Determination of α_s in the $SU(3)$ Yang-Mills theory

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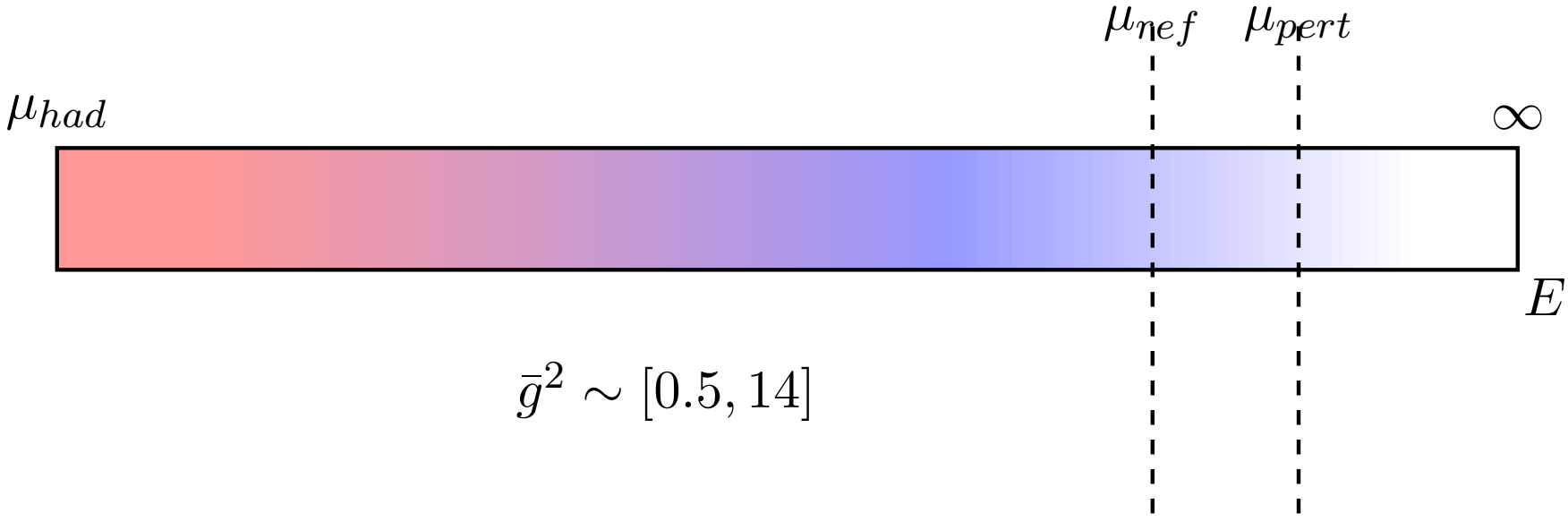
$$\Lambda_{\overline{MS}} = \frac{2c}{b_0}\Lambda_s$$

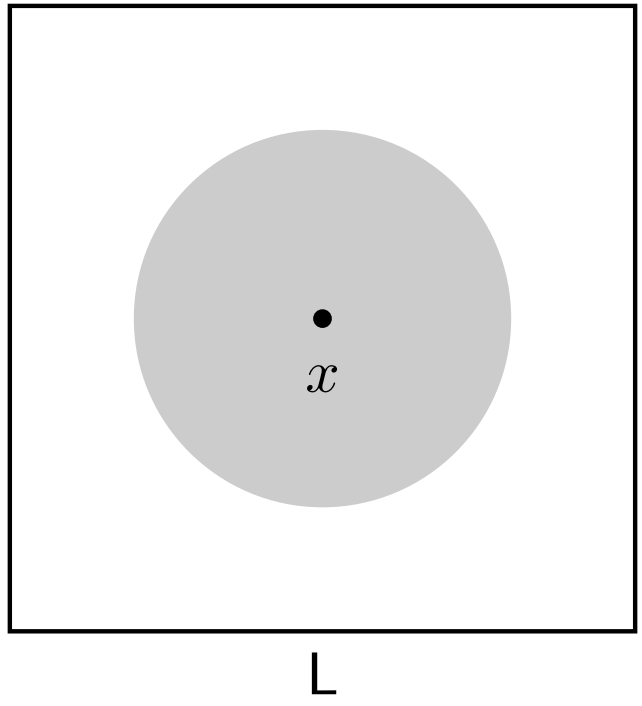
$$\bar{g}_s^2 = g^2_{\overline{MS}} + c g^4_{\overline{MS}} + O(g^6_{\overline{MS}})$$



