Validating a novel approach to identifying phase transitions

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Abstract

The Ising model serves as a model for simple magnetic systems and as a testing ground for the study of strongly-coupled systems. The model is exactly solvable in two dimensions and can be simulated with relatively small computing resources. We investigate the phase transition of the Ising model through a novel scaling procedure first proposed to explore the phase structure of a theory with four $SU(2)$ doublet fermions in Butt et al. [1]. We present a preliminary validation of this method by demonstrating that the procedure aligns with the known properties of the two-dimensional Ising model.

Motivation: Identifying phase transitions

Identifying the phase structure of strongly-coupled systems is a central challenge for nonperturbative field theory, with applications from condensed matter to theories beyond the standard model of particle physics.

Phase transitions can be identified through a variety of methods:

- discontinuities in physical quantities, such as magnetisation,
- changes in spectral quantities,
- analysing critical exponents.

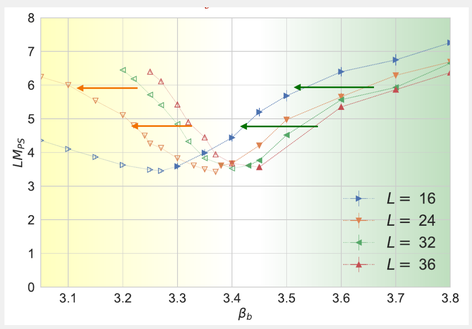
We study a new approach, based on the scaling behaviour of masses extracted from two-point functions.

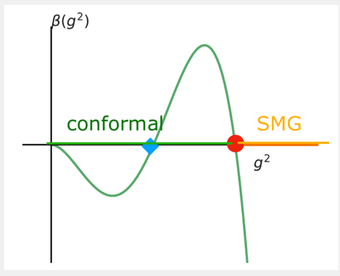
A scaling approach to phase transitions

Symmetric mass generation is a mechanism that does not break chiral symmetry, but generates mass through a nonvanishing vacuum state induced by a symmetric multifermion condensate. The authors of Ref. [1] studied the RG-invariant combination $M_{\mathrm{PS}}L$ to examine the existence of symmetric mass generation for a single massless staggered fermion in the fundamental representation of $SU(2)$. The RG flow of the bare coupling from UV to IR can be inferred from the change in coupling that is needed to keep $M_{\mathrm{PS}}L$ fixed as the volume is changed. For fixed $M_{\mathrm{PS}}L$:

- As aM_{PS} increases, the bare coupling moves away from a UV fixed point.
- As aM_{PS} increases, the bare coupling moves towards an IR fixed point.

This behaviour was indeed observed in Ref. [1] and illustrated below.





[LEFT] The phase transition of an $SU(2)$ theory with $N_f = 4$ massless fundamental fermions, from [1].
[RIGHT] A sketch of the merged fixed point in this diagram, from [2].

Ising model

The two-dimensional Ising model is a well-known and widely studied model for magnetic materials that provides an ideal model for testing nonperturbative computational techniques (see, for example, Ref. [3]):

- Exactly solvable
- Known phase transition
- Low computational simulation cost

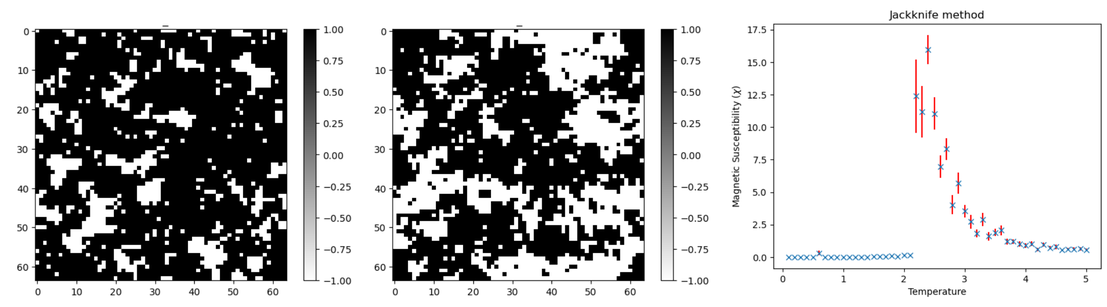
The Ising model Hamiltonian for spin s_i is defined by

$$H = -J \sum_{\langle ij \rangle} s_i s_j - B \sum_i s_i$$

where J is the coupling and B is the external magnetic field, which we set to zero. In the absence of the magnetic field, the Hamiltonian can be expressed in terms of the single dimensionless combination J/k_BT. The critical temperature is given by $k_BT_c/J = 2/\log(1+\sqrt{2}) \simeq 2.26$.

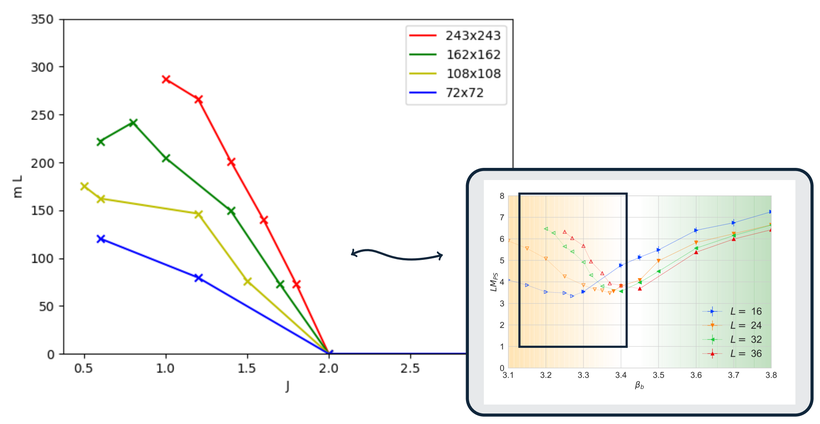
Numerical tests

We simulate square lattices up to size 243×243 (a size chosen for scaling reasons) via the Metropolis algorithm with thermalisation from both hot and cold starts and configurations saved every 100 Monte Carlo timesteps.



[LEFT] and [CENTER]: Sample plots of the lattices at $k_BT/J = 1$.
[RIGHT]: Sample plot of the magnetic susceptibility as a function of temperature in lattice units, demonstrating the presence of the phase transition.

Results



[MAIN] The phase transition of the two-dimensional Ising model, illustrated through the scaling behaviour of mL.
[INSET] Phase transition from [1].

The behaviour of mL as a function of temperature for four different volumes, demonstrating the flow away from the phase transition as the temperature is decreased at fixed mL, characteristic of a UV fixed point.

Summary

Scaling behaviour of correlators correctly identifies the known phase transition in the two-dimensional Ising model in the paramagnetic phase.

Future work:

- map the scaling behaviour of improved correlators in the ferromagnetic phase
- implement cluster algorithm and improved estimators
- increase statistics and lattice sizes

References

[1] Nouman Butt, Simon Catterall, and Anna Hasenfratz. *Symmetric Mass Generation with Four SU(2) Doublet Fermions*. *Phys. Rev. Lett.*, 134:031602, 2025.

[2] Anna Hasenfratz. *Symmetric Mass Generation: A New Paradigm*. ETH Theory Colloquium, 2025.

[3] Mark Newman and Gerard T. Barkema. *Monte Carlo Methods in Statistical Physics*. Clarendon Press, 1999.

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