T_{cc} pole trajectory

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in collaboration with

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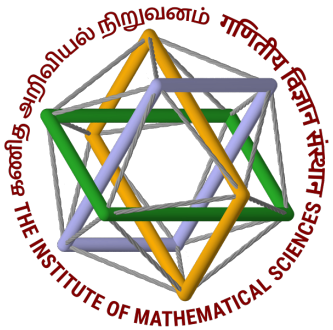


Introduction and Motivation

- LHCb discovered doubly charmed tetraquark T_{cc} just 0.36MeV below $D^0 D^{\star +}$ threshold. [R. Aaij *et al.*, Nat. Commun. 13, 3351 (2022).]
- Though lattice QCD clearly indicates the existence of deeply bound doubly bottom tetraquark $T_{bb}(bb\bar{u}\bar{d})$, the existence of doubly charm tetraquark $T_{cc}(cc\bar{u}\bar{d})$ is still unsettled.
- Inclusion of the diqauark-antidiquark operator.
- Worsening of the $D^\star$ plateau for lighter pion mass indicates the three body effects.

$$T_{cc}(cc\bar{u}\bar{d}) \to D\, D^\star \to D\, D\, \pi$$

- In this talk we report our on going study of the decay parameters of T_{cc} where we used RHQ action for the charm and clover action for the light quarks.

Hint from Singly Heavy Baryon

- To understand the role of diqauark-antidiquark operator consider the mass difference of $J^P = \frac{1}{2}^+$ singly heavy baryons Σ_Q and Λ_Q where $Q = b, c$; $\Sigma_b - \Lambda_b$ ∼191 MeV, $\Sigma_c - \Lambda_c \sim 167$ MeV.
- The interpolating operators are $(\Sigma_Q)_\alpha = \epsilon_{abc}(u^{aT} C\gamma_k d^b) Q^c_\alpha$ and $(\Lambda_Q)_\alpha = \epsilon_{abc}(u^{aT} C\gamma_5 d^b) Q^c_\alpha$. Both the operators have same quark content.
- With a pion mass $m_\pi \sim 500$ MeV these two operators generate a mass difference $\Sigma_Q - \Lambda_Q \sim 30$ MeV. As we reduce the pion mass these two operators produce significant mass splitting. [K. C. Bowler *et al.*, PRD 54, 3619 (1996).]
- For T_{cc} the diqauark-antidiquark operator is related to the Λ_Q operator by heavy quark-diqauark symmetry.

Heavy quark action

- We have used the relativistic heavy quark action for the charm quark

$$S = a^4 \sum_x \bar{\psi}(x) \left[m_Q + \left(\gamma_0 D_0 - \frac{a}{2} D_0^2 \right) + \zeta \left(\gamma_i D_i - \frac{a}{2} D_i^2 \right) - \frac{a}{4} c_P \, \sigma_{\mu\nu} F_{\mu\nu} \right] \psi(x)$$

- We used spin-average mass $\overline{M}_{D_s} = \frac{1}{4}\left(M_{D_s} + 3M_{D_s^*}\right)$, hyperfine splitting $\Delta M_{D_s} = M_{D_s^*} - M_{D_s}$ and velocity of light $E^2_{D_s^*}(\mathbf{p}) = c^2\mathbf{p}^2 + M^2_{D_s^*}$ for the tuning of RHQ action parameters $\{am_Q,\, c_P,\, \zeta\}$.
- We performed multivariate linear regression analysis

$$\begin{pmatrix} \overline{M}_{D_s} \\ \Delta M_{D_s} \\ c \end{pmatrix} = \mathbf{J} \cdot \begin{pmatrix} am_Q \\ c_P \\ \zeta \end{pmatrix} + \mathbf{A}$$

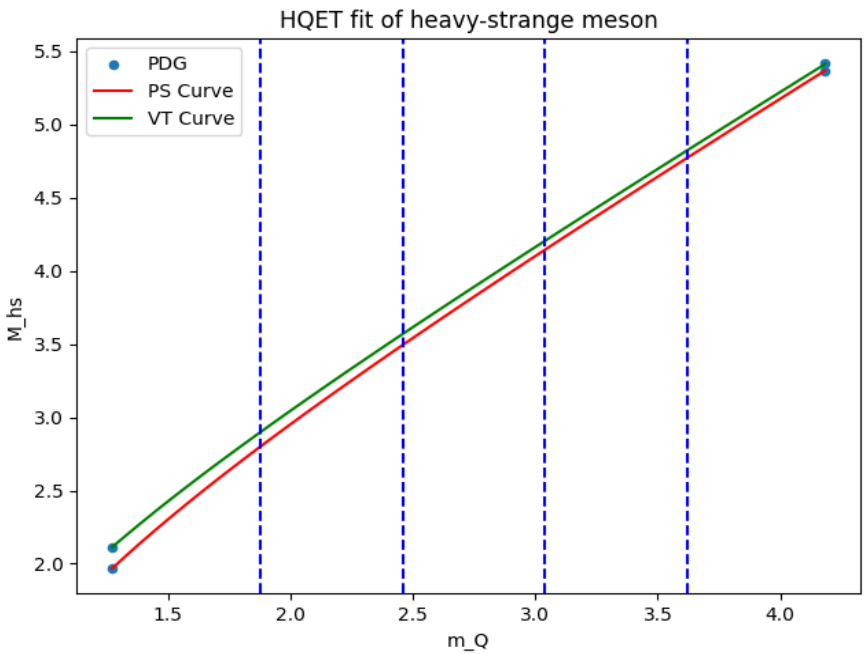
  The tuned parameters are obtained by matching $a\overline{M}^{\text{latt}}_{c\bar{c}} \approx 2076.2$ MeV, $a\Delta M^{\text{latt}}_{D_s} \approx 143.8$ MeV and $c^{\text{latt}} \approx 1$ and inverting the above relation.
- For strange and lighter quarks we employed the standard Wilson-Clover action. The strange quark mass is tuned to match the lattice value for fictitious $\eta_s(s,\bar{s})$ meson mass with 688.5 MeV. The light quark is varied from $m_\pi = m_{\eta_s}$ to $m_\pi \approx 470$ MeV.

