Charm tetraquarks investigations using CLS lattices

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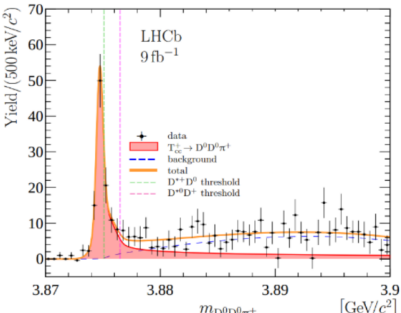
3^{rd} November, 2025
Lattice 2025

Meng, MP, *et al.* PRD 2411.06266
Prelovsek, MP, *et al.* PRD 2504.03473
Sadl, MP, *et al.* PRD 2406.09842

In collaboration with: Baru, Collins, Epelbaum, Guo, Leskovec, Meng, Ortiz-Pacheco, Prelovsek, Sadl, Vujmilovic, Yan

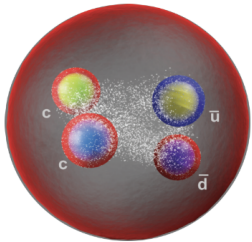
Doubly charm tetraquark: $\mathrm{T}_{\mathrm{cc}}^{+}$ [Revision]



LHCb: 2109.01038, 2109.01056

$$\delta m \equiv m_{\mathrm{T}_{\mathrm{cc}}^{+}} - (m_{\mathrm{D}^{*+}} + m_{\mathrm{D}^0})$$

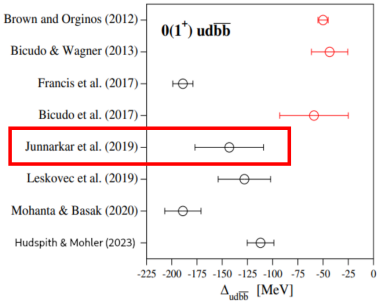
$$\begin{aligned} \delta m_{\mathrm{pole}} &= -360 \pm 40^{+4}_{-0}\ \mathrm{keV}/c^2\,, \\ \Gamma_{\mathrm{pole}} &= 48 \pm 2^{+0}_{-14}\ \mathrm{keV}\,. \end{aligned}$$

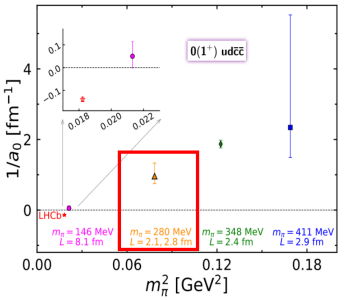


Courtesy: CERN Courier

- The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0D^+\pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.
  c.f. Ader&Richard PRD25(1982)2370
- Only two lattice calculations before discovery. No amplitude extraction.
  Cheung *et al.*HSC 1709.01417; Junnarkar, Mathur, MP, 1810.12285

$\mathbf{T_{bb}}$ and $\mathbf{T_{cc}}$ using lattice QCD: $\mathbf{I(J^P) = 0(1^+)}$





- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100MeV)$. Fig: Hudspith&Mohler 2023
  Red box: ILGTI work on QQ tetraquarks: Junnarkar, Mathur, MP PRD 2019
- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100keV)$. Fig: Lyu *et al.*PRL 2302.04505
  Red box: T_{cc} (RQCD) [PRL 2022].

  A handful of lattice determination of the DD^* amplitude.
- No concrete conclusions in the bottom-charm tetraquark sector.
  A summary of different lattice investigations → see review by Pedro Bicudo, 2212.07793
- All talks in this session on doubly heavy tetraquarks.

T_{cc} and DD^* scattering amplitude and parametrization

- For the DD^* system [total spin equals 1], and assuming only $l < 2$, we have a 3×3 diagonal t matrix.

$$(t_l^{(J)})^{-1} = \frac{2(\tilde{K}_l^{(J)})^{-1}}{E_{cm} p^{2l}} - i\frac{2p}{E_{cm}}, \quad (\tilde{K}_l^{(J)})^{-1} = p^{2l+1} \cot \delta_l^{(J)}$$

