Heavy hadron spectrum from 2+1+1 flavor MILC lattices

Sabiar Shaikh

NISER, Bhubaneswar

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Collaborators: Protick Mohanta, M. Padmanath, Subhasish Basak

Outline

1. Introduction
2. Action and Tuning
3. Simulation details
4. Hadron mass spectrum
5. Summary

Introduction

- **Mass spectra of heavy hadrons** containing one or more bottom quarks, including their **hyperfine splittings** and **mass differences**.
- Calculations are performed on **MILC's** $N_f = 2 + 1 + 1$ **HISQ gauge ensembles** at three lattice spacings.
- Valence quarks are simulated using:
  - NRQCD action for bottom quarks,
  - Anisotropic Clover action for charm quarks,
  - $O(a)$-improved Wilson–Clover for strange and light (up/down).
- Strange quark mass is tuned to a fictitious $\bar{s}\gamma_5 s$ of mass **687.2 MeV**.
- The bare bottom and charm quark masses are tuned using the **kinetic mass** and **pole mass** of corresponding heavy–strange mesons.
- Heavy hadron operators (parity $= +$ and spin $= \frac{1}{2}, \frac{3}{2}$) are constructed with all possible combinations of bottom, charm, strange, and light quarks to explore **ground-state masses** of bottom mesons and baryons.

Clover Action and Strange:

- We employ **Wilson–Clover action** for strange:

$$S_{\text{clover}} = \sum_n \bar{\psi}(n)\psi(n) - \kappa\Big[\sum_{n,\mu} \bar{\psi}(n)(1-\gamma_\mu)U_\mu(n)\psi(n+\hat{\mu}) + \sum_{n,\mu} \bar{\psi}(n)(1+\gamma_\mu)U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})\Big] - \kappa C_{\text{sw}} \sum_{n,\mu<\nu} \bar{\psi}(n)\sigma_{\mu\nu}F_{\mu\nu}\psi(n)$$

Here, $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, $\quad \kappa = \frac{1}{2(m+4)}$.

- The strange-quark hopping parameter κ_s is tuned using the **fictitious** η_s.
- From leading-order Chiral Perturbation Theory: Phys. Rev. D 101, 094503 (2020)

$$m_{\eta_s} = \sqrt{2m_K^2 - m_\pi^2} = 687.2 \text{ MeV},$$

where m_K and m_π are isospin-averaged masses.

Four component NRQCD action up to $O(v^4)$ for bottom:

- **The full Lagrangian**

$$\mathcal{L} = \psi^\dagger(D_4 + H_0 + \delta H)\psi \; + \; \xi^\dagger(D_4 - \overline{H_0} - \overline{\delta H})\xi$$

ψ, ξ are four component. Let $H = H_0 + \delta H$ and $\overline{H} = \overline{H_0} + \overline{\delta H}$. The EOM for $\psi^\dagger$:

$$D_4\psi(x,t) + H\psi(x,t) = 0$$
$$\Rightarrow \quad U_t(x,t)\psi(x,t+1) - \psi(x,t) + H\psi(x,t) = 0$$

- Green's function obeys: (showing how to evolve the propagator in time.)

$$U_t(x,t)G_\psi(x,t+1;0,0) - (1-H)G_\psi(x,t;0,0) = \delta_{x,0}\delta_{t,0}$$
$$\Rightarrow \quad G_\psi(x,t+1;0,0) = U_t^\dagger(x,t)(1-H)G_\psi(x,t;0,0)$$

- From renormalization considerations Phys. Rev. D 46, 4052 (1992):

$$G_\psi(x,t+1;0,0) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2}\right)U_t^\dagger(x,t)$$
$$\left(1-\frac{H_0}{2}\right)\left(1-\frac{\delta H}{2}\right)G_\psi(x,t;0,0)$$

- For stability purposes, we modify

$$G_\psi(x,t+1;0,0) = \left(1-\frac{\delta H}{2}\right)\left(1-\frac{H_0}{2n}\right)^n U_t^\dagger(x,t) \left(1-\frac{H_0}{2n}\right)^n \left(1-\frac{\delta H}{2}\right) G_\psi(x,t;0,0)$$

