Mixing of light meson and charmonium flavor singlets in $N_f = 3 + 1$ QCD

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Great interest in exotics: States such as the 0^{-+} glueball candidate $X(2370)$ are great study subjects for lattice calculations.

The issue with flavor singlets: With charm and light quarks in the sea, flavor-singlets of both types are in the **same** lattice symmetry channel.

A simple example: A correlation function of a charmonium operator gives us access to the charmonium energy eigenstates?

$$\begin{aligned} C_{\bar{c}c}(t) &= \langle \bar{c}(t)c(t) \cdot \bar{c}(0)c(0) \rangle_{\text{gauge}} \\ &= |\left\langle \eta' \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta' } t} + |\left\langle \eta'(2S) \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta'(2S)} t} + \ldots \\ &+ |\left\langle \eta_c \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta_c} t} + |\left\langle \eta_c(2S) \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta_c(2S)} t} + \ldots \\ &\overset{t\to\infty}{\approx} |\left\langle \eta' \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta' } t} \neq |\left\langle \eta_c \right| \bar{c}c \left| \Omega \right\rangle|^2 e^{-m_{\eta_c} t} \end{aligned}$$

OZI rule predicts $|\left\langle \eta' \right| \bar{c}c \left| \Omega \right\rangle|^2 << |\left\langle \eta_c \right| \bar{c}c \left| \Omega \right\rangle|^2$ but no **symmetry** enforces $\langle \eta' | \bar{c}c | \Omega \rangle|^2 = 0$.

Charmonium is not a ground state. **How can we get around this?**

Need a mixing matrix to account for **flavor-singlet mixing** and access states up the ladder of excitations, e.g. charmonium.

$$\begin{pmatrix} \left\langle \mathcal{O}_{\bar{q}q}(t)\bar{\mathcal{O}}_{\bar{q}q}(0)\right\rangle & \left\langle \mathcal{O}_{\bar{q}q}(t)\bar{\mathcal{O}}_{\bar{c}c}(0)\right\rangle \\ * & \left\langle \mathcal{O}_{\bar{c}c}(t)\bar{\mathcal{O}}_{\bar{c}c}(0)\right\rangle \end{pmatrix}$$

$\left\langle \mathcal{O}_{\bar{q}q}(t)\bar{\mathcal{O}}_{\bar{q}q}(0)\right\rangle$, $\left\langle \mathcal{O}_{\bar{c}c}(t)\bar{\mathcal{O}}_{\bar{c}c}(0)\right\rangle$:

- Involve quark-connected and quark-disconnected diagrams.
- "Implicit mixing": happens through quark-disconnected loops.

$\left\langle \mathcal{O}_{\bar{q}q}(t)\bar{\mathcal{O}}_{\bar{c}c}(0)\right\rangle$:

- Involves only quark-disconnected diagrams.
- "Explicit mixing": Both flavor contents explicitly appear.

Quark-disconnected contributions suffer from a severe **signal-to-noise** problem and are often **neglected** in charmonium.
We measure them directly in this work.

Our setup

9000 configurations with $N_f = 3 + 1$ Wilson fermions with non-perturbatively determined clover improvement in massive scheme + Lüscher-Weisz gauge action. 96×32^3 lattice with $a \approx 0.052$ fm, $m_\pi \approx 420$ MeV and close-to-physical charm quark mass. P. Fritzsch et al. J. High Energ. Phys. 2018, 25 (2018); R. Höllwieser et al. Eur. Phys. J. C 80, 349

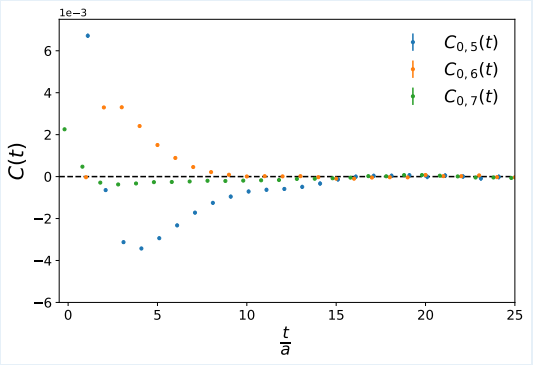
Light meson and charmonium operators are based on improved distillation to maximize overlaps onto states of interest. We use $N_v^{\text{Light}} = 100$ and $N_v^{\text{Charm}} = 200$. F. Knechtli, T. Korzec, M. Peardon, J. A. Urrea-Niño, Phys.Rev.D 106 (2022), M. Peardon et al. Phys.Rev.D 80 (2009)