- Using an effective range expansion near-threshold, we have

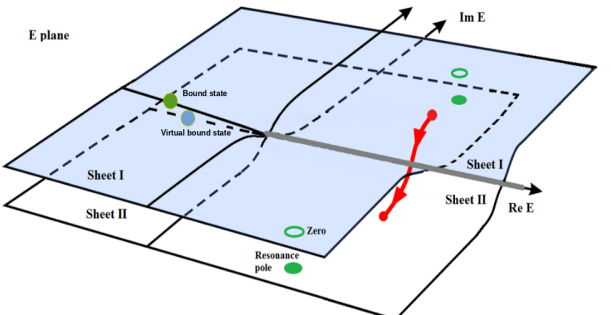
$$\tilde{K}^{-1} = \begin{bmatrix} \frac{1}{a_0^{(1)}} + \frac{r_0^{(1)} p^2}{2} & 0 & 0 \\ 0 & \frac{1}{a_1^{(0)}} + \frac{r_1^{(0)} p^2}{2} & 0 \\ 0 & 0 & \frac{1}{a_1^{(2)}} \end{bmatrix} \begin{matrix} J{=}1 \quad I{=}0 \\ J{=}0 \quad I{=}1 \\ J{=}2 \quad I{=}1 \end{matrix}$$

- Constraint on bound states:

$$p^{2l+1} cot(\delta_l) = -1^{\alpha} p^{2l} \sqrt{-p^2}$$

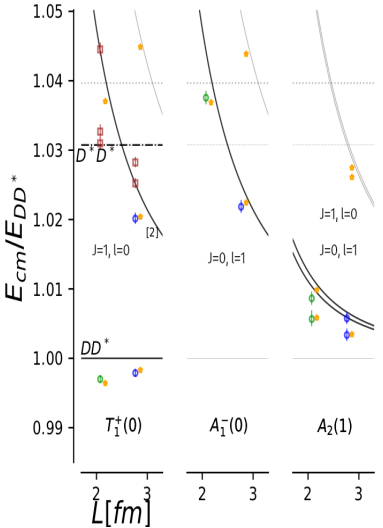
$$\alpha = 1(2) \text{ for a real (virtual) bound state.}$$

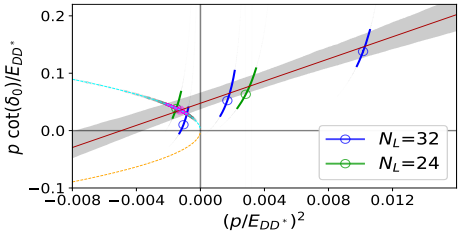
Virtual/bound states



- $T \propto (pcot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$. Virtual bound state is a pole in T with $p = -i|p|$.
- An example for virtual bound state: spin-singlet dineutron.
- Only subthreshold real solutions allowed in the first Riemann sheet. Solutions in the second Riemann sheet: non-normalizable $[Im(p) = -|Im(p)|]$.

$I = 0$ DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with an ERE: T_{cc}^+





MP, Prelovsek PRL 2022

- Fit quality: $\chi^2/d.o.f. = 3.7/5.$ $m_\pi \sim 280$ MeV
- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}
- Binding energy:
  $\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV.
- First evaluation of the DD^* amplitude in T_{cc} channel.

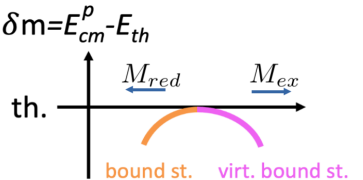
+/g refers to positive parity, -/u refers to negative parity.

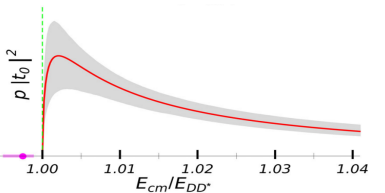
Our observations and inferences with ERE approach

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

- For $m_\pi > m_\pi^{phys}$, T_{cc} is expected to become a virtual bound state. At $m_\pi \sim$ 280 MeV, we indeed find a shallow virtual bound state.
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.





MP, Prelovsek PRL 2022

- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system. E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

Pion exchange interactions/left-hand cut: ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)

  $m_\pi > m_{D^*} - m_D \quad \Rightarrow \quad D^* \to D\pi$ is kinematicaly forbidden.

  $2 \to 2$ Generalized LQC: does not subthreshold lhc effects.

  Raposo&Hansen 2311.18793, Dawid *et al* 2303.04394, Hansen *et al* 2401.06609

  Sharpe Saturday 1200 plenary, Raposo Friday 1700 [Spectroscopy]

  ERE convergence fails at the nearest singularity.