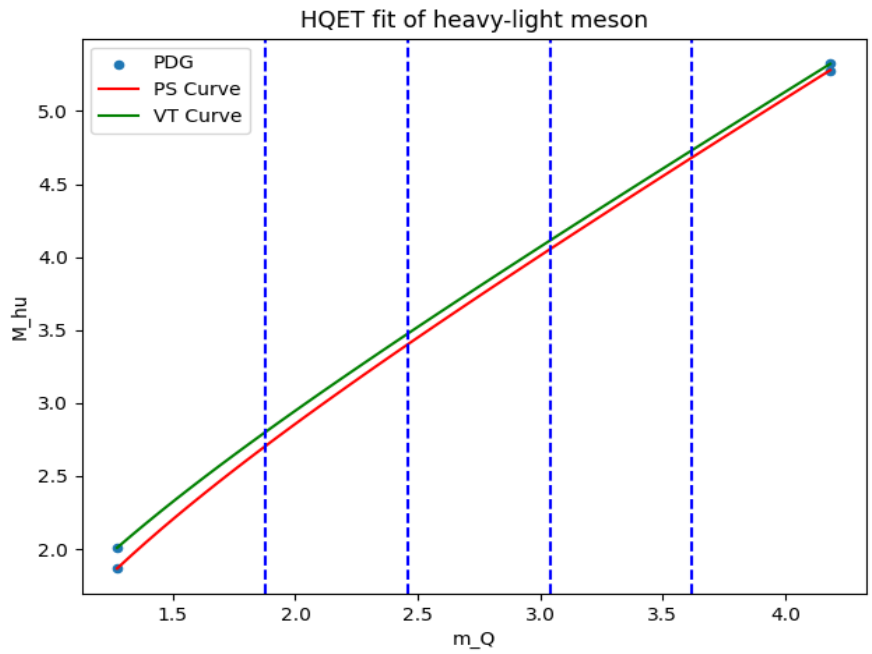
Heavy-light meson mass

In order to tune the RHQ action parameter we need both the heavy-strange pseudoscalar and vector meson masses. HQET provides formulae for masses of such mesons [Heavy Quark Physics, A Manohar & M Wise, 2000.] –

$$M_{hl} = m_h + \bar{\Lambda} - \frac{\lambda_1}{2m_h} - \frac{3\lambda_2}{2m_h}$$

$$M^*_{hl} = m_h + \bar{\Lambda} - \frac{\lambda_1}{2m_h} + \frac{\lambda_2}{2m_h}$$





Lattice setup

- We made use of $N_f = 2+1+1$ HISQ ensembles generated by MILC collaboration. Here we are showing results obtained on $L^3 \times T = 16^3 \times 48$ ensemble with lattice spacing $a \approx 0.15$ fm. No. of configuration used is 700. The tuned value of the RHQ action parameters are given below

Table: Parameters used in RHQ action.

Ensemble	m_Q	ζ	C_p	$\overline{M}_{D_s}$ (MeV)	ΔM_{D_s} (MeV)	c^{latt}
$16^3 \times 48$	0.6647	1.2488	2.5822	2075.96	143.8	1.00016

- The light κ used in our analysis and the corresponding pion masses are -

Table: Light κ table.