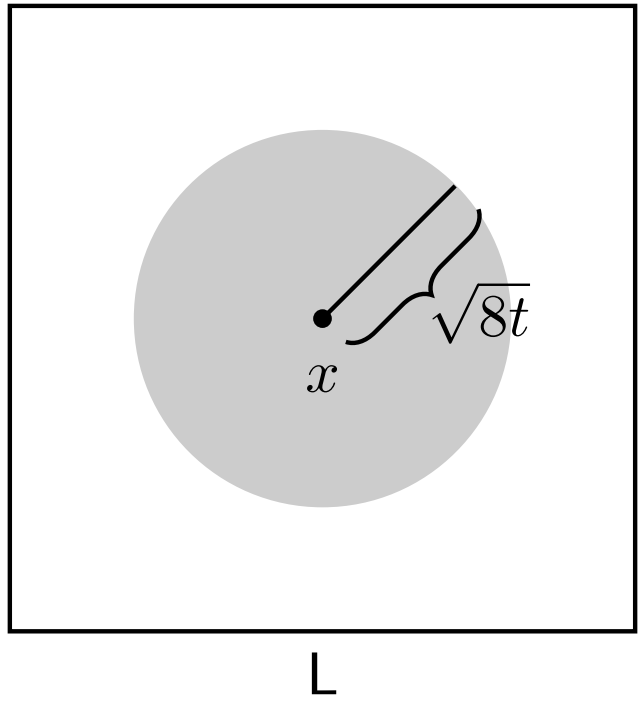
- (Dalla Brida and Ramos, 2019), (Dalla Brida, Höllwieser, et al., 2022)

Setup

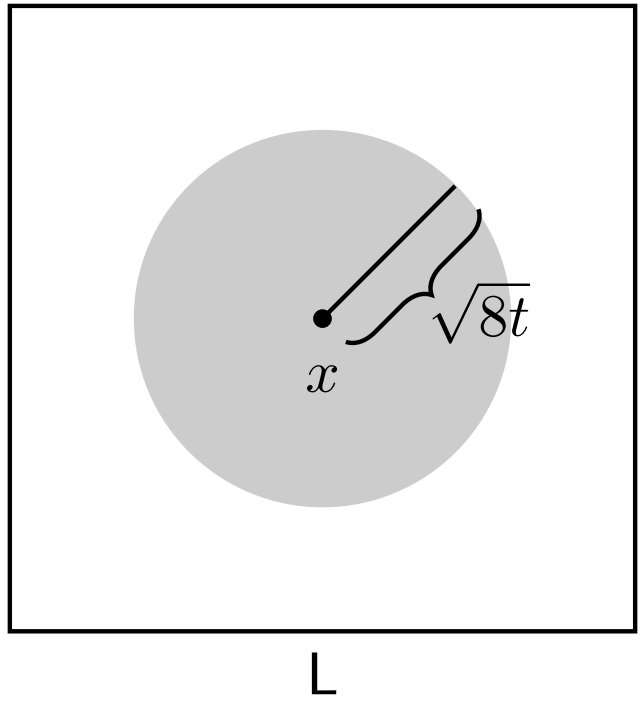
- finite volume scheme
- gradient flow
- translational invariant (TWBC)



$$\bar{g}^2 = \frac{1}{\mathcal{N}}\, t^2 \Big(\frac{4}{3} E_{\mathsf{pl}} - \frac{1}{3} E_{\mathsf{cl}} \Big)$$