  Left-hand cut in the DD^* system close below the DD^* threshold. Du *et al* 2303.09441

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):

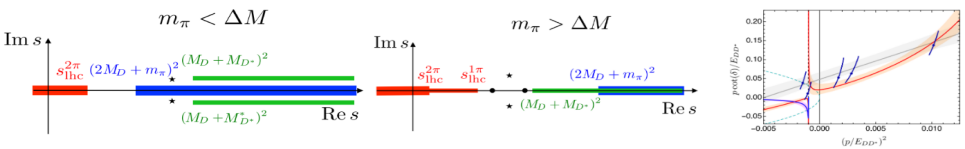


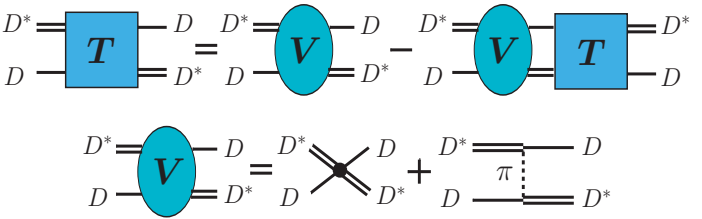
Figure taken from Du *et al* 2303.09441

Long range pion exchange interactions: the origin of left-hand singularity and cut.

Fits with a potential that incorporates the one pion exchange:

Virtual bound states ⇒ Virtual resonances

Solving Lippmann-Schwinger Equation for the DD^* amplitude



- The potential: a sum of short range and long range interactions

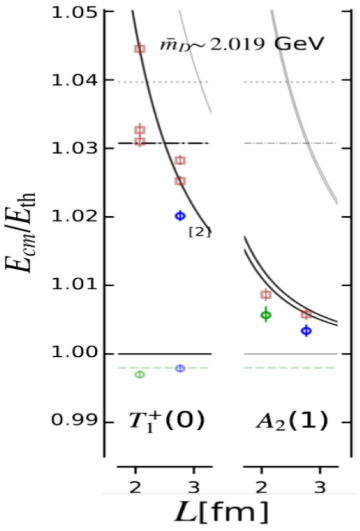
$$V(\boldsymbol{p},\boldsymbol{p}') = V_{\rm CT}(p,p') + V_\pi^S(p,p') \quad \text{with} \quad V_{\rm CT}(p,p') = 2c_0 + 2c_2(p^2+p'^2) + \mathcal{O}(p^4,p'^4)$$

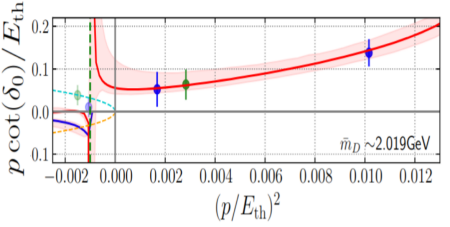
- The scattering amplitude $\qquad T^{-1} \propto p\cot\delta_0 - ip$

- The pion decay constant f_π and $DD^*\pi$ coupling g_c at $m_\pi \sim 280$ MeV following the 1-loop χPT.

Du *et al* 2303.09441

$I = 0$ DD^* scattering in $l = 0$ @ $m_c^{(h)}$ with EFT: T_{cc}^+



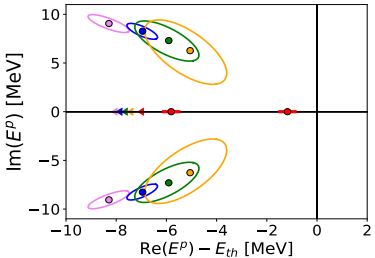


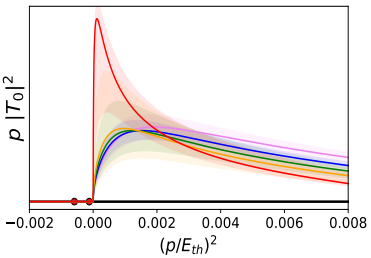
Collins, MP, *et al.* PRD 2024

- OPE pole at the *left-hand cut* branch point observed.
- Complex amplitudes along real axis along *left-hand cut*
- Hadronic poles observed as virtual resonance poles.
- Pure S-wave fits.
- E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$

+/g refers to positive parity, -/u refers to negative parity.