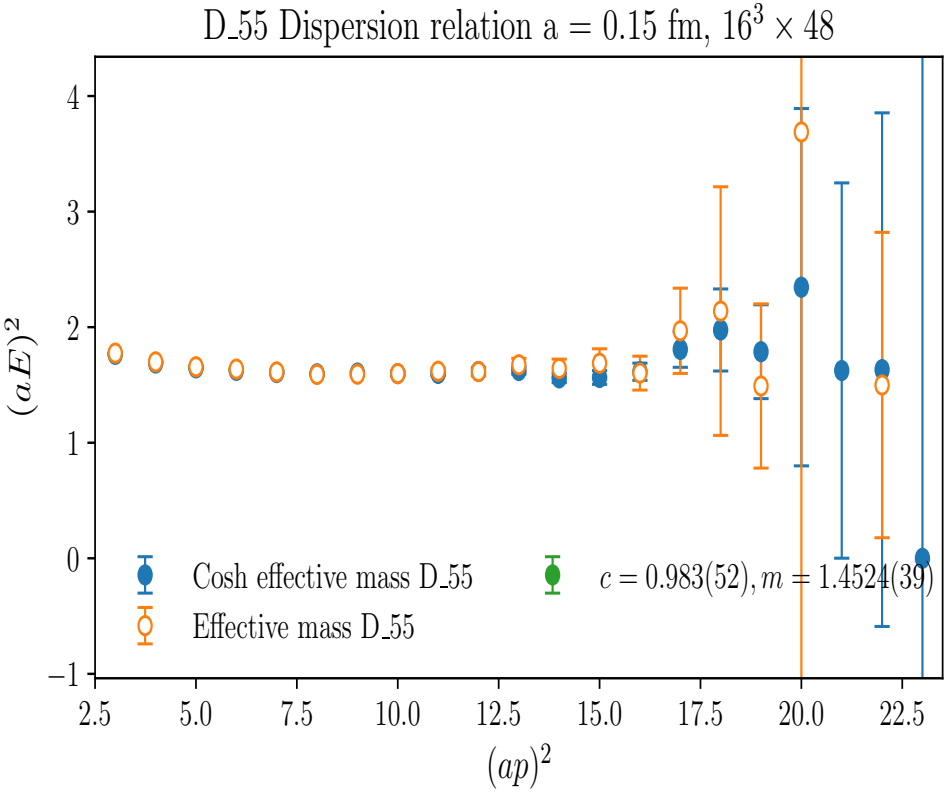
κ	0.14101	0.1412	0.1414	0.1418	0.1421	0.1422	0.1423
m_π(MeV)	690.1	660.1	634.2	568.2	515.7	494.1	472.9

- We also varied the heavy quark mass –

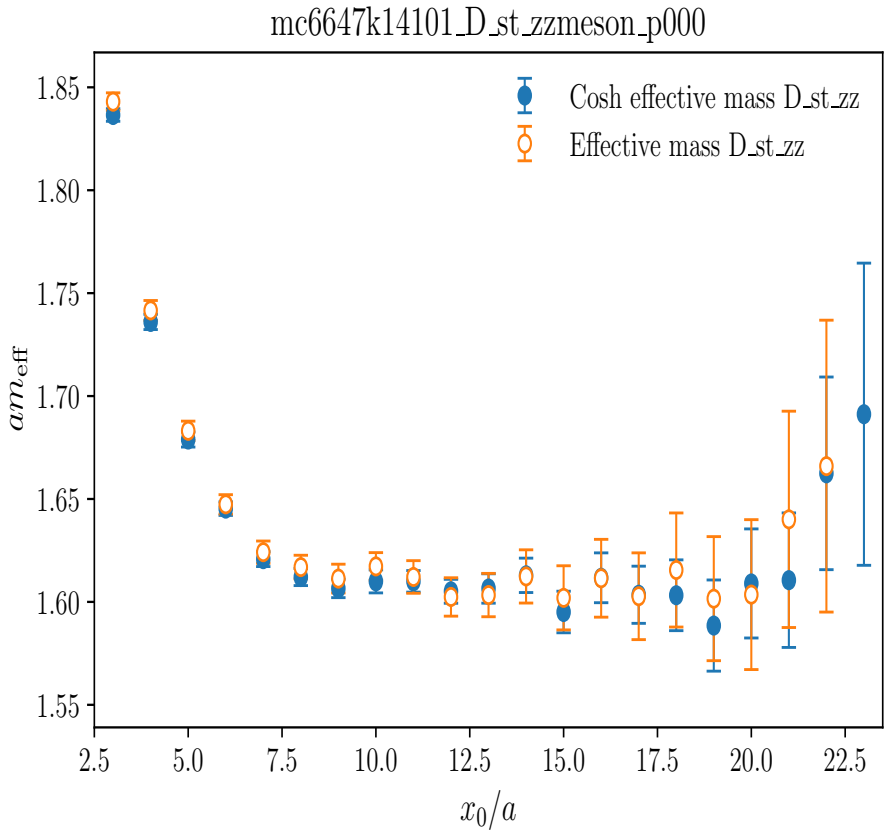
m_Q(GeV)	1.273	1.88	2.46	3.04	3.62	4.183
M_{hl}(MeV)	1967.7	2802.3	3494.6	4143.9	4770.9	5366.9
$M^\star_{hl}$(MeV)	2112.8	2901.4	3571.5	4207.3	4825.4	5415.4

We used the publicly available Lattice QCD codes MILC and Grid in our analysis.

