Precision charmonium spectroscopy on CLS ensembles: an update

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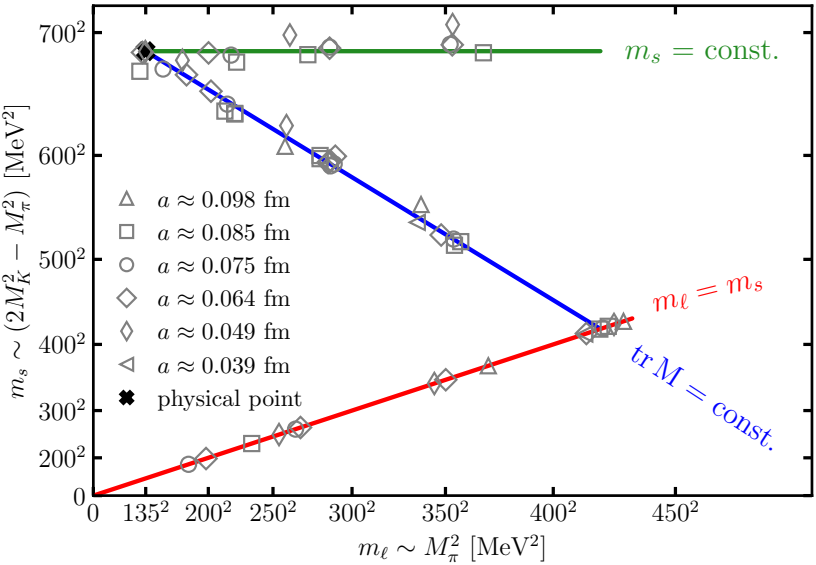
Motivation and scope

- Precision study of stable charmonia to set baseline for investigations of XYZ states.
- Compute low lying charmonium masses and decay constants from isoQCD quark-line connected correlation functions.
- Benchmark for studies including electromagnetic and/or annihilation effects.
- Cut-off effects: $O((m_c a)^2)$ for m_c but $a \not\ll m_c^{-1}$.
  Smaller for fine structure splittings $(m_c v)^2 \sim 0.3\, m_c^2$
  Even smaller for spin-independent splittings and $M_{\overline{\psi}} - 2M_{\overline{D}}$: $(m_c v^2)^2 \sim 0.1\, m_c^2$.

For the moment being, we only study $J = 0$ and $J = 1$ charmonia
(smeared interpolating operators without derivatives).

I will show results for M_{η_c} and $M_{J/\psi} - M_{\eta_c}$ on about 50 CLS gauge ensembles.