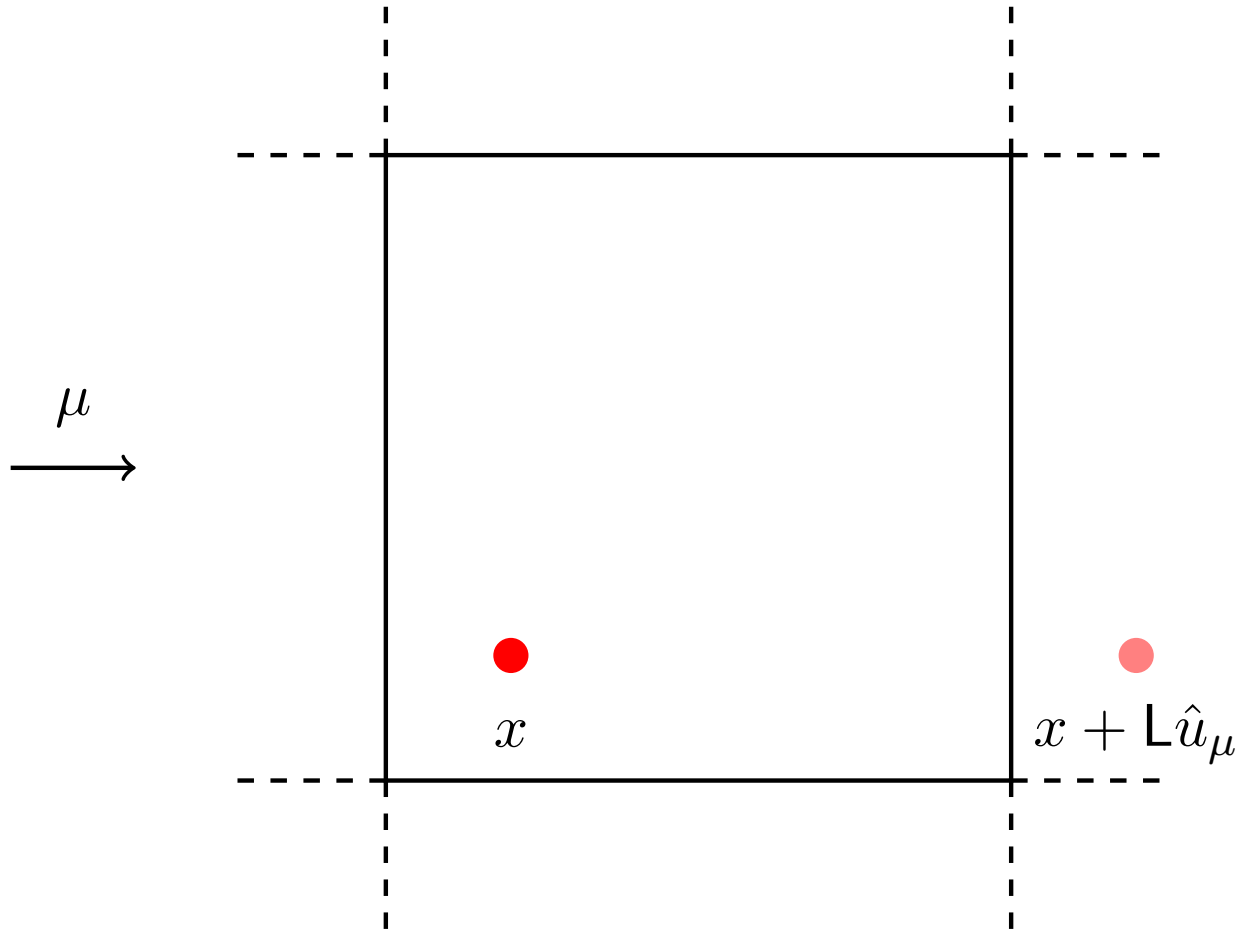
$$\bar{g}^2 = \frac{1}{\mathcal{N}}\, t^2 \Big(\frac{4}{3} E_{\mathsf{pl}} - \frac{1}{3} E_{\mathsf{cl}} \Big)$$



$$c = \frac{\sqrt{8t}}{L}$$

$$\bar{g}^2 = \frac{1}{\mathcal{N}}\, t^2 \Big(\frac{4}{3} E_{\mathsf{pl}} - \frac{1}{3} E_{\mathsf{cl}} \Big)$$

