

Hadron structure in the context of the EIC program

Constantia Alexandrou



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EXCELLENCE/0524/0459

Outline

* Introduction

- 📍 **Lattice QCD zero-temperature state-of-the-art simulations**

- 📍 **EIC science program for hadron structure**

* Precision lattice QCD hadron structure quantities (pion, kaon and nucleon)

- 📍 **Mellin moments of PDFs and GPDs:**

- ✦ **first and second Mellin moments**

- ✦ **higher than second Mellin moments**

* Direct computation of PDFs and GPDs (TMDs not covered, see 4 talks Friday afternoon)

Thanks to: Andrea Shindler, Artur Avkhadiev, Christian Zimmerman, Christopher Monahan, Dimitra Pefkou, Wayne Morris, Sara Collins, Wei Wang, Zhao Yong, for sending highlights

Lattice QCD - 45 years

$$\mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f$$

Fritzsch, Gell-Mann and Leutwyler, Phys. Lett. 47B (1973) 365

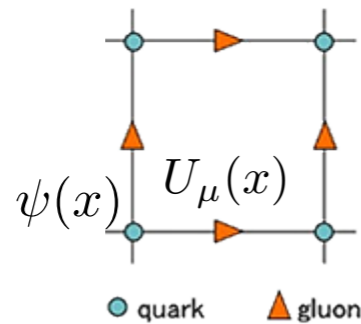
- **QCD defined on a lattice provides a non-perturbative regularization**

- **Unique properties:**

- ★ Confinement
- ★ Asymptotic freedom
- ★ Mass generation via interaction



Ken Wilson 1974

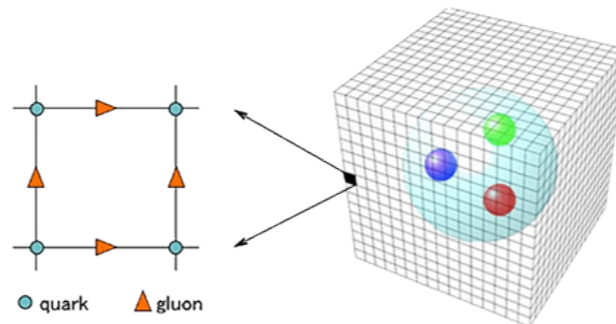


- **First simulation of SU(2) gauge ensembles**

- **Many algorithmic improvements:**



Mike Creutz 1980



- 1)
- 2) Domain decomposition 2004, Martin Lüscher
- 3) ...

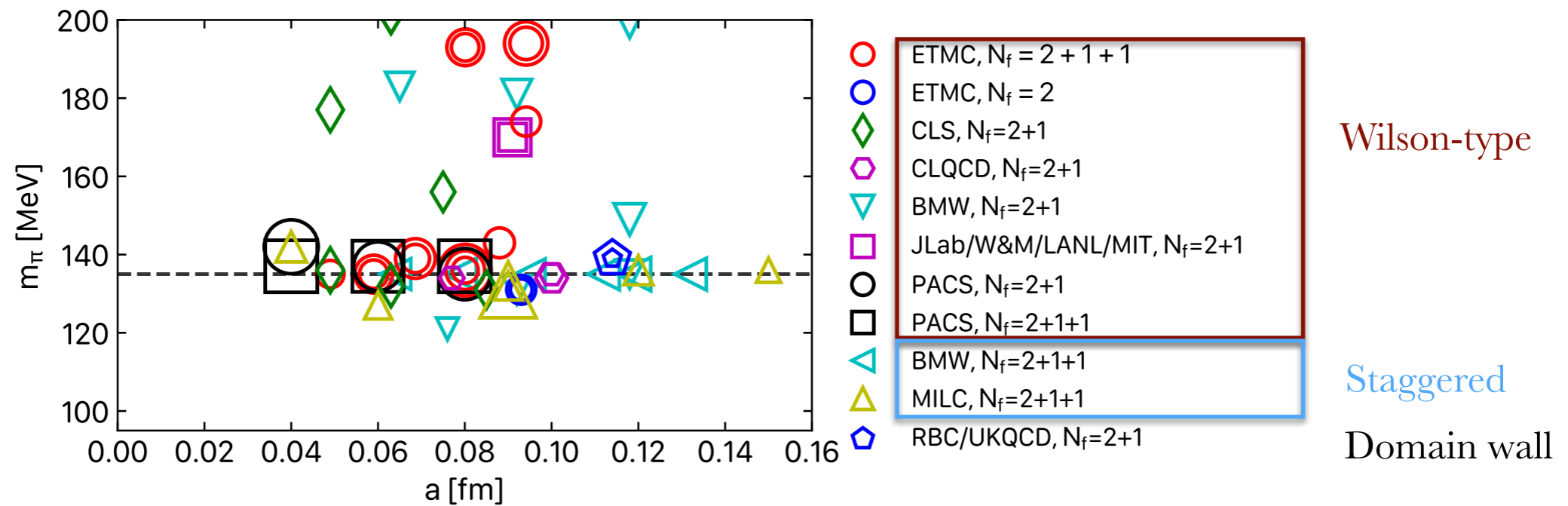


- **Modern era of lattice QCD**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \text{Det}(D_f[U]) \right) e^{-S_g[U]}$$

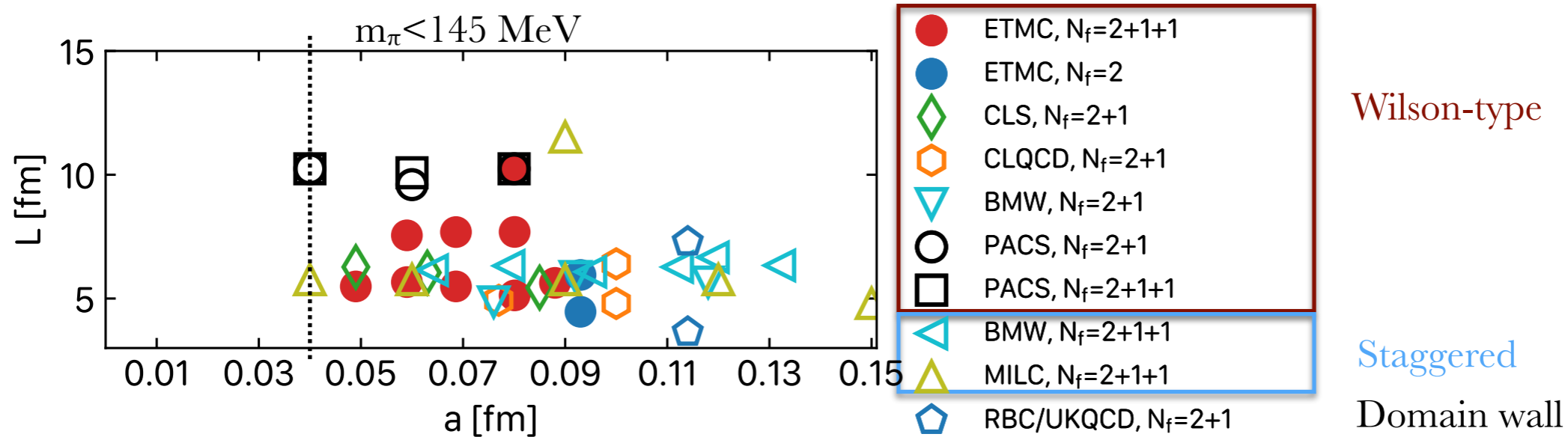
State-of-the-art gauge ensembles for hadron structure

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \text{Det}(D_f[U]) \right) e^{-S_g[U]}$$

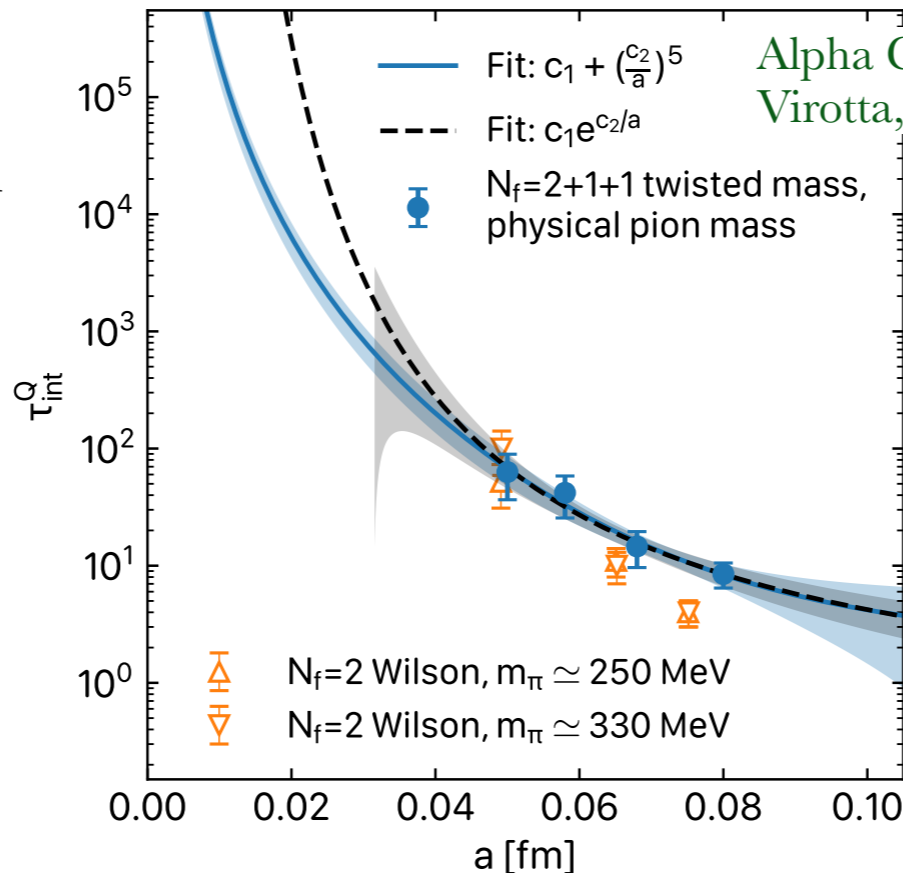


State-of-the-art gauge ensembles for hadron structure

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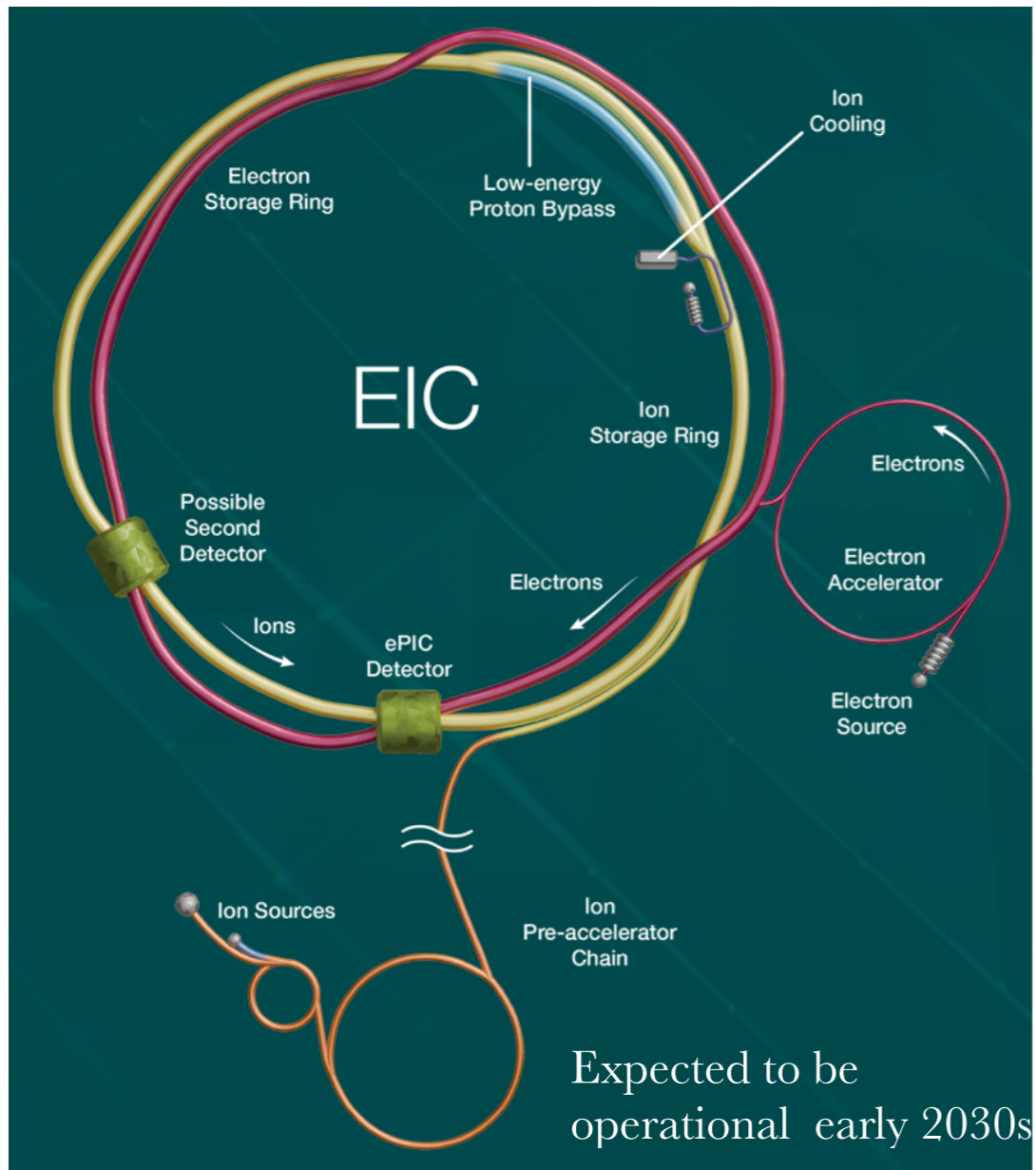


New ideas are needed to efficiently go below $\sim 0.04 \text{ fm}$



Alpha Collaboration: S. Schaefer, R. Sommer, F. Virota, Nucl. Phys. B845 (2011) 93

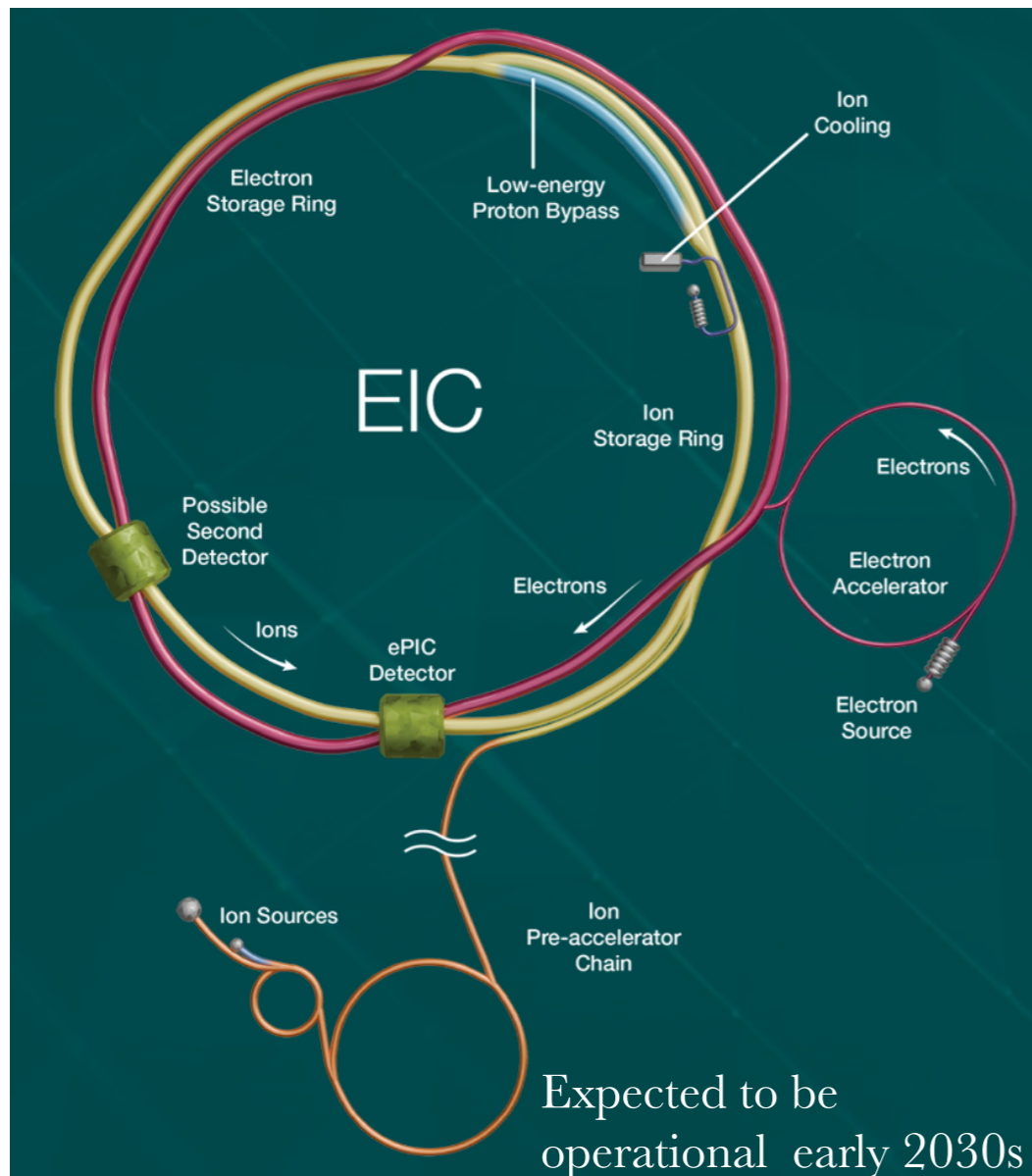
Electron Ion Collider



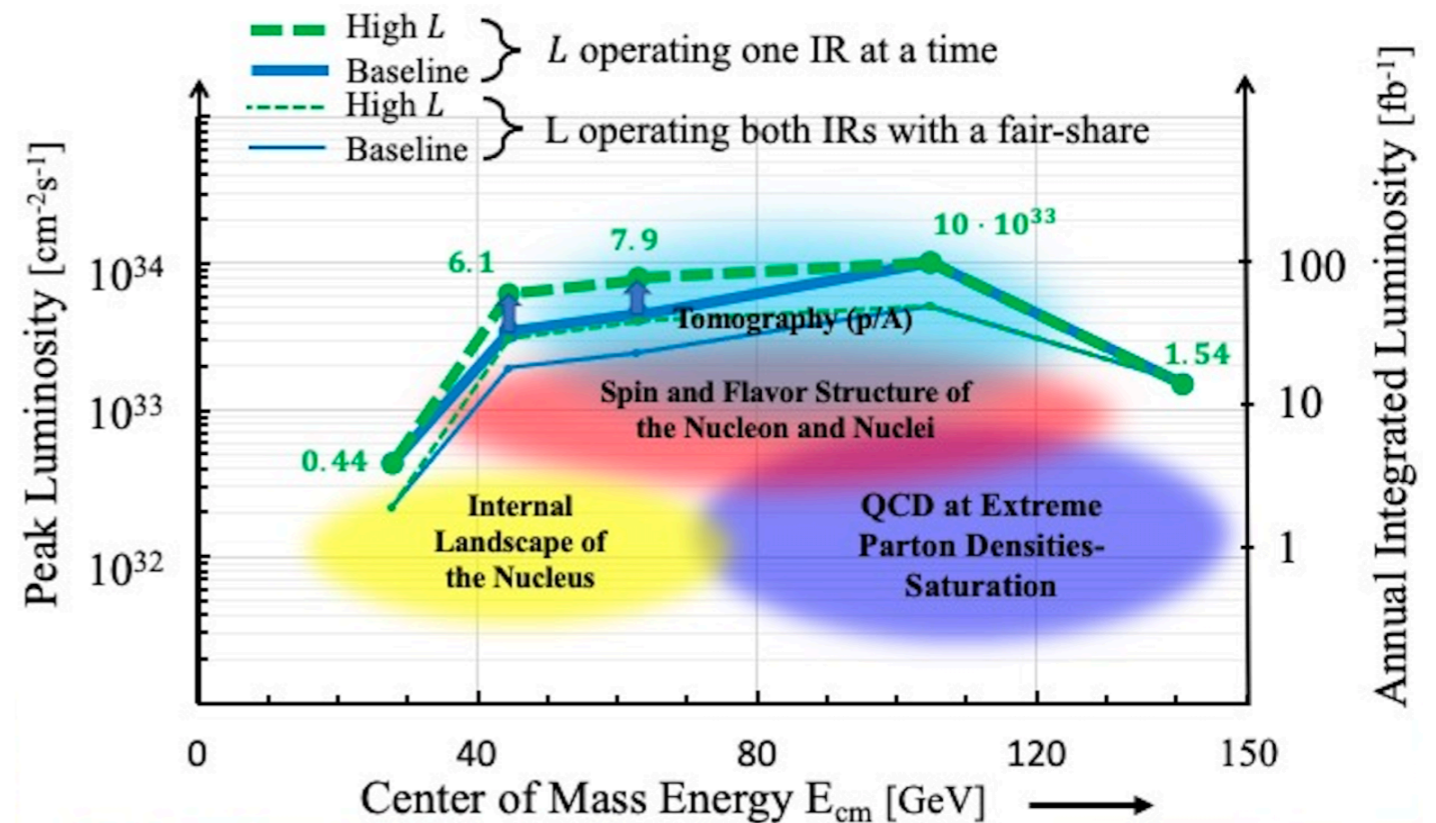
Key science questions:

- How do nucleon properties such as mass and spin emerge from partons and their underlying interactions?
- How are partons inside the nucleon distributed in both momentum and position space?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? **How do the confined hadronic states emerge from these quarks and gluons?** How do the quark-gluon interactions create nuclear binding?
- How does a dense nuclear environment affect the dynamics of quarks and gluons, their correlations, and their interactions? What happens to the gluon density in nuclei? Does it saturate at high energy, giving rise to gluonic matter or a gluonic phase with universal properties in all nuclei and even in nucleons?

