RTG 2575:
Rethinking
Quantum Field Theory

Importance of local tetraquark operators for the $T_{cc}(3875)^+$ spectrum

Based on arXiv:2510.26459

Andres Stump
Jeremy R. Green

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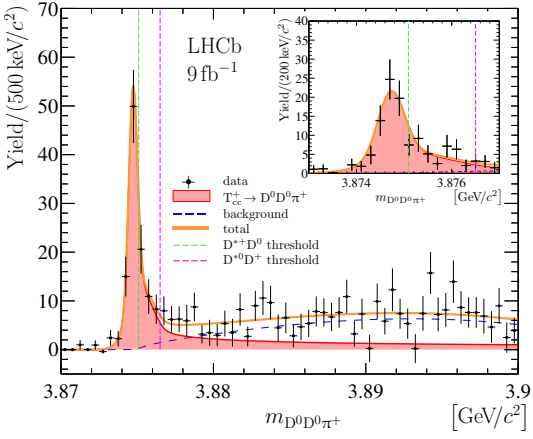
Humboldt-Universität zu Berlin

$T_{cc}(3875)^+$ tetraquark

- Exotic hadron discovered at LHCb
- Found in $D^0 D^0 \pi^+$ mass spectrum
  Just below $D^{*+} D^0$ threshold
- Minimal quark content $cc\overline{u}\overline{d}$, $I(J^P) = 0(1^+)$

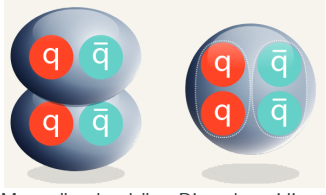


[Aaij et al. 2022]

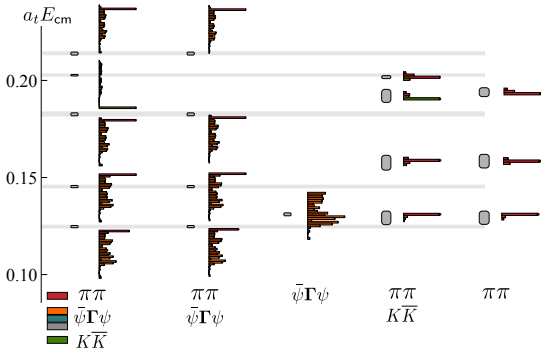
Finite-volume T_{cc} spectrum

How to obtain accurate finite-volume T_{cc} spectrum?

- Insufficiently diverse operator bases can lead to inaccurate spectrum
- Are bilocal $D^{(*)}D^*$ scattering operators enough?
- Do we need local tetraquark operators?



Meson "molecule" Diquark-antidiquark



ρ resonance: Diverse operator basis required
[Wilson et al. 2015]

Content

1. Local tetraquark operators in distillation
   - Distillation method
   - Position-space sampling method

2. Results for T_{cc}
   - Operator importance analysis
   - Finite-volume T_{cc} spectrum with and without local tetraquark operators
   - s-wave Lüscher analysis

Local tetraquark operators in distillation

Distillation method [Peardon et al. 2009]

Use Laplacian Heaviside (LapH) smearing for quark fields $\psi_{\mathrm{sm}}(t) = V(t)V(t)^\dagger\psi(t)$ with

$$V(t) = \left(v^{(1)}(t), v^{(2)}(t), \ldots, v^{(N_v)}(t)\right)$$

containing the N_v lowest eigenvectors of $\Delta(t)$

Properties:

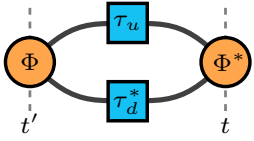
- $V(t)V(t)^\dagger$ is projector to subspace of lowest Laplace modes
- Can compute propagator $S = D^{-1}$ fully in this subspace ($4N_v \times 4N_v$ matrix called perambulator):

$$\tau(t',t) = V(t')^\dagger\, S(t',t)\, V(t)$$

- For constant smearing radius scale $N_v \propto V$
  [Morningstar et al. 2011]