Twisted boundary conditions



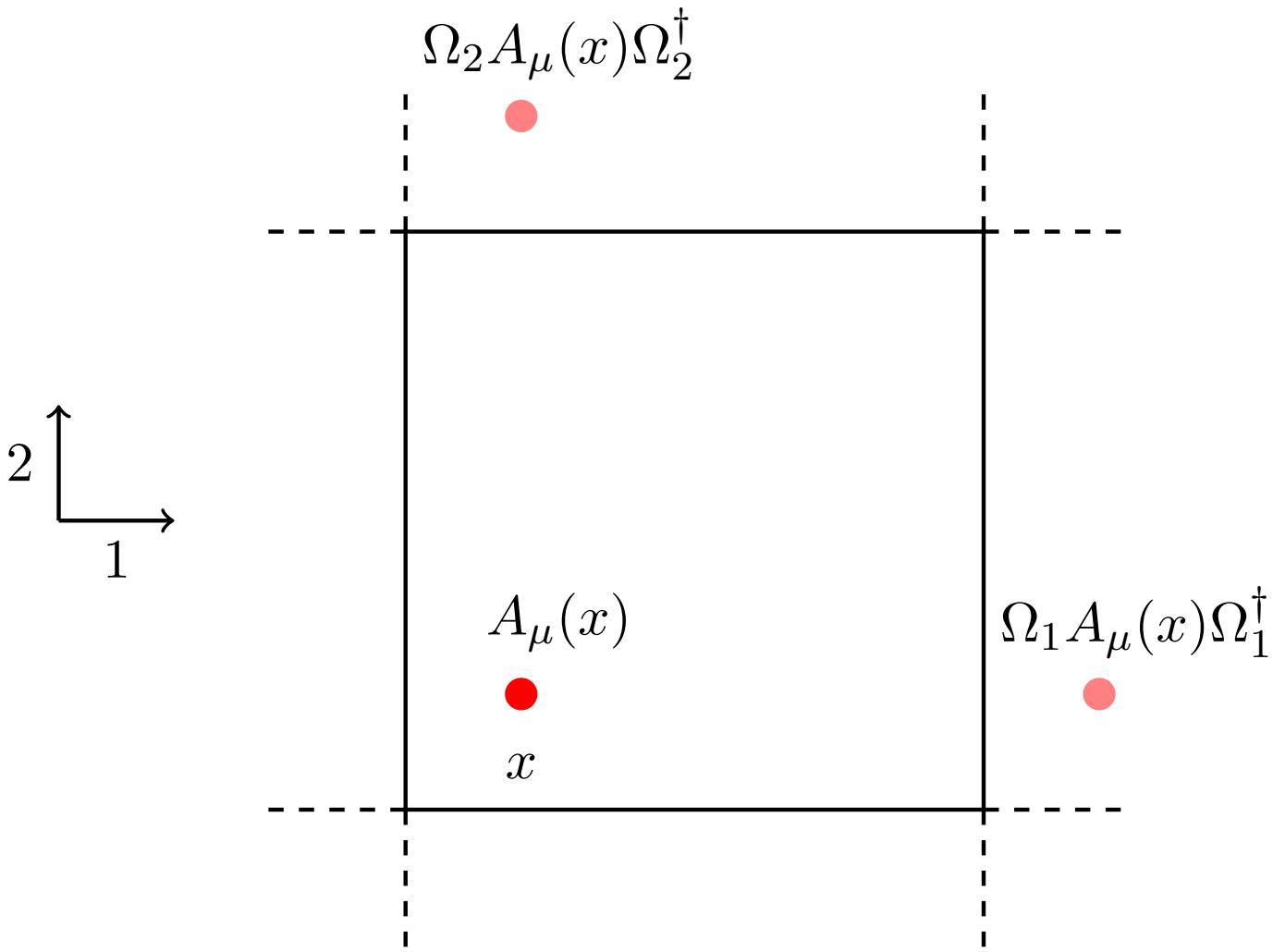
$$A_\nu(x) \longrightarrow \Omega_\mu(x)\, A_\nu(x)\, \Omega_\mu^{-1}(x) + \Omega(x)\partial_\mu\Omega(x)^{-1}$$

Periodic boundary conditions

- Degenerate minima of the action($\partial_{(\mu,\nu)} A = [A_\mu, A_\nu] = 0$)
- Quartic modes $\rightarrow \bar{g}^2$ non-analytic in $g^2_{\bar{MS}}$

- (Coste et al., 1985)

Twisted boundary conditions



- (Ramos, 2014)