CLS ensembles in the quark mass plane



$N_f = 2 + 1$

NP $O(a)$ improved Wilson fermions

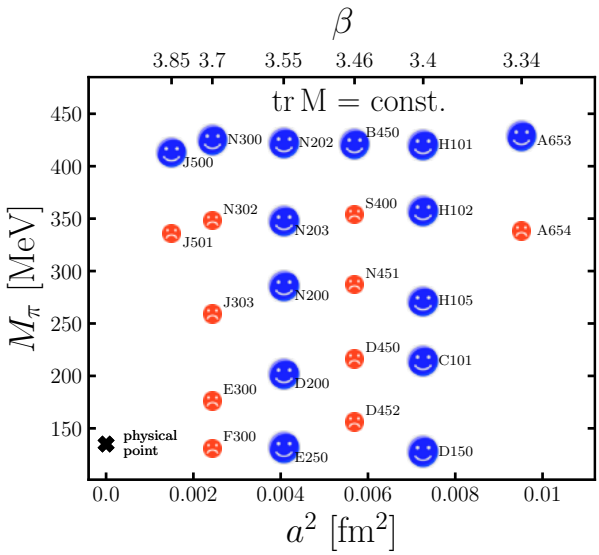
3 mass trajectories

6 lattice spacings

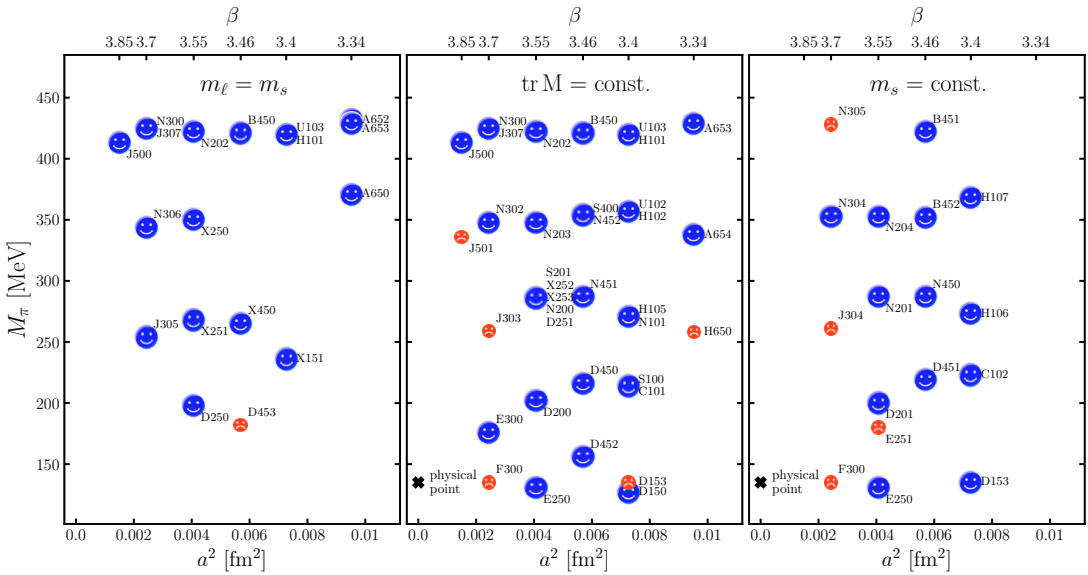
Many volumes, including $M_\pi L > 4$

Usually a few 1000 MDUs

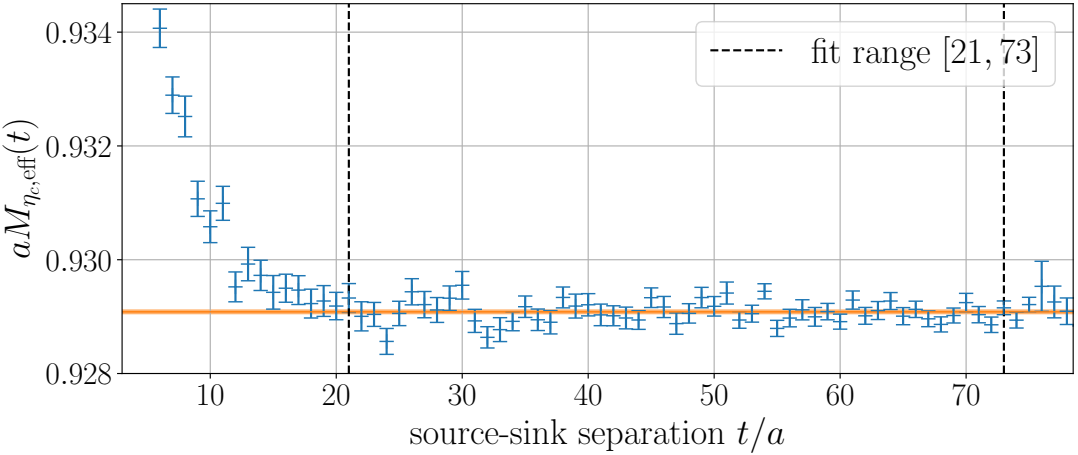
Analysis status at Lattice 2024



Ensembles used in this analysis



Example for a mass determination



$a \approx 0.064\,\mathrm{fm},\ M_\pi \approx 131\,\mathrm{MeV},\ V = 192 \cdot 96^3 a^4,\qquad aM(t) = \mathrm{arccosh}\left[\frac{C(t+a)+C(t-a)}{2C(t)}\right]$

Definitions and input

Scale set via $t_{0,\rm ph}^{-1/2} = 1.362(8)\,\text{GeV}$ [RQCD, 2211.03744], obtained from m_Ξ.