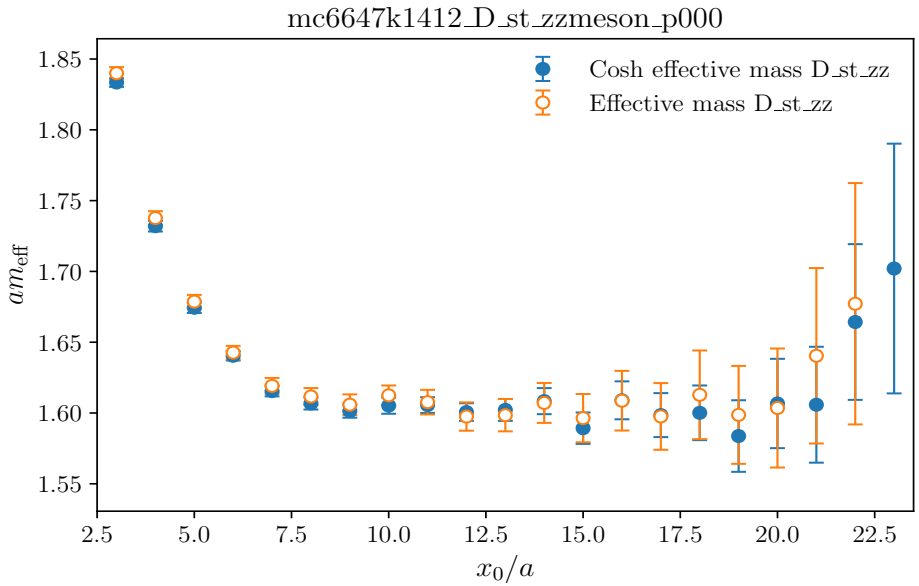
Dispersion relation



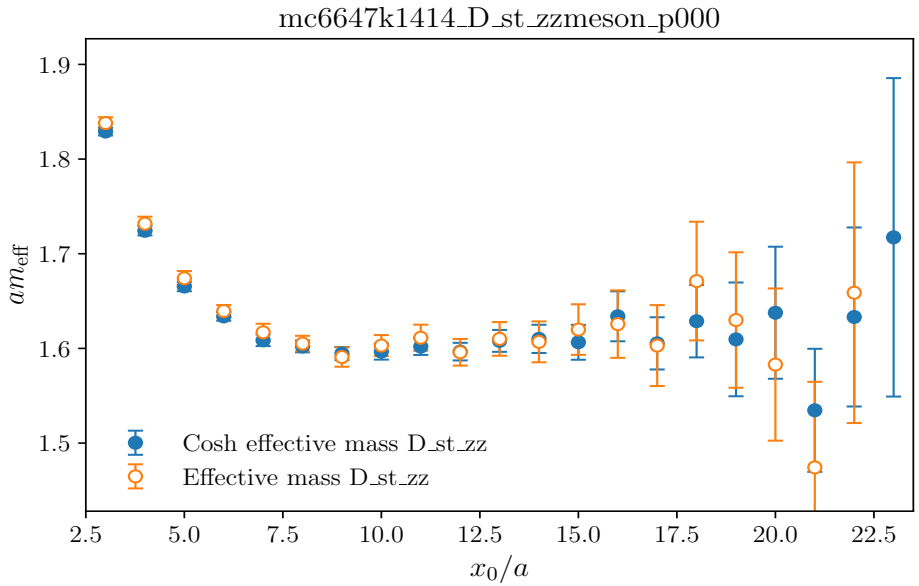
(a) D meson dispertion relation



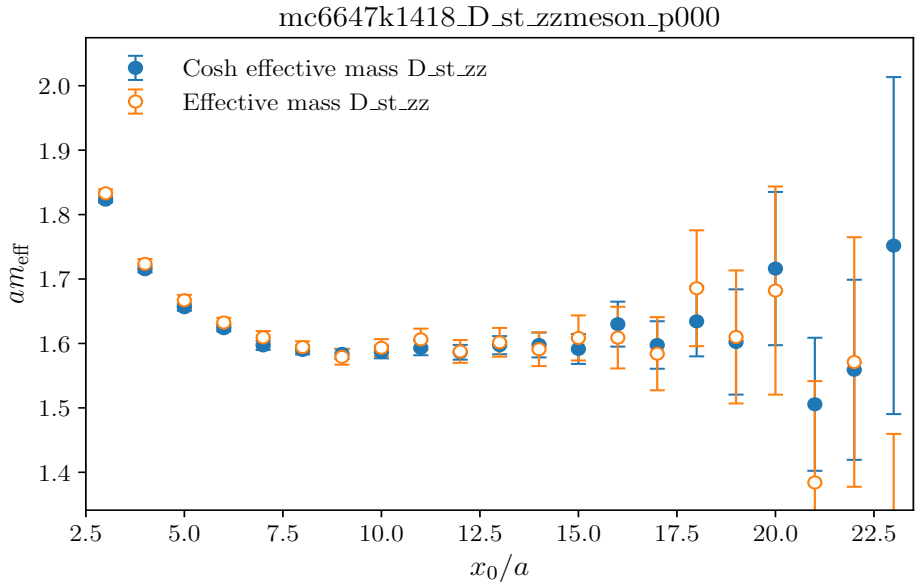
(b) $D^\star$ plateau



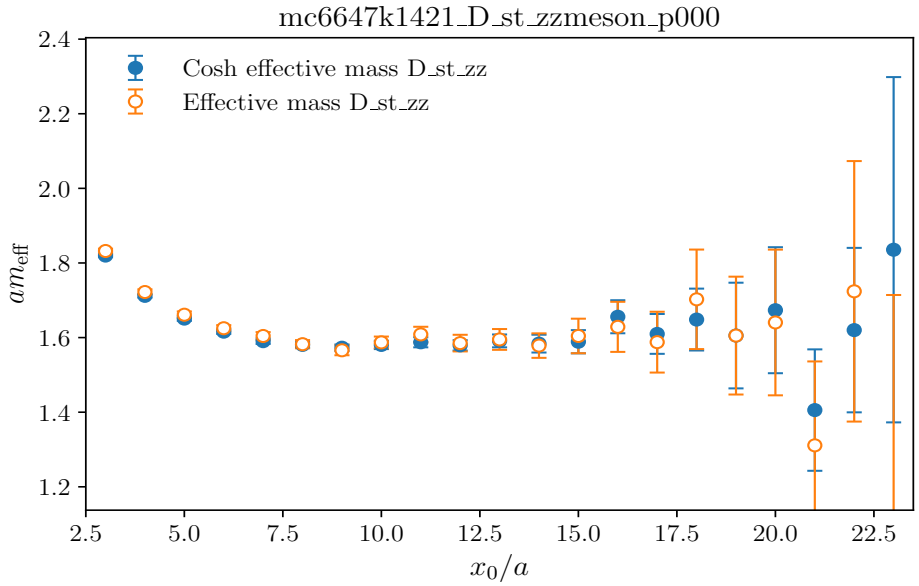
(a) κ=0.1412



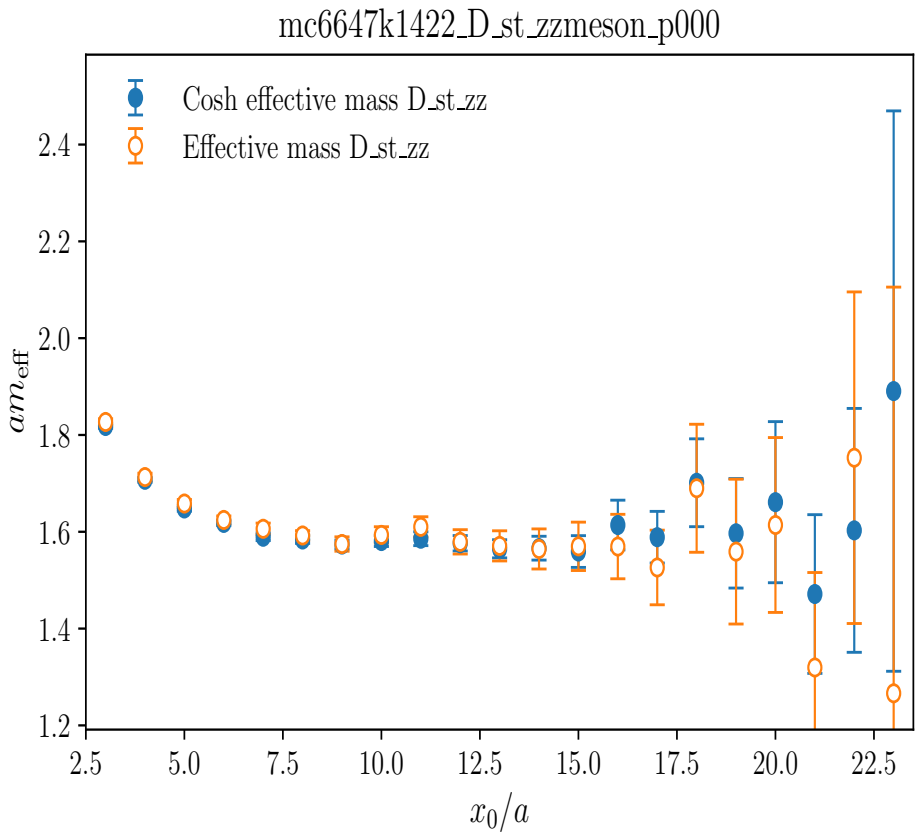
(b) κ=0.1414



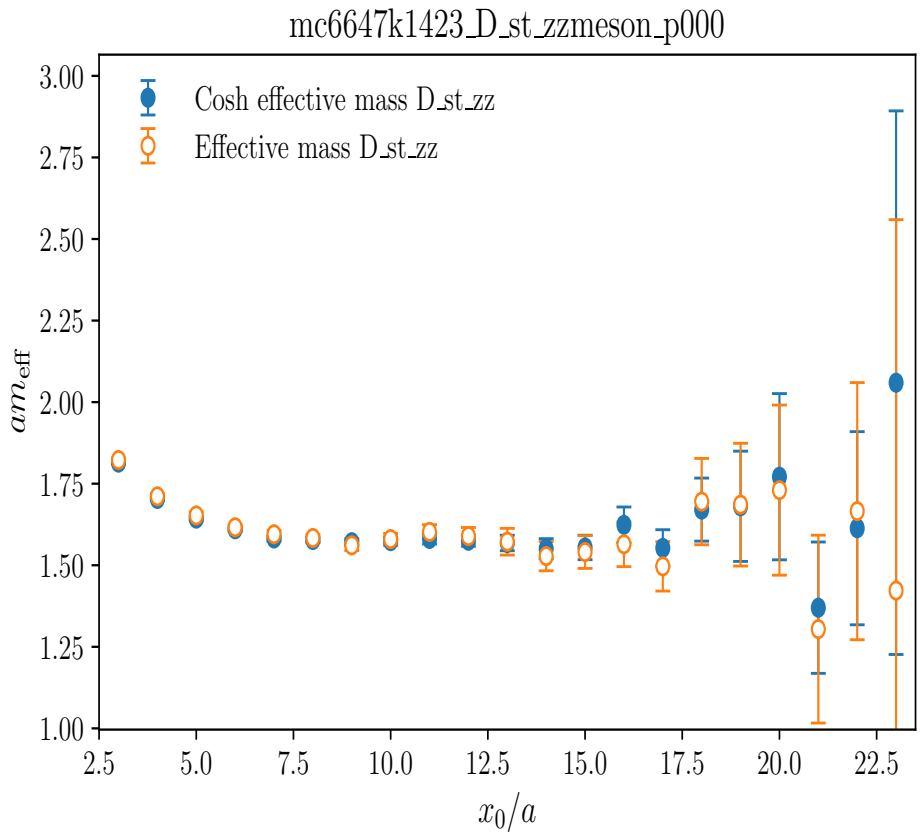
(c) κ=0.1418



(d) κ=0.1421



(a) κ=0.1422



(b) κ=0.1423

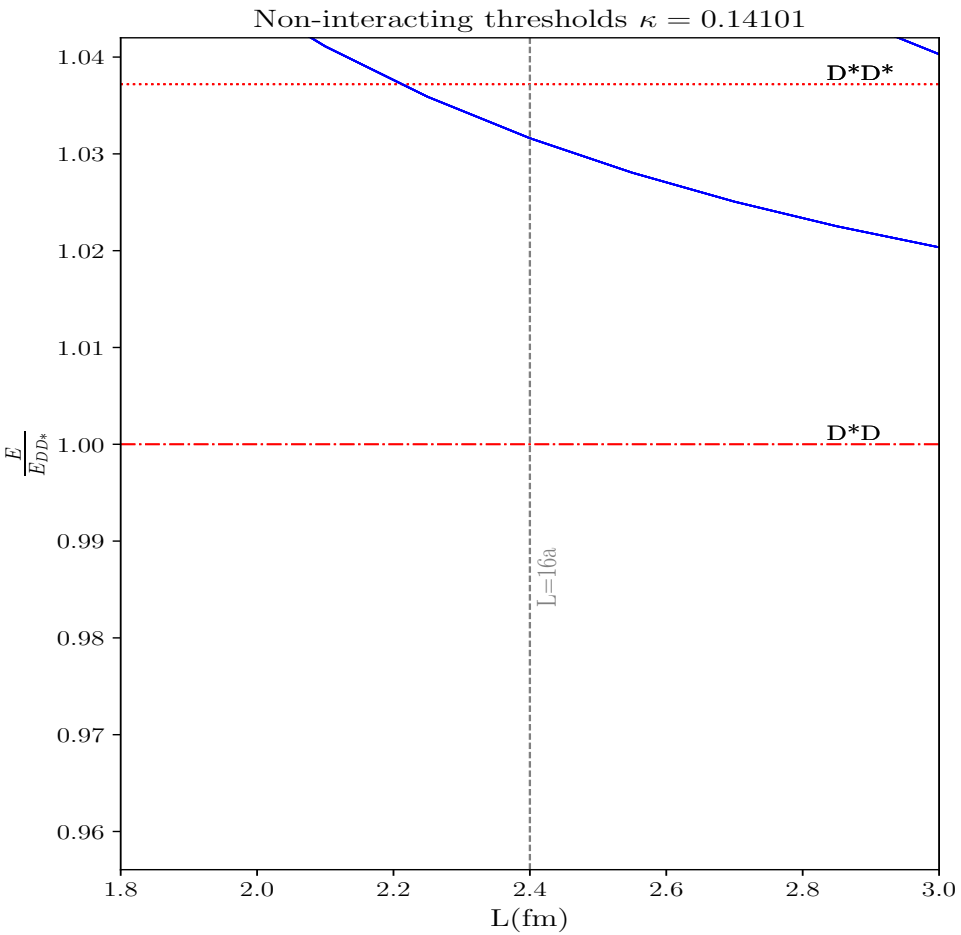