Target precision

$$\frac{\Delta \bar{g}^2}{\bar{g}^4} < 1.0\ \mathrm{e}^{-04}$$

- $\sim 5.0\ \mathrm{e}^{-04}$ in refs

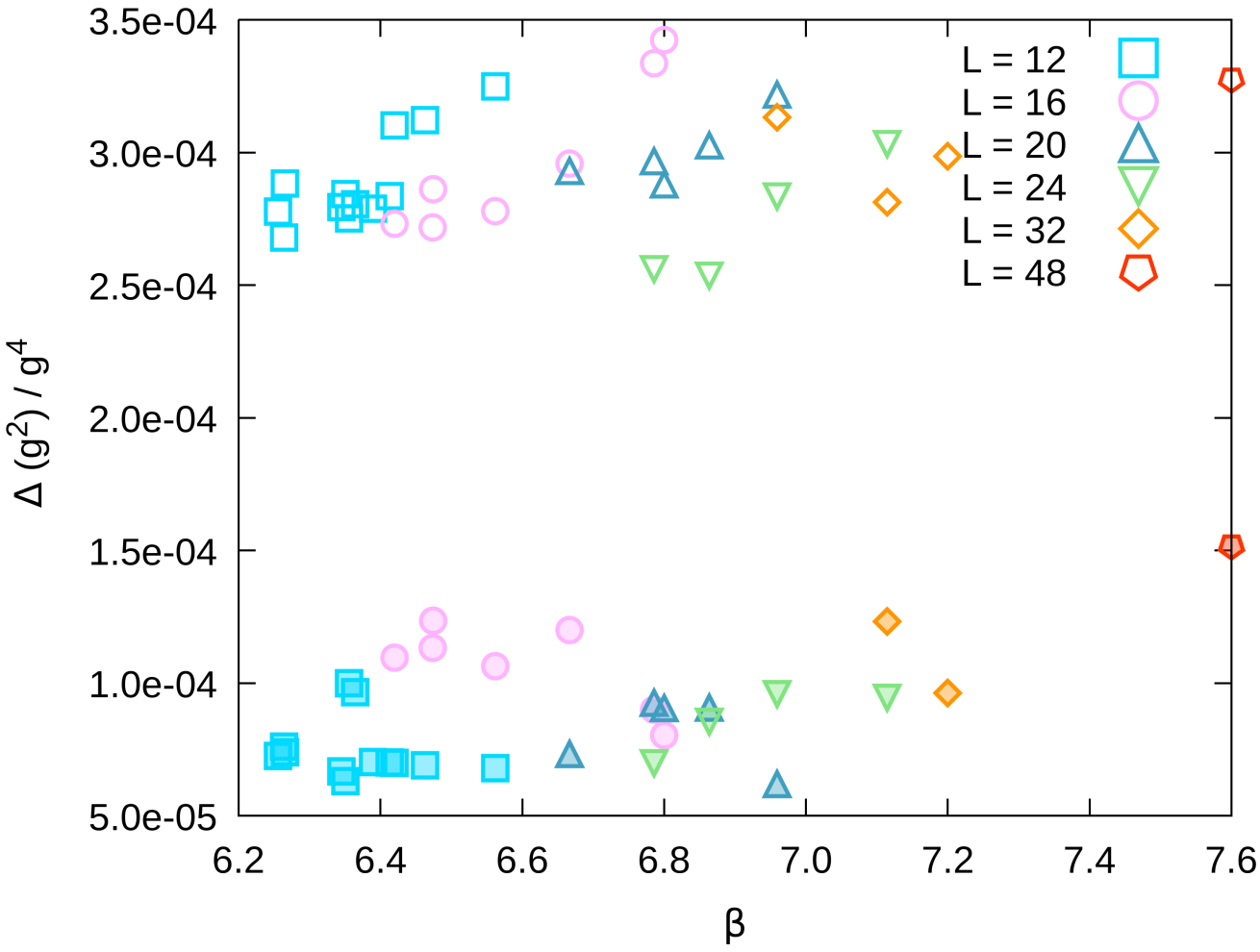
Mattia Dalla Brida & Alberto Ramos (2019).. "The gradient flow coupling at high-energy and the scale of SU(3) Yang–Mills theory".. in: *Eur. Phys. J. C* 79.8, p. 720. DOI: 10.1140/epjc/s10052-019-7228-z. arXiv: 1905.05147 [hep-lat].

Mattia Dalla Brida, Roman Höllwieser, et al. (2022).. "Determination of $\alpha_s(m_Z)$ by the non-perturbative decoupling method".. in: *Eur. Phys. J. C* 82.12, p. 1092. DOI: 10.1140/epjc/s10052-022-10998-3. arXiv: 2209.14204 [hep-lat].

