Light and strange quark masses with $N_f = 2 + 1$ Wilson fermions

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ALPHA Collaboration

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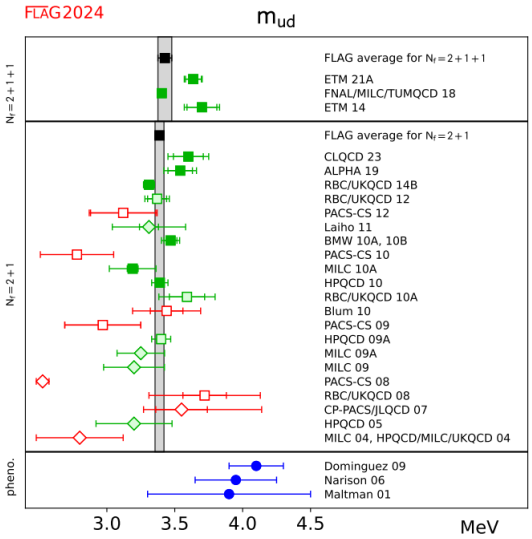
Outline

- Status of m_q
- Strategy of the computation
  - ➔ $N_f = 2 + 1$ CLS ensembles
  - ➔ Non-perturbative renormalization
  - ➔ Data analysis
- Preliminary results
  - ➔ m_{ud}
  - ➔ m_s
  - ➔ m_s/m_{ud}
- Conclusions

Status of m_{ud}



➔ FLAG Average:

$$m_{ud}(2\text{GeV}) = 3.387(39)\ \text{MeV}\ [1.2\%]$$

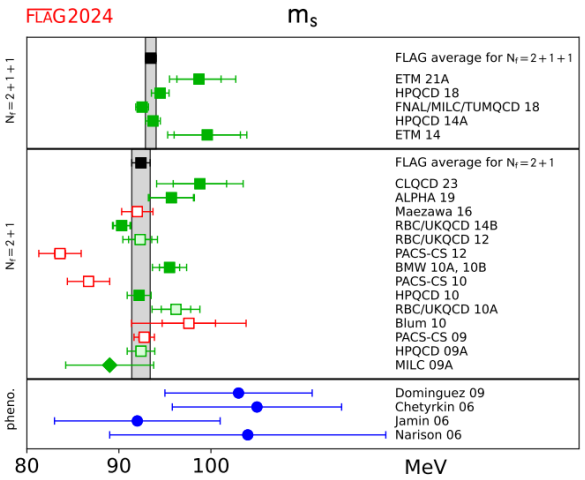
$$M_{ud}^{RGI} = 4.714(55)_m(46)_\Lambda\ \text{MeV}\ [1.5\ \%]$$

➔ `[ALPHA 19]`:

$$m_{ud}(2\text{GeV}) = 3.54(15)\ \text{MeV}\ [4.2\ \%]$$

$$M_{ud}^{RGI} = 4.70(19)\ \text{MeV}\ [4.1\ \%]$$

Status of m_s



➔ FLAG Average:

$$m_s(2\text{GeV}) = 92.4(1.0)\ \text{MeV}\ [1.1\%]$$

$$M_s^{RGI} = 128.5(1.4)_m(1.2)_\Lambda\ \text{MeV}\ [1.4\ \%]$$

➔ `[ALPHA 19]`:

$$m_s(2\text{GeV}) = 95.7(3.5)\ \text{MeV}\ [3.6\ \%]$$

$$M_s^{RGI} = 127.0(4.5)\ \text{MeV}\ [3.5\ \%]$$

Setup

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  - ➔ Open boundary conditions in time on most ensembles
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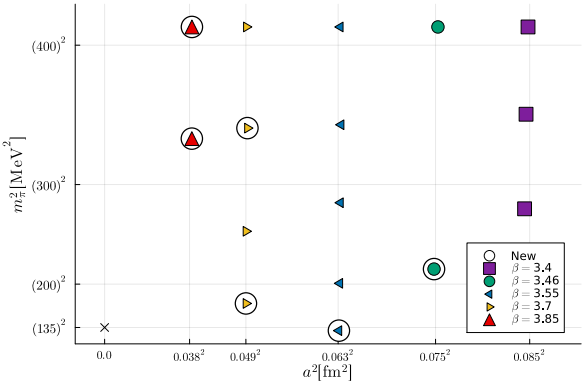
  - ➔ Five lattice spacings $[0.038, 0.085]$ fm and $m_\pi \in [130, 420]$ MeV
- Combined analysis with new scale-setting determination [2510.20450]
  - ➔ Edinburgh Consensus + f_K^{isoQCD} [2411.04268]
  - ➔ $\sqrt{t_0}$ with 0.5% precision
  - ➔ Mixed-action setup with twisted mass fermions in the valence
    (Light quark mass determination in progress)