Pole positions and scattering rate [EFT]

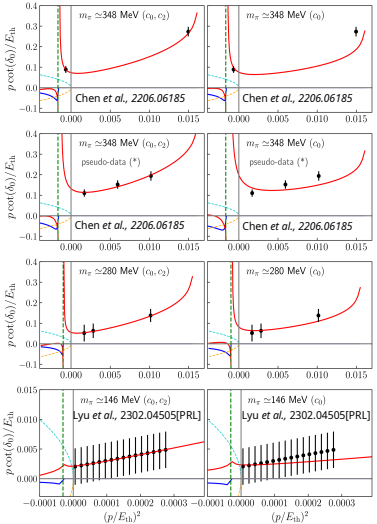




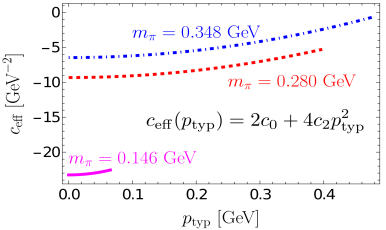
- Subthreshold resonance pole pair moving towards the real axis with increasing m_c.
- Collide on the real axis below threshold and turn back-to-back.
  At the heaviest m_c: virtual bound poles [in Red]
- With increasing m_c, subthreshold resonance poles evolves to become a pair of virtual bound poles.
- Enhancement in the DD^* scattering rate ($p|T_0|^2$).
  E_{th} refers to $E_{DD^*} = M_D + M_{D^*}$

Collins, MP, *et al.* PRD 2024

m_π dependence of the T_{cc} pole [EFT]



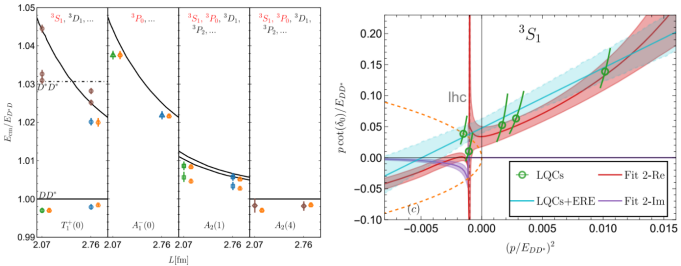
Collins, MP, *et al.* PRD 2024



- Qualitative study of m_π dependence using $V_{\rm CT}(p, p') = 2c_0 + 2c_2(p^2 + p'^2)$
- Two parameter fit (c_0, c_2) [left] and a single parameter fit (c_0, with $c_2 = 0$) [right].
- Resonance poles at $m_\pi \sim$348 and $\sim$280 MeV. Shallow virtual bound poles at $m_\pi =$146 MeV.
- Stronger attraction for lighter m_π. [$c_{\rm eff}$] stronger binding in T_{cc} for lighter pions.
- $m_\pi =$146 MeV: HALQCD. Lyu *et al.* 2302.04505
  $m_\pi =$348 MeV: Chen *et al.* 2206.06185

Work around to LQC: A plane-wave approach and modified LQC

- An effective field theory incorporating OPE with a plane wave basis expansion [PlW].



Meng *et al.* 2312.01930

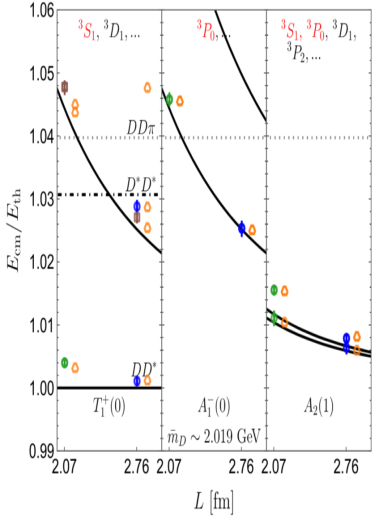
Virtual bound states ⇒ Virtual resonances [m_π ∼280 MeV]

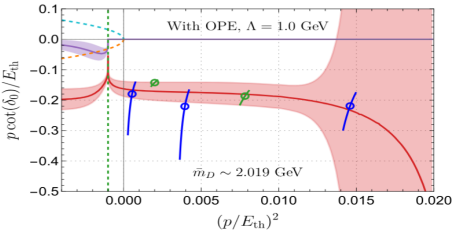
- Modified 3-particle (Lüscher) Quantization Condition:

Dawid, Hansen, Romero-Lopez, Raposo, Sharpe 2023-2024

- Relevant talks: Sharpe Saturday 1200 plenary, Raposo Friday 1700 [Spectroscopy]
- Several followup investigations and proposals. *c.f.* Talks at Chiral Dynamics workshop: link