Using this scale, we define the isoQCD physical quark mass point ($\{m_q\} \to \{m_{q,\rm ph}\}$) as

$$M_\pi = 135\,\text{MeV}, \quad M_K = 494\,\text{MeV}, \quad M_{\overline{D}} = \frac{2M_D + M_{D_s}}{3} = 1899.4\,\text{MeV}.$$

Define $\quad \overline{M}^2 = \dfrac{2M_K^2 + M_\pi^2}{3} \approx 2B_0\overline{m}, \quad \delta M^2 = 2\left(M_K^2 - M_\pi^2\right) \approx 2B_0(m_s - m_\ell).$

To leading non-trivial order in ChPT $M_{\overline{\psi}}$ and $M_{\overline{D}}$ are independent of δM^2.

Quantities in lattice units: $\mathbf{M}$, $\mathbf{t}_0$ etc. Then for $m_q \to m_{q,\rm ph}$, $L \to \infty$, $a \to 0$:

$$\mathbf{M}_\psi(\{m_q\}, L, a)\sqrt{\mathbf{t}_0(\{m_q\}, a)} \longrightarrow M_\psi\sqrt{t_{0,\rm ph}}, \quad \mathbf{t_0}(\{m_q\}, a)a^2 \longrightarrow t_{0,\rm ph} \quad \text{etc.}$$

Use t_0^* (defined at the point where $M_\pi = M_K$ and $12t_0^*M_\pi^2 = 1.11$) for the continuum limit extrapolation:

$$\frac{1}{\mathbf{t}_0^*} = \frac{a^2}{t_0^*}.$$

Extrapolation strategy

Ideally, simulate at fixed values of the $O(a)$ improved coupling $g^2 = g_0^2[1 + b_g(g_0^2)a\overline{m}]$.

Instead, we simulate at fixed $\beta = 6/g_0^2$ values, thereby changing the lattice spacing by an $O(a)$ effect between points with different $\mathrm{Tr}\, M = 3\overline{m}$.

To cancel this, we must extrapolate dimensionless ratios of quantities, e.g., $\mathbf{M}_1/\mathbf{M}_2$ or $\mathbf{M}\sqrt{\mathbf{t}_0}$.

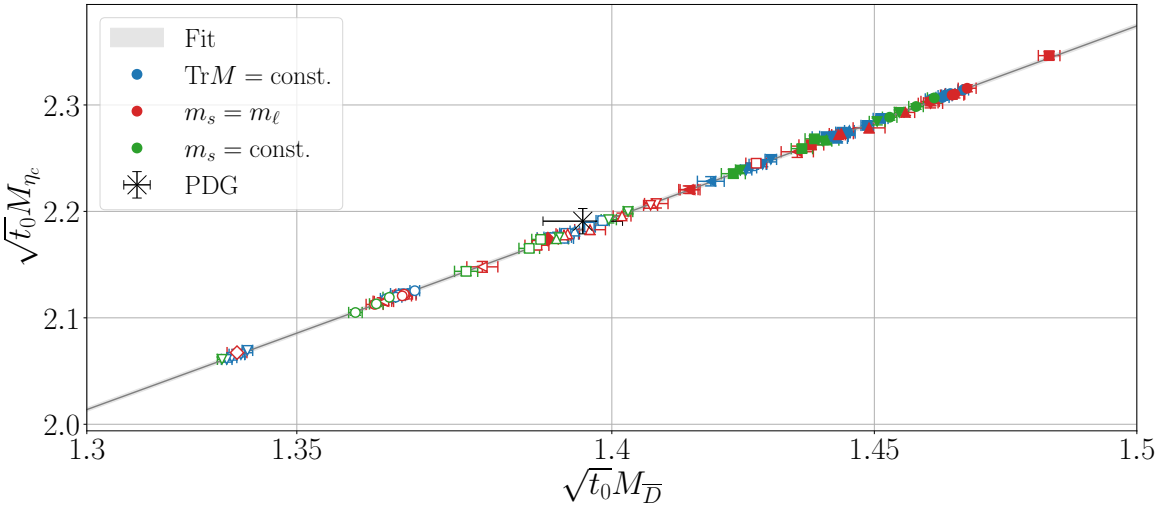
Therefore, we carry out a global fit to the charmonium mass data of the type