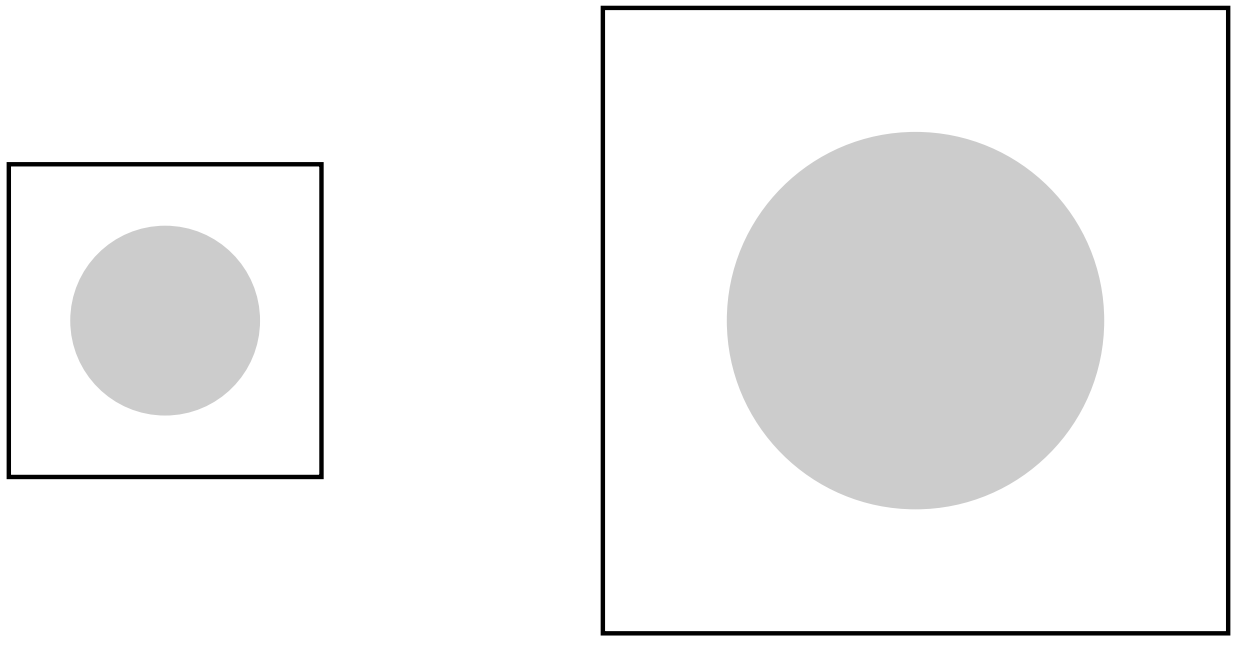
Finite lattice spacing

$\frac{L}{a}$	β	$\bar{g}^2(L)$	$\bar{g}^2_{TWBC}(L)$
12	6.2556	5.6616(89)	5.3402(21)
12	6.2643	5.5995(84)	5.2781(21)
12	6.2654	5.5876(90)	5.2687(21)
12	6.3451	5.0759(72)	4.7818(15)
12	6.3509	5.0329(72)	4.7525(14)
12	6.3560	5.0078(69)	4.7263(22)
12	6.3642	4.9594(69)	4.6848(21)
12	6.3894	4.8288(65)	4.5601(15)
12	6.4133	4.7133(63)	4.4477(14)
12	6.4200	4.6821(68)	4.4218(14)
12	6.4630	4.4928(63)	4.2418(12)
12	6.5619	4.1143(55)	3.8897(10)
16	6.4200	6.0206(99)	5.6452(35)
16	6.4740	5.5929(85)	5.2538(31)
16	6.4741	5.5766(89)	5.2552(34)
16	6.5619	5.0554(71)	4.7531(24)
16	6.6669	4.5412(61)	4.2850(22)
16	6.7859	4.0969(56)	3.8700(13)
16	6.8000	4.0449(56)	3.8273(12)
20	6.6669	5.4190(86)	5.0971(19)
20	6.7859	4.7887(68)	4.4967(19)
20	6.8000	4.7153(64)	4.4388(18)
20	6.8637	4.4531(60)	4.1906(16)
20	6.9595	4.0977(54)	3.87140(93)
24	6.7859	5.5530(79)	5.2255(19)
24	6.8637	5.1006(66)	4.7942(20)
24	6.9595	4.6378(61)	4.3725(18)
24	7.1146	4.0599(50)	3.8391(14)
32	7.1146	5.926(11)	4.6816(27)
32	7.2000	4.9887(70)	4.3226(18)
48	7.6000	4.5933(63)	4.0688(25)

- Dalla Brida, Höllwieser, et al., 2022


3.5e-04
3.0e-04
2.5e-04
2.0e-04
1.5e-04
1.0e-04
5.0e-05
Δ (g²) / g⁴
6.2
6.4
6.6
6.8
7.0
7.2
7.4
7.6
β
L = 12
L = 16
L = 20
L = 24
L = 32
L = 48