Electron Ion Collider



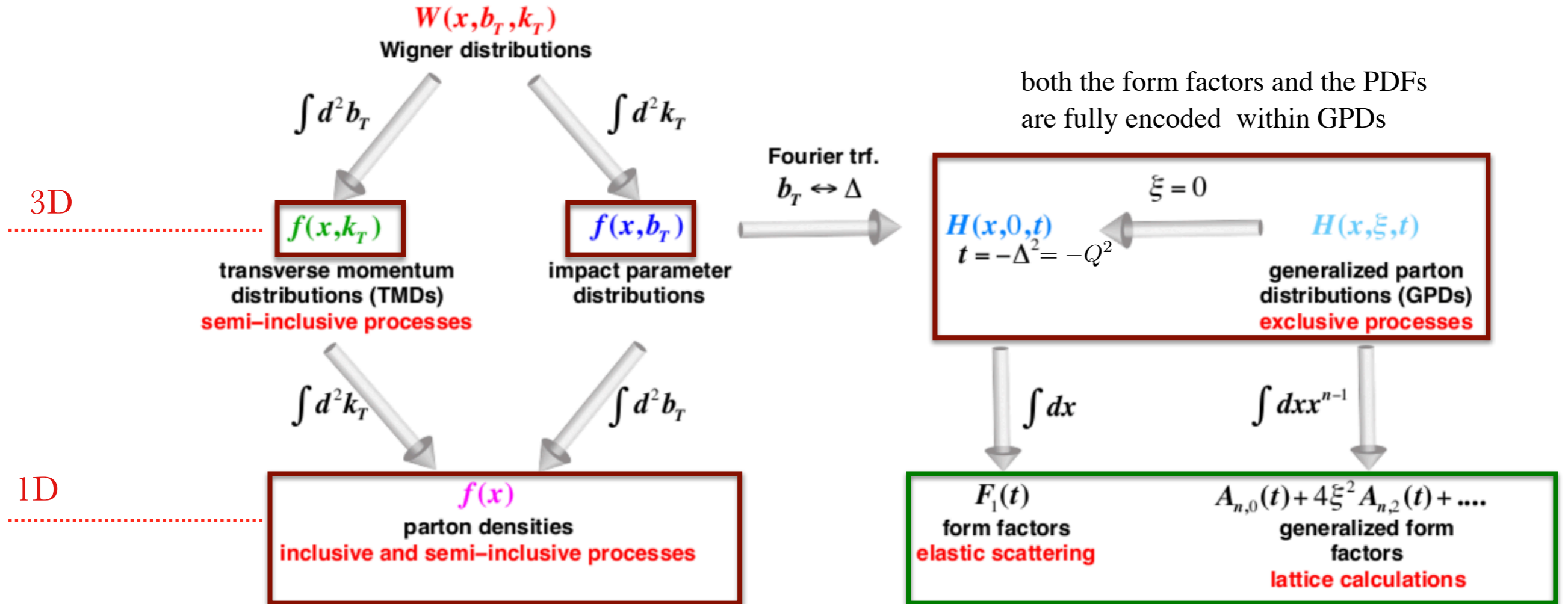
- High lepton and light-ion beam polarization ($\sim 70\%$)
- Availability of ion beams from deuterons to the heaviest nuclei



Key science questions:

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- How are partons inside the nucleon distributed in both momentum and position space?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? **How do the confined hadronic states emerge from these quarks and gluons? ...**
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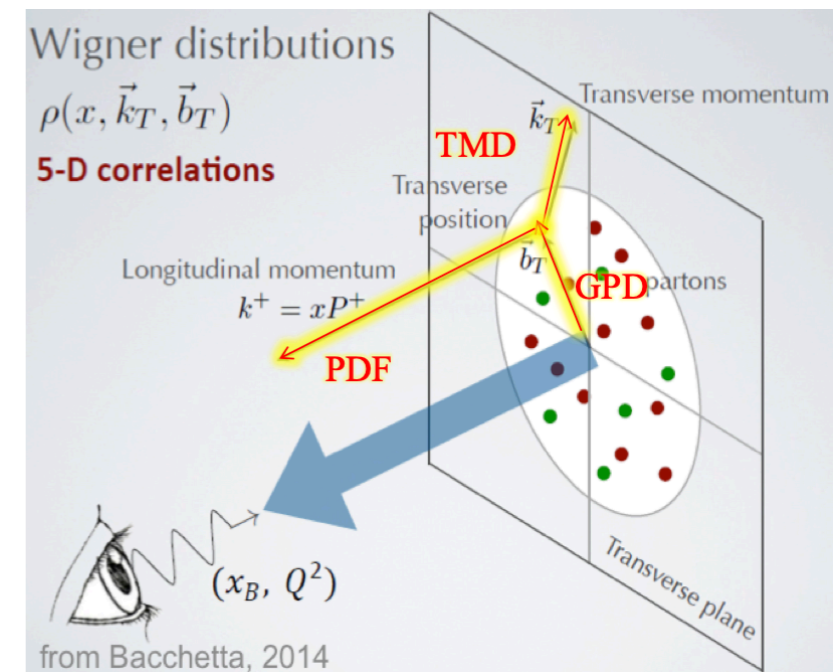
Hadron tomography



both the form factors and the PDFs are fully encoded within GPDs

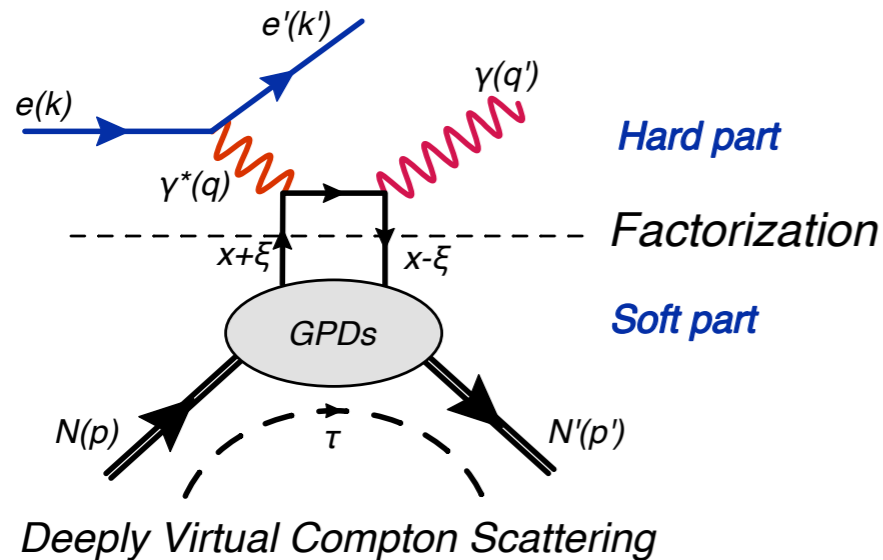
Studies in lattice QCD since the 1990s
EIC report: A. Arcardi *et al.* arXiv:1212.1701

- ✱ Parton Distribution Functions (PDFs) \rightarrow momentum and spin distributions of quarks and gluons.
- ✱ Generalised Parton Distributions (GPDs) \rightarrow correlation between the transverse position and longitudinal momentum of the partons.
- ✱ Transverse Momentum Dependent (TMDs) PDFs \rightarrow link the longitudinal and transverse momenta of the partons.

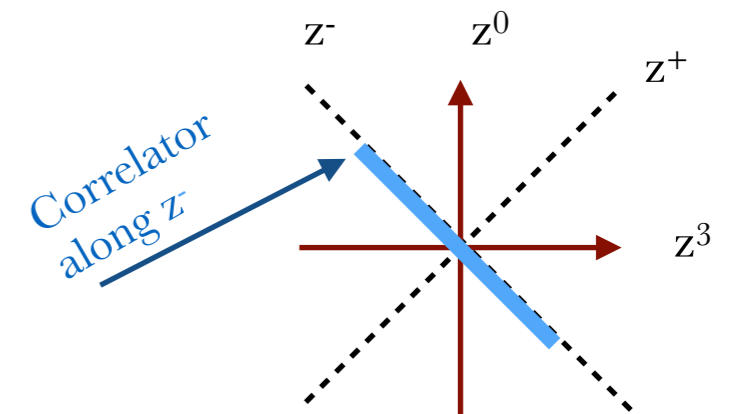


Generalised Parton Distributions (GPDs)

- * High energy scattering processes: Factorization into a hard partonic subprocess, calculable in perturbation theory, and a universal non-perturbative parton distribution



- D. Mueller *et al.*, Fortschr. Phys. 42, 101 (1994)
- A. V. Radyushkin, PLB 380, 417; PLB385, 333 (1996); PRD56, 5524 (1997)
- X. Ji, PRL. 78, 610; PRD55, 7114 (1997); J. Phys. G24, 1181 (1998)



- Probes 3D quark distribution
—> Replace final photon with J/ ψ for the 3D gluon distribution

Spin 1/2: The unpolarized GPDs of the nucleon

$$F_{\gamma^+}(x, \xi, \tau) = \frac{1}{2P^+} \left[H(x, \xi, \tau) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, \tau) \bar{u}(p') \frac{i\sigma^{+\alpha} (p'_\alpha - p_\alpha)}{2m} u(p) \right]$$

- $P^+ = \frac{p'^+ + p^+}{2}$
- $\tau = -t = -Q^2 \equiv (p' - p)^2$
- $\xi = \frac{p^+ - p'^+}{2P^+}$: skewness

* **Forward limit: PDFs** e.g. $H_{\gamma^+}(x, 0, 0) = q(x)$, $x > 0$; $H_{\gamma^+}(x, 0, 0) = -\bar{q}(x)$, $x < 0$

* Moments of GPDs are generalized form factors

$$\int_{-1}^1 dx x^{n-1} H_{\gamma^+}(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i A_{ni}(\tau) + \text{mod}(n, 2)(2\xi)^n C_{n0}(\tau)]$$

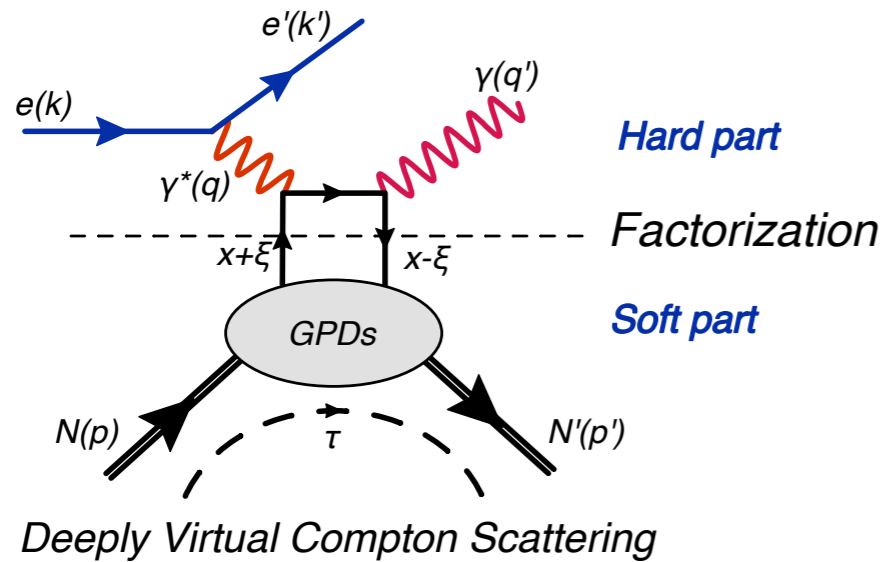
$$\int_{-1}^1 dx x^{n-1} E_{\gamma^+}(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i B_{ni}(\tau) - \text{mod}(n, 2)(2\xi)^n C_{n0}(\tau)]$$

Ph. Hagler, Phys. Rept. 490 (2010) 49

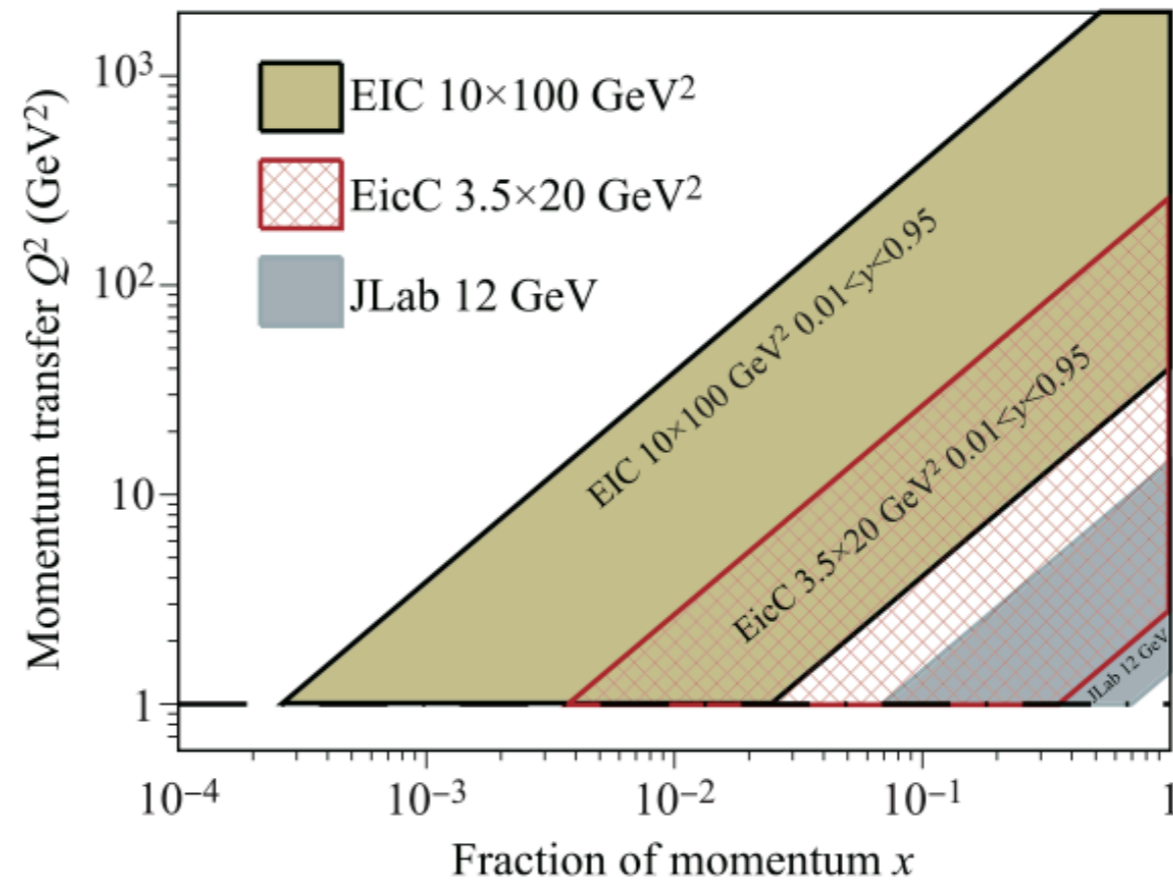
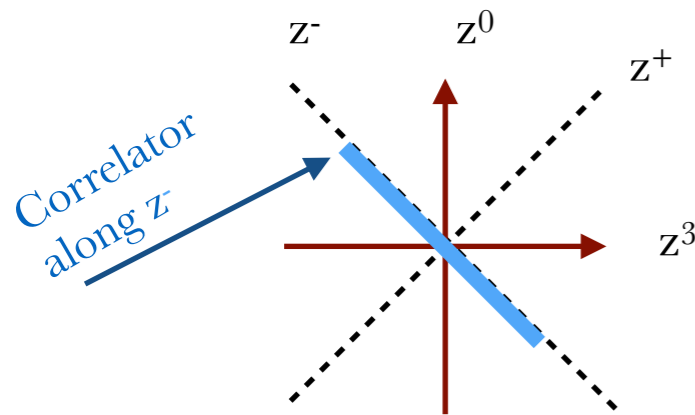
n=1 : Dirac and Pauli FFs $A_{10}(\tau) = F_1(\tau)$, $B_{10}(\tau) = F_2(\tau)$,

Generalised Parton Distributions (GPDs)

- * High energy scattering processes: Factorization into a hard partonic subprocess, calculable in perturbation theory, and a universal non-perturbative parton distribution



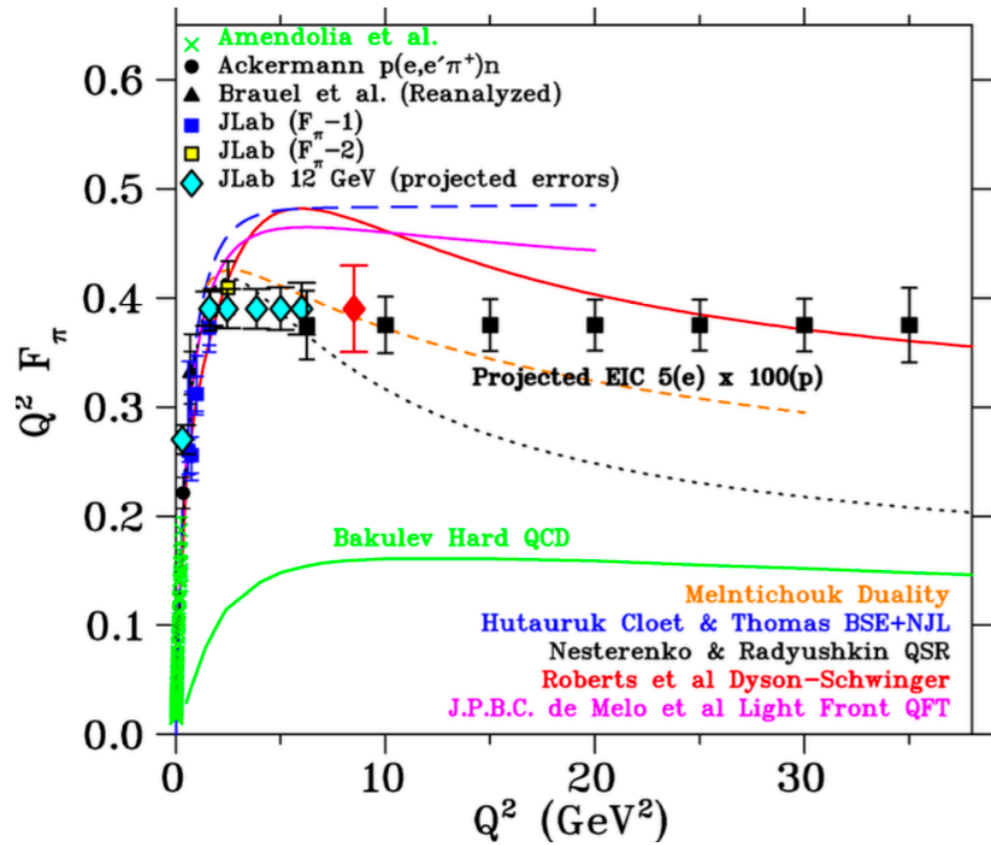
- D. Mueller *et al.*, Fortschr. Phys. 42, 101 (1994)
- A. V. Radyushkin, Phys. Lett. B380, 417 (1996), hep-ph/9604317
- A. V. Radyushkin, Phys. Lett. B385, 333 (1996), hep-ph/9605431
- A. V. Radyushkin, Phys. Rev. D56, 5524 (1997), hep-ph/9704207
- X. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249
- X. Ji, Phys. Rev. D55, 7114 (1997), hep-ph/9609381
- X. Ji, J. Phys. G24, 1181 (1998), hep-ph/9807358



EIC complements JLab accessing the low x-region probing flavor and quark-antiquark asymmetries

Examples of EIC hadron quantities (1D structure): form factors

✳ Charges, form factors (FFs) of pion, kaon and proton where EIC will access Q^2 up to 40 GeV^2



✳ Need to reach large momentum transfers

- Breit frame allows larger momentum transfers

S. Syritsyn et al, PoS LATTICE2024 (2025) 340 & EINN2025

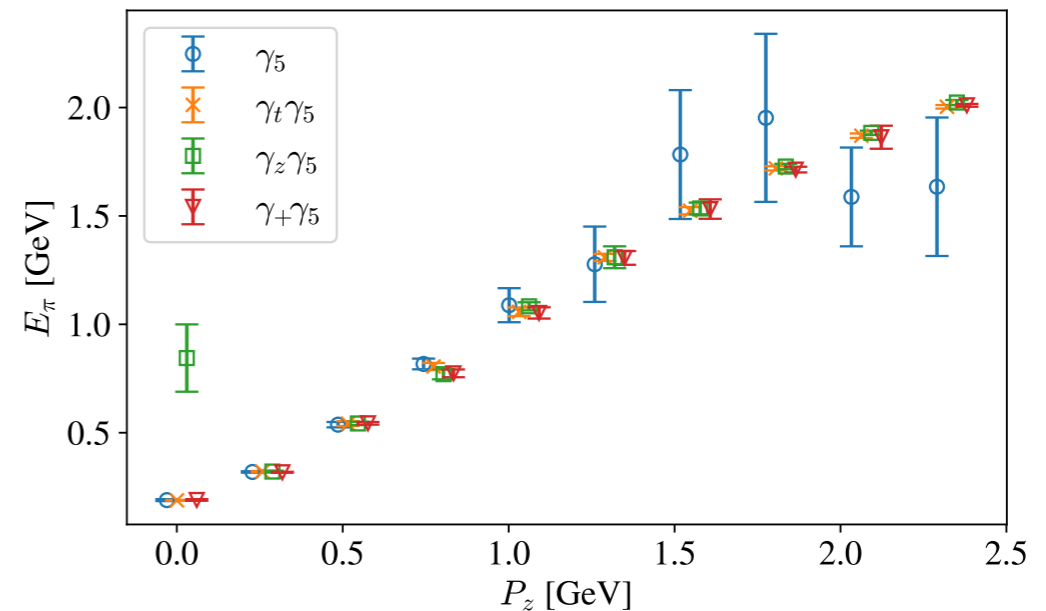
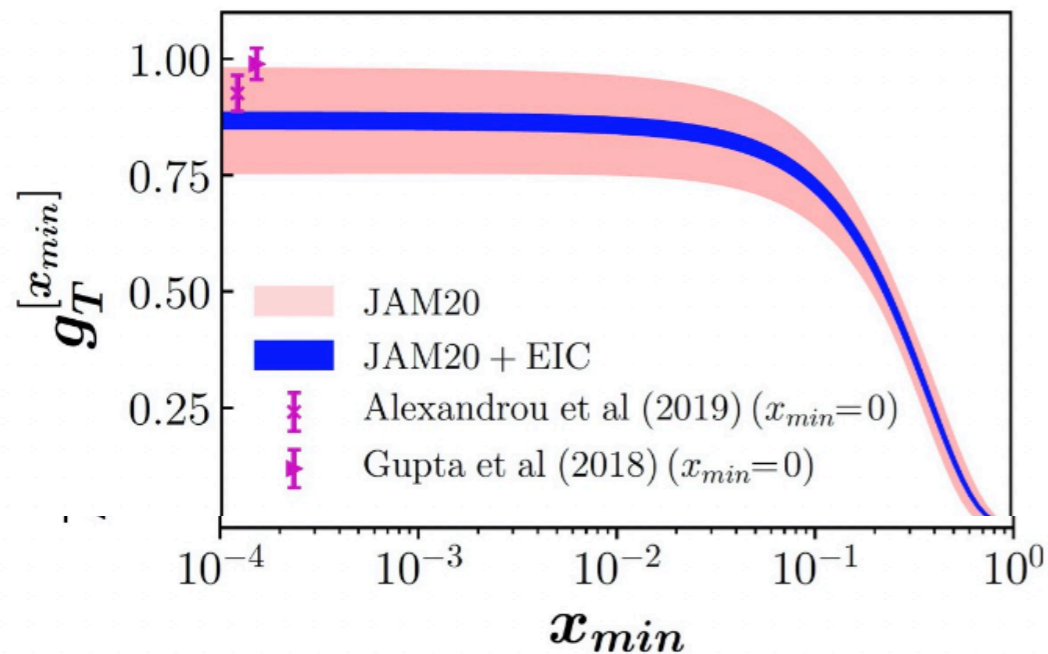
- Reduction of noise-to-signal