We focus on S-wave, i.e. 0^{-+} and 1^{--}. More details can be found in Phys. Rev. D 112, 074502.

A closer look at the explicit mixing in 0^{-+}

There is **statistically significant** signal for $\left\langle \mathcal{O}_{\bar{q}q}(t)\bar{\mathcal{O}}_{\bar{c}c}(0)\right\rangle$:

- $\langle\Omega|\,\bar{c}c\,|n\rangle\,\langle n|\,\bar{q}q\,|\Omega\rangle \neq 0$ for at least one energy eigenstate $|n\rangle$.
- First hint for non-trivial mixing.
- Sign change due to mixed overlap coefficients.

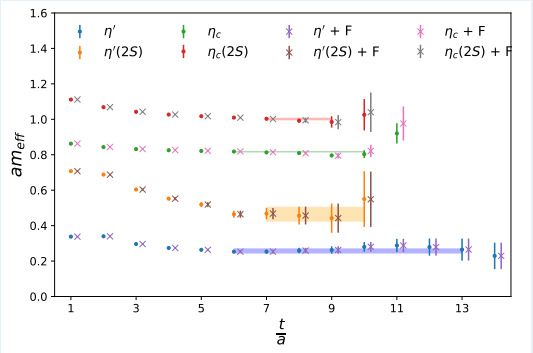


Does this non-zero mixing affect the spectrum?

We solve 3 GEVPs:

- Light meson only: η', $\eta'(2S)$.
- Charmonium only: η_c, $\eta_c(2S)$
- Full mixing: $\eta' + F$, $\eta_c + F$, etc...

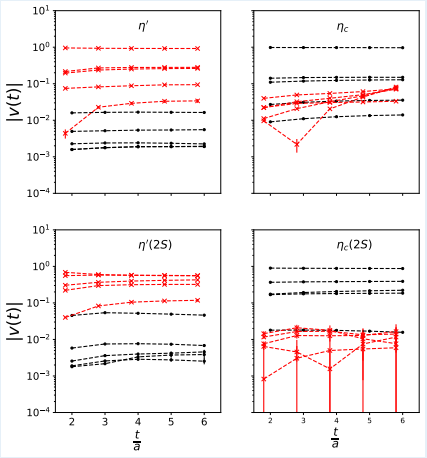
The spectrum remains stable, OZI prediction of small mixing seems to hold. This is a **direct** measurement.



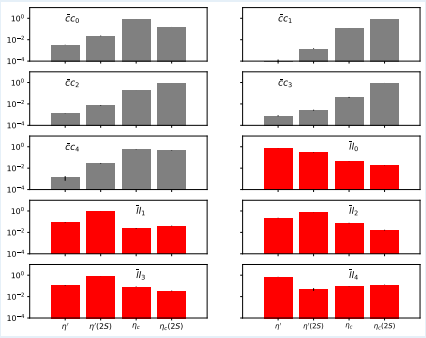
Contrary to 0^{++} case presented by us in Lattice 2024, where **implicit mixing** resolved a light state with charmonium operators. 2502.04977.

How do the operators mix?

GEVP Vectors



Overlaps



5 light meson and 5 **charmonium** operators.

The GEVP vectors are related to the overlaps $\langle n| \hat{O}^\dagger |\Omega\rangle$. Phys. Rev. D 82, 034508 (2010).