- The EOM for $\xi^\dagger$:

$$\begin{aligned} & D_4\xi(x,t) - \overline{H}\xi(x,t) = 0 \\ \Rightarrow \quad & \xi(x,t) - U_t^\dagger(x,t-1)\xi(x,t-1) - \overline{H}\xi(x,t) = 0 \\ \Rightarrow \quad & U_t^\dagger(x,t-1)G_\xi(x,t-1;0,0) - (1-\overline{H})G_\xi(x,t;0,0) = \delta_{x,0}\delta_{t,0} \\ \Rightarrow \quad & G_\xi(x,t-1;0,0) = U_t(x,t-1)(1-\overline{H})G_\xi(x,t;0,0) \end{aligned}$$

- Here, n is the factor introduced to ensure numerical stability at small am_b, where $n > 3/2m_b$. Phys. Rev. D 101, 094503 (2020)

- The heavy-quark and heavy-antiquark Hamiltonians: Phys. Rev. D 85, 054509 (2012)

$$
\begin{aligned}
H_0 &= -\frac{\Delta^2}{2m_b} = \overline{H_0} \\
\delta H_1 &= -c_1 \frac{(\Delta^2)^2}{8m_b^3} = \overline{\delta H_1} \\
\delta H_2 &= c_2 \frac{i}{8m_b^2}(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla) = -\overline{\delta H_2} \\
\delta H_3 &= -c_3 \frac{1}{8m_b^2}\sigma.(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}) = -\overline{\delta H_3} \\
\delta H_4 &= -c_4 \frac{1}{2m_b}\sigma \cdot \tilde{\mathbf{B}} = \overline{\delta H_4} \\
\delta H_5 &= c_5 \frac{\Delta^4}{24m_b} = \overline{\delta H_5} \\
\delta H_6 &= -c_6 \frac{(\Delta^2)^2}{16nm_b^2} = \overline{\delta H_6}
\end{aligned}
$$

- ∇ is symmetric lattice derivative, and Δ^2, Δ^4 are the lattice discretization of the continuum $\sum_i D_i^2$, $\sum_i D_i^4$. $c_1, c_2, c_3, ..., c_6 = 1$ (tree level).

Bottom-Quark Tuning

- The b-quark mass is tuned using the **kinetic mass** of the B_s:

$$E(\mathbf{P}) = M_1 + \frac{\mathbf{P}^2}{2M_2}, \qquad Phys.Rev.D85, 054509(2012)$$

  where M_1 is the **pole mass** and M_2 is the **kinetic mass**.

- Tuning condition is set such that the $M_2(B_s)$ matches, $M_{B_s}^{\mathrm{PDG}} = 5366.93$ MeV.

- The **hyperfine splitting**: $\Delta_{\mathrm{HFS}} = M_{B_s^*}^{\mathrm{rest}} - M_{B_s}^{\mathrm{rest}} = 48.5$ MeV (PDG), and gives consistent results using pole mass at zero momentum.

- The coefficients $c_1, c_2, c_3, c_5, c_6, n$ and tadpole factor u_0 are taken from HPQCD results. Phys. Rev. D 85, 054509 (2012)

- The bare parameters m_b and c_4 are tuned to reproduce the PDG spin-averaged mass and hyperfine splitting.

Relativistic Heavy Quark (RHQ) Action and Charm tuning

- The **RHQ action** is employed for simulating the charm quark. It modifies the standard Wilson–Clover action by introducing an **anisotropy factor** ν:

$$\frac{S_{\mathrm{RHQ}}}{m_c + 1 + 3\nu} = \sum_n \bar{\psi}(n)\psi(n) - \kappa_Q \Big[\sum_n \bar{\psi}(n)(1-\gamma_0)U_0(n)\psi(n+\hat{0}) + \sum_n \bar{\psi}(n)(1+\gamma_0)U_0^\dagger(n-\hat{0})\psi(n-\hat{0}) + \nu \sum_{n,i} \bar{\psi}(n)(1-\gamma_i)U_i(n)\psi(n+\hat{i}) + \nu \sum_{n,i} \bar{\psi}(n)(1+\gamma_i)U_i^\dagger(n-\hat{i})\psi(n-\hat{i}) \Big] - \kappa_Q C_P \sum_{n,\mu<\nu} \bar{\psi}(n)\sigma_{\mu\nu}F_{\mu\nu}\psi(n).$$

- Here $\kappa_Q = \dfrac{1}{2(m_c + 1 + 3\nu)}$.
- The parameters $\{m_c,\ C_P,\ \nu\}$ are tuned by matching:
  - Spin-averaged meson mass,
  - Hyperfine splitting,
  - Velocity of light ($c^2 \approx 1$).