$$\mathbf{M}_\psi = \frac{1}{\sqrt{\mathbf{t}_0}}\left\{A_0 + c_c\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} + \bar{c}\,\overline{\mathbf{M}}^2\mathbf{t}_0 + c_{c2}\mathbf{M}_{\overline{D}}^2\mathbf{t}_0 + \frac{c_a}{\mathbf{t}_0^*} + \frac{c_{ac}}{\mathbf{t}_0^*}\mathbf{M}_{\overline{D}}\sqrt{\mathbf{t}_0} + \ldots\right\}.$$

The charmonium mass is then obtained from the dimensionless fit parameters A_0, c_c, $\bar{c}$ and c_{c2}, setting t_0, $M_{\overline{D}}$, M_π and M_K to their physical values.

Two charm quark masses on each ensemble, mostly bracketing the physical one.

Charm quark mass interpolation

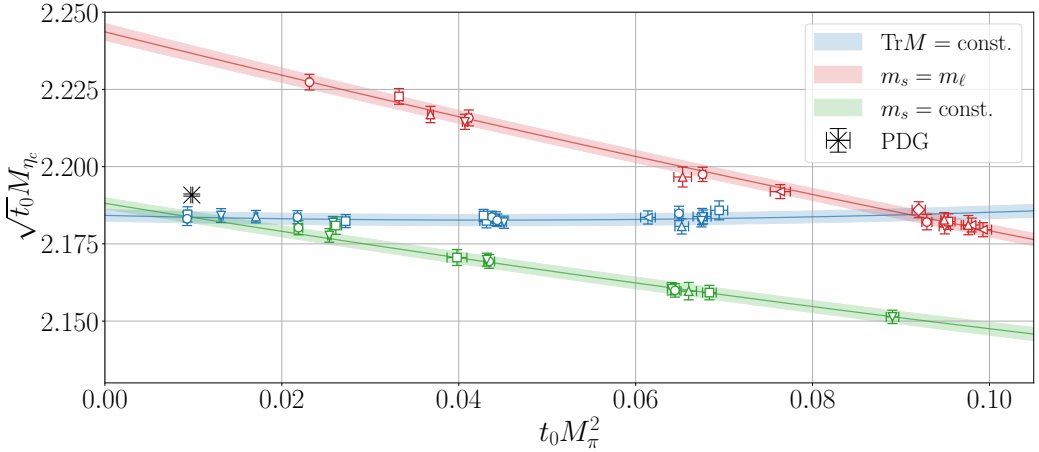


Smaller mass: open, Larger mass: solid.

Fit details

- On each ensemble, correlations between $\mathbf{M}_\pi$, $\mathbf{M}_K$ and the $\mathbf{M}_{\overline{D}}$ and $\mathbf{M}_\psi$ for the two charm quark masses are taken into account: 6×6 covariance matrices (5×5 for $m_s = m_\ell$).
- "x-errors" are included.
- $\mathbf{t}_0$ enters for each ensemble and $\mathbf{t}_0^*$ for each β-value (priors with errors).
- $t_{0,\mathrm{ph}}^{-1/2} = 1.362(8)\,\mathrm{GeV}$ is used in the end to define the physical point and convert into GeV.
- Cuts: $M_\pi < 450\,\mathrm{MeV}$, $L > 2.3\,\mathrm{fm}$, $LM_\pi > 3,5$.
- 6–15 fit parameters, 94 data points.
  Best fit in terms of χ^2/d.o.f. has 12 (10) parameters for M_{η_c} ($M_{J/\psi} - M_{\eta_c}$).
- AIC model average with weights $w_i \propto \exp\left(-\frac{1}{2}(\chi_i^2 + 2k_i)\right)$ for model i with k_i parameters.