Effects on hyperfine splitting

We check for the η_c mass shift when including **implicit** and **explicit** mixing in **2 ways:**

1. Get mass plateau for all cases and calculate mass difference.

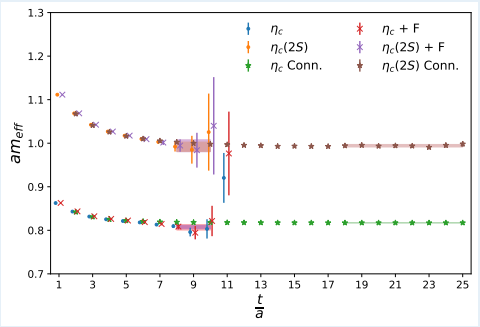
- ✓ Direct measurement.
- × Relies on reaching plateau before errors grow too much.

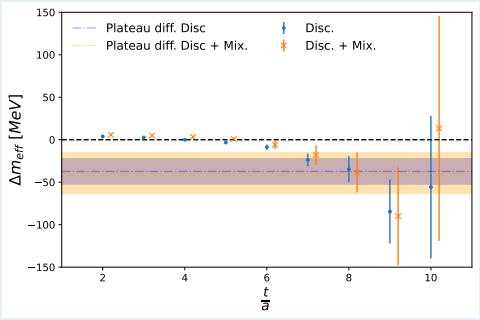
2. Get effective mass from ratio

$$R(t) = \frac{C_{\text{Full}}(t)}{C_{\text{Conn}(t)}} \approx Ae^{-(m_{\text{Full}} - m_{\text{Conn.}})t}$$

- ✓ Correlated difference benefiting from all points.
- × Exponential form is only an approximation due to $C_{\text{Conn}(t)}$ not having a spectral decomposition.

Previous studies determined sign from the slope of $R(t) - 1$. Phys. Rev. D 70, 034506 (2004), JHEP 2004, 004 (2004).





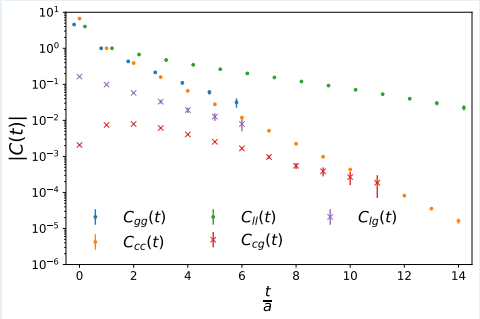
Implicit mixing: $-37(15)$ MeV, Both mixing: $-39(24)$ MeV. Ratio approach is consistent.

- ✓ Consistent with $20 - 30$ MeV negative shift in previous studies Phys. Rev. D 70, 034506 (2004), JHEP 2004, 004 (2004). as well as negative sign from NRQCD perturbation theory. Phys. Rev. D 75, 054502 (2007)
- ! Recent **indirect** measurement had **positive** shift $\approx 7.3(1.2)$ MeV. Phys. Rev. D 102, 054511 (2020) and one **direct** measurement had positive $3 - 4$ MeV but at $N_f = 2$. Ch. Phys. C 46, 043102 (2022)

What comes next?

Possible 0^{-+} glueball candidate $X(2370)$: we should include glueball-like operators. We use 3D Wilson loops described in Phys. Rev. D 110, 054515 (2024).

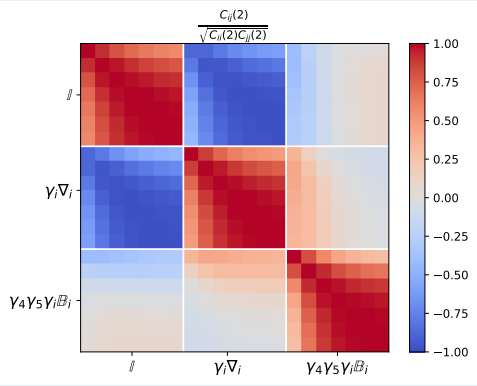
- ✓ Statistically non-zero signal **but** is lost very early.
- × Signal stops before any of the mass plateaus we use.
- × Need multi-level: **See L. Barca's talk at 2:30 PM**.



