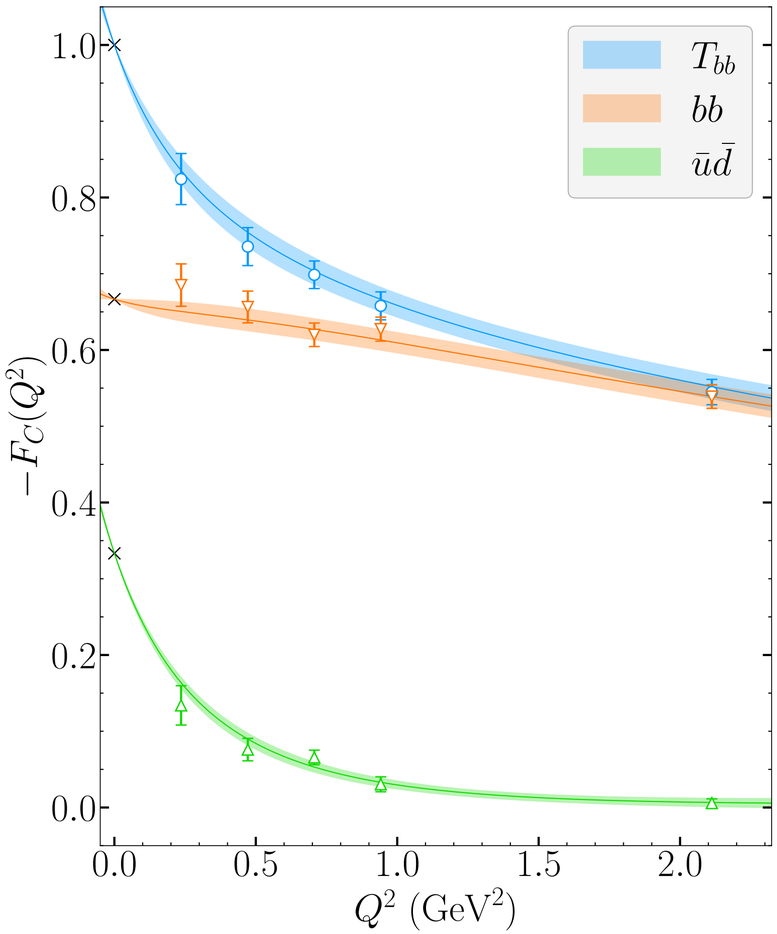


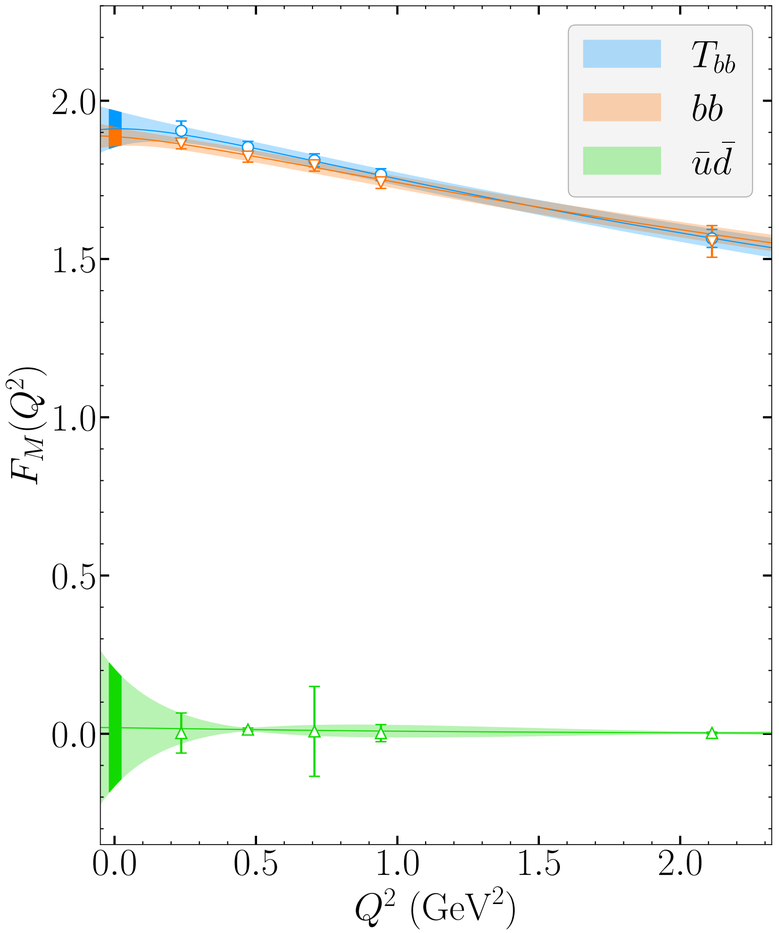


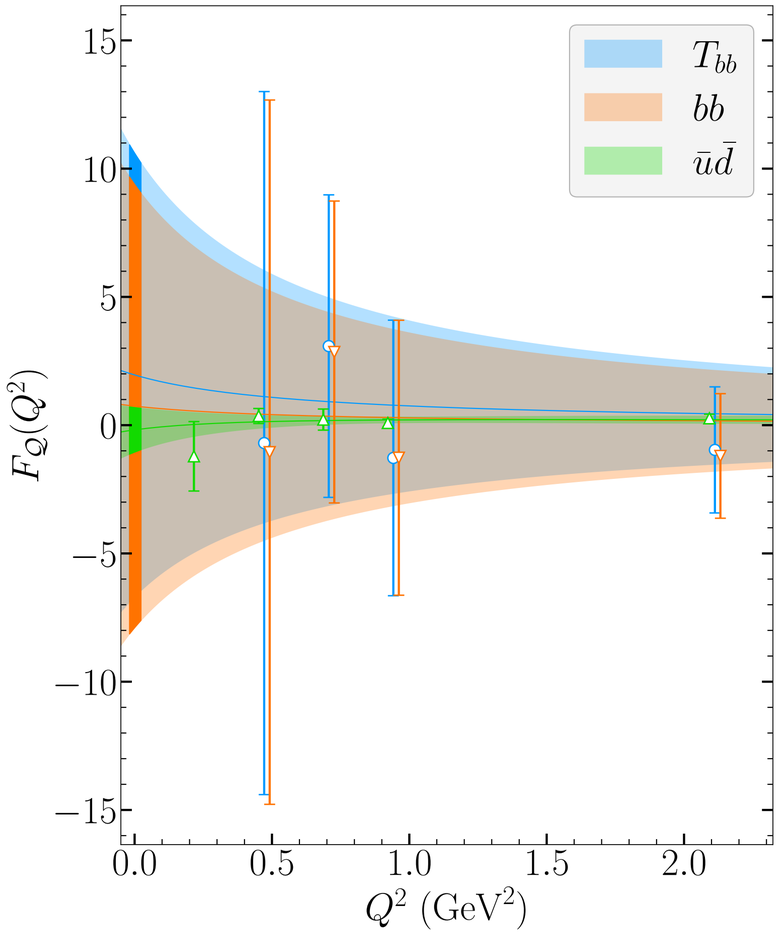
$$\circledcirc \{[bb]^{l_{bb}=0,s_{bb}=1}_{\bar{c}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}_{c}\}^{J^P=1^+}_{l_{12}=0}$$