▶ Momentum smearing

G. S. Bali, B. Lang, B. U. Musch and A. Schäfer, Phys. Rev. D 93, 094515 (2016)

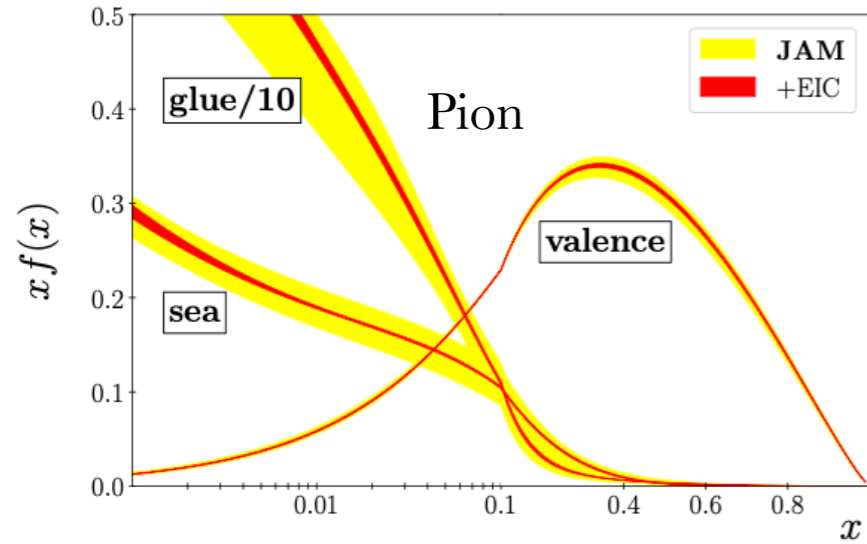
▶ Better interpolating fields

R. Zhang et al., Phys.Rev.D 112 (2025) 5, L051502

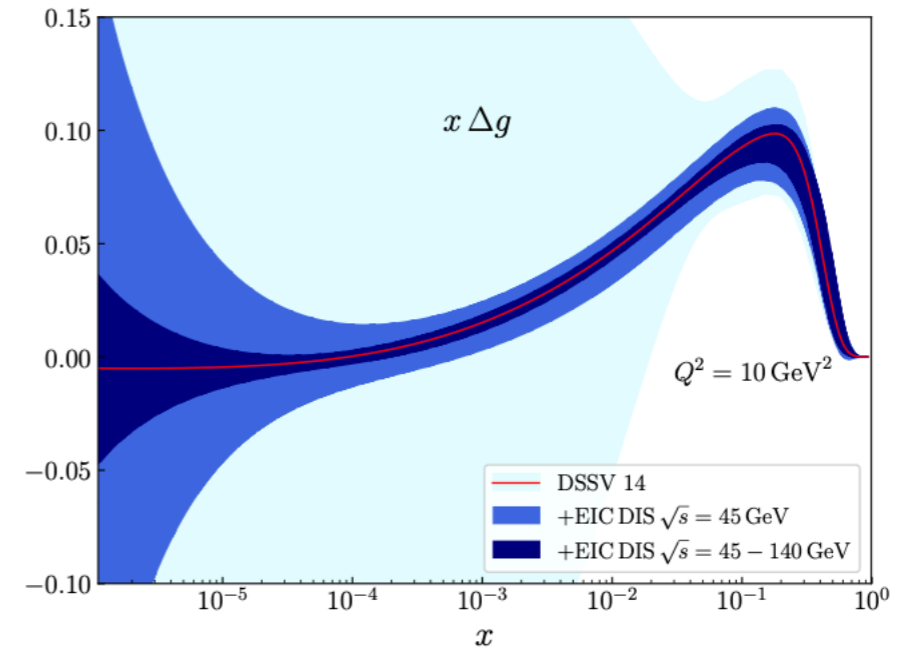


Examples of EIC hadron quantities (1D structure): PDFs

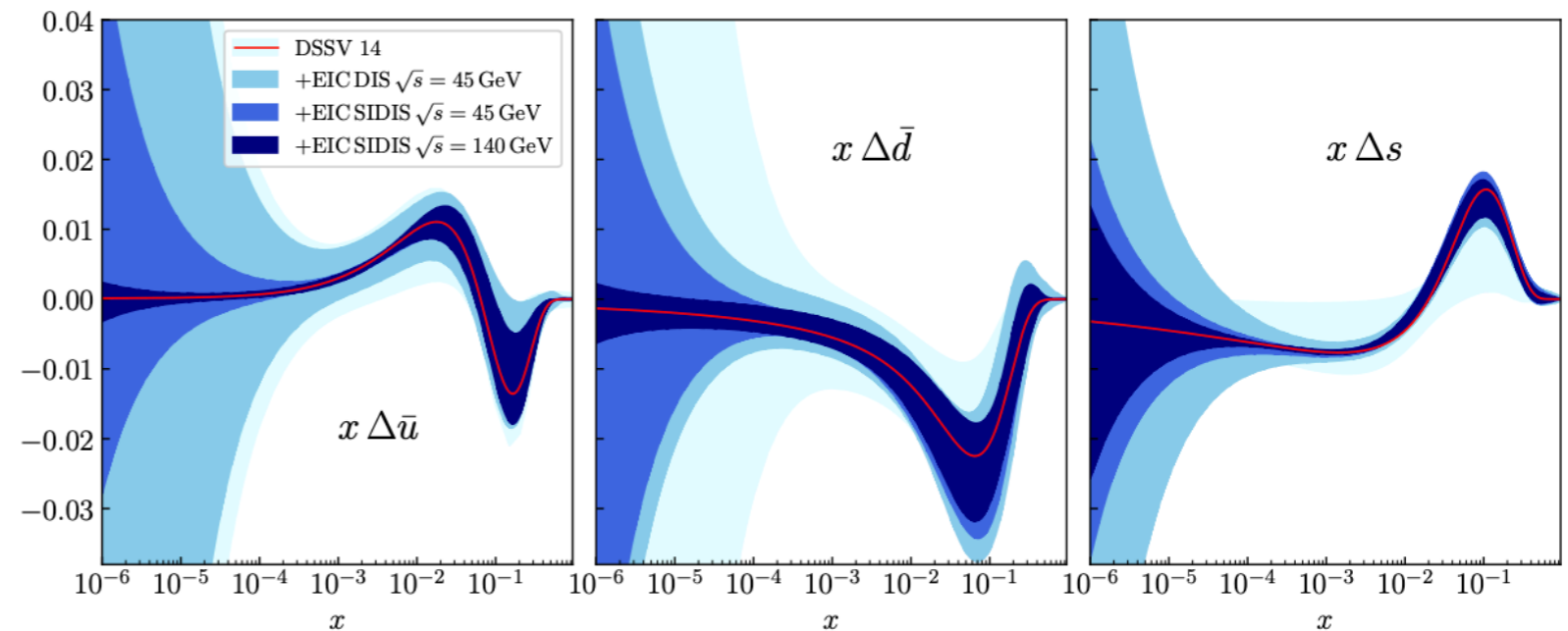
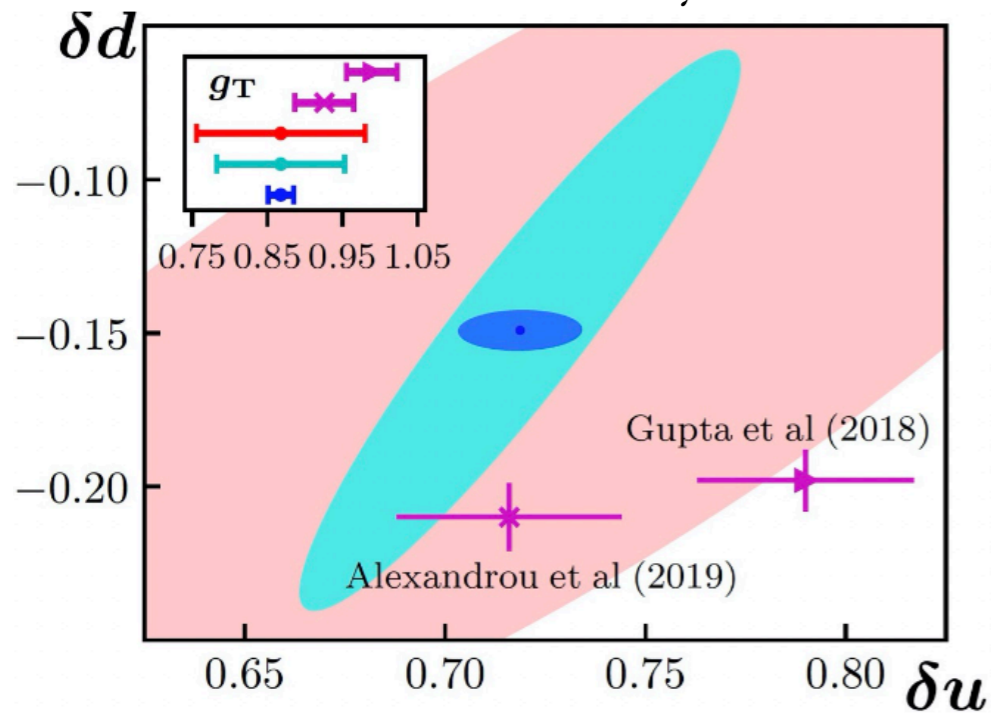
✳ Twist-2 PDFs with EIC accessing $x < 0.1$



Nucleon helicity



Nucleon transversity

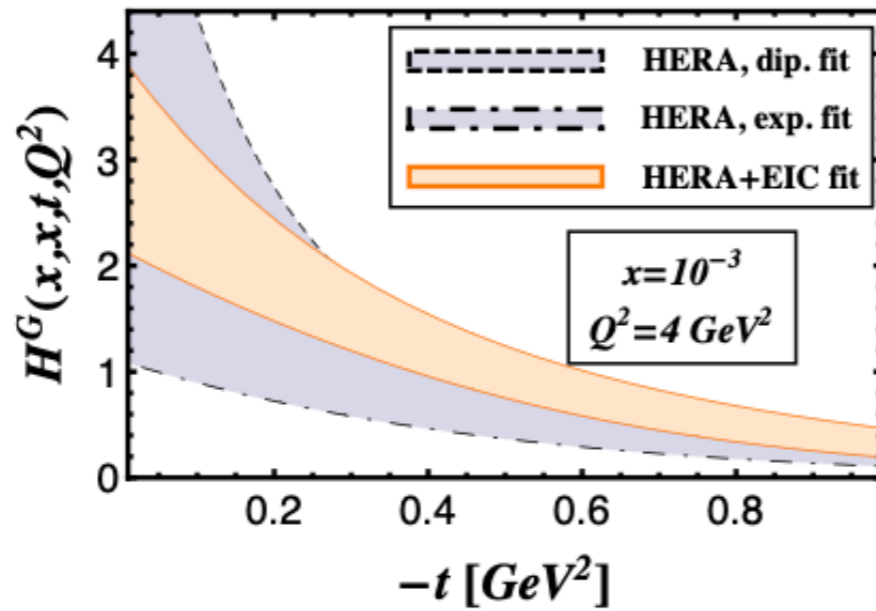


EIC Yellow report: [arXiv:2103.05419](https://arxiv.org/abs/2103.05419)
 EIC White paper: [arXiv:2211.15746](https://arxiv.org/abs/2211.15746)

✳ Higher twist PDFs and moments probing multiparton correlations, e.g. twist-3 PDFs such as the scalar $e(x)$, transversity $g_T(x)$ and the d_2 term

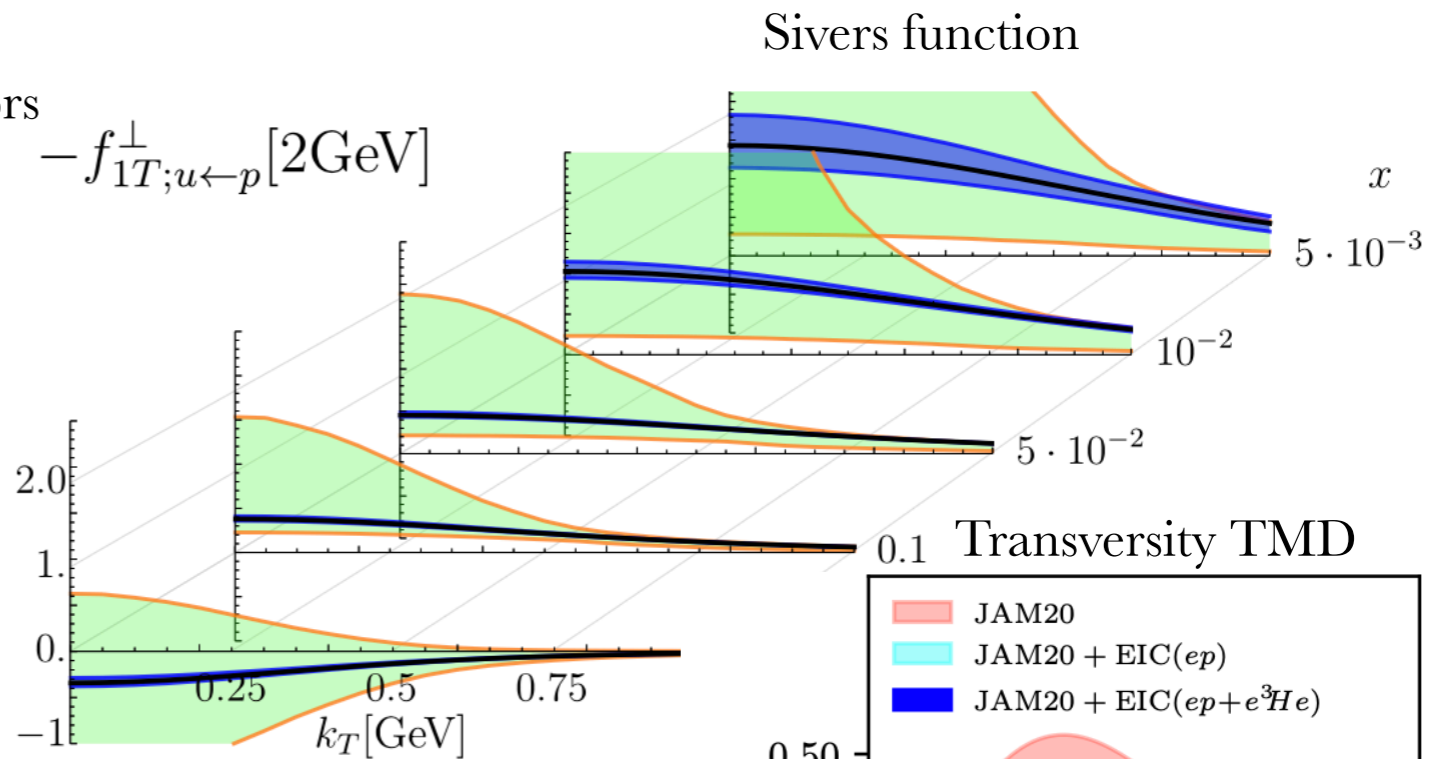
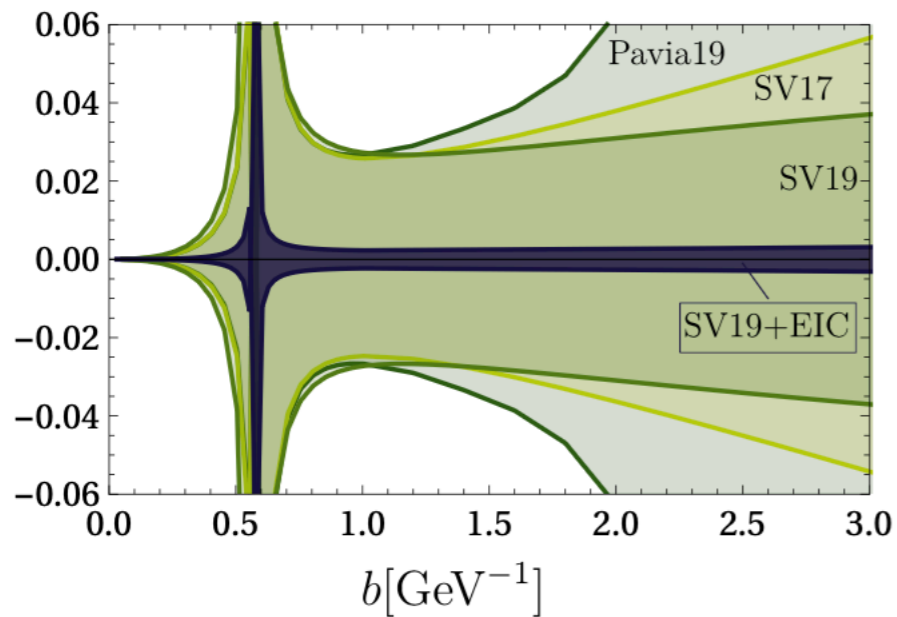
Examples of EIC hadron quantities (3D structure)

* GPDs

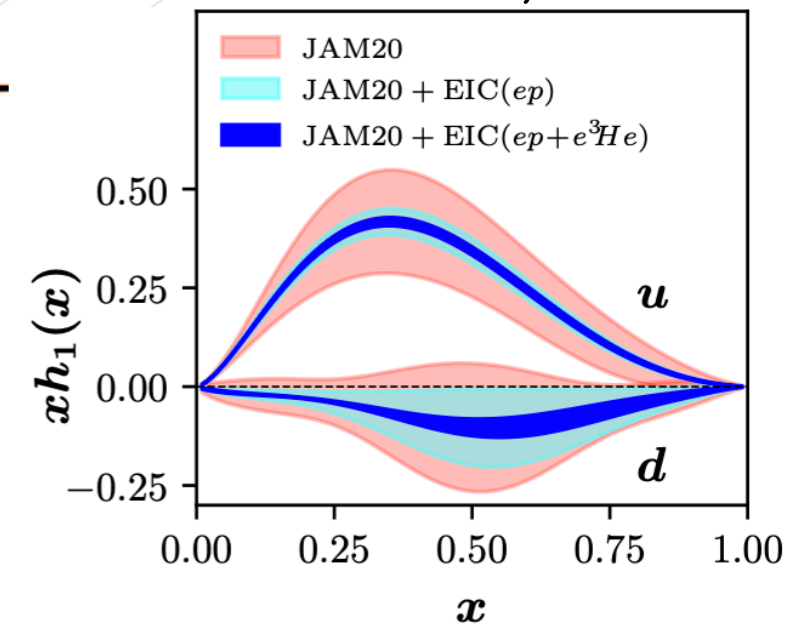


* TMDs

Collin-Soper (CS) kernel: Factor of 10 reduction of errors



EIC Yellow report: [arXiv:2103.05419](https://arxiv.org/abs/2103.05419)
EIC White paper: [arXiv:2211.15746](https://arxiv.org/abs/2211.15746)



Mellin moments - Precision era of lattice QCD in hadron structure

- How do hadron properties emerge from partons and their underlying interactions?