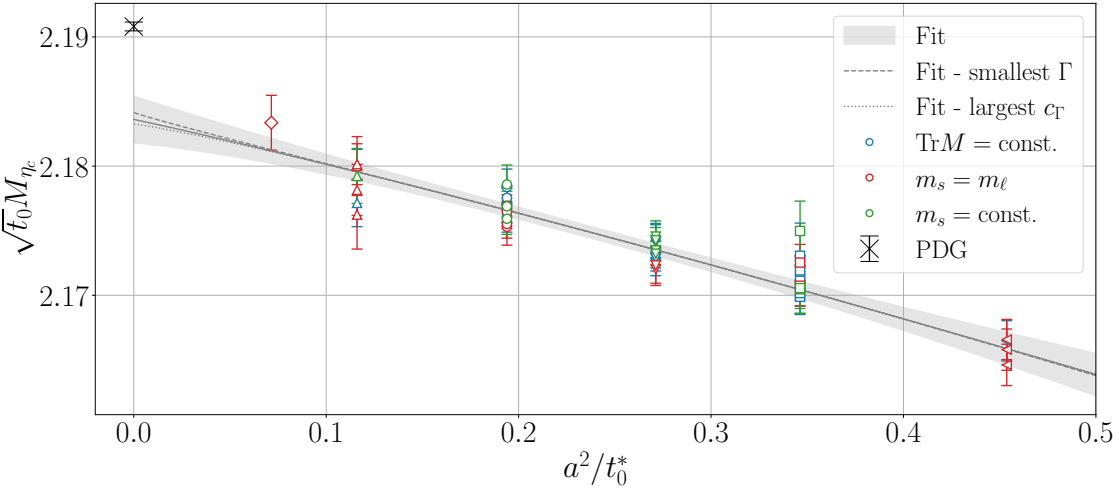
Pion mass dependence of the η_c mass (fit with largest AIC weight)



Error smaller than that of $\sqrt{t_{0,\rm ph}}$ because charm quark mass set via $\mathbf{M}_{\overline{D}} = M_{\overline{D}}\sqrt{t_{0,\rm ph}/\mathbf{t}_0}$.

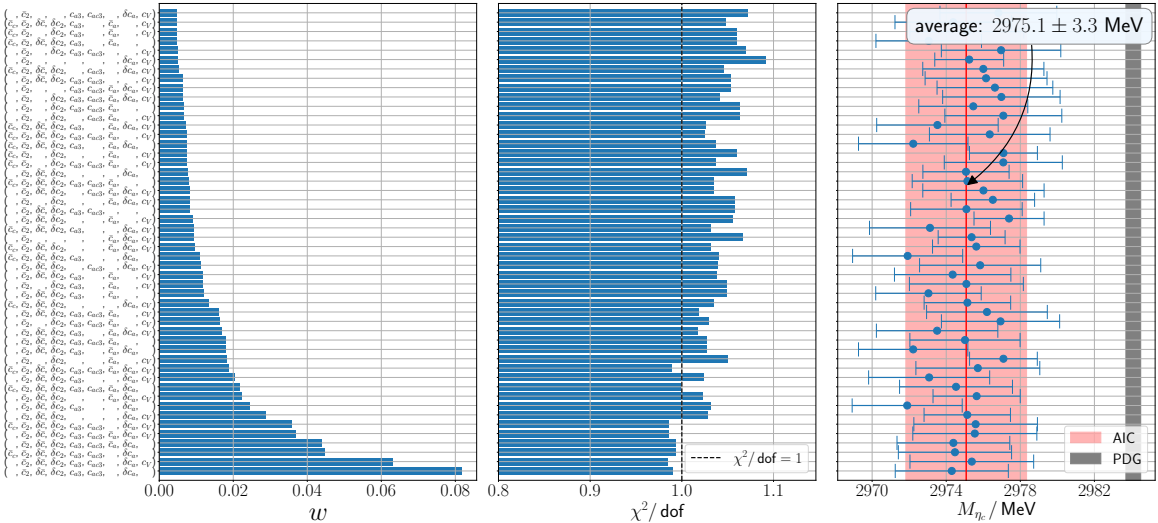
$\curvearrowright$ cancellations: the difference $M_{\eta_c} - 2M_{\overline{D}}$ approximately scales with $\sqrt{t_{0,\rm ph}}$.