$I = 1$ DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with EFT+PlW



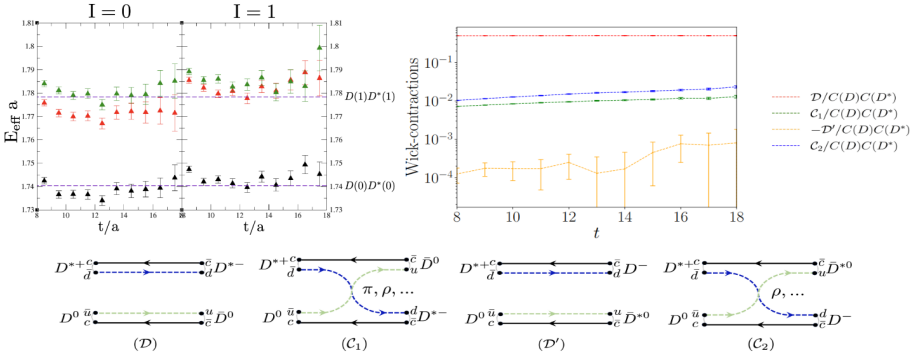


Meng, MP, *et al.* PRD 2411.06266

- Fit quality: $\chi^2/d.o.f. = 2.5/6.$ $m_\pi \sim 280$ MeV
- Fit parameters: $a_0^{(1)} = -0.30(4)$ fm & $r_0^{(1)} = -0.89(55)$ fm
- No poles observed in the accessible region.
- First evaluation of the $I = 1$ DD^* amplitude. More recent investigation: Lachini *et al.* 2505.01363

+/g refers to positive parity, -/u refers to negative parity.

Isovector $\mathbf{T_{cc}}$: Weak repulsion



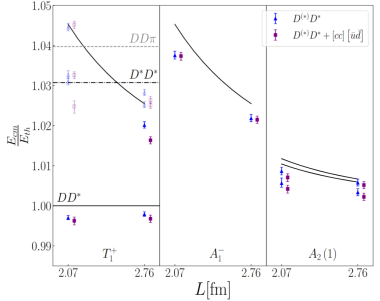
$$C_I^{DD^*}(t) = \mathcal{D} - \mathcal{C}_1 + (-)^{I+1}(\mathcal{D}' - \mathcal{C}_2)$$

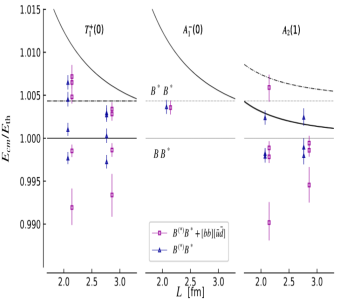
- $\mathcal{C}_2$ determining the isospin effects. Weakly repulsive interactions.

Meng, MP, *et al.* PRD 2411.06266

- Consistent with observations reported in Chen *et al.*, 2206.06185 [PLB], Shi *et al.* arXiv:2502.07438

Effect of local $[QQ]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: $I = 0$ DD^*/BB^* T_{QQ} Spectra