Step scaling function



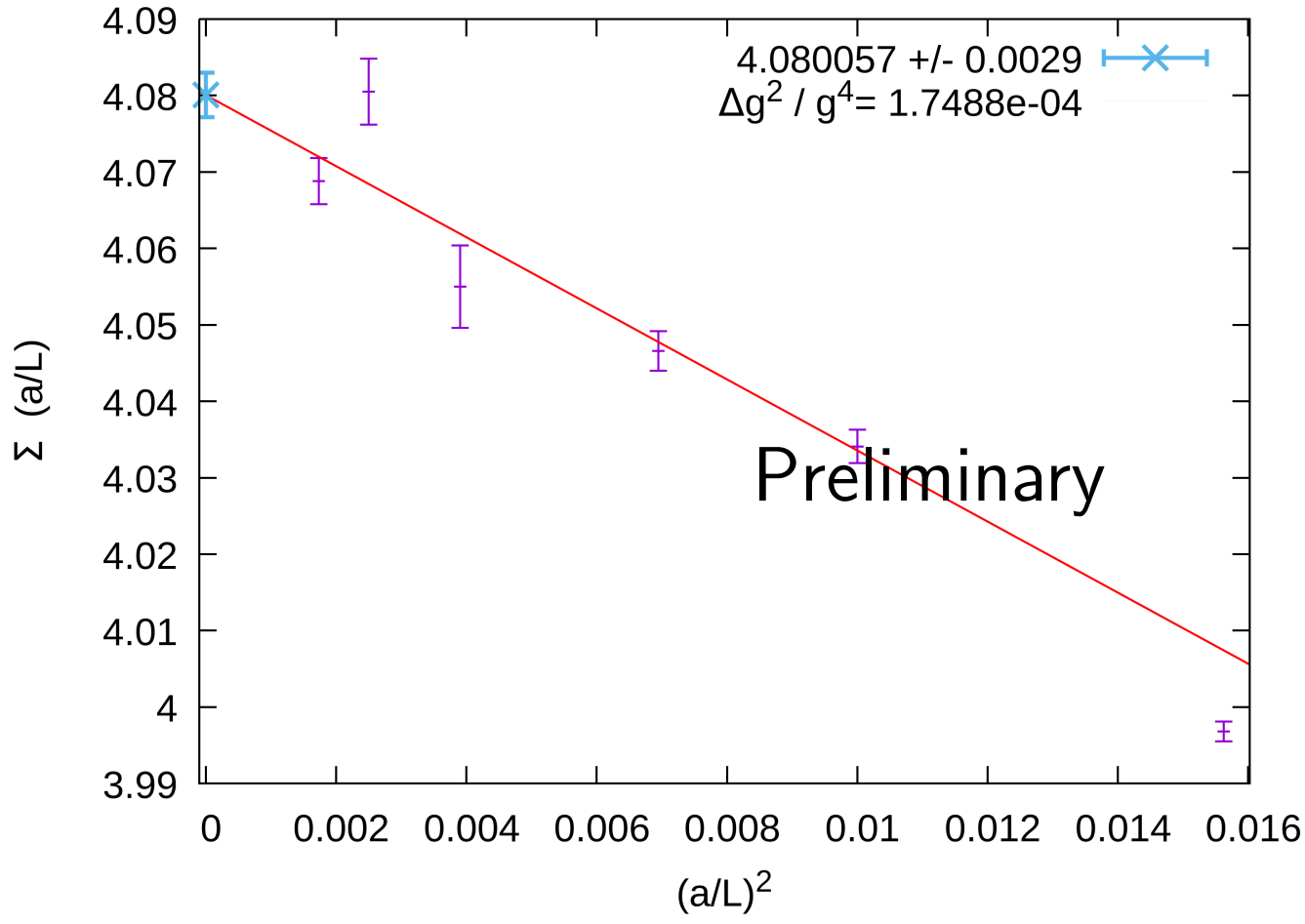
$$u = \bar{g}^2(L) \longrightarrow \sigma(u) = \bar{g}^2(2L)$$

Continuum limit

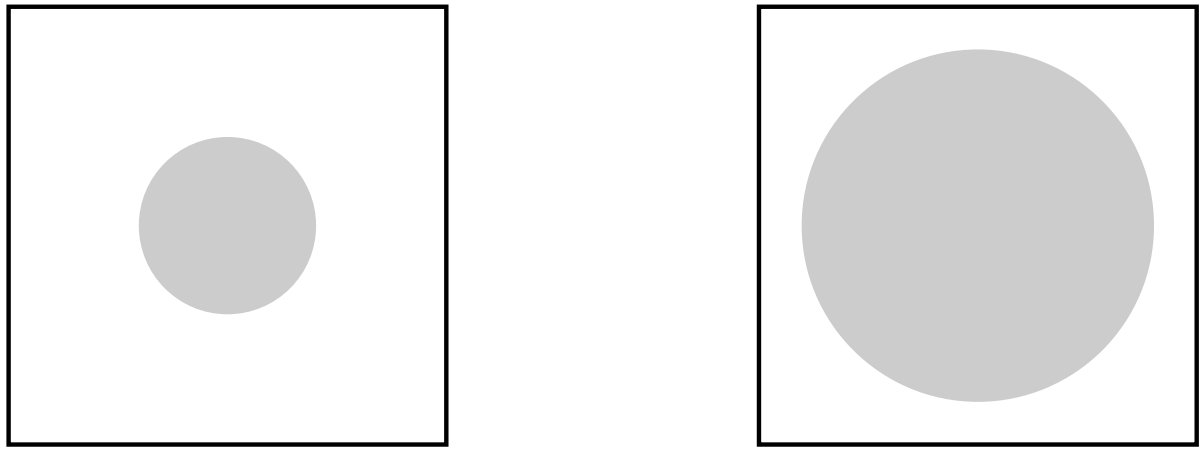
$\frac{L}{a}$	β	$\bar{g}^2(L)$	$\bar{g}^2(2L)$
8	6.7468	2.82072(39)	3.99714(98)
10	6.9086	2.82118(41)	4.0355(20)
12	7.0480	2.81979(67)	4.0451(23)
16	7.2728	2.8196(12)	4.0531(48)
20	7.4517	2.81859(91)	4.0764(39)
24	7.6000	2.82053(75)	4.0688(25)