EIC yellow report, arXiv:2103.05419

- * Light-cone matrix elements cannot be computed using a Euclidean lattice formulation
- * Expansion of light-cone operator leads to a tower of local twist-2 operators \longrightarrow connected to moments that can be computed in lattice QCD
- * Readily accessible in lattice QCD from matrix elements of local operators with computations in early 90s

G. Martinelli and Ch. Sachradja Phys. Lett. B217 (1989) 319

Computation of lower Mellin moments of GPDs

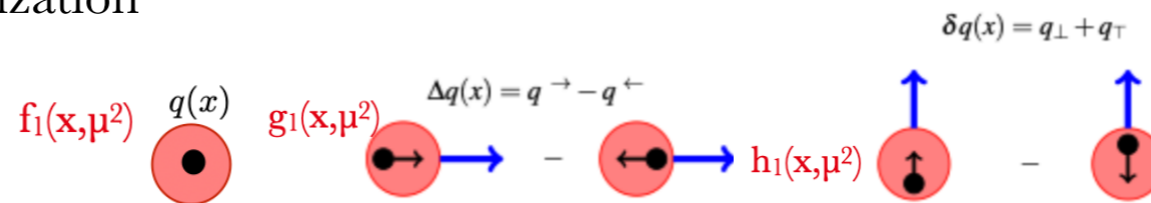
- * Light-cone matrix elements cannot be computed using a Euclidean lattice formulation
- * Expansion of light-cone operator leads to a tower of local operators \rightarrow connected to moments that can be computed in lattice QCD
- * Readily accessible in lattice QCD from matrix elements of local operators with computations in early 90s
- * To leading order we have the twist-2 operators

$$\begin{aligned} \mathcal{O}_V^{\mu_1 \mu_2 \dots \mu_n} &= \bar{\psi} \gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \psi && \xrightarrow{\text{unpolarized}} && \langle x^{n-1} \rangle_q = \int_0^1 dx x^{n-1} [q(x) - (-1)^{n-1} \bar{q}(x)] \\ \mathcal{O}_A^{\mu_1 \mu_2 \dots \mu_n} &= \bar{\psi} \gamma_5 \gamma^{\{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \psi && \xrightarrow{\text{helicity}} && \langle x^{n-1} \rangle_{\Delta q} = \int_0^1 dx x^{n-1} [\Delta q(x) + (-1)^{n-1} \Delta \bar{q}(x)] \\ \mathcal{O}_T^{\rho \mu_1 \mu_2 \dots \mu_n} &= \bar{\psi} i \sigma^{\rho \{\mu_1} i \overleftrightarrow{D}^{\mu_2} \dots i \overleftrightarrow{D}^{\mu_n\}} \psi && \xrightarrow{\text{transversity}} && \langle x^{n-1} \rangle_{\delta q} = \int_0^1 dx x^{n-1} [\delta q(x) - (-1)^{n-1} \delta \bar{q}(x)] \end{aligned}$$

$$q = q_{\downarrow} + q_{\uparrow}, \quad \Delta q = q_{\downarrow} - q_{\uparrow}, \quad \delta q = q_{\top} + q_{\perp}$$

Made traceless and $\{\}$ denotes symmetrization

Spin-1/2



		Γ		
		γ^+	$\gamma^+ \gamma_5$	$i \sigma^{+j} \gamma_5$
Pol.	U	H		E_{\top}
	L		\tilde{H}	\tilde{E}_{\top}
	T	E	\tilde{E}	$H_{\top}, \tilde{H}_{\top}$

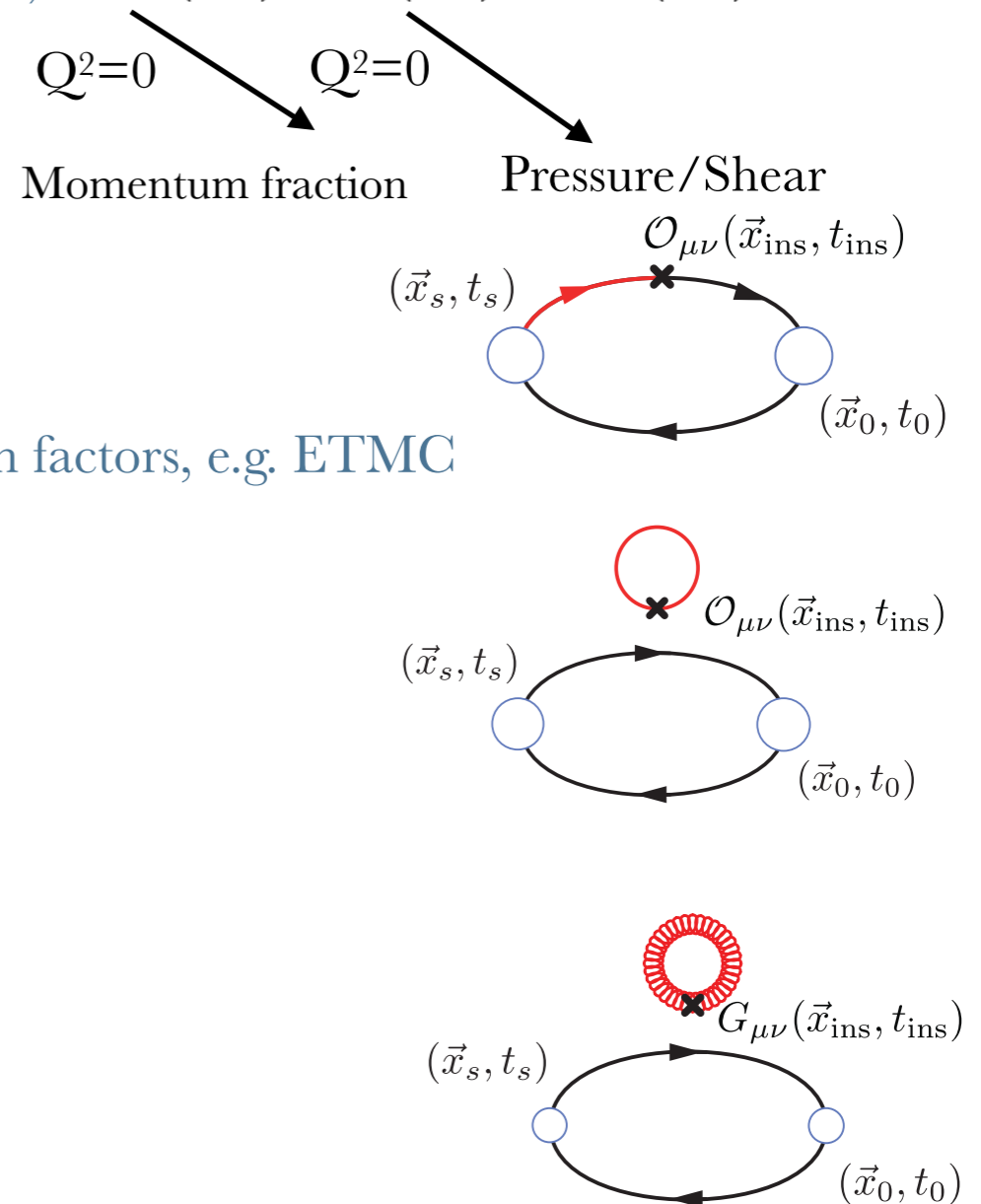
- * Matrix element of first and second Mellin moments give information on the mass and spin of the nucleon

Spin-0 particles: pion, kaon

✱ Spin-0

- Matrix elements of local operators (n=1) \rightarrow vector and tensor charges, g_V, g_T and form factors: $F_V(Q^2), F_T(Q^2)$
- Can also consider the scalar operator in relation to the mass \rightarrow scalar charge g_S and FF: $F_S(Q^2)$
- Matrix elements of first derivative operators (n=2) or the energy and momentum tensor (quarks and gluons) \rightarrow unpolarized and tensor generalized form factors (GFFs): $A_{20}(Q^2), C_{20}(Q^2), B_{T20}(Q^2)$

E.g. for unpolarized $\langle h(\vec{p}) | \bar{T}_{\mu\nu}^{q,g} | h(\vec{p}) \rangle = 2p_\mu p_\nu \langle x \rangle_{q,g}$



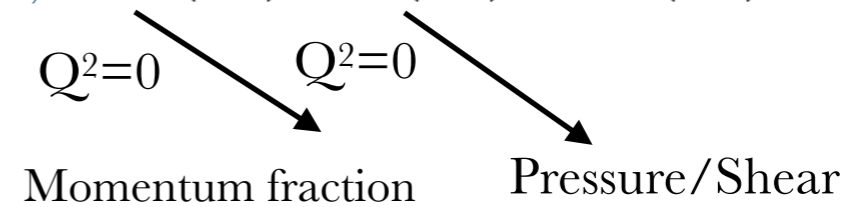
- Most studies are for the pion EM form factor
- Surprising few results for pion tensor and scalar and all kaon form factors, e.g. ETMC for one ensemble

Spin-0 and 1/2 particles: pion, kaon, nucleon

✱ Spin-0

- Matrix elements of local operators (n=1) \rightarrow vector and tensor charges, g_V, g_T and form factors: $F_V(Q^2), F_T(Q^2)$
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✱ Spin-1/2

- Matrix elements of local operators (n=1) \rightarrow vector, axial and tensor charges, g_V, g_A, g_T and form factors and $g_S, F_S(Q^2)$

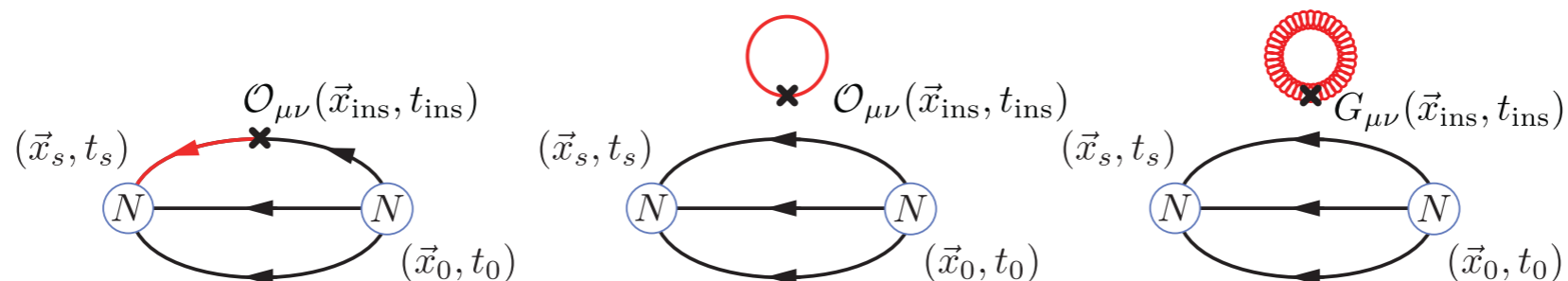
Helicity gives intrinsic quark spin: $\Delta\Sigma_{q+}(\mu^2) = \int_0^1 dx [\Delta q(x, \mu^2) + \Delta \bar{q}(x, \mu^2)] = g_A^q$

EM nucleon form factors studied since the 50s: $F_1(Q^2), F_2(Q^2)$

- Matrix elements of first derivative operators (n=2) \rightarrow unpolarized, helicity and tensor generalized form factors (GFFs)

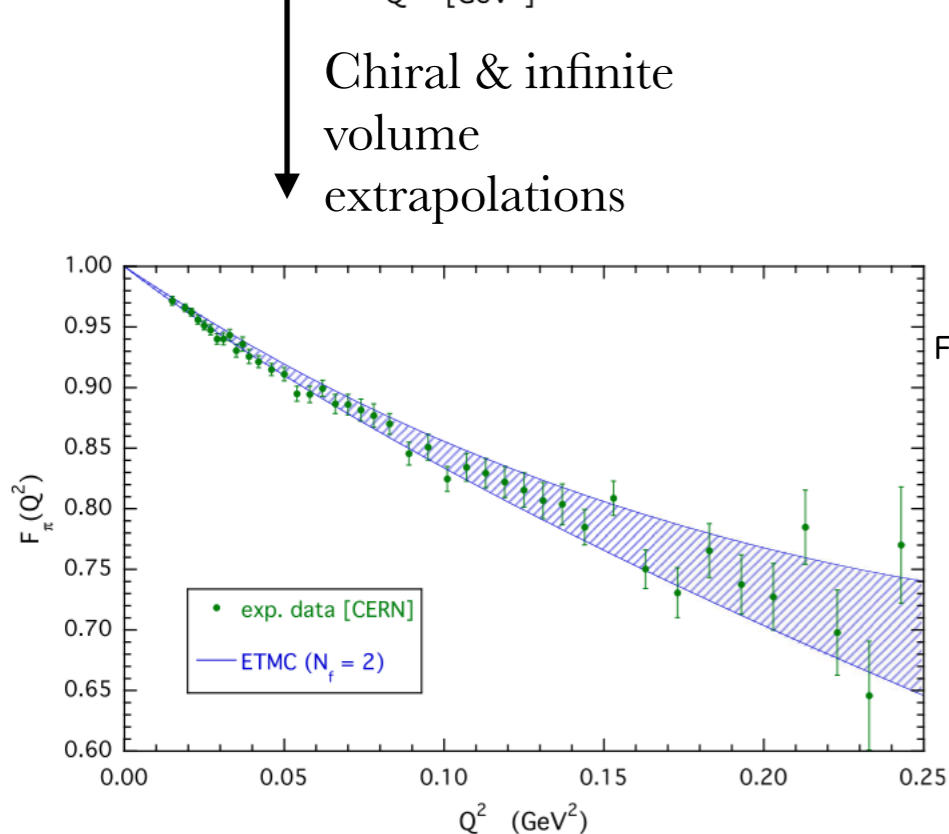
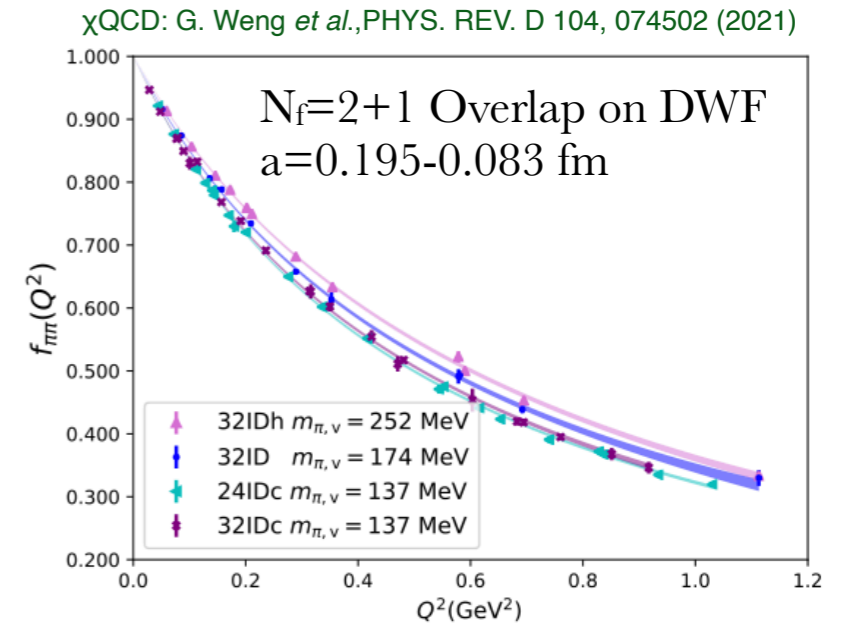
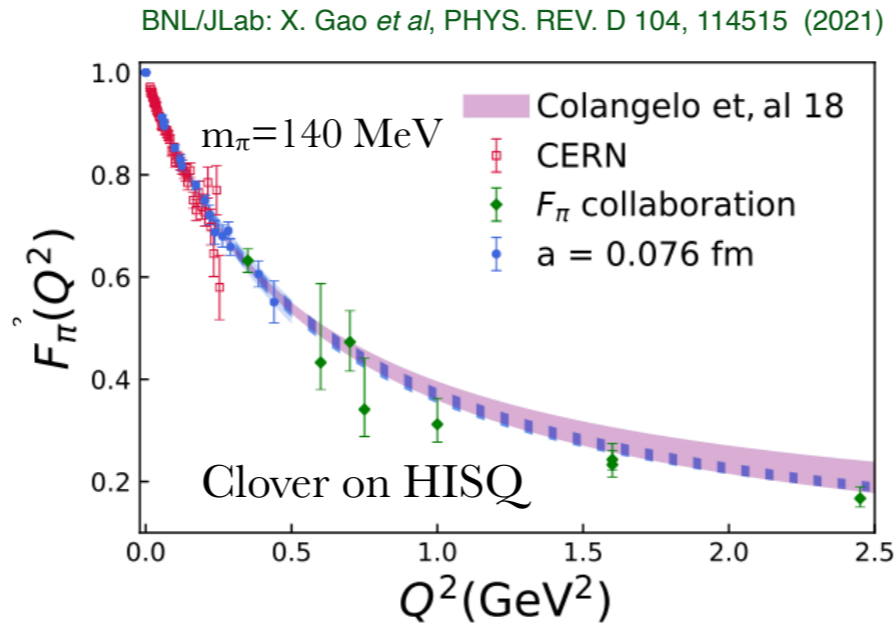
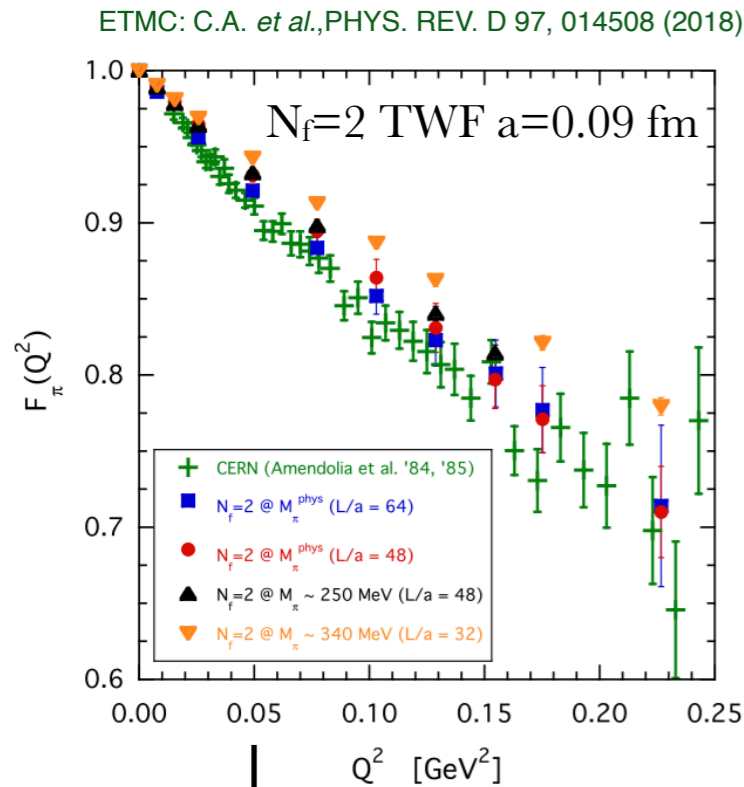
Moments: $\langle x \rangle_q = A_{20}(0), \quad \langle x \rangle_{\Delta q} = \tilde{A}_{20}(0), \quad \langle x \rangle_{\delta q} = A_{20}^T(0)$ and spin: $J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$

Spin and momentum sums: $\sum_q [\frac{1}{2}\Delta\Sigma_q + L_q] + J_g = \frac{1}{2}, \quad \sum_q \langle x \rangle_q + \langle x \rangle_g = 1$

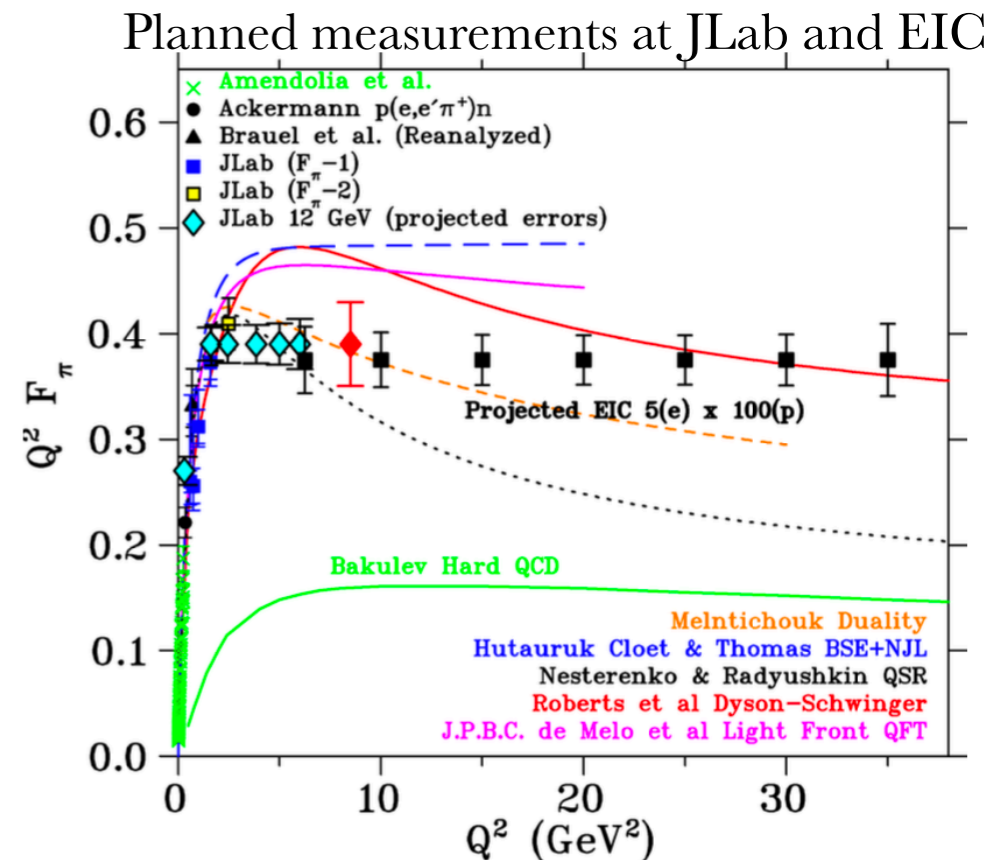
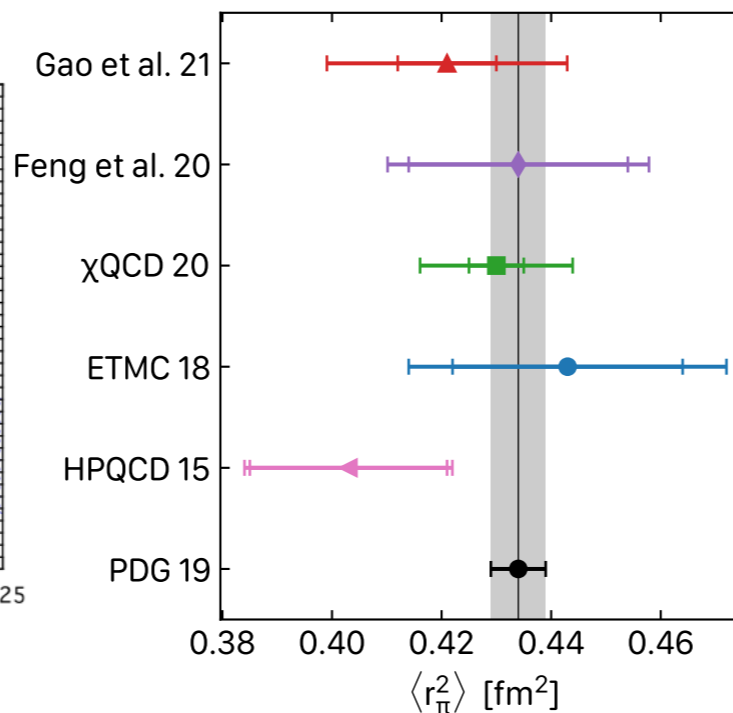


Vector form factor of the pion

✱ Three recent computations that include at least one ensemble with physical pion mass



$$\langle r^2 \rangle_\Gamma = - \frac{6}{F_\Gamma(0)} \left. \frac{\partial F_\Gamma(Q^2)}{\partial Q^2} \right|_{Q^2=0}$$