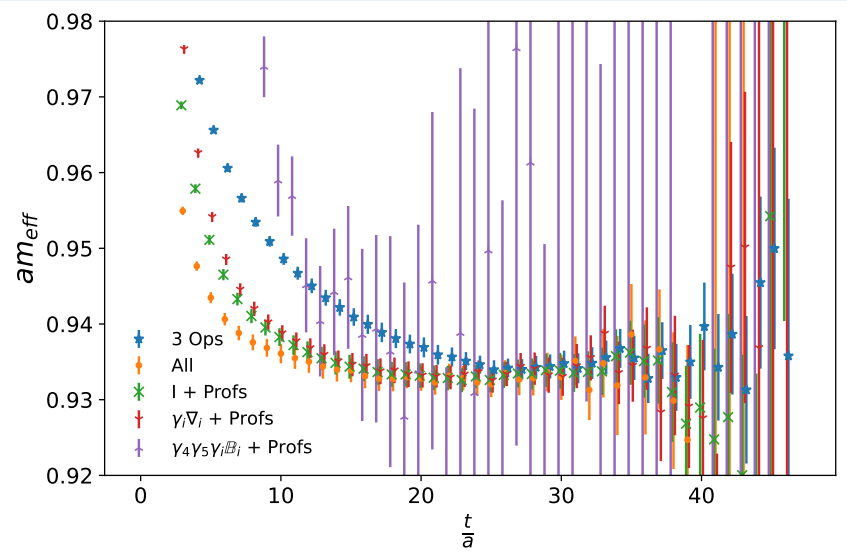
Extending the basis of operators

- ✓ Distillation profiles help us fully exploit one operator $\bar{q}\Gamma q$...
- ! but also can be combined with multiple Γ, e.g. HadSpec's wide basis of derivative-based operators. Phys. Rev. D 77, 034501 (2008)

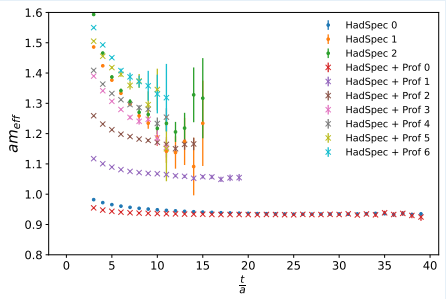
Connected-only A_1^{++} charmonium

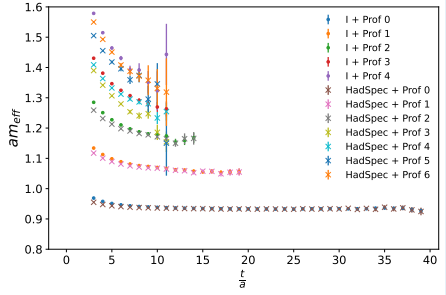


- Derivative-based operators can "remember" different continuum J: Very useful to resolve excitations!
- ✓ Profiles cheaply increase basis size and help!
- ✓ 7 profiles $\times$ 3 Γ = 21, comparable with HadSpec's complete basis.



- ✓ One (good) $\Gamma + 7$ profiles is **better** than 3 Γ's and standard distillation.
- ✓ 3 $\Gamma + 7$ profiles gives the best result at very little additional cost.





! 3 Γ's with standard distillation are too few and cannot resolve excitations well.

✓ 3 $\Gamma's$ with 7 profiles each fully exploits each operator and allows to resolve several excitations, including possible higher J.

✓ $\Gamma = \mathbb{I}$ with 7 profiles gives similarly clean results...

× ...but can **miss** higher excitations.

Conclusion: combine the approaches to get the benefit of both.

Conclusions

- Light meson and charmonium operators create states which overlap with light meson and charmonium states, i.e. they don't decouple. Direct measurement instead of OZI-based assumption.
- The mixing is very small and the 0^{-+} spectrum remains mostly stable with or without these effects. Contrary to scalar channel.
- Mixing effects shift down the η_c mass by 39(24) MeV. Consistent with some studies but needs higher precision!
- Additional mixing with glueball-like operators is non-zero but these operators are particularly noisy. Need multi-level!
- Using distillation profiles together with basis of multiple meson operators drastically improves the effective mass extraction. Improvement extends to other single- and multi-hadron operators.