Tuning of RHQ Parameters

- The spin-averaged D_s meson mass: $\bar{M}_{D_s} = \frac{1}{4} M_{D_s} + \frac{3}{4} M_{D_s^*}$
- The PDG spin-averaged mass of the D_s

$$\bar{M}_{D_s}^{(\text{exp})} = 2.076 \text{ GeV (PDG)}.$$

- **Clover coefficient** C_P is tuned such that the **hyperfine splitting**

$$\Delta M_{D_s} = M_{D_s^*} - M_{D_s}$$

matches the PDG value $\Delta M_{D_s}^{(\text{exp})} = 143.8$ MeV (PDG)

- **Anisotropy parameter** ν is adjusted to satisfy the relativistic dispersion relation:

$$E^2(\vec{p}) = E^2(0) + \vec{p}^{\,2},$$

ensuring the speed of light $c^2 \simeq 1$ on the lattice.

- The tuning is performed using the smallest nonzero momentum

$$|\vec{p}| = \frac{2\pi}{L}.$$

Bottom baryon operators

- We have used NRQCD for *bottom* and anisotropic clover action for *charm* and clover action for *lighter* quarks.
- Triple bottom
  1. $\left(\mathcal{O}_k^{hhh}\right)_\alpha = \epsilon_{abc}\left(Q^{a\,T} C\gamma_k Q^b\right) Q_\alpha^c,$

     $Q = \theta(N_t/2 - t)\begin{pmatrix}\psi_h \\ 0\end{pmatrix} + \begin{pmatrix}0 \\ \chi_h\end{pmatrix}\theta(t + N_t/2)$
- Double bottom
  1. $(\mathcal{O}_k^{hhl})_\alpha = \epsilon_{abc}\left(Q^{a\,T} C\gamma_k Q^b\right) l_\alpha^c$
  2. $(\mathcal{O}_k^{hlh})_\alpha = \epsilon_{abc}\left(Q^{a\,T} C\gamma_k l^b\right) Q_\alpha^c$
  3. $(\mathcal{O}_5^{hlh})_\alpha = \epsilon_{abc}(Q^{a\,T} C\gamma_5 l^b) Q_\alpha^c$
- Single bottom
  1. $(\mathcal{O}_5^{hl_1l_2})_\alpha = \epsilon_{abc}(l_1^{a\,T} C\gamma_5 l_2^b) Q_\alpha^c$
  2. $(\mathcal{O}_k^{hl_2l_1})_\alpha = \epsilon_{abc}(Q^{a\,T} C\gamma_k l_2^b) l_{1\alpha}^c$
  3. $(\mathcal{O}_5^{hl_2l_1})_\alpha = \epsilon_{abc}(Q^{a\,T} C\gamma_5 l_2^b) l_{1\alpha}^c$

Simulation Details

Ensembles: Phys. Rev. D 85, 054509 (2012)

Name	$L^3 \times T$	a (fm)	No. of configs.
l1648f211b580m013m065m838a	$16^3 \times 48$	0.15	700
l2464f211b600m0102m0509m635a	$24^3 \times 64$	0.12	700
l3296f211b630m0074m037m440e	$32^3 \times 96$	0.09	50*

*It's ongoing. We have to add more configurations.

Point Source:

Strange quark tuning (κ_s):

We set $C_{sw} = u_0^{-3}$ PRD 85, 114506 (2012),
u_0 values are taken from PRD 82, 074501 (2010).

$L^3 \times T$	κ_s	C_{sw}
$16^3 \times 48$	0.14101	1.598
$24^3 \times 64$	0.140015	1.552
$32^3 \times 96$	0.138613	1.497

Tuning parameters

Charm quark tuning m_c, C_p, ν:

$L^3 \times T$	m_c	C_p	ν
$16^3 \times 48$	0.6647	2.5822	1.2488
$24^3 \times 64$	0.2033	1.9264	1.2168
$32^3 \times 96$	-0.556	0.36	1.52

Bottom quark tuning m_b, c_4:

$L^3 \times T$	m_b	c_4
$16^3 \times 48$	3.8167	1.4456
$24^3 \times 64$	2.2498	1.0336
$32^3 \times 96$	1.44	0.9

Unitarity in four component NRQCD

- Effective masses from the two parities match within statistical errors over the plateau region. It confirms unitarity occurs in the four-component NRQCD.