$\frac{L}{a}$	u	$\Sigma(u, \frac{a}{L})$
8	2.82053	3.9968(13)
10	2.82053	4.0341(22)
12	2.82053	4.0466(26)
16	2.82053	4.0550(54)
20	2.82053	4.0805(43)
24	2.82053	4.0688(30)

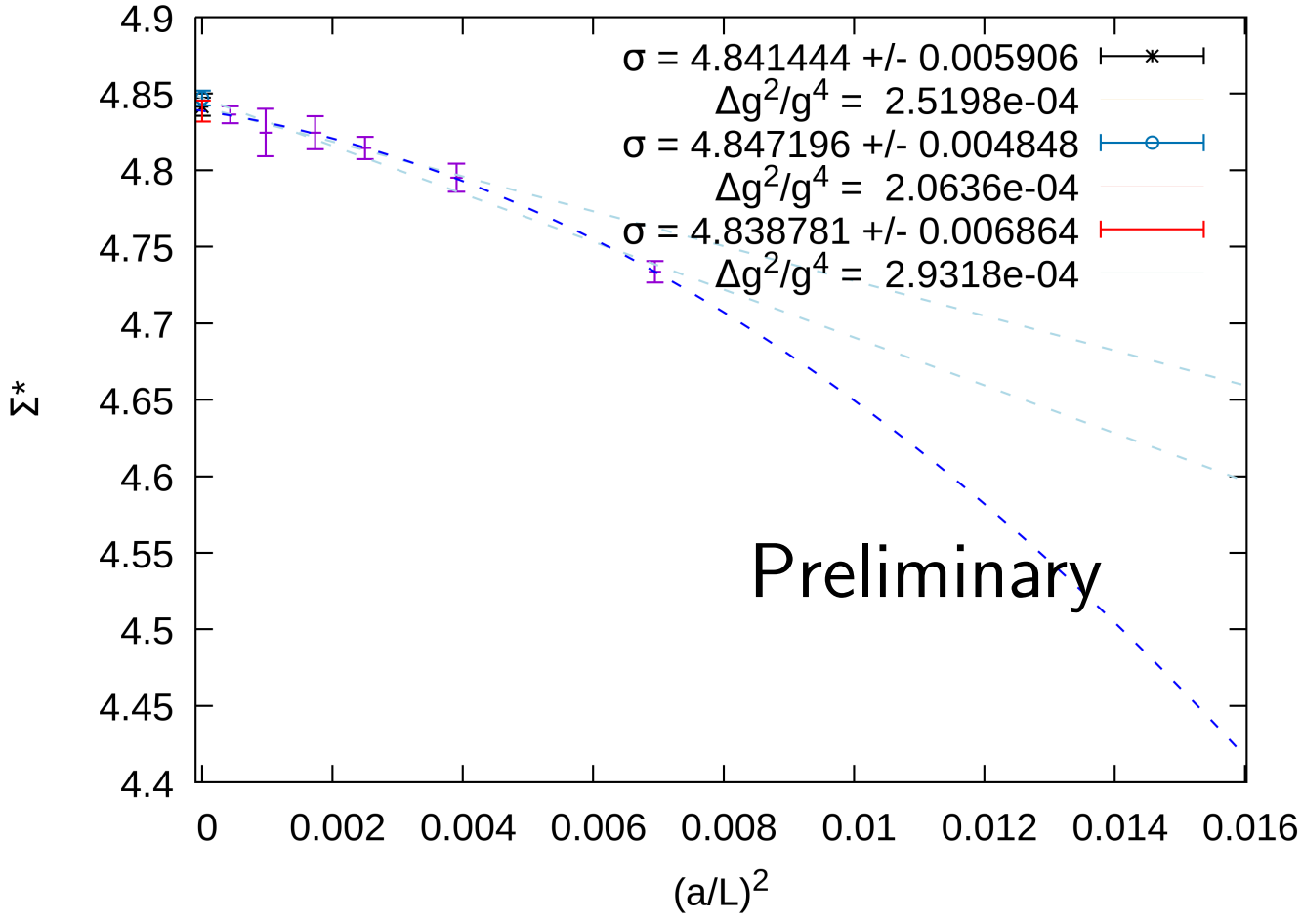
Continuum limit



Step scaling function $\mathcal{J}_1$



$$u = \bar{g}^2_{c=0.2}(L) \longrightarrow \sigma(u) = \bar{g}^2_{c=0.4}(L)$$

- (Nada and Ramos, 2021)

Continuum limit