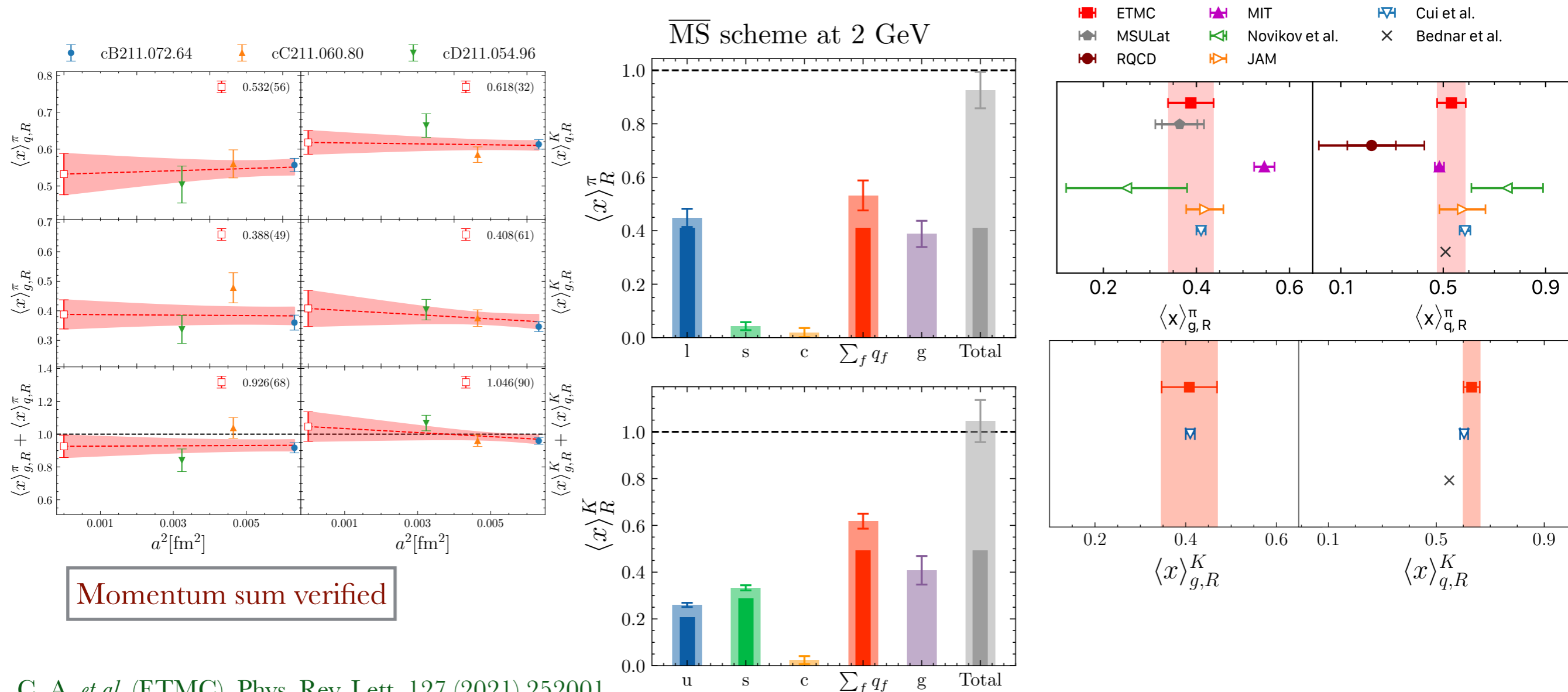
Pion and Kaon momentum fractions

✳ ETMC: continuum limit taken directly at physical pion mass

$N_f = 2 + 1 + 1$, Twisted – mass fermions : $64^3 \times 128, a = 0.080$ fm, $m_\pi = 140$ MeV

$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV

$96^3 \times 192, a = 0.057$ fm, $m_\pi = 141$ MeV



C. A. *et al.* (ETMC), Phys. Rev. Lett. 127 (2021) 252001

C. A. *et al.* (ETMC), Phys. Rev. Lett 134 (2025) 131902

✳ MIT group computed GFFs of the pion using a Clover ensemble with $m_\pi \sim 170$ MeV, $a \sim 0.09$ fm

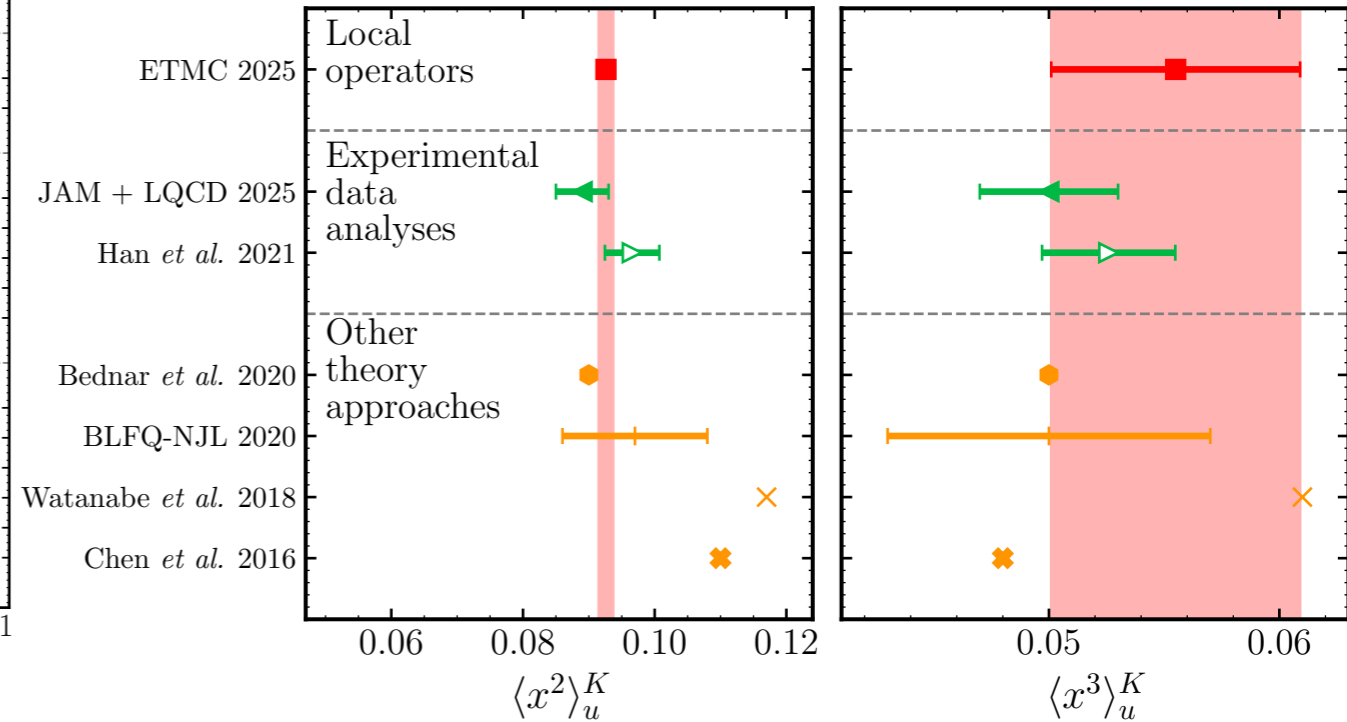
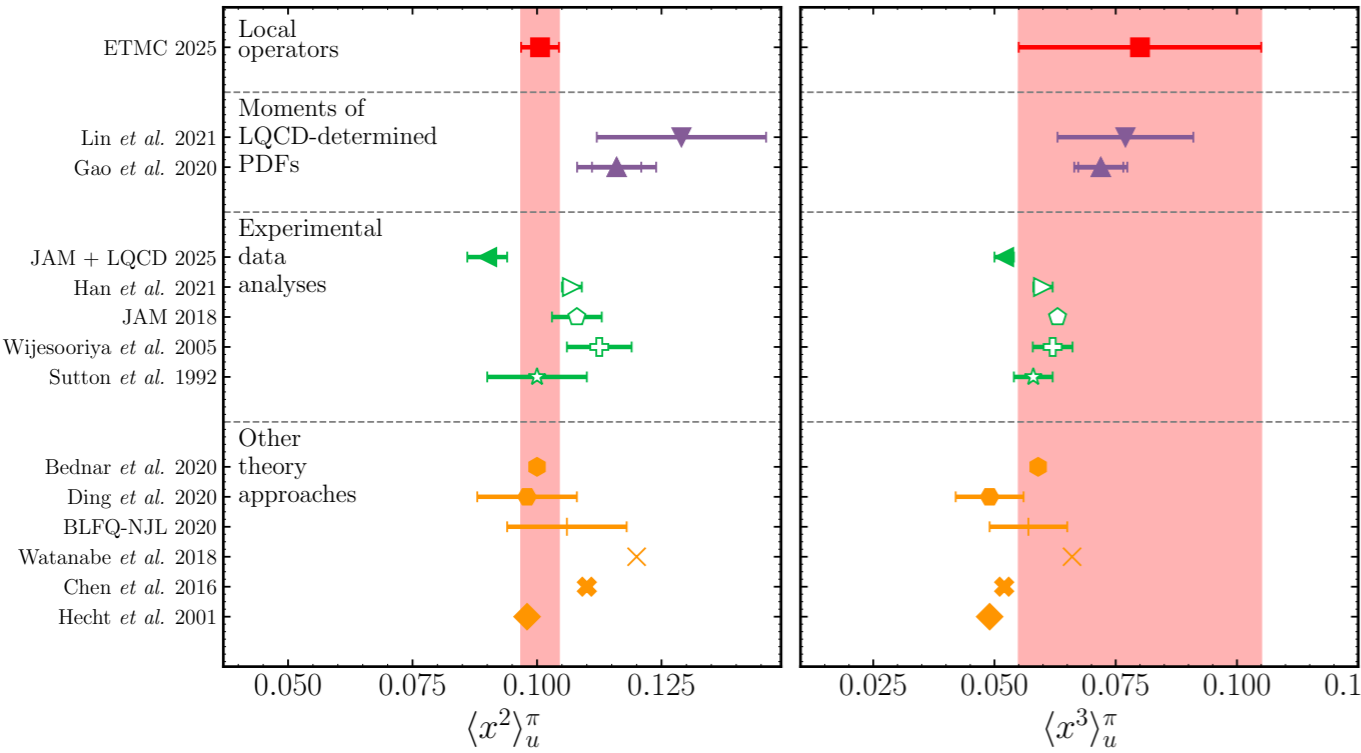
→ confirmed the momentum sum

D. C. Hackett *et al.*, Phys. Rev. D 108 (2023) 114504

Higher Mellin moments for pion and kaon

Pion

Kaon



✳ Third and fourth Mellin moment

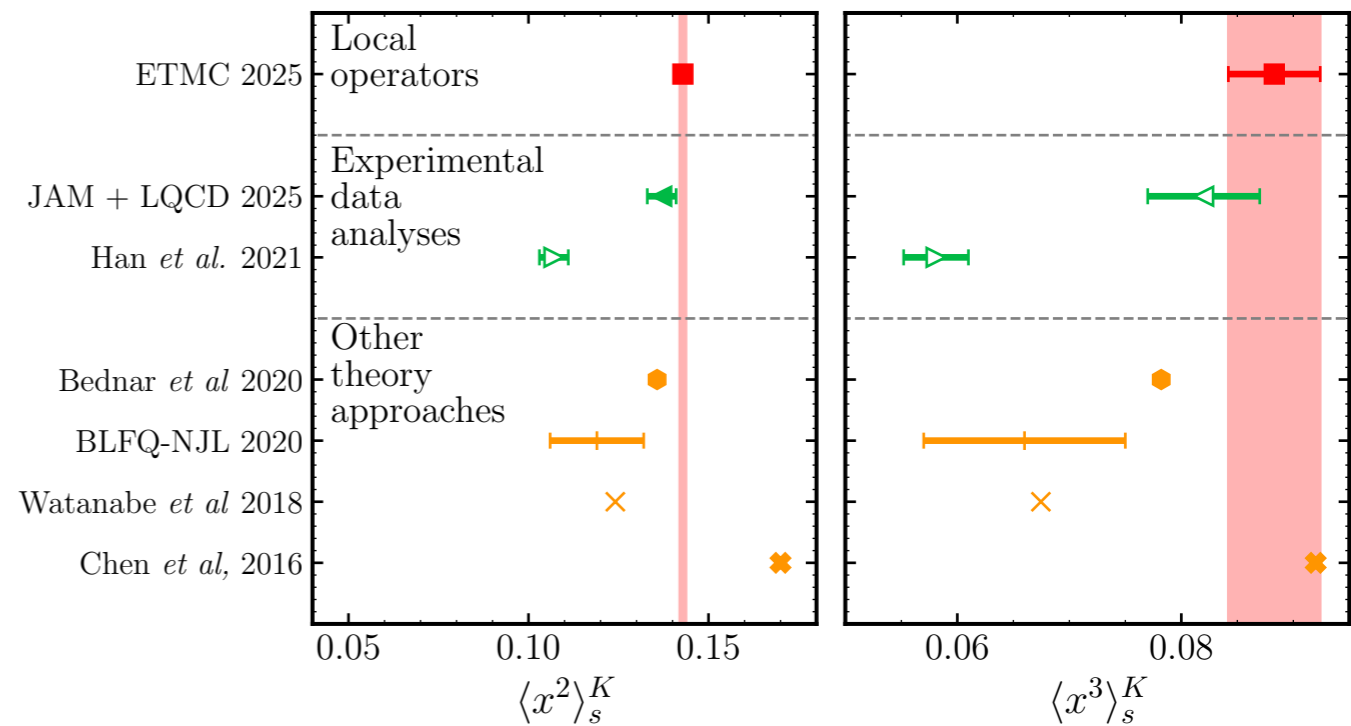
$N_f = 2 + 1 + 1$, Twisted mass fermions

$64^3 \times 128$, $a = 0.08$ fm, $m_\pi = 140$ MeV

$$\frac{\langle x \rangle_u^K}{\langle x \rangle_s^K} = 0.715(5), \quad \frac{\langle x^2 \rangle_u^K}{\langle x^2 \rangle_s^K} = 0.647(8), \quad \frac{\langle x^3 \rangle_u^K}{\langle x^3 \rangle_s^K} = 0.632(67)$$

- Results point to the strange quark PDF having its support at larger x than the up quark PDF in the kaon

- SU(3) symmetry breaking is more pronounced for higher moments



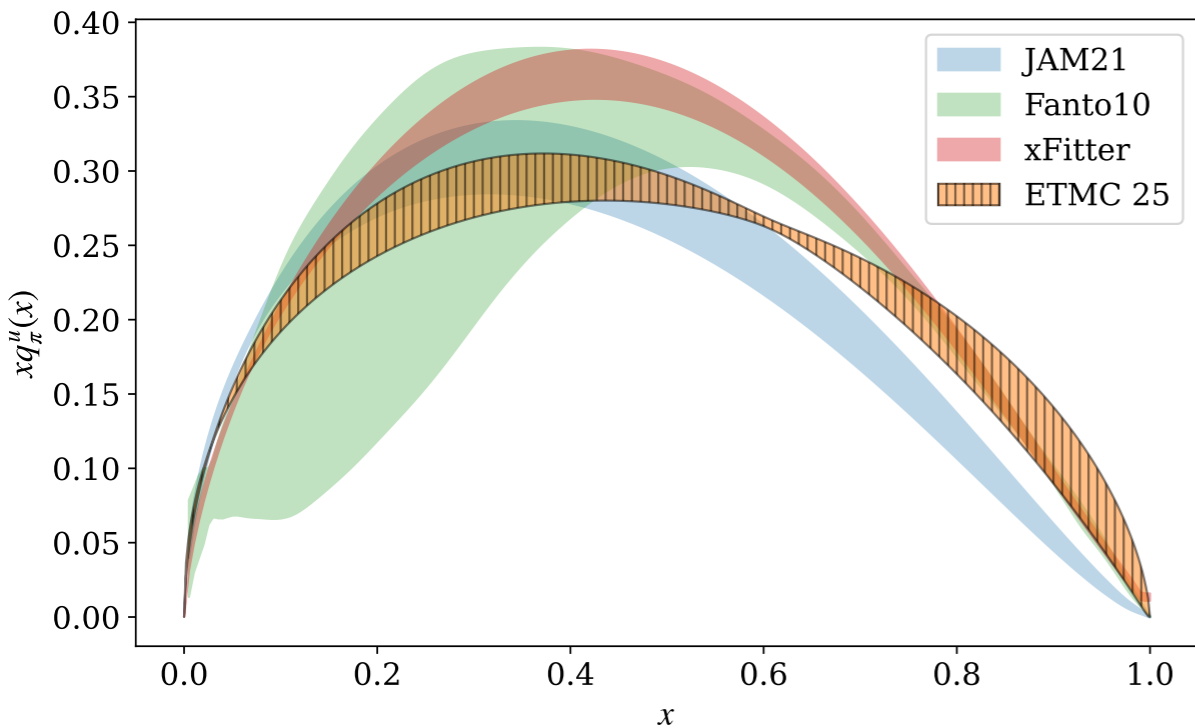
Reconstruction of PDFs using moments

✳ Use moments up to $\langle x^3 \rangle$

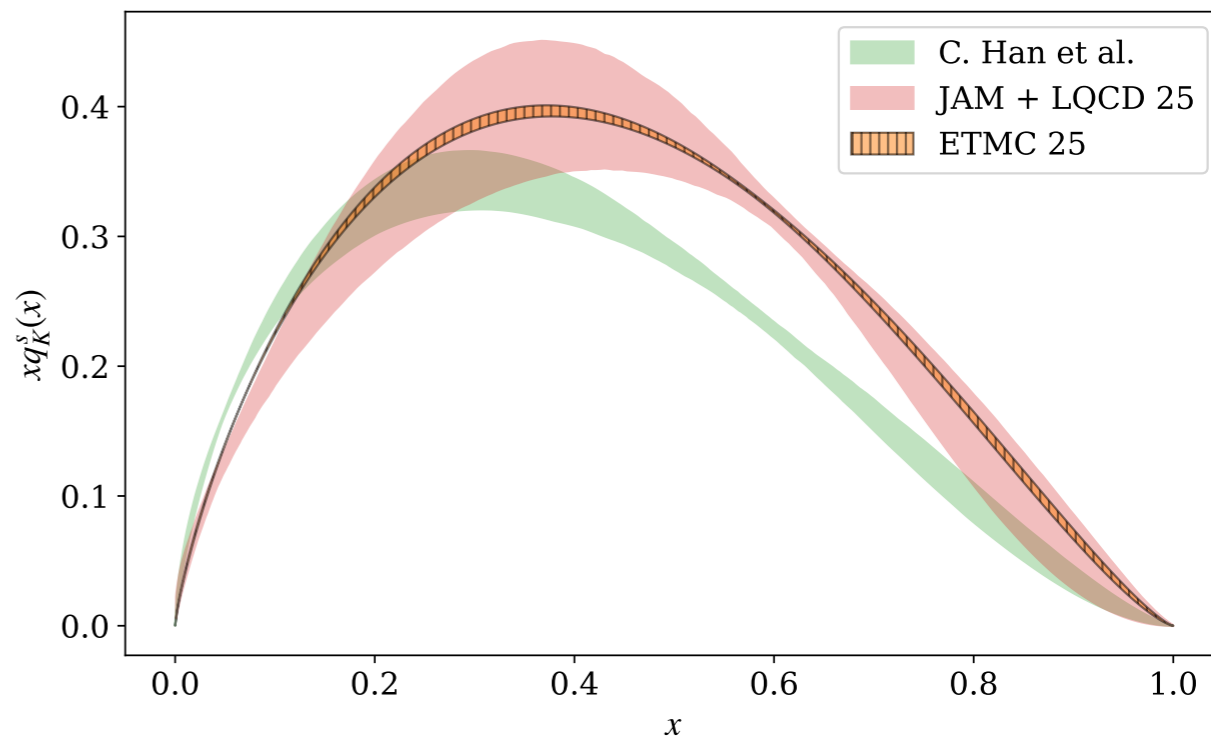
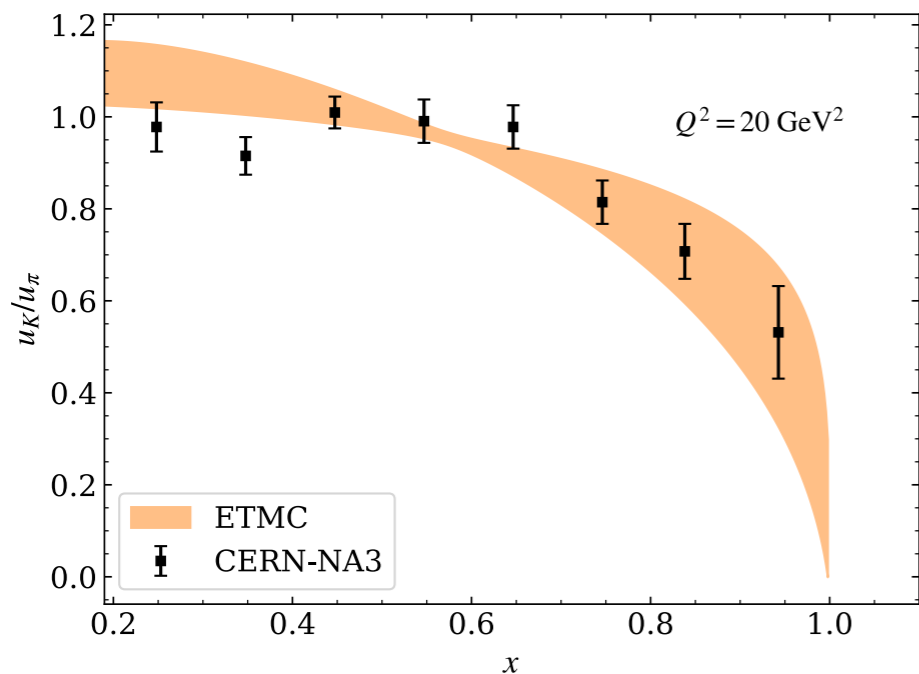
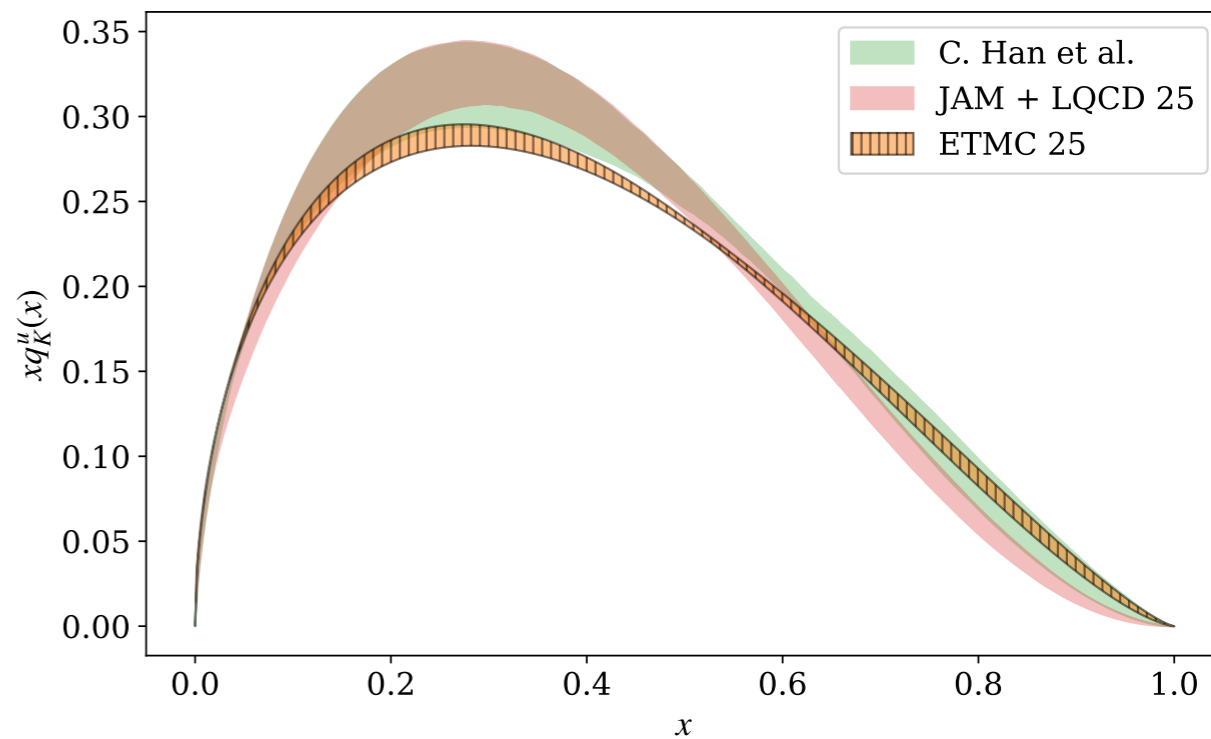
	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
u^π	0.2194(22)	0.1021(34)	0.079(25)
u^K	0.2151(12)	0.0925(11)	0.0557(54)
s^K	0.30081(67)	0.14312(86)	0.0881(41)

$$q_{\pi,K}^f(x) = N x^\alpha (1-x)^\beta (1 + \gamma \sqrt{x}) \quad \langle 1 \rangle_M = \int_0^1 q_M(x) = 1$$