$$c \overset{?}{\to} 3,\bar{6}$$

Results: T_{bb} form factors







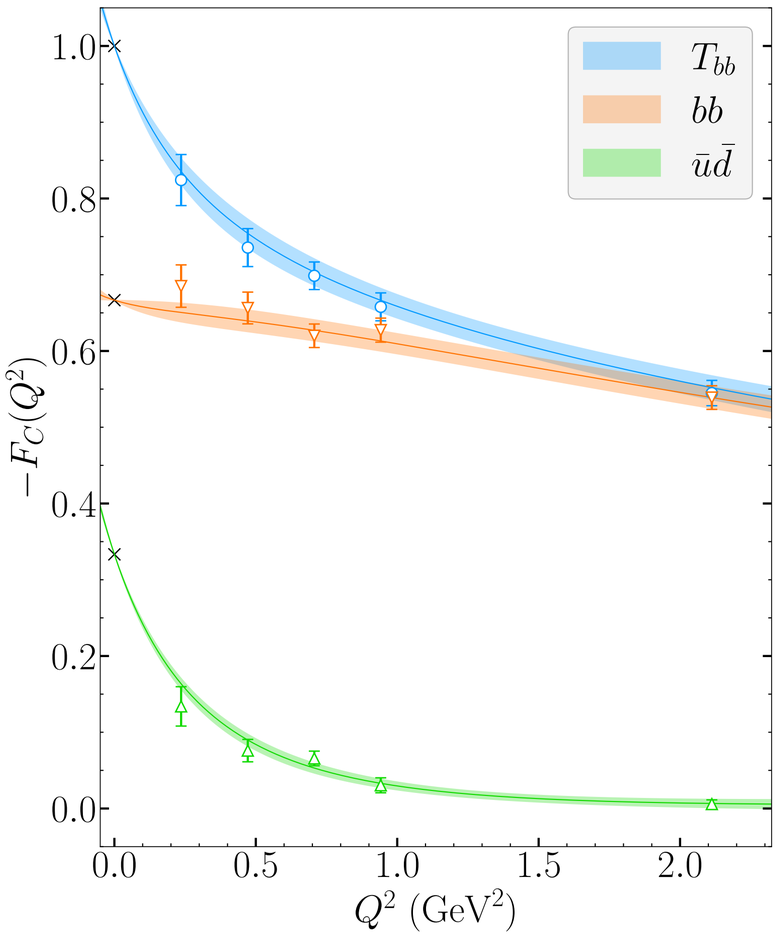
- $\{[bb]^{l_{bb}=0,s_{bb}=1}_{\bar{c}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}_{c}\}^{J^P=1^+}_{l_{12}=0}$