E.g. for pion two-point function we get

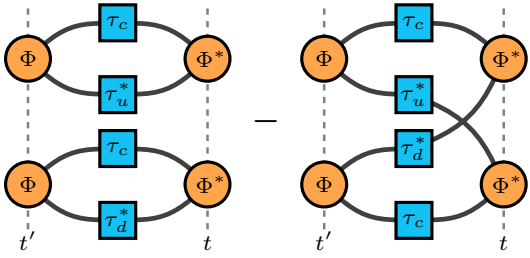
$$C(t',t) = \left\langle \mathrm{tr}\left[\Phi(t') \cdot \tau_u(t',t) \cdot \Phi(t)^\dagger \cdot \tau_d(t',t)^\dagger\right]\right\rangle_G$$



$D^{(*)}D^*$ scattering operators:

$$\mathcal{O}^{DD^*}(t) \sim (\overline{u}\gamma_5 c)(\boldsymbol{p}_1, t)\ (\overline{d}\gamma_i c)(\boldsymbol{p}_2, t)$$

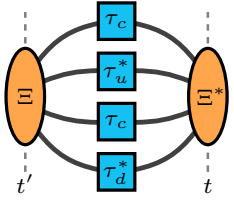
- Matrix multiplications in LapH subspace
  $\rightarrow$ $N_v^3 \propto V^3$ cost scaling
  $\rightarrow$ Affordable!



Local tetraquark operators:

$$\mathcal{O}^T(t) \sim (cc\bar{u}\bar{d})(\boldsymbol{p}, t)$$

- Diquark-antidiquark or $D^{(*)}D^*$ spin/color structure
- Contraction involving rank-4 tensor Ξ
  $\rightarrow$ $N_v^5 \propto V^5$ cost scaling
  $\rightarrow$ **Expensive**, especially in large volumes!



Position-space sampling for local tetraquark operators

Usual order of contraction produces rank-4 tensor
$\rightarrow$ V^5 cost scaling

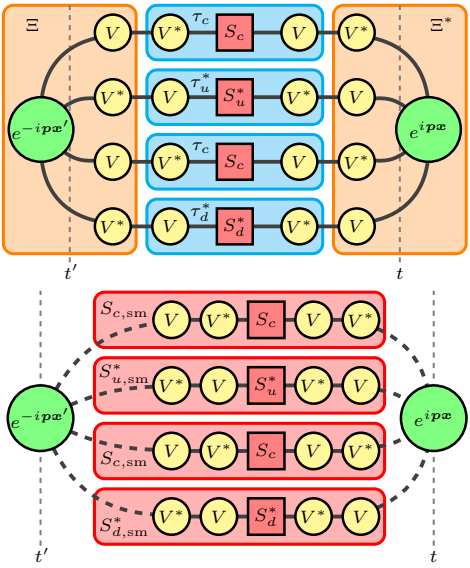
Can we do better?

- Explicitly compute smeared propagators

$$S_{f,\,\mathrm{sm}}(t',t) = V(t') \cdot \tau_f(t',t) \cdot V(t)^\dagger$$

  $\rightarrow$ V^3 scaling
- Position-space sampling for momentum projection:
  - Sum only over regular sparse grids
  - Random shift at sink and source: sampling from full lattice
    $\rightarrow$ **Unbiased estimator**

$\rightarrow$ Position-space sampling makes tetraquark operators affordable in distillation!



Results for T_{cc}

Lattice Setup

CLS gauge ensembles (B450, N202):

- $N_f = 3$ Wilson clover fermions
- $SU(3)$-flavour-symmetric point with physical $\frac{1}{3}(m_u + m_d + m_s)$
- $m_\pi \approx 420$ MeV
- Same action for valence charm quark
- c mass tuned to get physical $\frac{1}{3}(m_{D^0} + m_{D^+} + m_{D_s^+})$

Operator importance analysis for T_{cc}

Operator basis (rest frame T_1^+):

- 9 bilocal $D^{(*)}D^*$ operators
  - Organized in groups

$$D^{(*)}(\boldsymbol{k}^2)D^*(\boldsymbol{k}^2)\ \{N_{\text{ops}}\} \qquad \text{(integer momentum } \boldsymbol{k} = \frac{L}{2\pi}\boldsymbol{p}\text{)}$$

- 3 tetraquark operators ($T\ \{3\}$)
  - Diquark-antidiquark ($(\overline{\mathbf{3}}_c \otimes \mathbf{3}_c)_{\mathbf{1}_c}$ color irrep)
  - local DD^* and D^*D^*