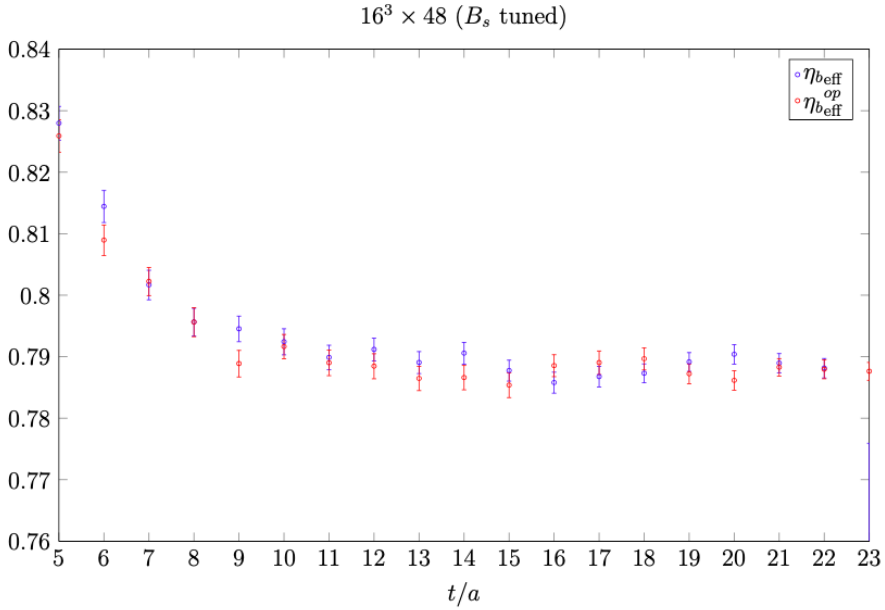


Figure: Effective mass vs t

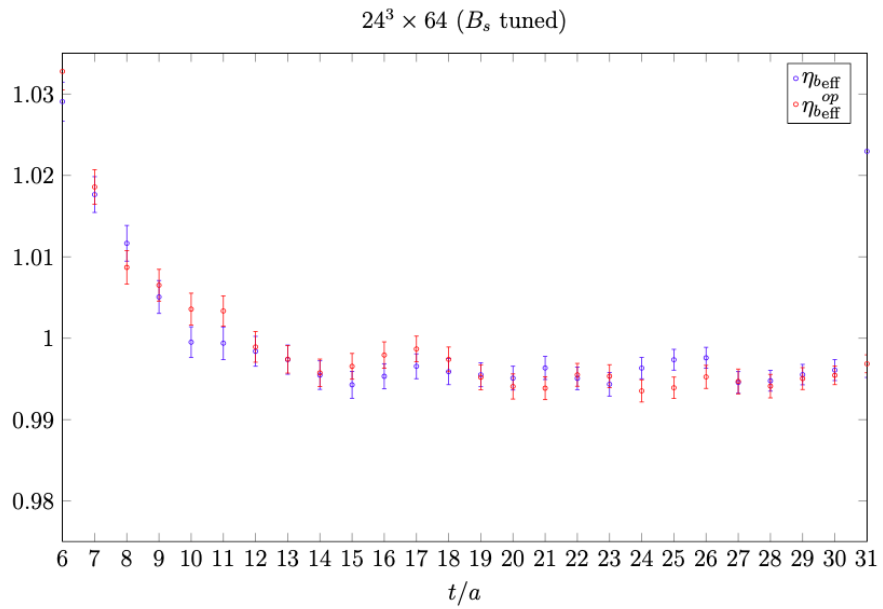


Figure: Effective mass vs t

Mass difference and offset

Charm tuning using D_s and D_s^* (RHQ):

$L^3 \times T$	$\bar{D}_s$ (MeV)	ΔD_s (MeV)	c
$16^3 \times 48$	2075.96	143.79	1.0
$24^3 \times 64$	2076.27	143.81	1.0
PDG	2076.24	143.8	1.0

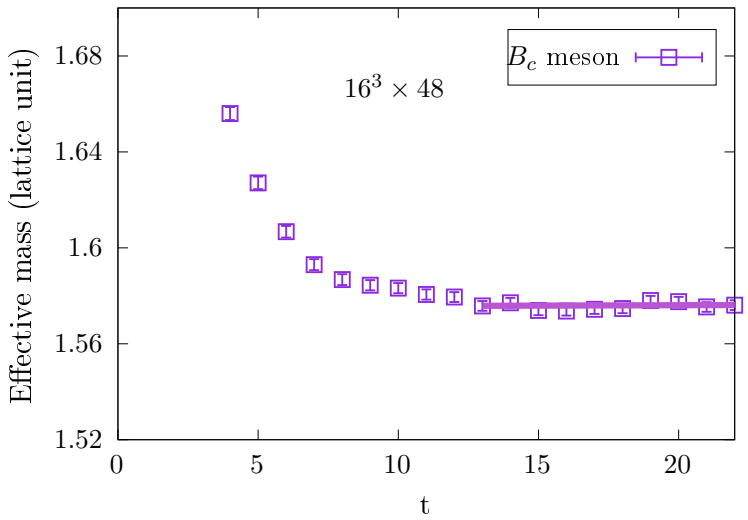
Bottom tuning using B_s and B_s^* (NRQCD):