The η_c mass: continuum limit (fit with largest AIC weight)



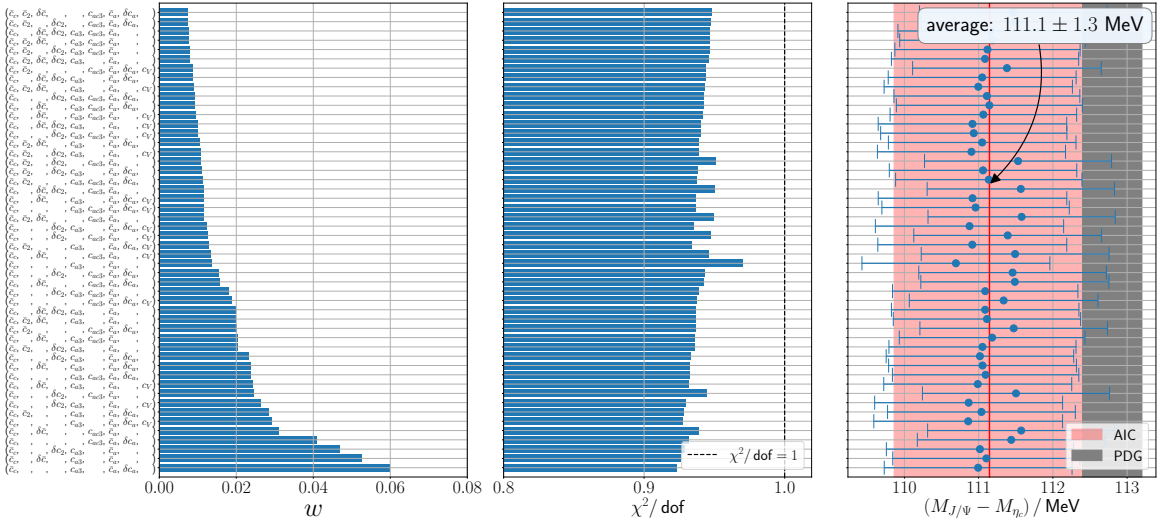
Additional models: $a^2 \mapsto g^{2\Gamma} a^2$ with either smallest Γ or term with dominant c_Γ coefficient.
[N Husung, P Marquard, R Sommer, 2111.02347]