Variational method to extract energies [Blossier et al. 2009]:

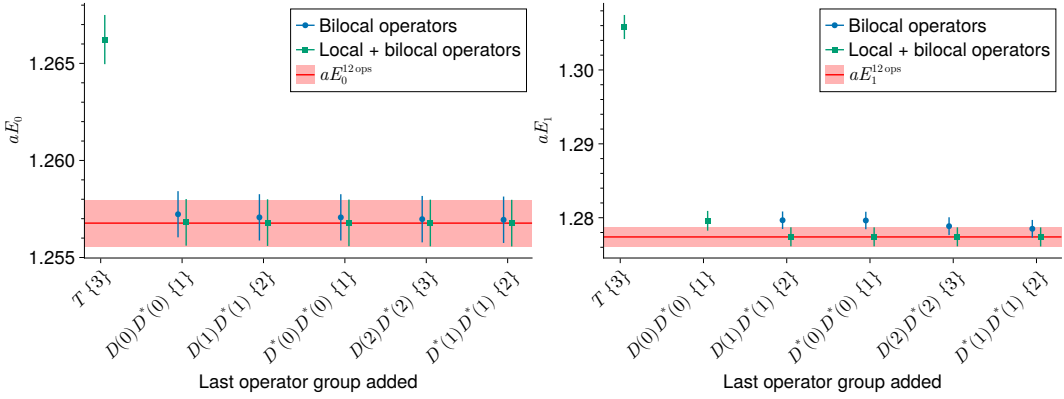
- Plateau fits to effective energy difference

$$\Delta E_{\text{eff},n}(t) = E_{\text{eff},n}(t) - m^D_{\text{eff}}(t) - m^{D^*}_{\text{eff}}(t)$$

(where all effective energies have plateaued individually)

$\rightarrow$ Significant error cancellation

Operator importance analysis for T_{cc}

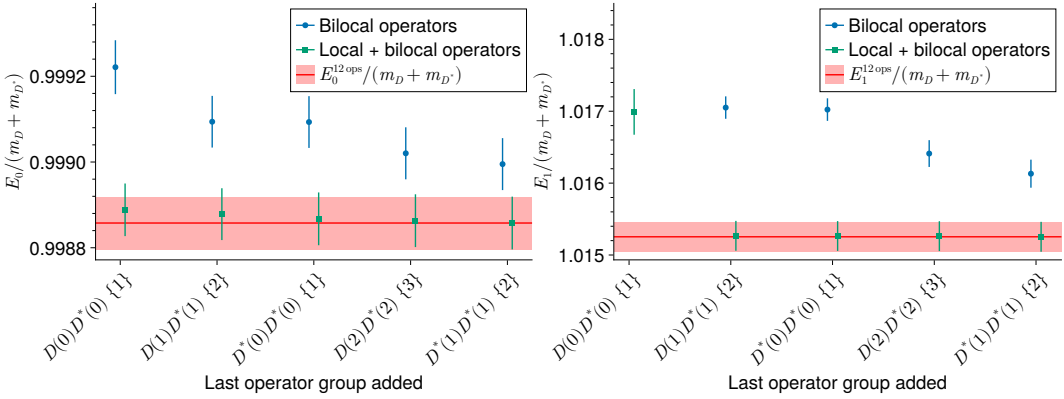


- Negligible effect for ground state
- $\sim 1\ \sigma$ shift in first excited state

In Lüscher analysis: Correlation with non-interacting energies is relevant!

$\rightarrow$ Look at $E_n/(m_D + m_{D^*})$

Operator importance analysis for T_{cc} (ratio with threshold energy)



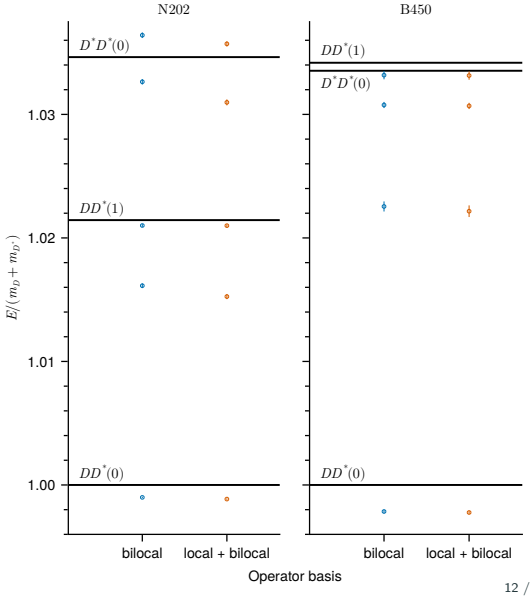
- Large error cancellation in ratio
- Significant difference between result from bilocal and local + bilocal basis
  - Fast convergence when including local operators
  - Step-like decreases without them