Interpolating Operators

- We therefore used the following operators in our analysis

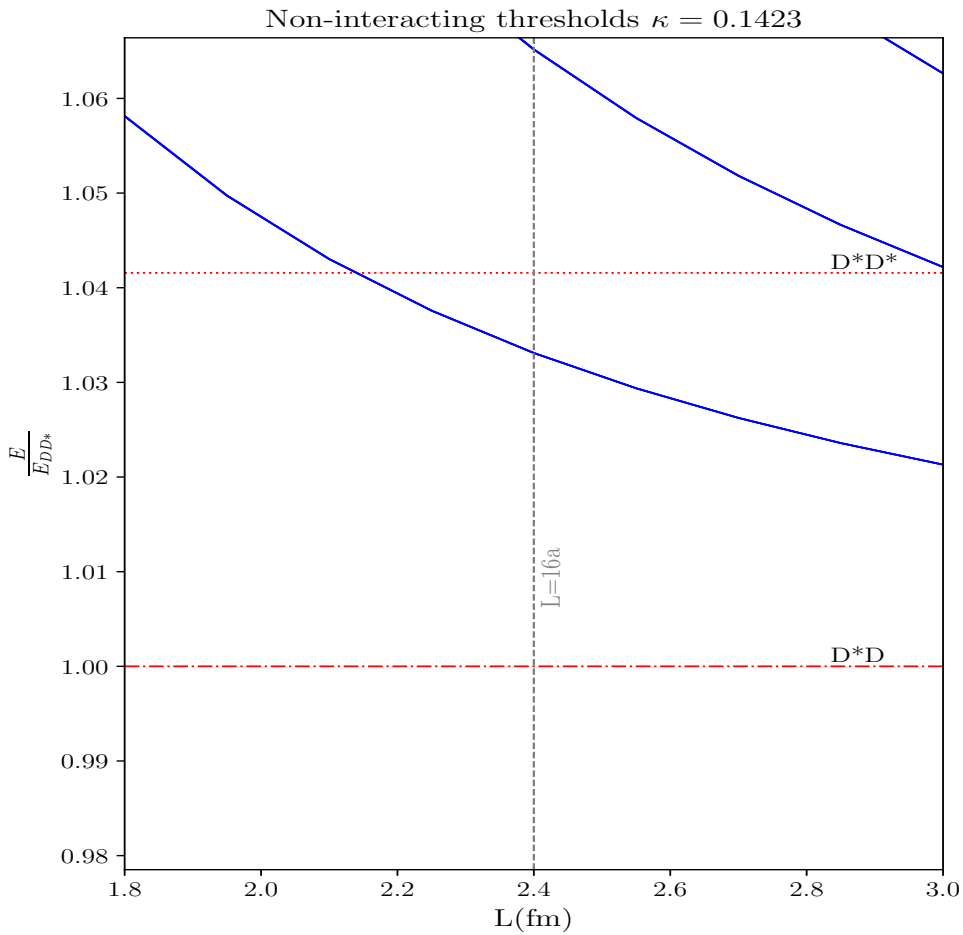
$$
\begin{aligned}
\mathcal{D}(x) &= \Big[c(x)^{a\,T} C\gamma_k\, c(x)^b\Big]\ \Big[\bar{u}(x)^a\, C\gamma_5\, \bar{d}(x)^{b\,T}\Big] \\
\mathcal{M}_1(x) &= \big[\bar{d}(x)^a\, \gamma_k\, c(x)^a\big]\ \Big[\bar{u}(x)^b\, \gamma_5\, c(x)^b\Big] \\
\mathcal{M}_2(x) &= \epsilon_{kij}\,\big[\bar{d}(x)^a\, \gamma_i\, c(x)^a\big]\ \Big[\bar{u}(x)^b\, \gamma_j\, c(x)^b\Big] \\
\mathcal{S}(t;\mathbf{p}_1,\mathbf{p}_2) &= \sum_{\mathbf{x}} \big[\bar{d}(x)^a\, \gamma_k\, c(x)^a\big]\ e^{i\mathbf{p}_1\cdot\mathbf{x}} \times \sum_{\mathbf{y}} \Big[\bar{u}(y)^b\, \gamma_5\, c(y)^b\Big]\ e^{i\mathbf{p}_2\cdot\mathbf{y}}
\end{aligned}
$$

- We performed our analysis in the centre-of-mass frame, therefore the $D^\star$ and D meson in the scattering operator $\mathcal{S}$ are given back-to-back momentum $\mathbf{p}_1 + \mathbf{p}_2 = 0$.
- We then calculated the correlator matrix required for GEVP analysis up to $\mathbf{p}^2 = 1$.
- Except for the $\mathcal{C}_{\mathcal{SS}}(t;\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_4) = \big\langle \mathcal{S}(t;\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{S}(0;\mathbf{p}_3,\mathbf{p}_4)^\dagger \big\rangle$ correlators we need to compute only the point-all propagators. For the computation of $\mathcal{C}_{\mathcal{SS}}$ we used the one end trick.