Ensembles

Name	β	a[fm]	T/a	L/a	$\frac{m_\pi}{\mathrm{MeV}}$	$\frac{m_K}{\mathrm{MeV}}$
H101	3.4	0.085	96	32	426	426
H102			96	32	360	446
H105			96	32	286	470
H400	3.46	0.075	96	32	429	429
D450			128	64	221	483
N202	3.55	0.063	128	48	418	418
N203			128	48	350	448
N200			128	48	288	470
D200			128	64	204	488
E250			192	96	131	497
N300	3.70	0.049	128	48	427	427
N302			128	48	350	457
J303			192	64	261	481
E300			192	96	177	499
J500	3.85	0.038	192	64	418	418
J501			192	64	339	453



[1411.3982][1608.08900][2510.20450]

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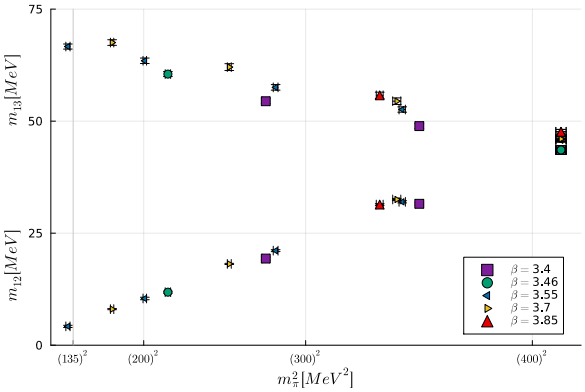

[1411.3982][1608.08900][2510.20450]

Computation of m_q

Dimensionless quantities for the masses

$$\phi_2 = 8t_0 m_\pi^2, \quad \phi_K = 8t_0 m_K^2, \quad \phi_4 = 8t_0(m_K^2 + \frac{1}{2}m_\pi^2)$$

$$\phi_{12} = \sqrt{8t_0}m_{12} = \sqrt{8t_0}m_{ud}, \qquad \phi_{13} = \sqrt{8t_0}m_{13} = \sqrt{8t_0}\frac{(m_{ud} + m_s)}{2}$$

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PCAC quark masses improvement

$$m_{ij}^I = \frac{Z_A}{Z_P} m_{ij} \left(1 + a(\tilde{b}_A - \tilde{b}_P) a m_{ij} + \left(\frac{\overline{b}_A - \overline{b}_P}{Z r_m} - (\tilde{b}_A - \tilde{b}_P)\frac{r_m - 1}{r_m N_f}\right) a Z r_m Tr(M)\right)$$

$$m_{ij} = \frac{\tilde{\partial}_0 \sum_{\vec{y},\vec{x}} \left(\langle A_0(x) P(y)\rangle + a c_A \tilde{\partial}_0 \langle P(x) P(y)\rangle\right)}{2 \sum_{\vec{y},\vec{x}} \langle P(x) P(y)\rangle}$$

$(\tilde{b}_A - \tilde{b}_P) \rightarrow$ [1906.03445] (LCP-1), $\qquad c_A \rightarrow$[1502.04999]

Renormalization

$$M_i = \frac{M_i}{\overline{m}_i(\mu_{pt})} \frac{\overline{m}_i(\mu_{pt})}{\overline{m}_i(\mu_{had})} \lim_{a \to 0} Z_m(g_0^2, a\mu_{had}) m_i(g_0^2)$$

- $M_i/\overline{m}_i(\mu_{had})$: RG running to a hadronic scale, $\mu_{had} \sim 230\ \mathrm{MeV}$. [1802.05243]
  - → Rel. precision $\sim 1\%$, improvements are planned
- $Z_m(g_0^2, a\mu_{had})$: Renormalization factor in the same scheme as the RG running
- $m_i(g_0^2)$: Bare masses in large volume simulations