Finite-volume T_{cc} spectrum with and without local tetraquark operators

Rest frame T_1^+ spectrum

- Qualitative agreement (no level missed)
- Significant shifts in various energy levels
- Smaller effect in smaller volume (B450)

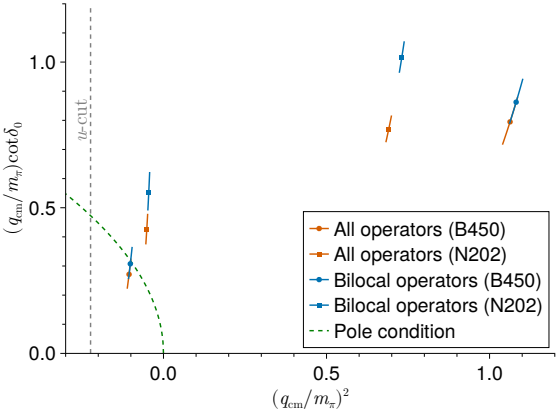
→ **Considerable systematic error** without local operators!



s-wave Lüscher analysis

s-wave DD^* scattering phase shift:

- Single-channel s-wave Lüscher analysis
- Shifts also appear in scattering phase shift



Conclusion

- Position-space sampling makes local tetraquark operators affordable in distillation
- Qualitative agreement between finite-volume T_{cc} spectra from bilocal and local + bilocal operator basis
- However, when including local tetraquark operators:
  - Fast convergence of energy levels when enlarging operator basis
  - Significant shifts in several energy levels
- Not including local operators in a T_{cc} analysis can lead to significant systematic errors

Thank you for your attention!

References

References

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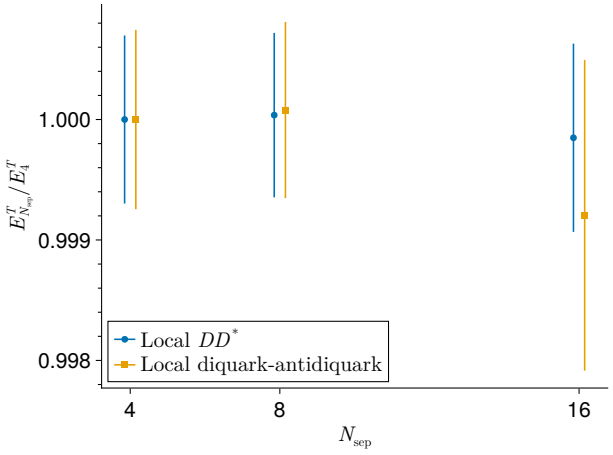
Backup slides

N_{sep} analysis



Plateau values $E^T_{N_{\text{sep}}}$ from plateau fits to the effective energy of the local DD^* and diquark-antidiquark two-point functions vs. the point separation N_{sep}. The plateau values are normalized with the $N_{\text{sep}} = 4$ value E^T_4.

CLS gauge ensembles

Ensemble	$N_s^3 \times N_t$	β	a [fm]	$\kappa_u = \kappa_d = \kappa_s$	κ_c	N_{cfg}	N_{src}	N_v	N_{sep}
B450	$32^3 \times 64$	3.46	0.0749	0.136890	0.126243	1612	8	32	8
N202	$48^3 \times 128$	3.55	0.0633	0.137000	0.128423	899	8	68	8

- $N_f = 3$ Wilson clover fermions
- $SU(3)$-flavour-symmetric point with physical $\frac{1}{3}(m_u + m_d + m_s)$
- $m_\pi \approx 420$ MeV
- Same action for valence charm quark
- c mass tuned to get physical $\frac{1}{3}(m_{D^0} + m_{D^+} + m_{D_s^+})$