Pion



Kaon



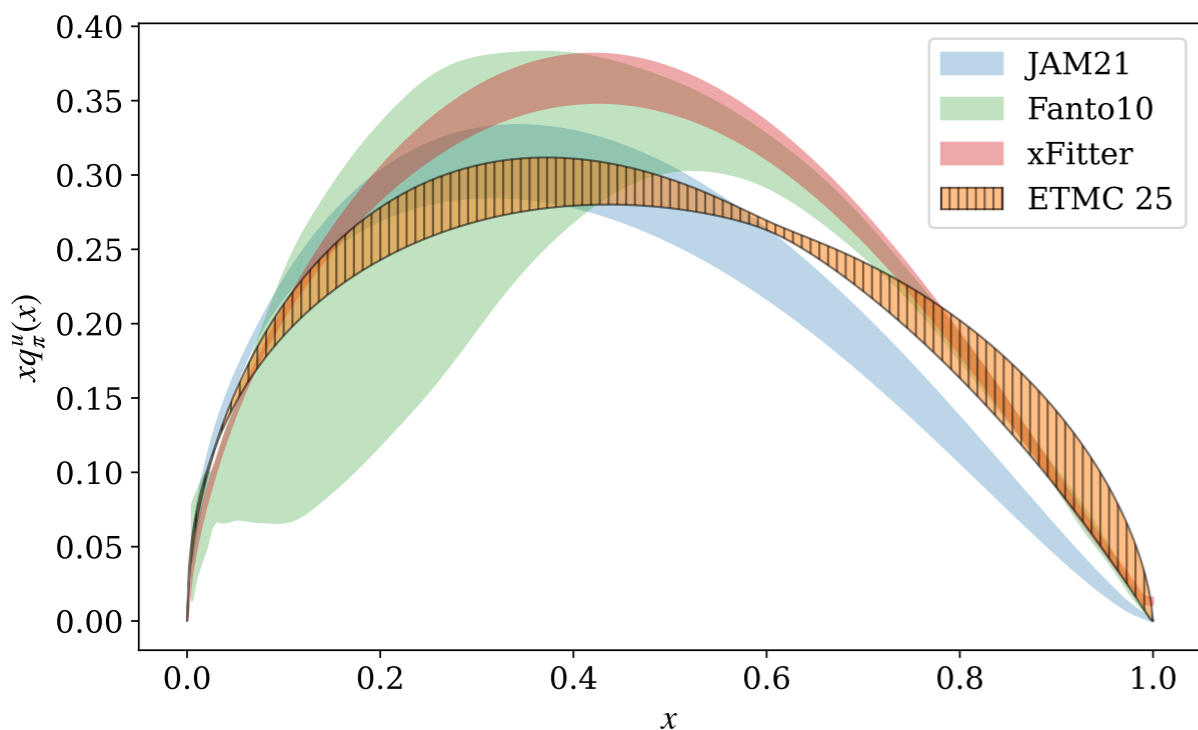
Reconstruction of PDFs using moments

✱ Use moments up to $\langle x^3 \rangle$

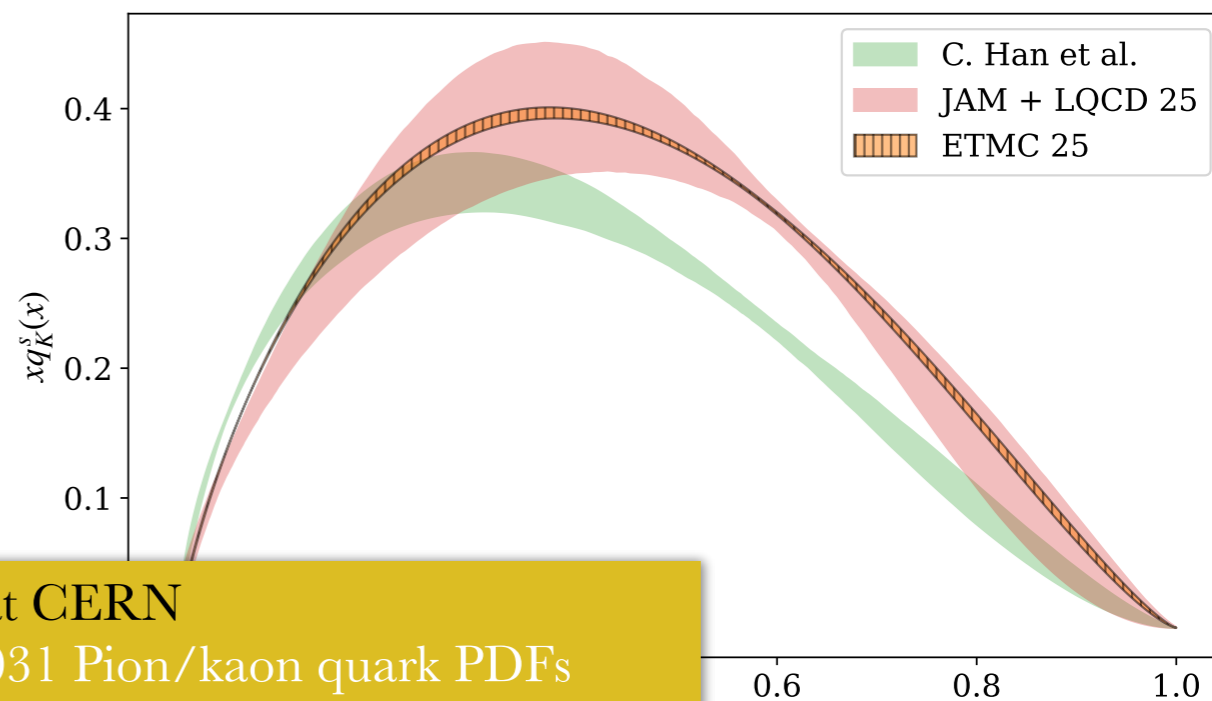
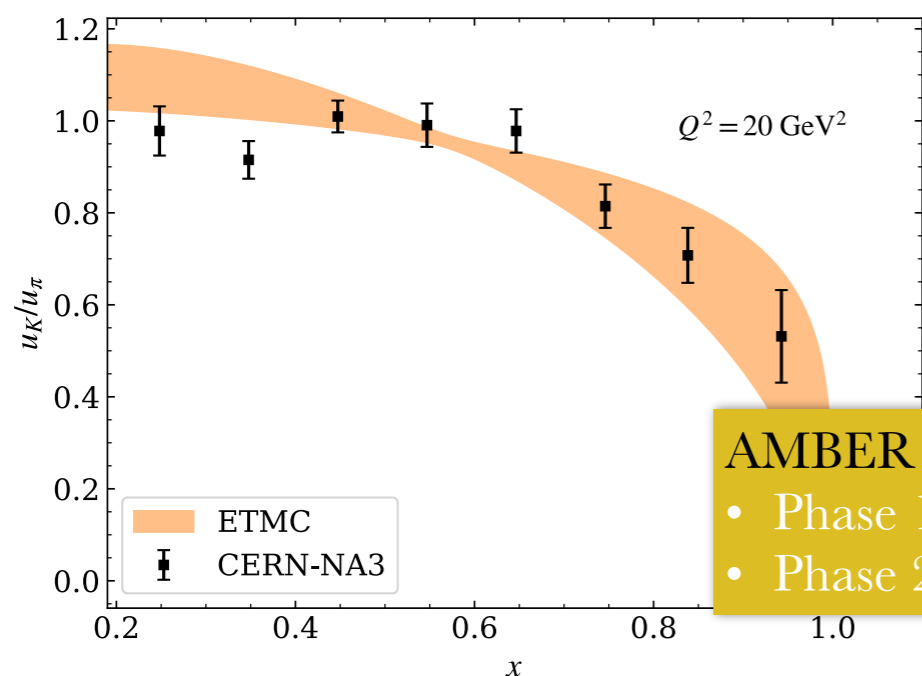
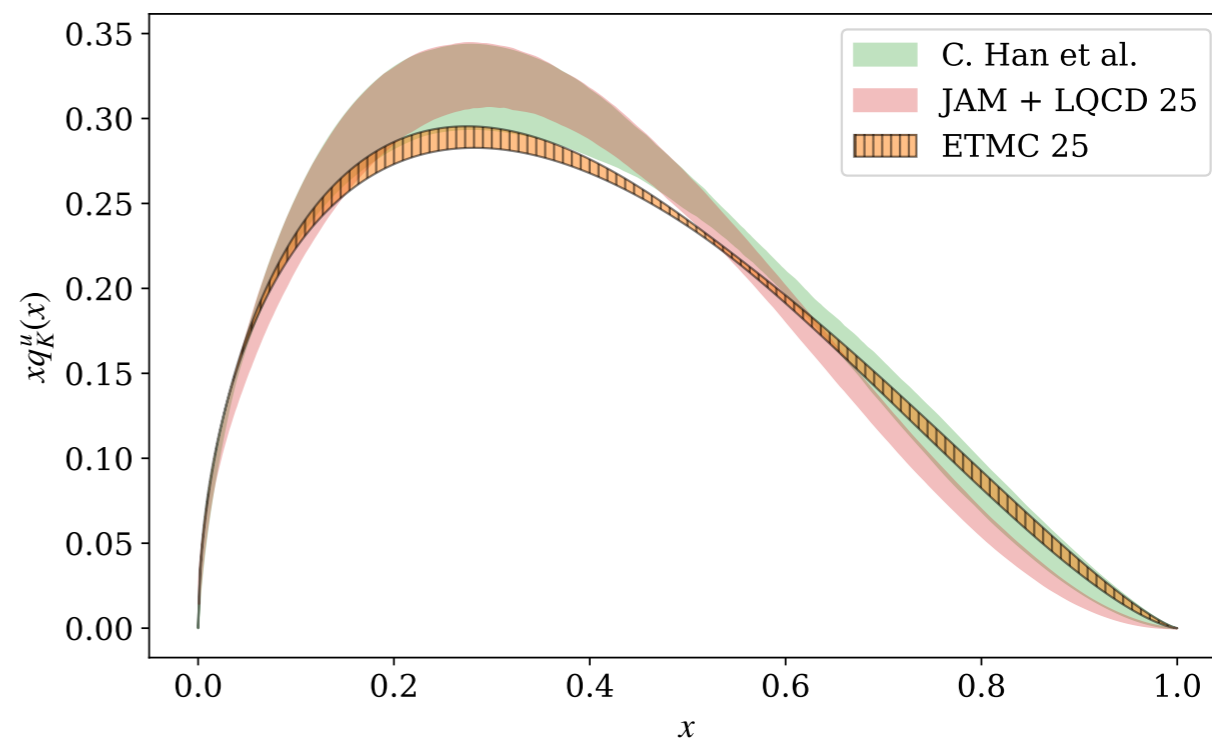
q_f^M	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
q_u^π	0.2157(24)	0.1005(36)	0.080(25)
q_u^K	0.2115(14)	0.0920(11)	0.056(5)
q_s^K	0.2958(7)	0.1430(8)	0.088(4)

$$q_{\pi,K}^f(x) = N x^\alpha (1-x)^\beta (1 + \gamma \sqrt{x}) \quad \langle 1 \rangle_M = \int_0^1 q_M(x) = 1$$

Pion



Kaon



AMBER experiment at CERN

- Phase 1: 2023—>2031 Pion/kaon quark PDFs
- Phase 2: 2031—>2041 kaon quark and gluon PDFs

Beyond the fourth moment

✳ Standard techniques allow computation of up to the fourth Mellin moment with increased statistics needed as the order of the moment increases.

✳ New ideas may enable computation of higher order moments, e.g. using:

- Wilson flow

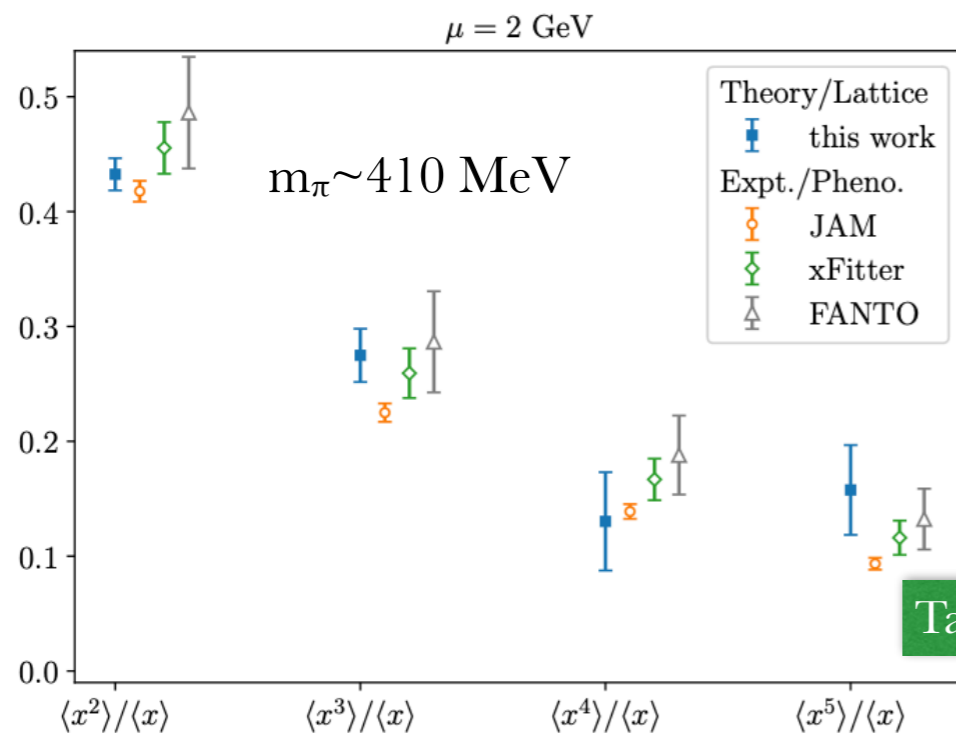
A. Shindler, *Phys.Rev.D* 110 (2024) 5, L051503, arXiv:2311.18704

- Heavy-quark OPE (HOPE) method for computing moments of the light-core distribution amplitude

HOPE collaboration: W. Detmold *et al.* *Phys.Rev.D* 105 (2022) 3, 034506, arXiv:2103.09529

W. Detmold *et al.*, arXiv:2509.04799

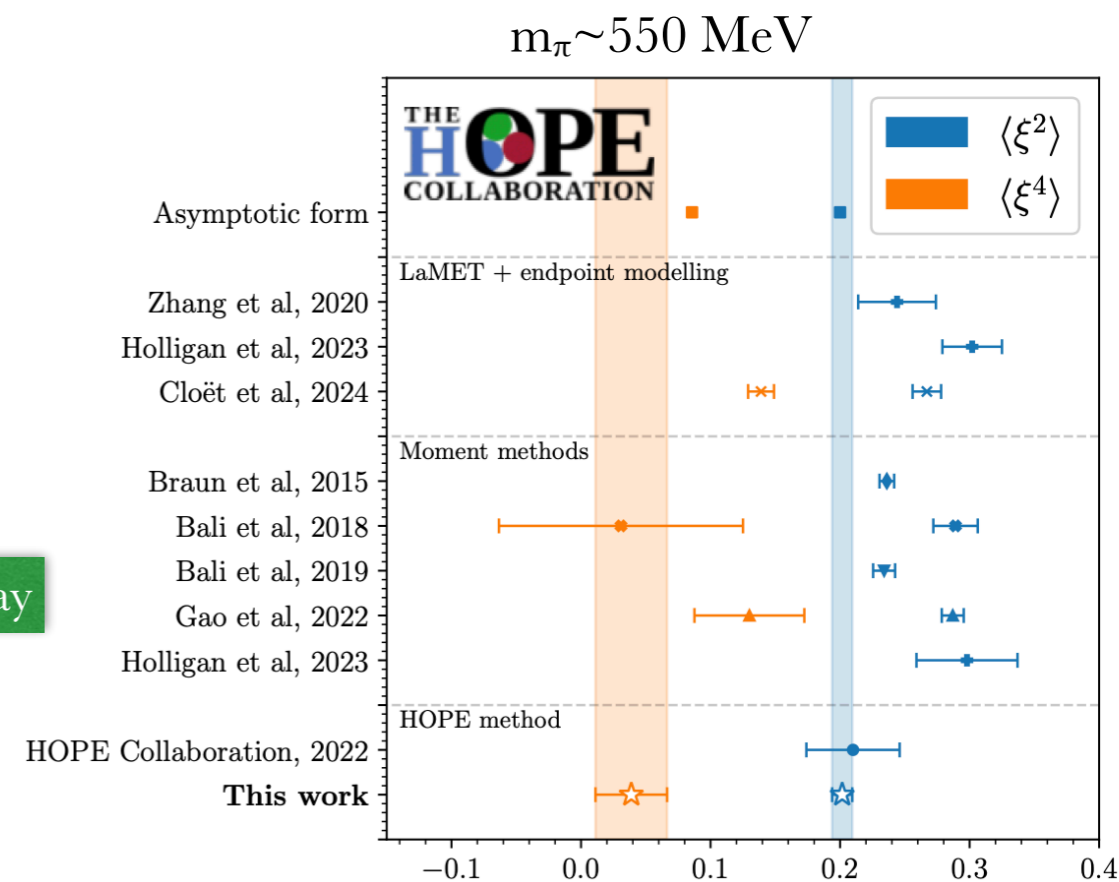
with a proof of concept presented in the pion case for both approaches for heavy pion mass.