Chiral-continuum extrapolations

NLO SU(3) χPT

$$\phi_{12} = \frac{\phi_2}{2B_0\sqrt{8t_0}}\left(1 - \frac{16}{8t_0 f_0^2}(2L_8 - L_5)\phi_2 - \frac{32}{8t_0 f_0^2}(2L_6 - L_4)\phi_4 - \frac{1}{24\pi^2 8t_0 f_0^2}\left[\frac{3}{2}\phi_2 ln\left(\frac{\phi_2}{8t_0\Lambda_\chi^2}\right) - \frac{1}{2}\phi_\eta ln\left(\frac{\phi_\eta}{8t_0\Lambda_\chi^2}\right)\right]\right)$$

$$\phi_{13} = \frac{2\phi_4 - \phi_2}{2B_0\sqrt{8t_0}}\left(1 - \frac{8}{8t_0 f_0^2}(L_5 - 2L_8)\phi_2 - \frac{16}{8t_0 f_0^2}(4L_6 - 2L_8 - 2L_4 - L_5)\phi_4 - \frac{1}{24\pi^2 8t_0 f_0^2}\phi_\eta ln\left(\frac{\phi_\eta}{8t_0\Lambda_\chi^2}\right)\right)$$

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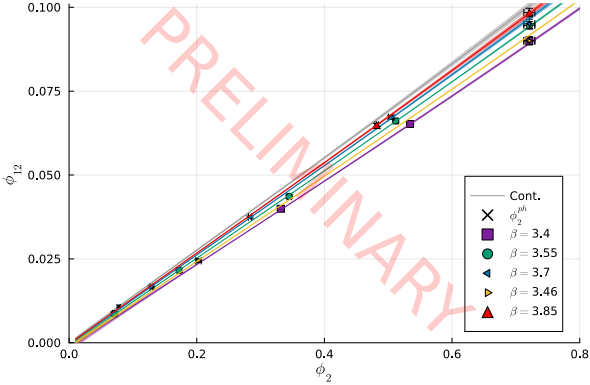
Fit ansatze

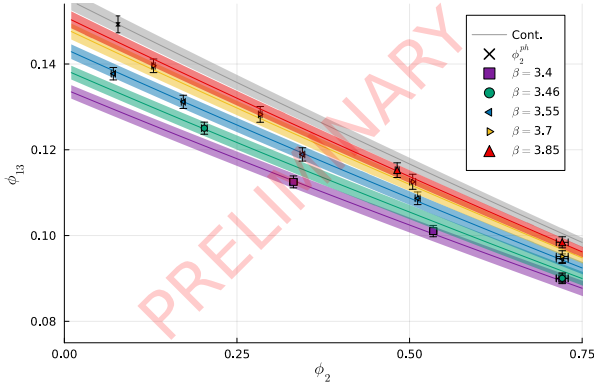
$$\phi_{12} = \phi_2\left[p_1 + p_2\phi_2 + p_3 K\left(\mathcal{L}_2 + \mathcal{L}_\eta\right)\right] + \frac{a^2}{8t_0}(p_4 + p_5\phi_2) \qquad \phi_{13} = \phi_K\left[p_1 + p_2\phi_K + \frac{2}{3}p_3 K\mathcal{L}_\eta\right] + \frac{a^2}{8t_0}(p_6 + p_7\phi_2)$$

$$\mathcal{L}_2 = \phi_2 log\phi_2, \qquad \mathcal{L}_\eta = \phi_\eta log\phi_\eta, \qquad \phi_\eta = \frac{4\phi_4}{3} - \phi_2, \qquad K = (8t_0 16\pi^2 f_{\pi K}^2)^{-1}$$

Chiral-continuum extrapolations

$$\phi_{12} = \phi_2 \left[p_1 + p_2\phi_2 + p_3 K \left\{ \phi_2 log\phi_2 + \phi_\eta log\phi_\eta \right\} \right] + \frac{a^2}{8t_0} \left(p_4 + p_5\phi_2 \right), \qquad \phi_{13} = \phi_K \left[p_1 + p_2\phi_K + \frac{2}{3} p_3 K \phi_\eta log\phi_\eta \right] + \frac{a^2}{8t_0} \left(p_6 + p_7\phi_2 \right)$$





Chiral-continuum extrapolations

Following [1911.08025], it is useful to fit to:

→ LO cutoff effects cancel. Mostly chiral effects.

$$\frac{\phi_{12}}{\phi_{13}} = \frac{2\phi_2}{2\phi_4 - \phi_2}\left[1 + \frac{p_2}{p_1}\left(\frac{3}{2}\phi_2 - \phi_4\right) - K(\mathcal{L}_2 - \mathcal{L}_\eta)\right] + \frac{a^2}{8t_0}(2\phi_4 - 3\phi_2)\,(p_4 + p_5\phi_2)$$

$$\frac{4\phi_{13}}{2\phi_4 - \phi_2} + \frac{\phi_{12}}{\phi_2} = 3p_1 + 2p_2\phi_4 + p_3K(\mathcal{L}_2 + \mathcal{L}_\eta) + \frac{a^2}{8t_0}\,(p_6 + p_7\phi_2)$$

→ Chiral effects are highly suppressed. Mostly cutoff effects.

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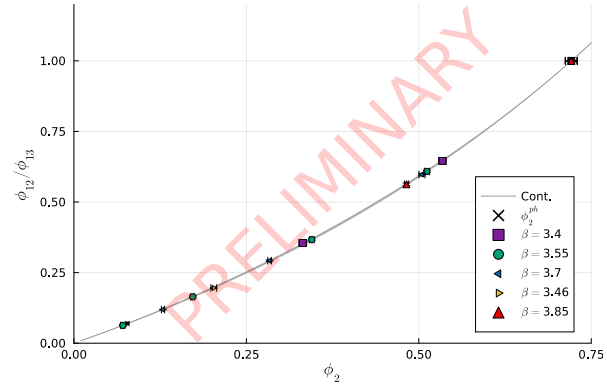

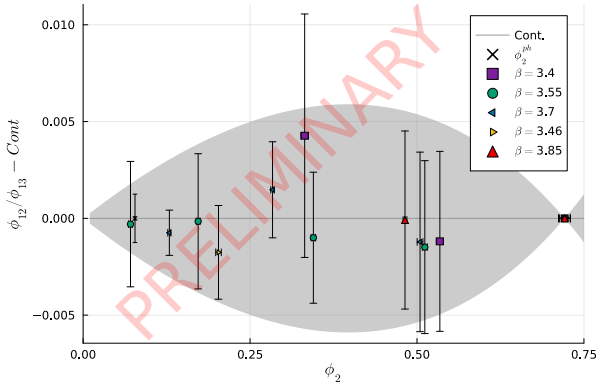

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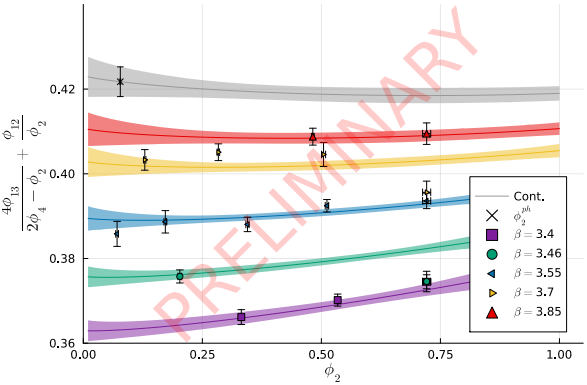

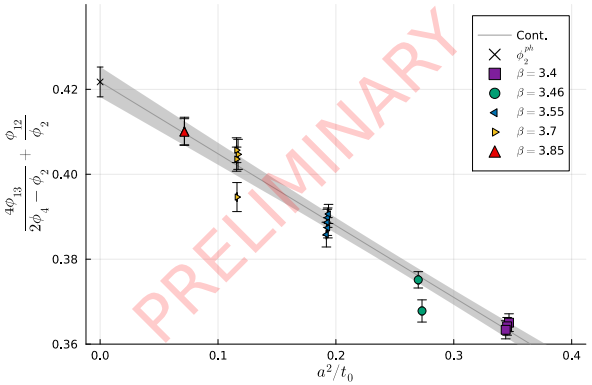

Systematic uncertanties

Set of models:

- ➔ [Tay] : Taylor expansion around the symmetric point
- ➔ [χptm] : Fit the χPT expression for the quark masses
- ➔ [χptr] : Fit the χPT expression for the ratios above

Cutoff effects:

- ➔ $[a^2]$: a^2/t_0
- ➔ $[a^2\phi]$: $a^2/t_0 + \phi_2 a^2/t_0$

Extraction of masses:

- ➔ [A] : Fixed plateau
- ➔ [B] : Model averaged plateau

Cuts in data:

- ➔ $[m_\pi < 420\mathrm{MeV}]$
- ➔ $[\beta > 3.4]$: $a \leq 0.075$ fm

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Model Average [2008.01069][2302.06550][2510.20450]

$$\langle \mathcal{O} \rangle_{MA} = \sum_i \mathcal{O}^{(i)} W^{(i)} \qquad \sigma^2_{syst} = \langle \mathcal{O}^2 \rangle_{MA} - \langle \mathcal{O} \rangle^2_{MA}$$

Takeuchi Information Criterion [Takeuchi, 76] $\qquad W^{(i)} = \mathcal{N} exp\left(-\frac{1}{2}(\chi^2 - 2\langle \chi^2 \rangle)\right)$

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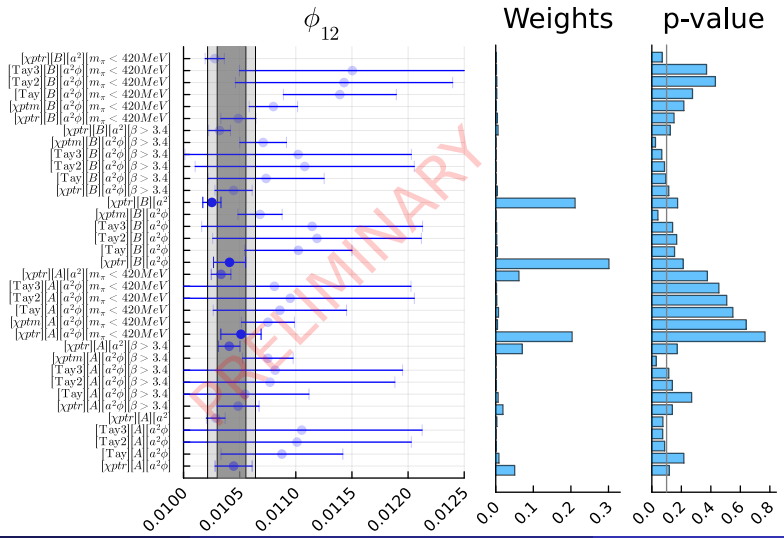

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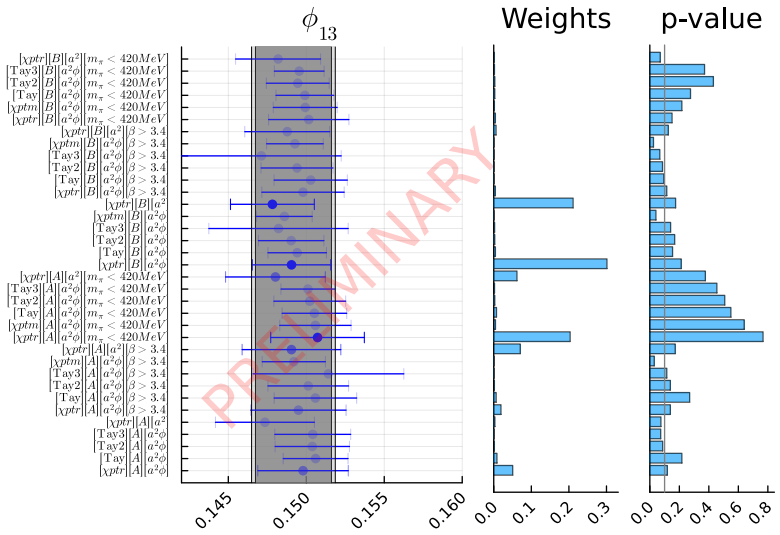
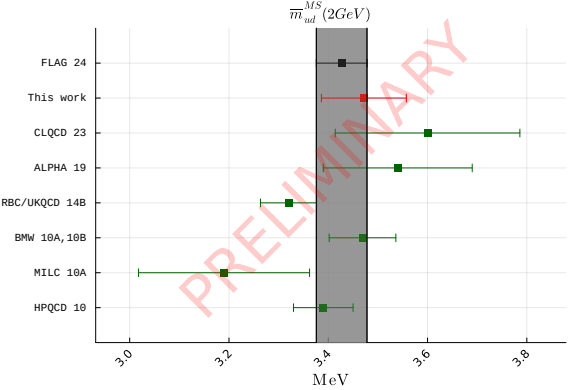
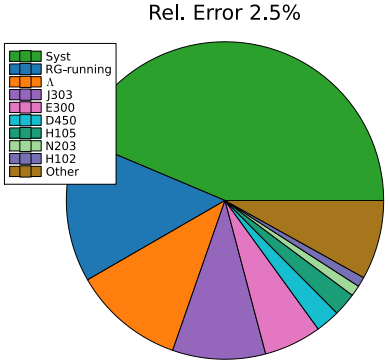
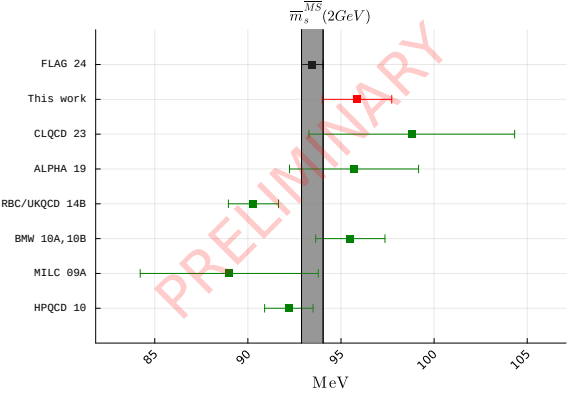