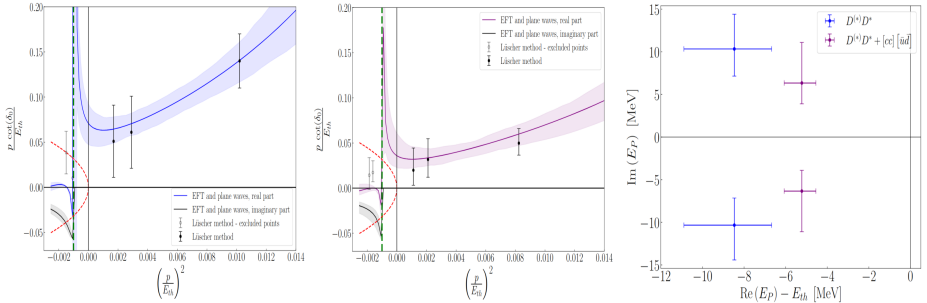




Prelovsek, MP, *et al.* PRD 2504.03473

- Bilocal meson-meson $[1_c \otimes 1_c \to 1_c]$ interpolators:
  $O^{M_1M_2}(\boldsymbol{P}) = \sum_k A_k M_1(\boldsymbol{p}_{1k}) M_2(\boldsymbol{p}_{2k}), \quad \boldsymbol{P} = \boldsymbol{p}_{1k} + \boldsymbol{p}_{2k}$
- Local diquark-antidiquark $[\bar{3}_c \otimes 3_c \to 1_c]$ interpolators: $O^{4q}(\boldsymbol{P}) = [Q\Gamma_a Q]_{\bar{3}_c}[\bar{u}\Gamma_b \bar{d}]_{3_c}(\boldsymbol{P})$
- Left: DD^* Ground state energies nearly unaffected.
  First excitation in rest frame T_1^+ irrep gets significant downward shift.
- Right: BB^* spectra shows a significant change, already on the ground state energy estimates.
  Bottom quark approximated with a relativistic Wilson clover action.
  Caution: Uncontrolled discretization errors.

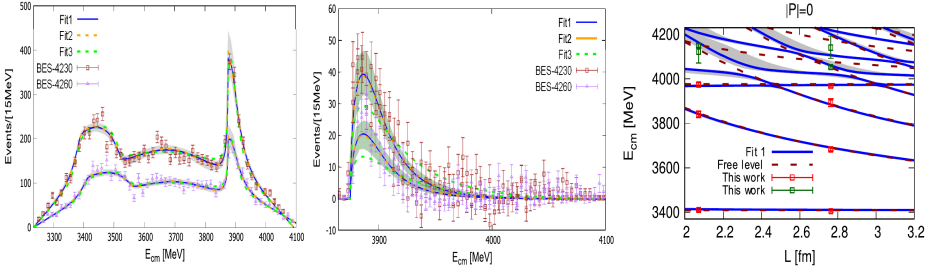
Effect of local $[cc]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: $I = 0$ DD^* Amplitude (T_{cc})



Prelovsek, MP, *et al.* PRD 2504.03473

- Amplitude extraction performed only for the charm sector.
- Left: Amplitude with purely $O^{M_1 M_2}(\boldsymbol{P})$.
- Right: Amplitude with full basis: $O^{M_1 M_2}(\boldsymbol{P})$ plus $O^{4q}(\boldsymbol{P})$.
- Potentially strong effects on the effective range and shape parameters. Enhanced binding.
- No amplitude extraction performed for bottom sector.
  Need for a multichannel analysis of the BB^*-B^*B^* system.

Isovector axialvector charmonium-like states from LFT and EFT



- First study of isovector charmonium-like states using multiple volume and inertial frames.
- Similated FV levels either consistent with noninteracting scenario or have mild -ve energy shifts. Most prominent in $D\bar{D}^*$ FV levels.
- Lattice QCD ensembles : CLS Consortium
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $m_D \sim$ 1927 MeV, $a \sim 0.086$ fm
- Highlights
  - $\Rightarrow$ Virtual poles below $D\bar{D}^*$ threshold in both charge parities.
  - $\Rightarrow$ Coupled $J/\psi\pi$-$D\bar{D}^*$ scattering as a function of m_π with an EFT framework.
    Results suggest EFT reasonably reconciles experimental lineshapes and FV energy levels
  - $\Rightarrow$ First study of s-wave $J/\psi\pi$ phase shifts using lattice.
- Our (RQCD) recent publication on charmonium:

Sadl, Collins, Guo, MP, Prelovsek, Yan, PRD 2406.09842.

Summary

- Reported our recent investigations on doubly charm tetraquarks, particularly addressing near threshold poles.
- Isovector doubly charm tetraquark: Weak repulsion.
  Meng, MP, *et al.* PRD 2411.06266
- Effect of local $[QQ]_{\bar{3}_c}[\bar{u}\bar{d}]_{3_c}$ operators: charm and bottom sectors
  Prelovsek, MP, *et al.* PRD 2504.03473
- Isovector axialvector charmonium-like states from LFT and EFT
  Sadl, MP, *et al.* PRD 2406.09842

Other related talks:

- Shrimal, MP, *et al.* PRD 2025: Mon 1700 [Spectroscopy]
  Lattice study of $cc\bar{u}\bar{s}$ tetraquark channel in $D^{(*)}D_s^{(*)}$ scattering
- Ivan Vujmilovic, *et al.* 2025: Wed 1050 [Spectroscopy]
  Electromagnetic form factors and structure of the T_{bb} tetraquark from lattice QCD

Thank you