$\mathbf{p}^2$=1 plot



(a) κ=0.14101



(b) κ=0.1423

- $\mathbf{p}^2 = 1$ plot with box size aL in fm. At lattice spacing a=0.15 fm, $D(\mathbf{p})D^\star(-\mathbf{p})$ point lies below the first inelastic level $D^\star D^\star$. i.e. we don't need to consider p^2 =2 on $16^3 \times 48$ lattice.

One End Trick

- We used the so called one-end-trick in the computation of $\mathcal{C}_{\mathcal{SS}}(t)$ –

$$\begin{aligned}
\mathcal{C}_{\mathcal{SS}}(t;\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_4) &= \left\langle \mathcal{S}(t;\mathbf{p}_1,\mathbf{p}_2)\,\mathcal{S}(0;\mathbf{p}_3,\mathbf{p}_4)^\dagger \right\rangle \\
&= \sum_{\mathbf{x},\mathbf{y},\mathbf{z}} e^{i(\mathbf{p}_1\cdot\mathbf{x}+\mathbf{p}_2\cdot\mathbf{y}-\mathbf{p}_4\cdot\mathbf{z})}\ \mathrm{Tr}\Big[\Big\{\gamma_5\mathcal{G}_d(t,\mathbf{x};0)^\dagger\gamma_5\Big\}\Big(\gamma_k\mathcal{G}_c(t,\mathbf{x};0)\gamma_k\Big)\Big] \\
&\qquad \times \mathrm{Tr}\Big[\mathcal{G}_c(t,\mathbf{y};0,\mathbf{z})\Big\{\gamma_5\mathcal{G}_u(0,\mathbf{z};t,\mathbf{y})\gamma_5\Big\}\Big] \\
&\quad - \sum_{\mathbf{x},\mathbf{y},\mathbf{z}} e^{i(\mathbf{p}_1\cdot\mathbf{x}+\mathbf{p}_2\cdot\mathbf{y}-\mathbf{p}_4\cdot\mathbf{z})}\ \mathrm{Tr}\Big[\Big\{\gamma_k\gamma_5\mathcal{G}_d(t,\mathbf{x};0)^\dagger\gamma_5\gamma_k\Big\}\Big(\mathcal{G}_c(t,\mathbf{x};0,\mathbf{z}) \\
&\qquad \gamma_5\mathcal{G}_u(0,\mathbf{z};t,\mathbf{y})\,\gamma_5\mathcal{G}_c(t,\mathbf{y};0)\Big)\Big]
\end{aligned}$$

- We implemented complex $\mathbb{Z}(2)\times\mathbb{Z}(2)$ random numbers at timeslice $t=0$ and inverted the RHQ action or the clover action as needed. The first term can be written as

$$\begin{aligned}
&= \frac{1}{N}\sum_n\sum_{\mathbf{x}} e^{i(\mathbf{p}_1\cdot\mathbf{x})}\Big\{\gamma_k\gamma_5\mathcal{G}_d(t,\mathbf{x};0)^\dagger\gamma_5\gamma_k\Big\}^{c_1c_2}_{s_1s_2}\Big(\mathcal{G}_c(t,\mathbf{x};0)\Big)^{c_2c_1}_{s_2s_1} \\
&\times \sum_{\mathbf{y}} e^{i(\mathbf{p}_2\cdot\mathbf{y})}\Big(\phi^n_c(\mathbf{y},t)\Big)^{c_3}_{s_3}\Big(\phi^n_u(\mathbf{y},t)^\dagger\Big)^{c_3}_{s_3}
\end{aligned}$$

Where

$$\Big(D_c(r,x)\Big)^{c_1c_2}_{s_1s_2}\Big(\phi^n_c(x)\Big)^{c_2}_{s_2} = \delta_{r_0,0}\Big(\Xi(\mathbf{r})[n]\Big)^{c_1}_{s_1}$$

$$\Big(D_u(r,x)\Big)^{c_1c_2}_{s_1s_2}\Big(\phi^n_u(x)\Big)^{c_2}_{s_2} = \delta_{r_0,0}\Big(\Xi(\mathbf{r})[n]\Big)^{c_1}_{s_1}e^{i(\mathbf{p}_4\cdot\mathbf{r})}$$

Summary and Status

- We report our on going study of T_{cc} pole on N_f = 2+1+1 HISQ ensembles with RHQ heavy and Wilson-Clover light quarks.
- We varied the pion mass (light κ) from $\sim$ 688 MeV to $\sim$ 470 MeV.
- We varied the heavy quark mass from $M_{hs} \sim D_s$ to $M_{hs} \sim B_s$.
- We are currently analysing the data.

Thank You