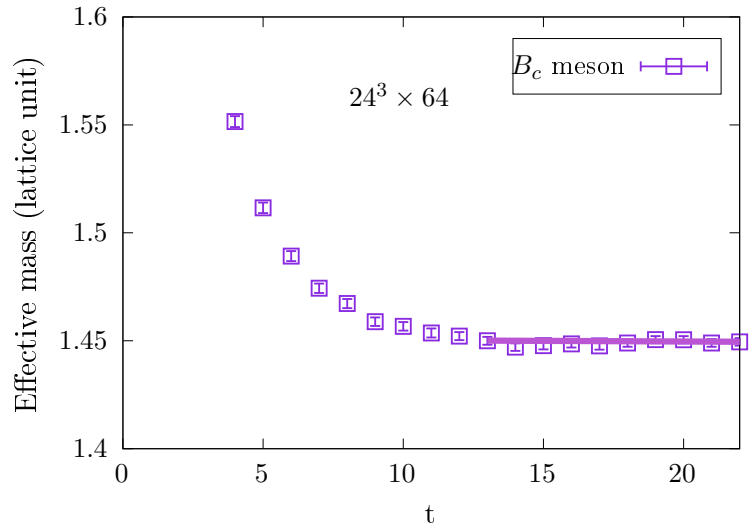
$L^3 \times T$	B_s kinetic mass (MeV)	ΔB_s (MeV)
$16^3 \times 48$	5367.27	48.49
$24^3 \times 64$	5366.92	48.5
PDG	5366.93	48.5

$L^3 \times T$	B_s Pole Mass (MeV)	B_s Kinetic Mass (MeV)	Offset (MeV)
$16^3 \times 48$	1170	5367	4197
$24^3 \times 64$	1435	5367	3932

B_c meson mass

- B_c contains both a bottom and a charm quark; its mass is sensitive to the tuning of both the NRQCD (b) and RHQ (c) actions simultaneously.
- The effective mass signal is reasonable.