Model averaging for the η_c mass

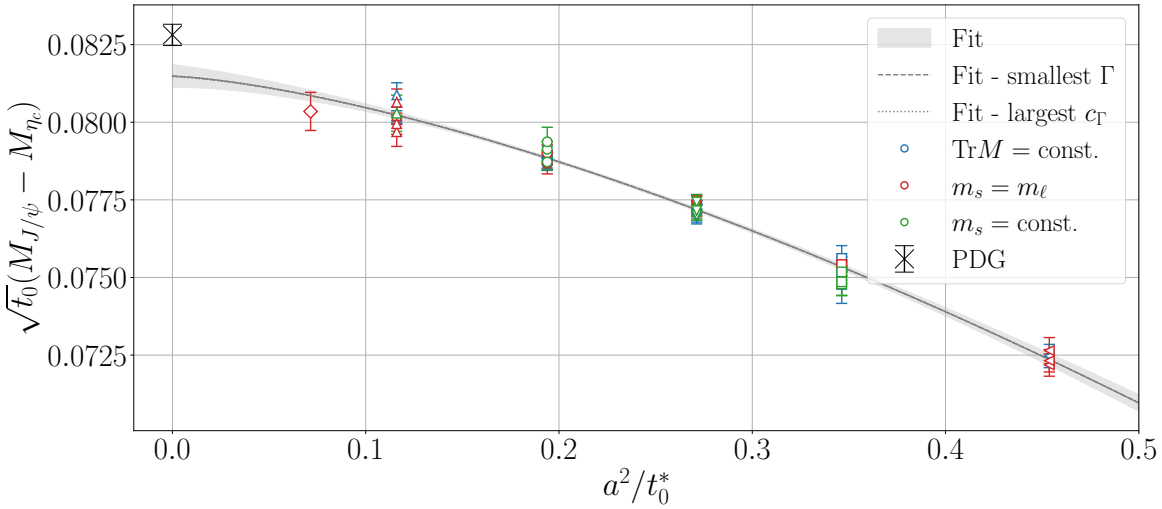


Also $M_{J/\psi}$ smaller than experiment for the isoQCD charm mass definition $M_{\overline{D}} = 1899.4\,\mathrm{MeV}$.

Model averaging for the $1S$ splitting

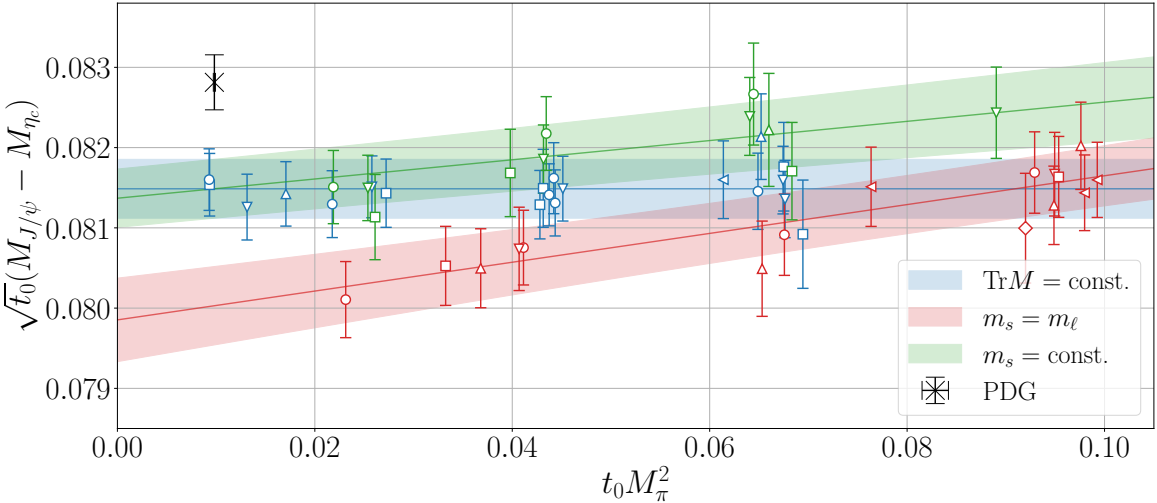


The $1S$ fine structure splitting: continuum limit



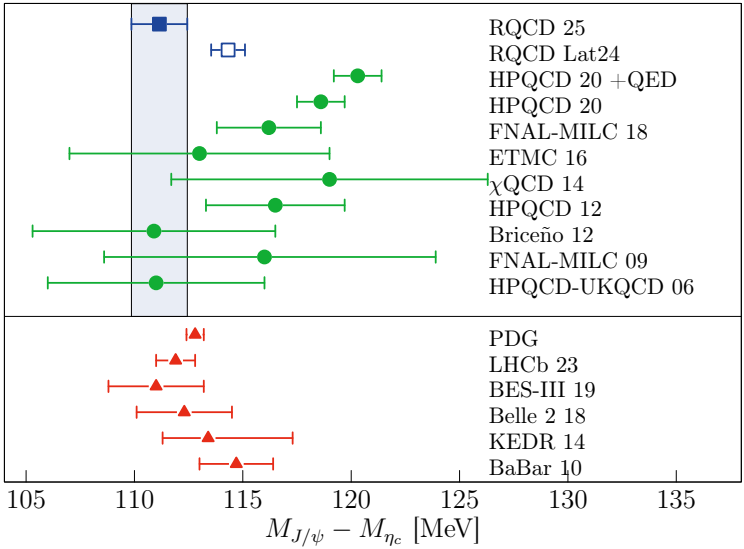
Fits with largest weights shown. Uncertainty of physical point definition ($t_{0,\mathrm{ph}}$) not included!