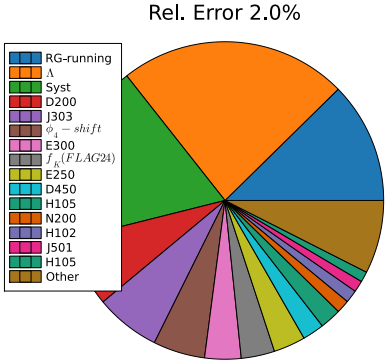
Results $\overline{m}_{ud}$



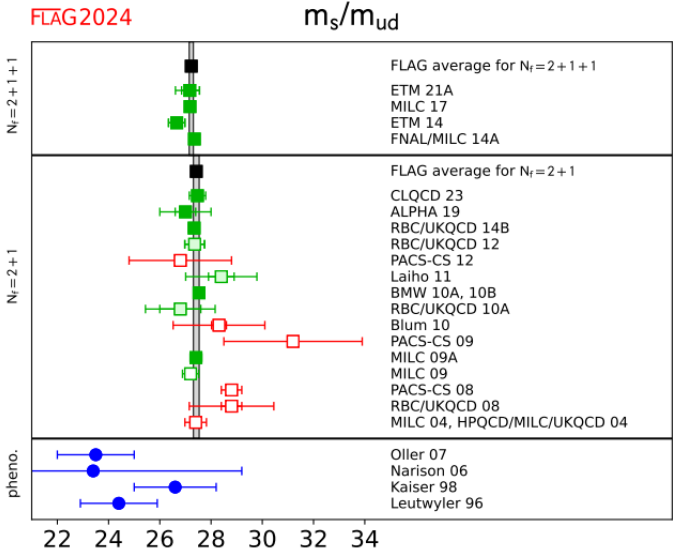


Results $\overline{m}_s$





Status of $\overline{m}_s/\overline{m}_{ud}$



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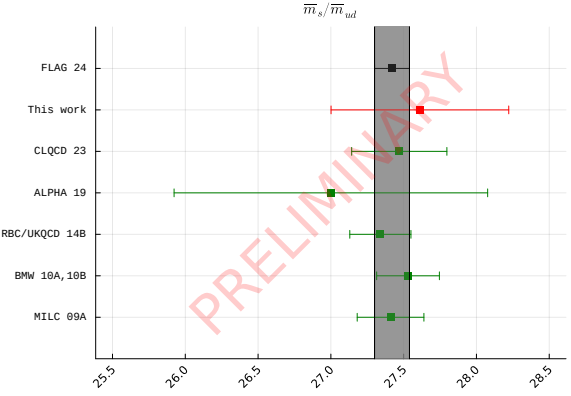
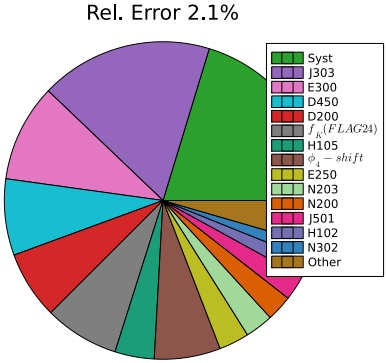