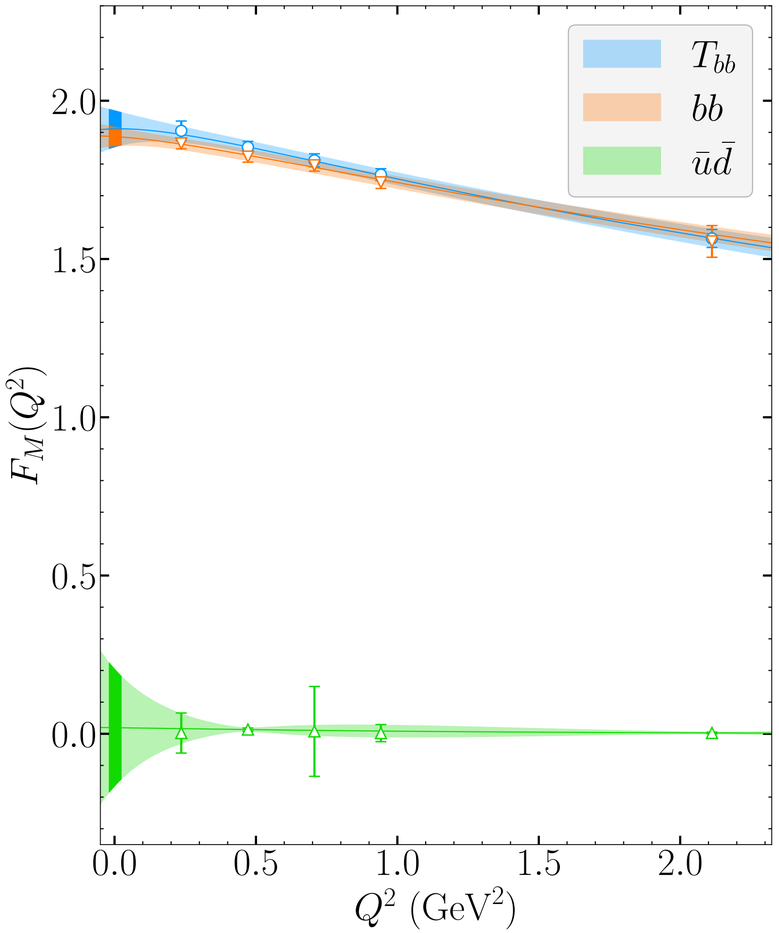
$c \overset{?}{\to} 3,\bar{6}$

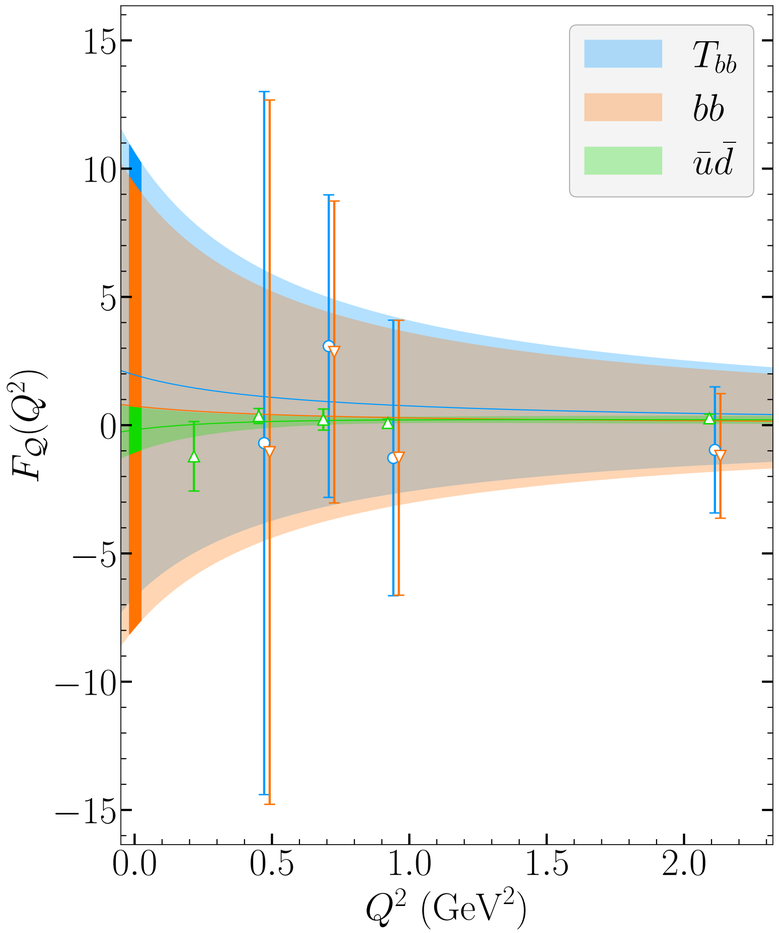
- color config. constrained by Pauli principle

- $(-1)^{s_{bb}+l_{bb}+c} = 1, \quad (-1)^{s_{\bar{u}\bar{d}}+l_{\bar{u}\bar{d}}+c} = -1 \quad \longrightarrow \quad c = 3$

Results: T_{bb} form factors





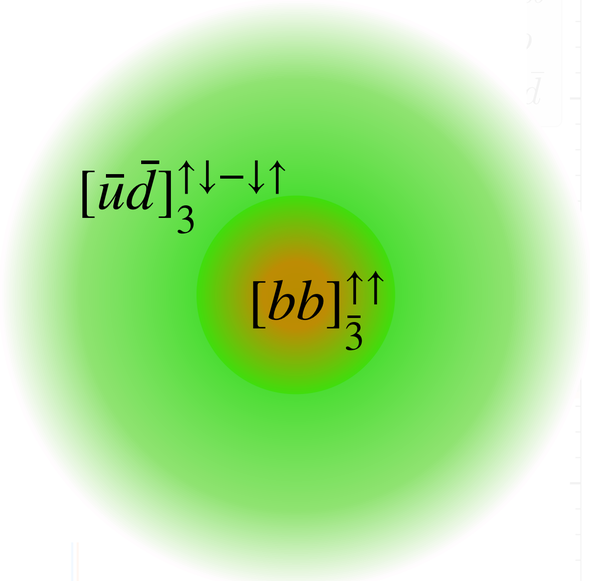


- $\{[bb]^{l_{bb}=0,s_{bb}=1}_{\bar{3}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}_{3}\}^{J^P=1^+}_{l_{12}=0}$