The $1S$ fine structure splitting: pion mass dependence

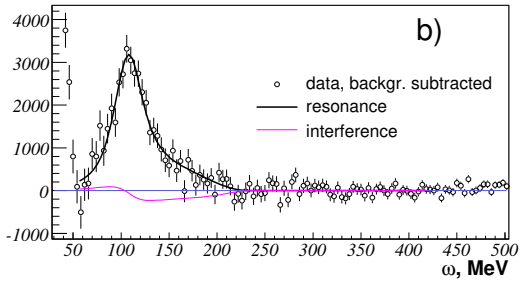


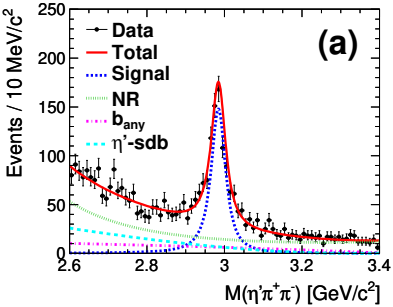
Fits with largest weights shown. Uncertainty of physical point definition ($t_{0,\mathrm{ph}}$) not included!

The fine structure splitting in isoQCD. Connected two-point function only.



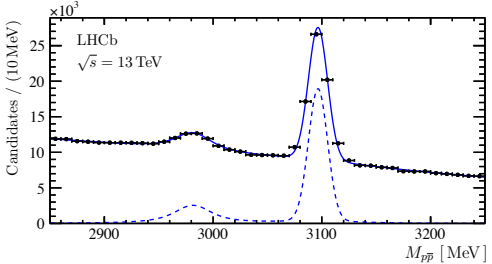
Experimental status

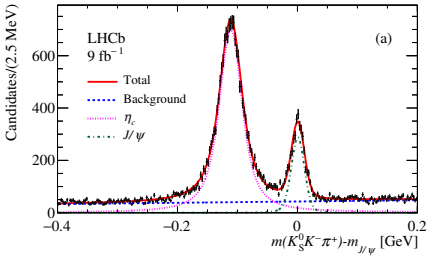




[KEDR, 1406.7644]

[Belle, 1805.03044]





[LHCb, 1911.03326]

[LHCb, 2304.14891]

Summary and outlook

- From potential models and [HPQCD, 2005.01845], one would expect an increase of the $1S$ fine structure splitting by up to 2 MeV, due to electromagnetism.
- Given the decay width $\Gamma_{\eta_c} = 30.5(5)\,\mathrm{MeV}$, the agreement with experiment may be surprising.
  $M_{\eta_c} = 2975(3)\,\mathrm{MeV}$ vs. $2984.1(4)\,\mathrm{MeV}$ (PDG) (including EM, the difference is $2\,\sigma$).
- Not straightforward to include annihilation diagrams: the axial anomaly may decrease the splitting but many decay channels and mixing with other flavour-singlet 0^{-+} states.
- Sea quark effects are very significant (as expected): the quenched splitting of $77(2)(6)\,\mathrm{MeV}$ [QCD-TARO, hep-lat/0307004] becomes $111.1(1.3)\,\mathrm{MeV}$ in isoQCD.

Future plans

- Add a few ensembles.
- Compute masses and decay constants of other charmonium states.