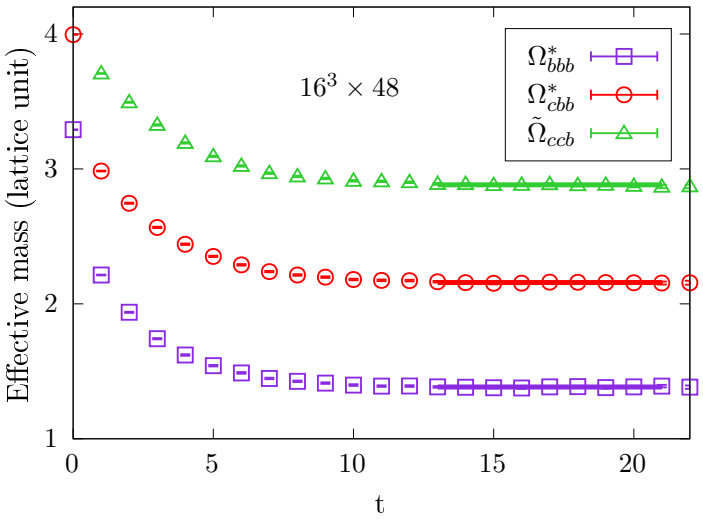


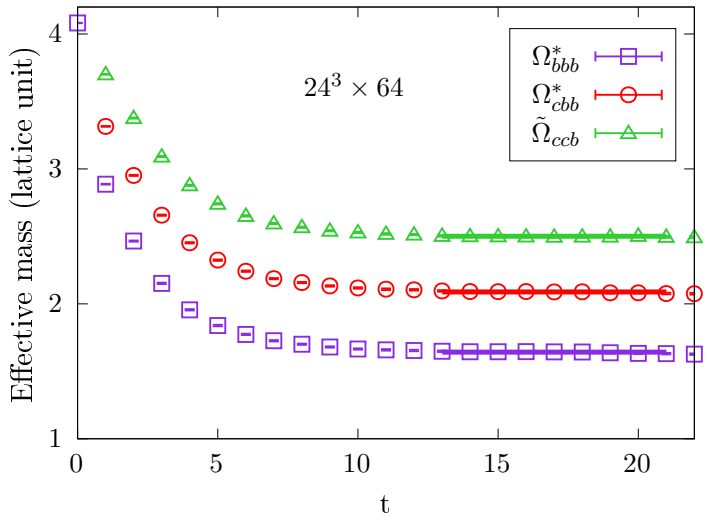


$L^3 \times T$	B_c (MeV)
$16^3 \times 48$	6276
$24^3 \times 64$	6314
PDG	6275

Baryon Effective mass

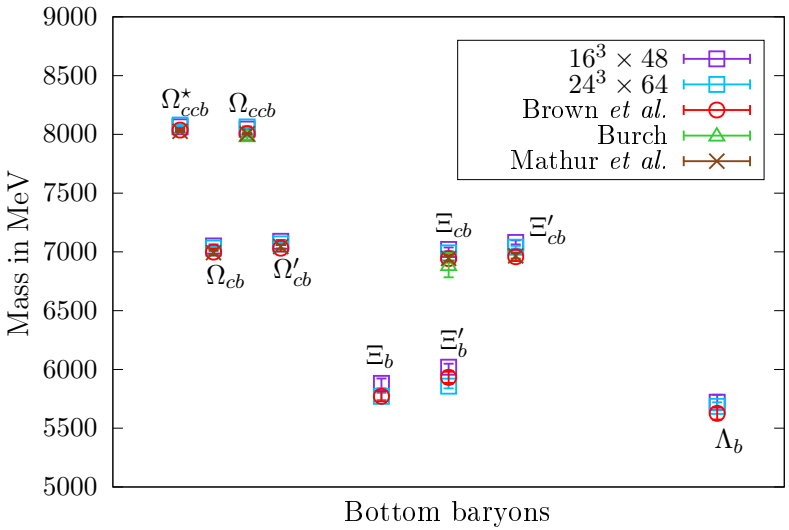
- The effective mass signal is reasonable.

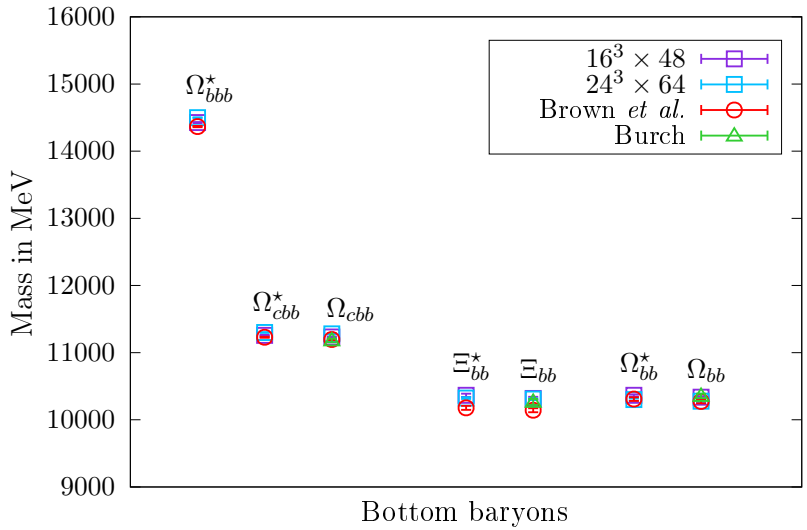




Mass spectrum

- Our results ($16^3 \times 48$ and $24^3 \times 64$) are consistent with other lattice results.





Summary

- Lattice QCD study of **heavy hadron mass spectra** with one or more bottom quarks using **2+1+1 flavor HISQ ensembles** from MILC.
- Calculations will be performed at **three lattice spacings** for **continuum extrapolation** and to control discretization errors.
- A combination of actions was employed:
  - NRQCD for bottom quarks
  - RHQ (Relativistic Heavy Quark) for charm quarks
  - Clover for strange and light quarks
- The quark masses were tuned using known hadron masses: η_s, D_s, B_s.
- Focus on **ground-state masses**; future work includes **hyperfine splittings** and **mass differences**.
- Our results are consistent with other lattice results, confirming the reliability of our mixed-action setup.

Thank You