Summary

- first glimpse of T_{bb} structure via its EM form factors
- extracted ground-state electromagnetic matrix elements
  - five values of $Q^2 = -(p_2 - p_1)^2$
  - four states: T_{bb}, B, B^*, π
    - obtained all available EM form factors for each state
- lattice three-pt data + Pauli sufficiently constrain structure

$$\{[bb]^{l_{bb}=0,s_{bb}=1}_{\bar{3}}[\bar{u}\bar{d}]^{I=0,l_{\bar{u}\bar{d}}=0,s_{\bar{u}\bar{d}}=0}_{3}\}^{J^P=1^+}_{l_{12}=0}$$

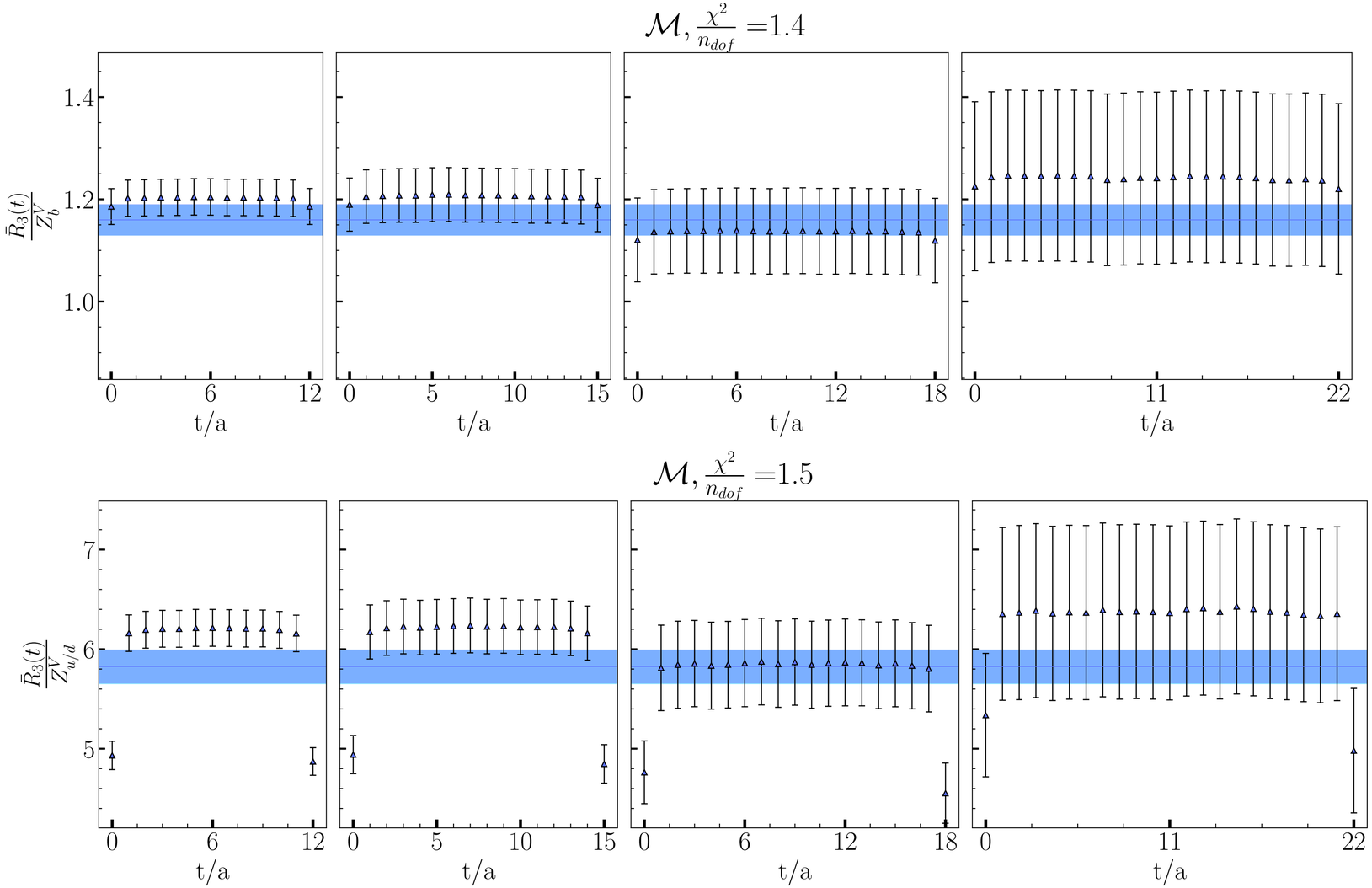


Backup: vector current renormalization

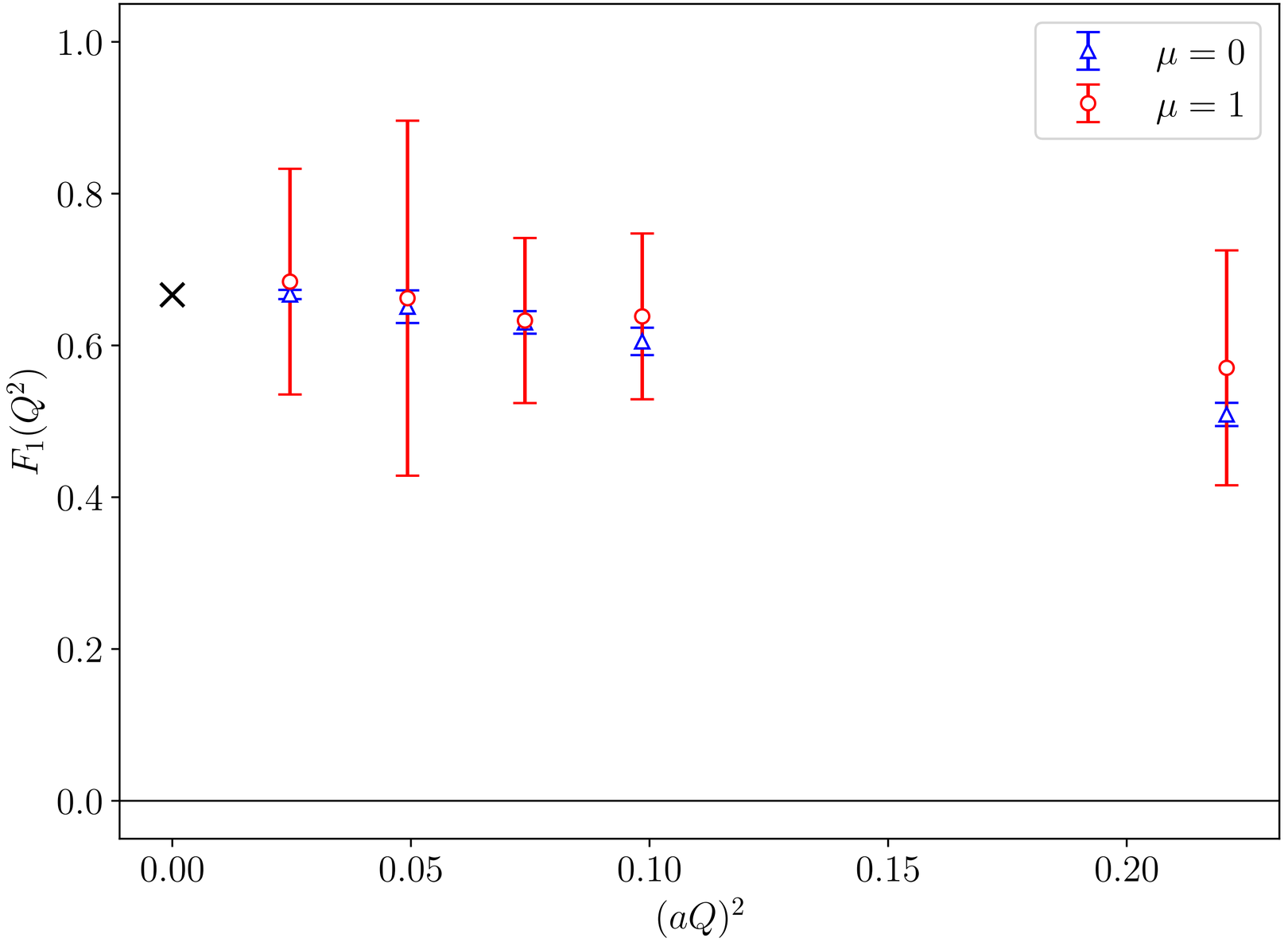
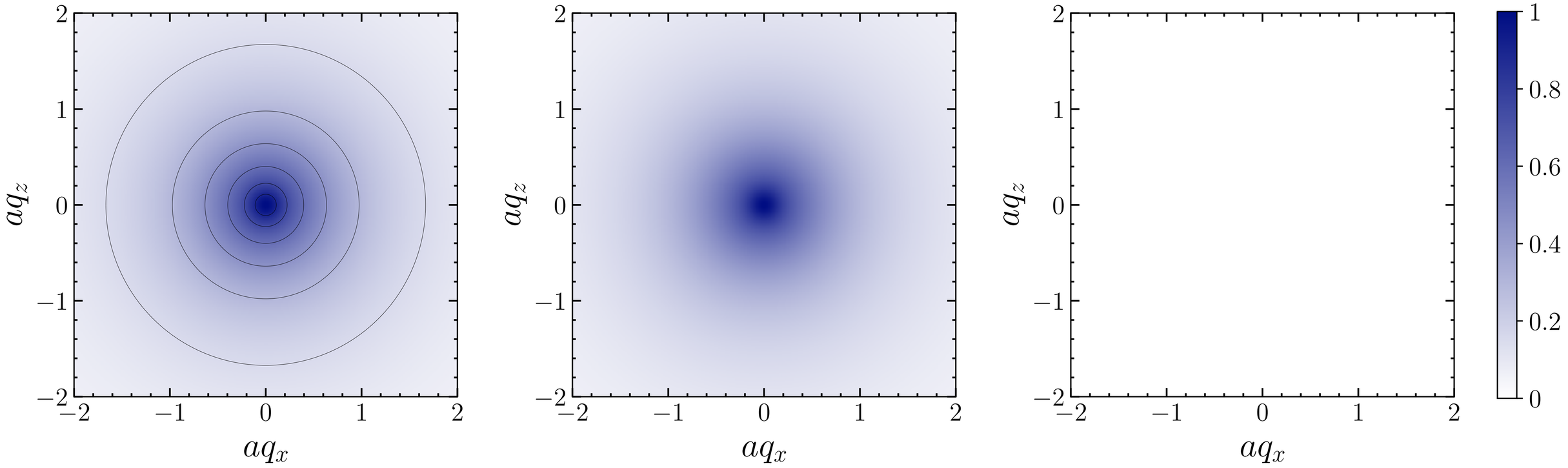
$$\hat{j}^{\mu}_{EM} = Z^{V}_{u/d}\left(\frac{2}{3}\bar{u}\gamma^{\mu}u - \frac{1}{3}\bar{d}\gamma^{\mu}d\right) - Z^{V}_{b}\cdot\frac{1}{3}\bar{b}\gamma^{\mu}b$$

- renormalization conditions:
  - $F_C^{b,T_{bb}} \equiv -\frac{2}{3}$
  - $F_C^{\bar{u},T_{bb}} \equiv -\frac{2}{3}$

- $Z_b^V = 3.97(11), \quad Z_{u/d}^V = 0.725(16)$



Backup: current renormalization anisotropy



Backup: T_{bb} quadrupole contribution to charge density $\rho(\vec{r})$