Talk by D. Pefkou, Tuesday

A. Francis *et al.* *PoS LATTICE2024* (2025) 336;

A. Francis *et al.* arXiv:2510.26738; arXiv:2509.02472



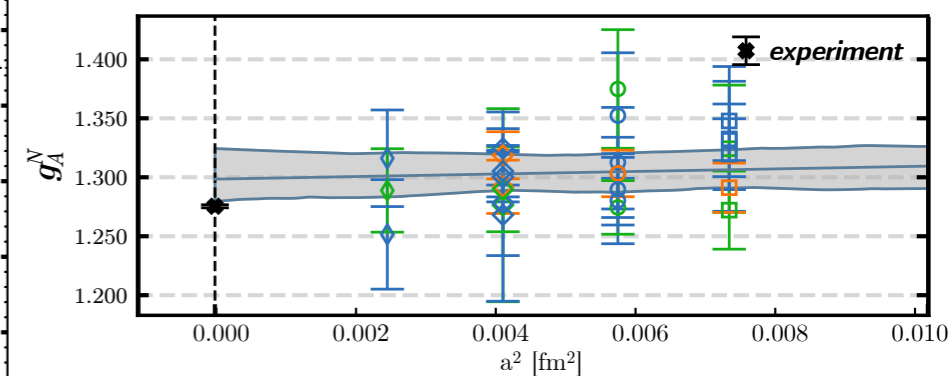
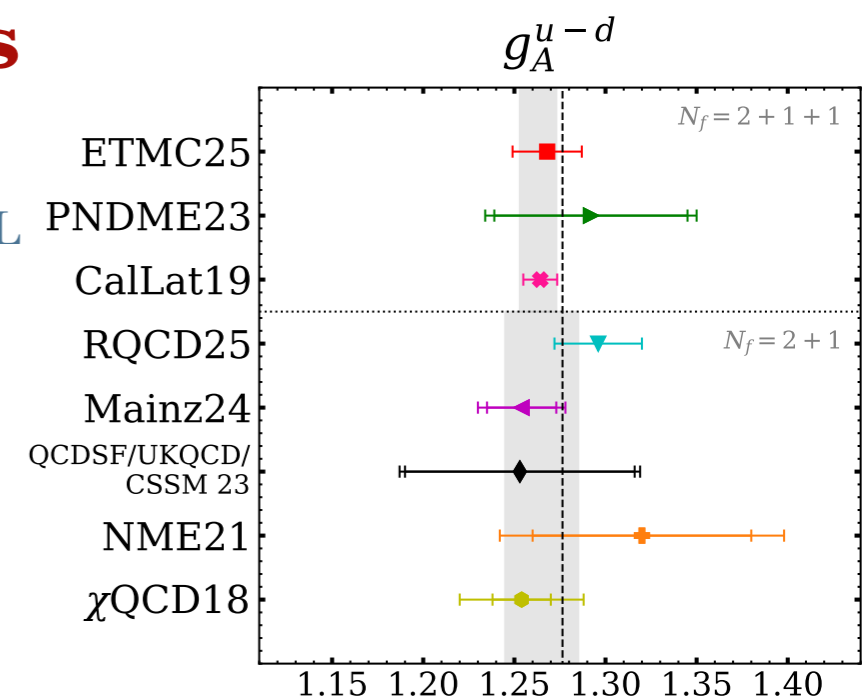
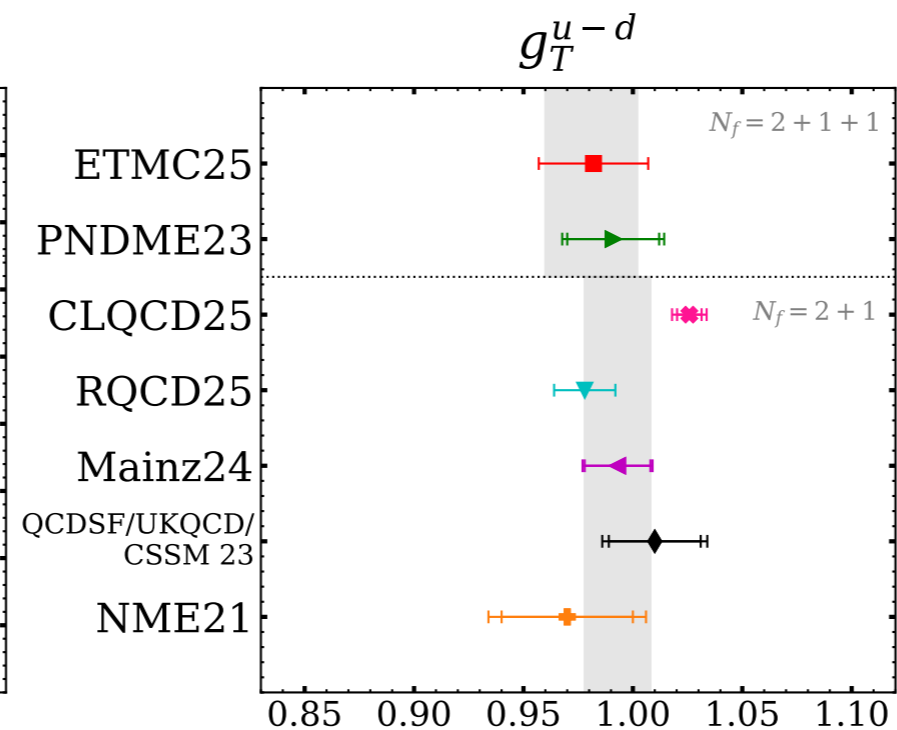
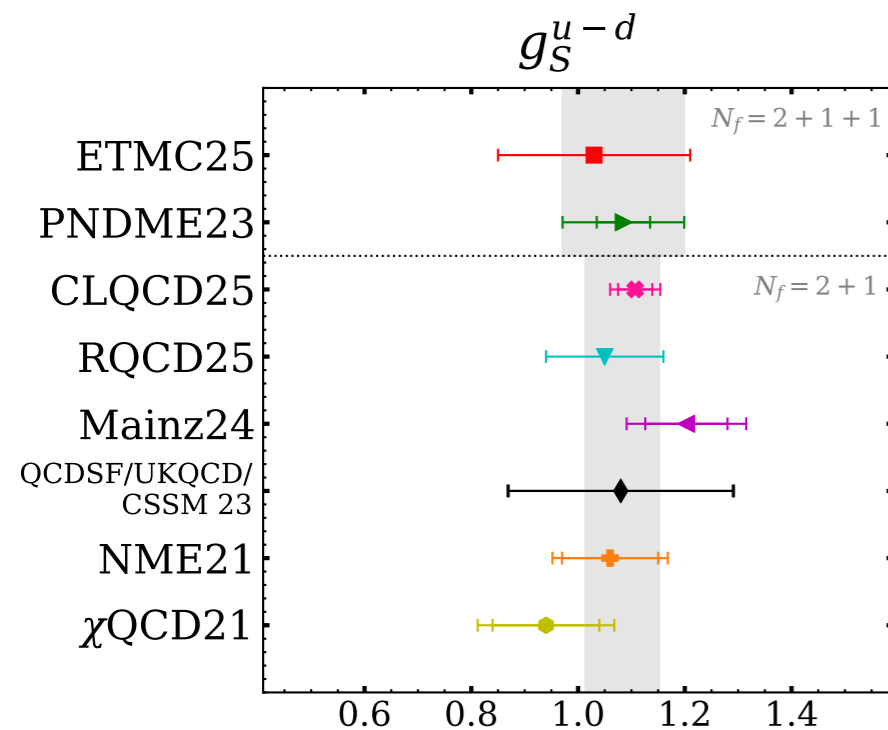
Talk by S.-P. Alex Chang, Tuesday

✳ Compute directly PDFs/GPDs and take moments

Nucleon isovector charges

✿ Nucleon isovector charges are well-studied by many collaborations: Recent results by

- RQCD: 47 CLS ensembles with $6 a \sim (0.038-0.098)$ fm, $m_\pi \sim (480-130)$ MeV and multiple L
- ETMC: 4 ensembles with $4 a \sim (0.08-0.05)$ fm and $m_\pi \sim 140$ MeV
- CLQCD: 16 Clover ensembles with $4 a \sim (0.105-0.052)$ fm, $m_\pi \sim (340-134)$ MeV and multiple L



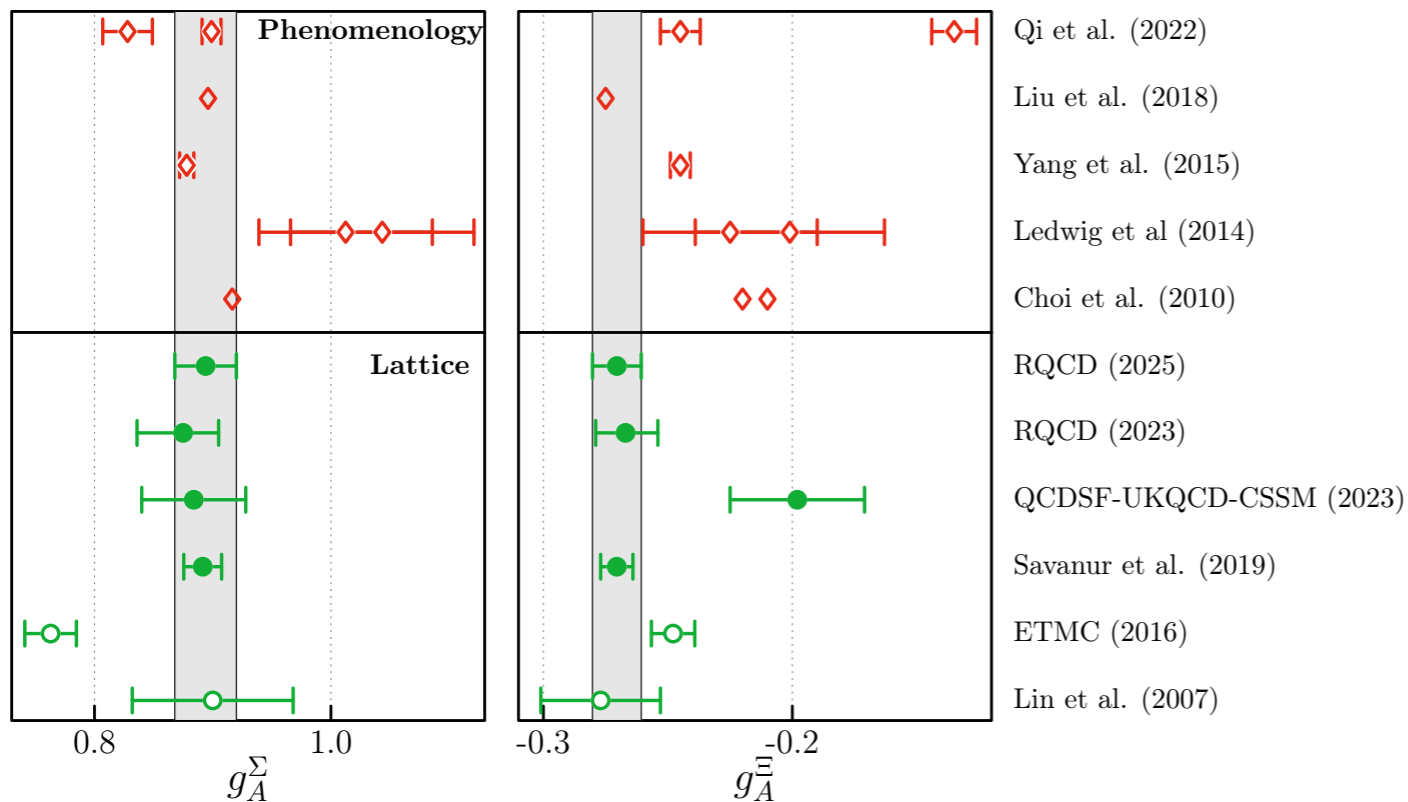
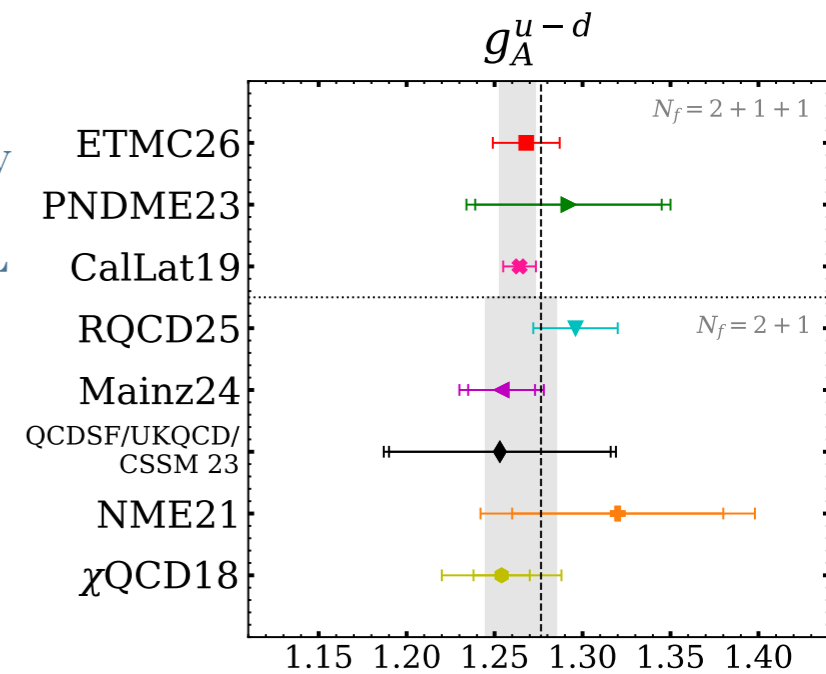
Talk by Masato Nagatsuka, Wednesday

Baryon isovector charges

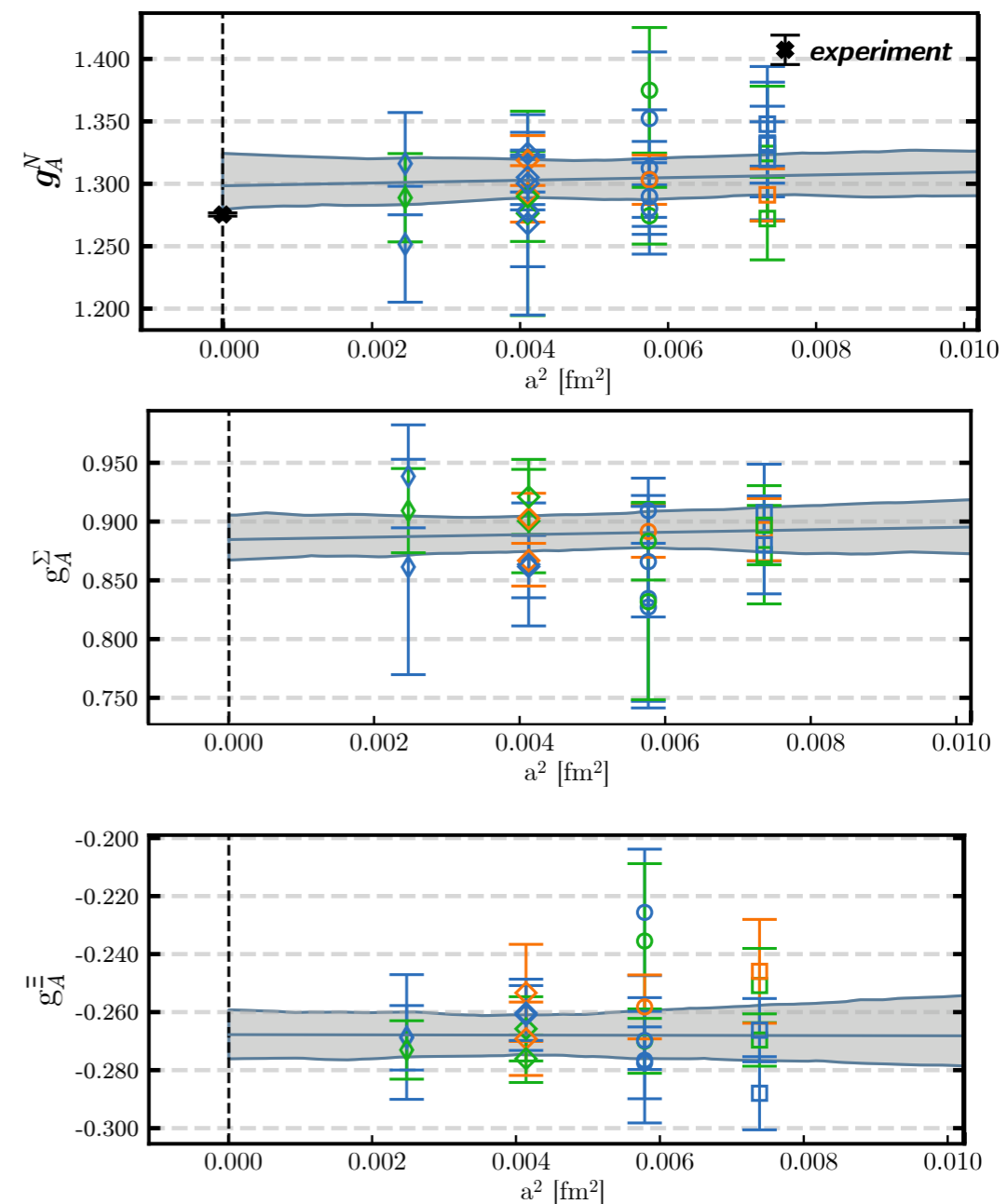
✿ Nucleon isovector charges are well-studied by many collaborations: Recent results by

- RQCD: 47 CLS ensembles at $6 a \sim (0.038-0.098)$ fm, $m_\pi \sim (480-130)$ MeV and multiple L
- ETMC: 4 ensembles at $4 a \sim (0.08-0.05)$ fm and $m_\pi \sim 140$ MeV
- CLQCD: 16 Clover ensembles at $4 a \sim (0.105-0.052)$ fm, $m_\pi \sim (340-134)$ MeV and multiple L

✿ RQCD has new results for isovector charges and momentum fractions for Σ and Ξ

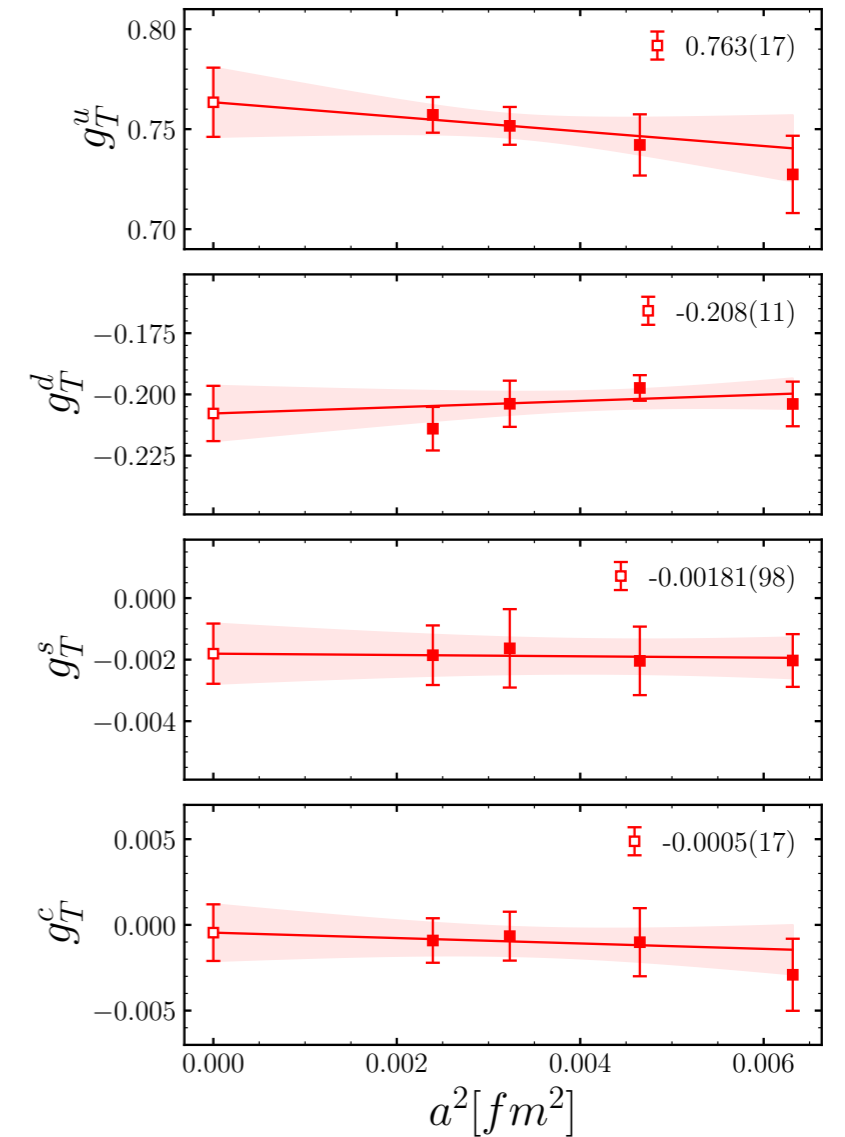
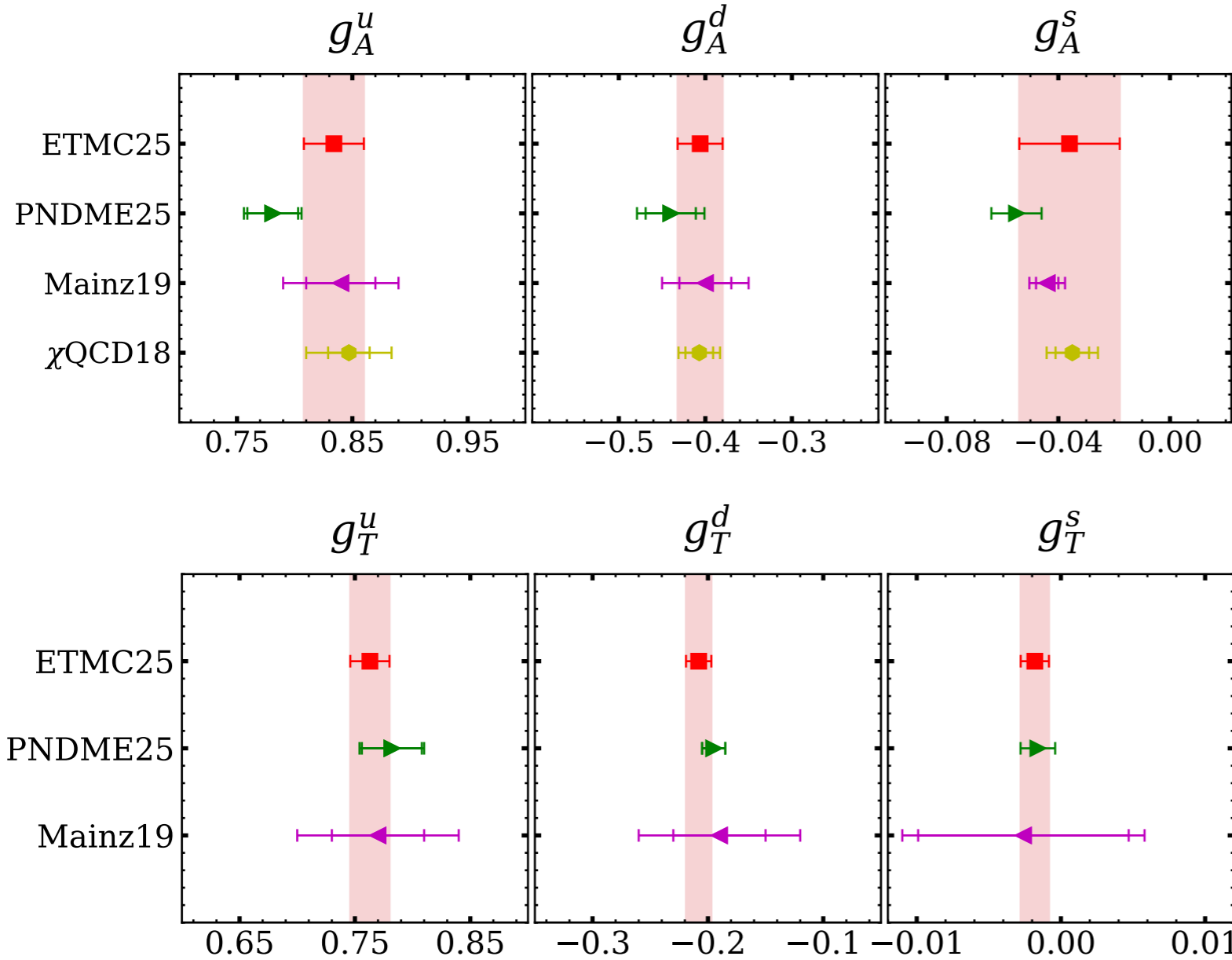