Conclusions

- Preliminary quark mass results in agreement with previous lattice determinations and FLAG24 average [2411.04268]. Relative precision:

	m_{ud}	m_s	m_s/m_{ud}
This work [PRELIMINARY]	2.5%	2.0%	2.1%
[ALPHA 19]	4.2%	3.6%	4.0%
FLAG24	1.2%	1.1%	0.4%

- Work in progress:
  - ➔ Extended study of systematic effects.
  - ➔ Inclusion of twisted mass data using the mixed-action setup.
  - ➔ Improvement on the RG running.

Thank you!

Backup

RG Running

We start with the RG equations

$$\mu\frac{d}{d\mu}\overline{m}_i(\mu) = \tau(\overline{g}(\mu))\overline{m}_i(\mu) \qquad \mu\frac{d}{d\mu}\overline{g}(\mu) = \beta(\overline{g}(\mu))$$

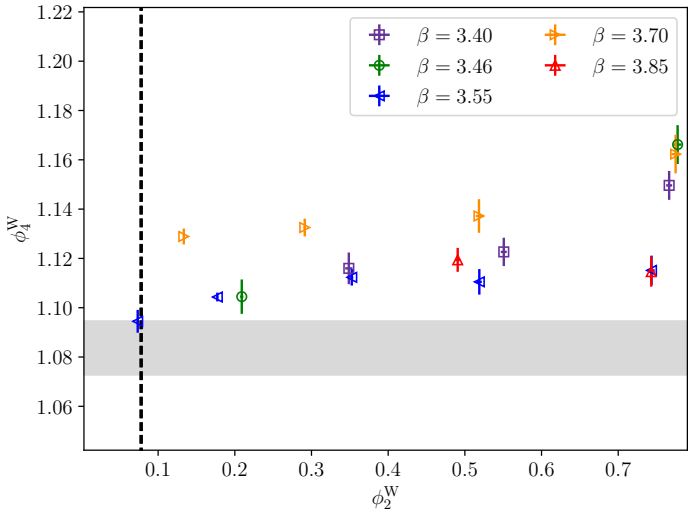
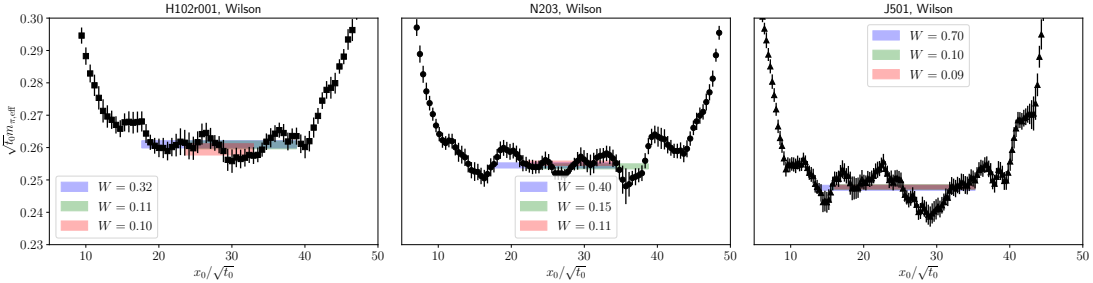
where

$$\beta(g) \underset{g\to 0}{\sim} -g^3(b_0 + b_1 g^2 + ...) \qquad \tau(g) \underset{g\to 0}{\sim} -g^2(d_0 + d_1 g^2 + ...)$$

We can now define the RGI mass

$$M_i = \overline{m}_i(\mu)\left(2b_0\overline{g}^2(\mu)\right)^{-\frac{d_0}{2b_0}} exp\left(-\int_0^{\overline{g}(\mu)} dg\left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right)$$

ϕ_4 shift


$\beta = 3.40$
$\beta = 3.46$
$\beta = 3.55$
$\beta = 3.70$
$\beta = 3.85$
ϕ_4^{W}
ϕ_2^{W}
1.22
1.20
1.18
1.16
1.14
1.12
1.10
1.08
1.06
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Plateau determination