RQCD: G. Bali *et al.*, Phys.Rev.D 108 (2023) 034512;
S. Collins communication

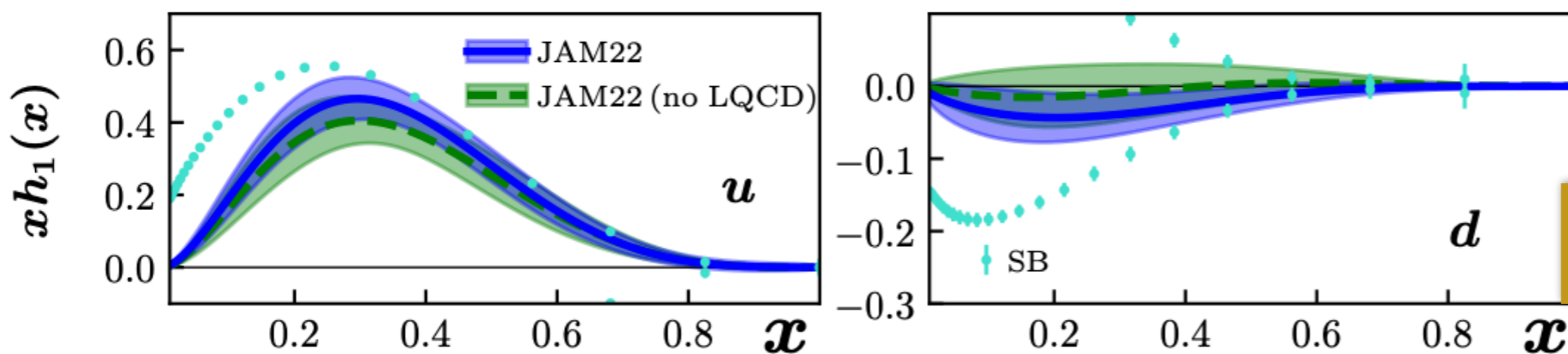


Nucleon charges

ETMC: continuum limit taken directly at physical pion mass



✳ Tensor charge provides input for phenomenology

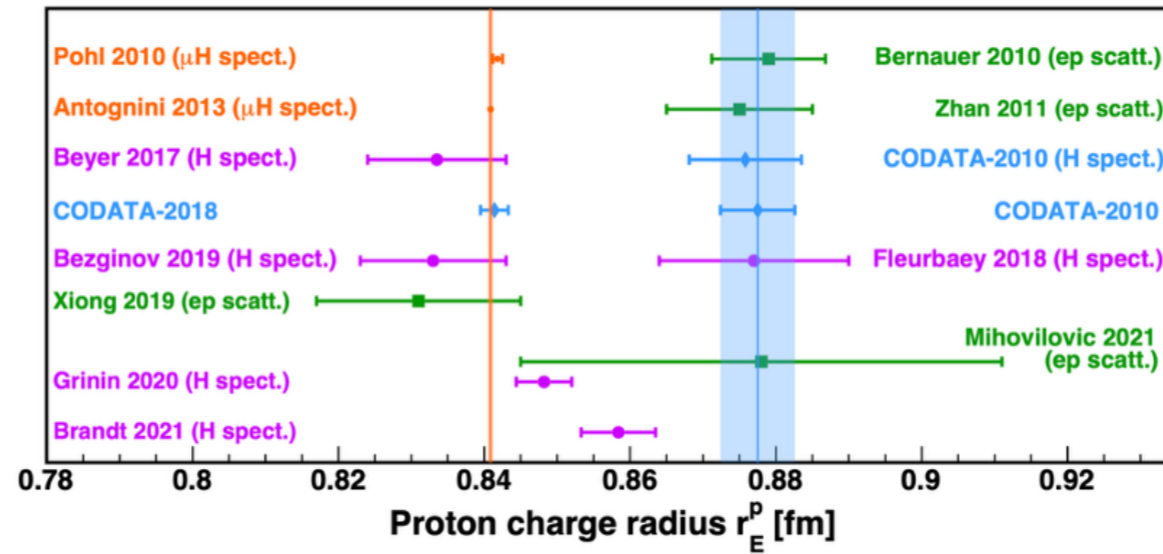


SoLID@JLab will reduce uncertainties

R. Milner EINN2025

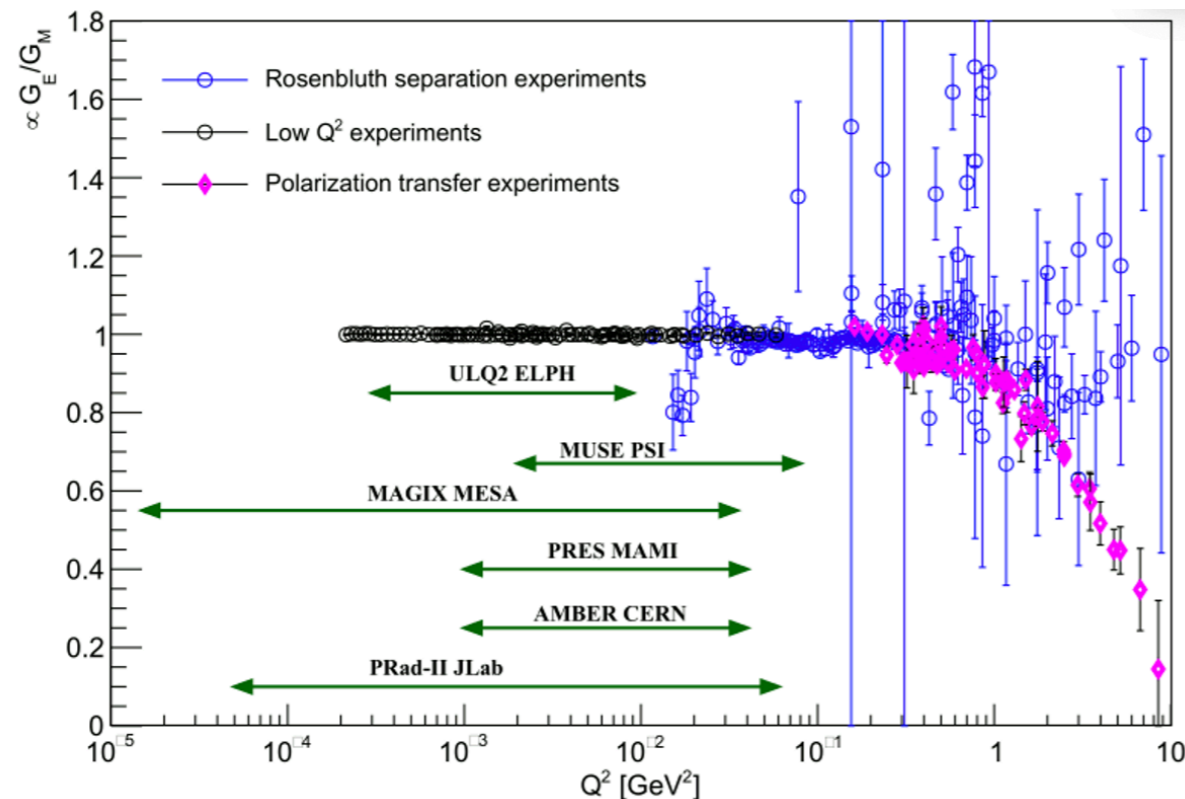
Nucleon EM form factors

- Electron-proton scattering since the 50s, however still surprises



W. Xiong, Ch. Peng, Universe 9 (2023) 4, 182 , arXiv:2302.13818

- EIC will measure EM FFs up to Q^2 40 GeV²



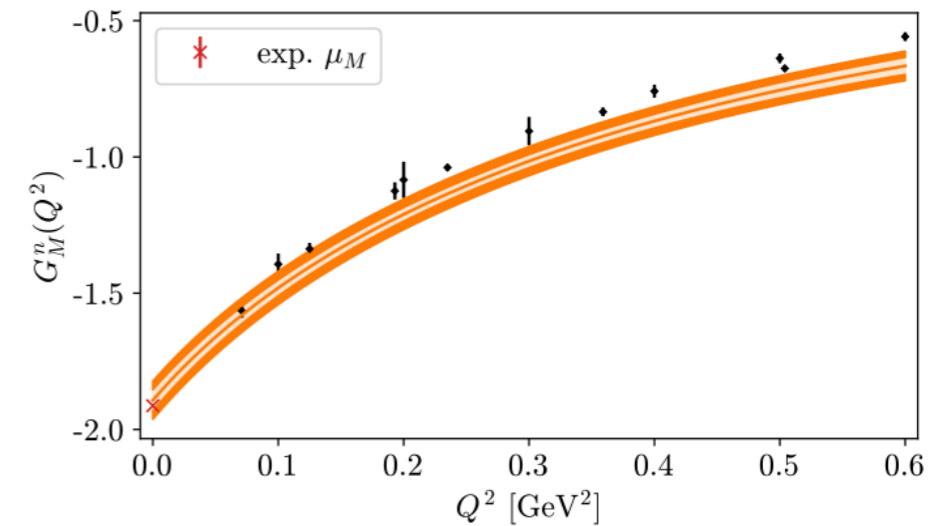
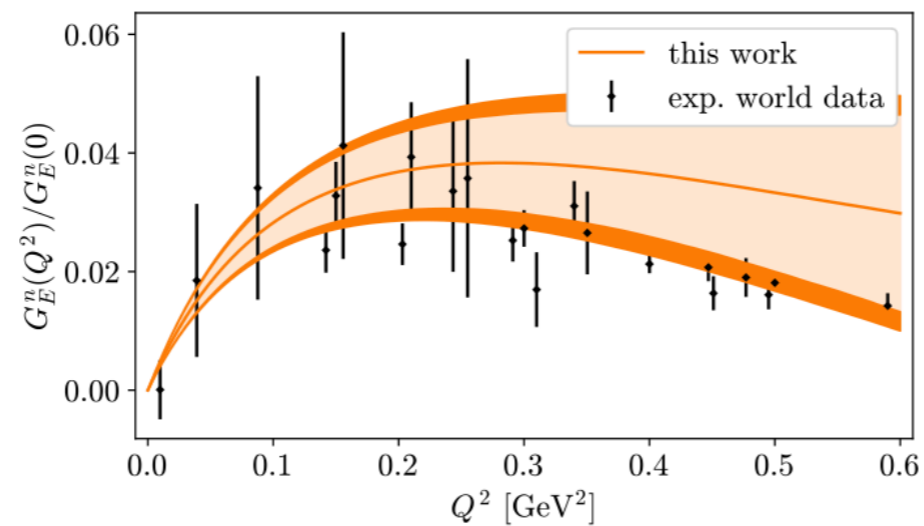
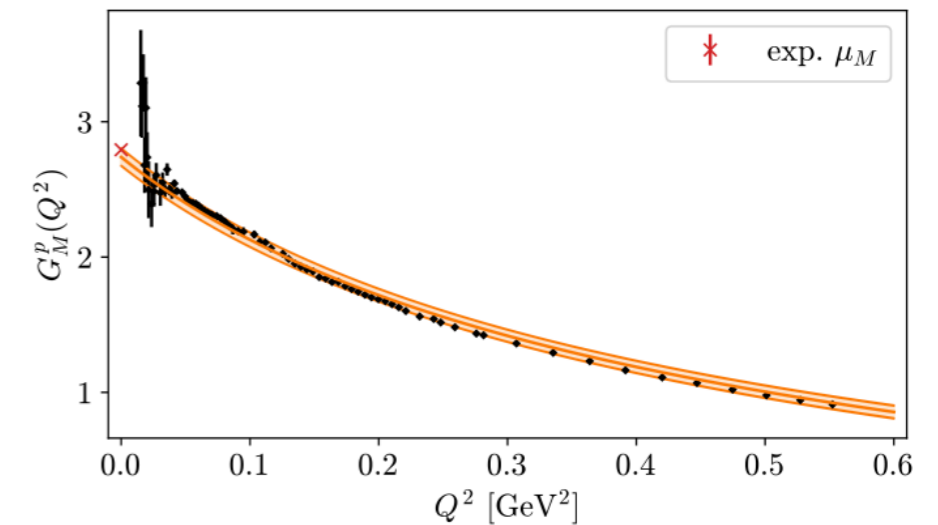
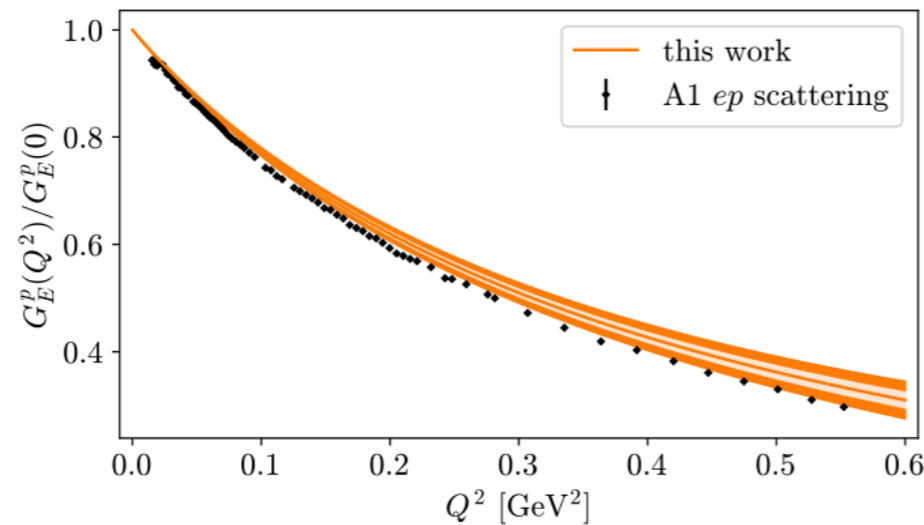
At high Q^2 the difference between Rosenbluth-separation experiments and the polarisation-transfer ones is more evident

Nucleon EM form factors

- Electron-proton scattering since the 50s
- Still surprises

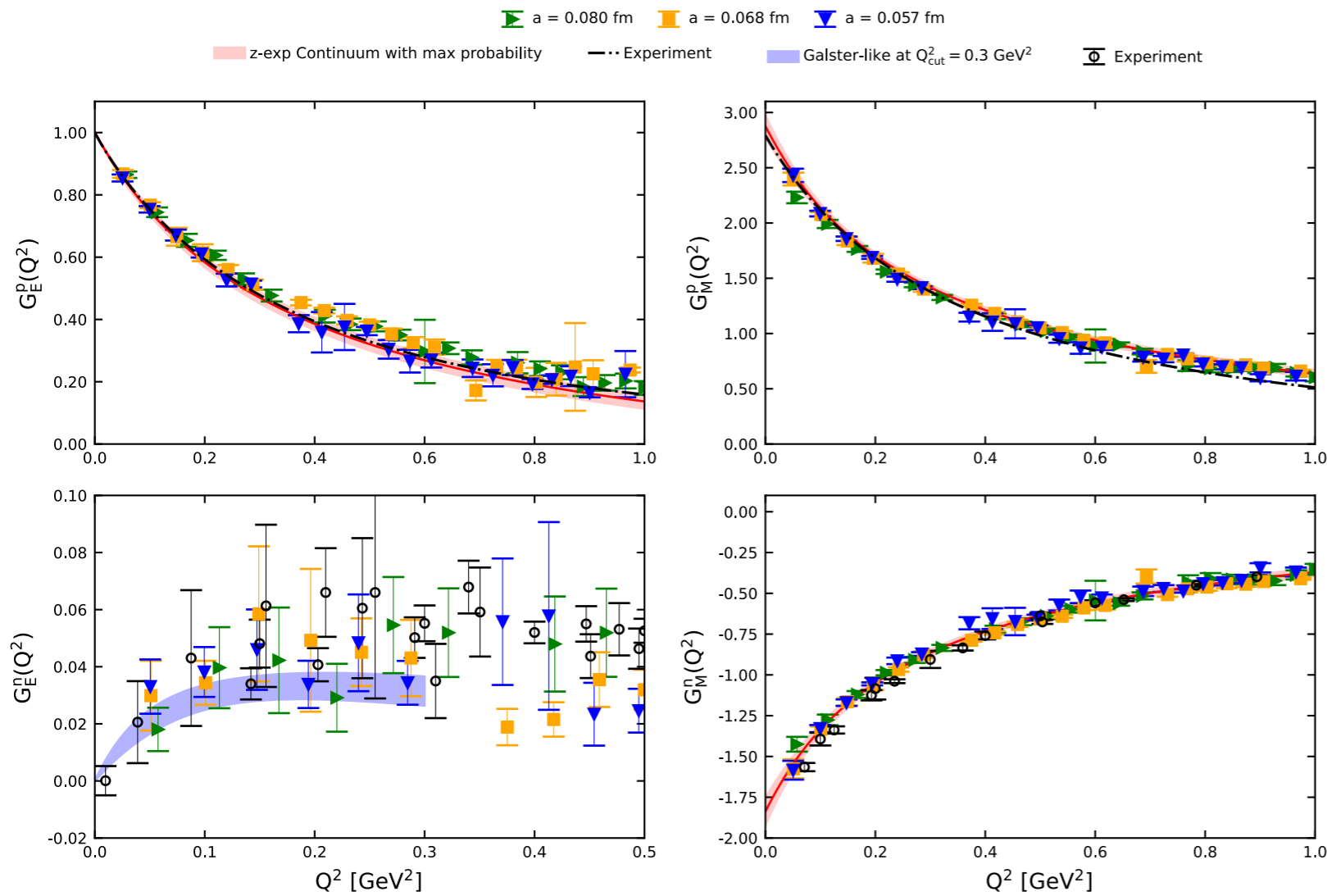


✳ Mainz results using 6 CLS ensembles at 4 lattice spacings with one ensemble with physical m_π



- Use covariant baryon chiral perturbation theory to model the Q^2 dependence of the form factor
- Fit simultaneously the Q^2 -dependence with the extrapolation to the physical m_π , $a = 0$, $L \rightarrow \infty$

Nucleon EM and axial form factors by ETMC



Two-fit forms for Q^2 dependence:

- Model independent z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k(Q^2)$$

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} + t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} + t_0}}$$

t_{cut} = particle production threshold and $t_0=0$

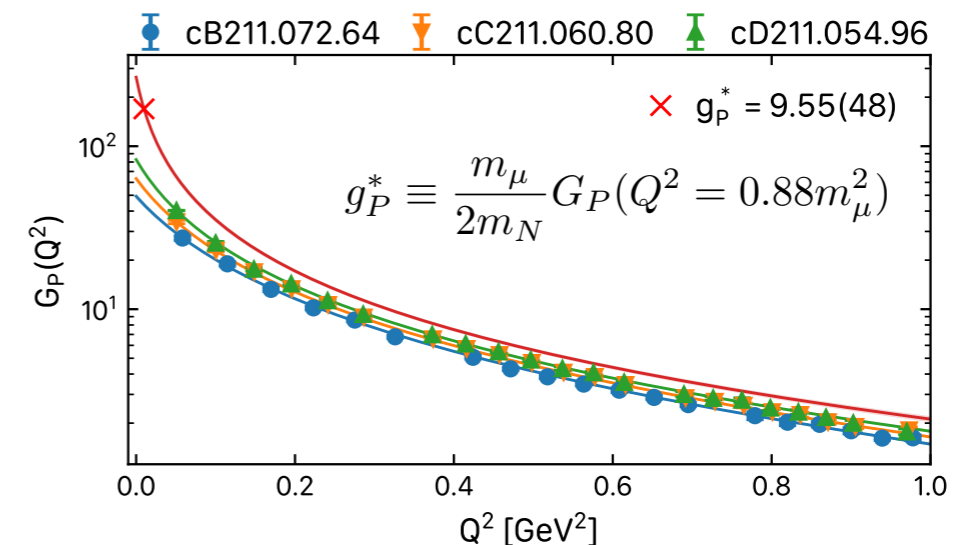
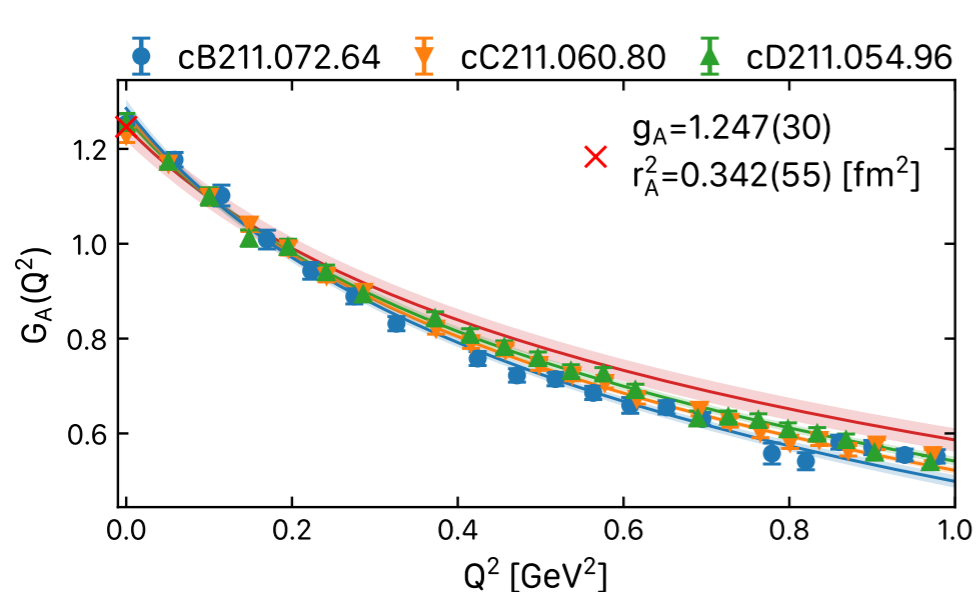
- Dipole or Galster form

$$G(Q^2) = \frac{g}{\left(1 + \frac{Q^2}{M^2}\right)^2}$$

$$G_E^n(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71\text{GeV}^2}\right)^2}$$

Talk by B. Prasad, Thursday

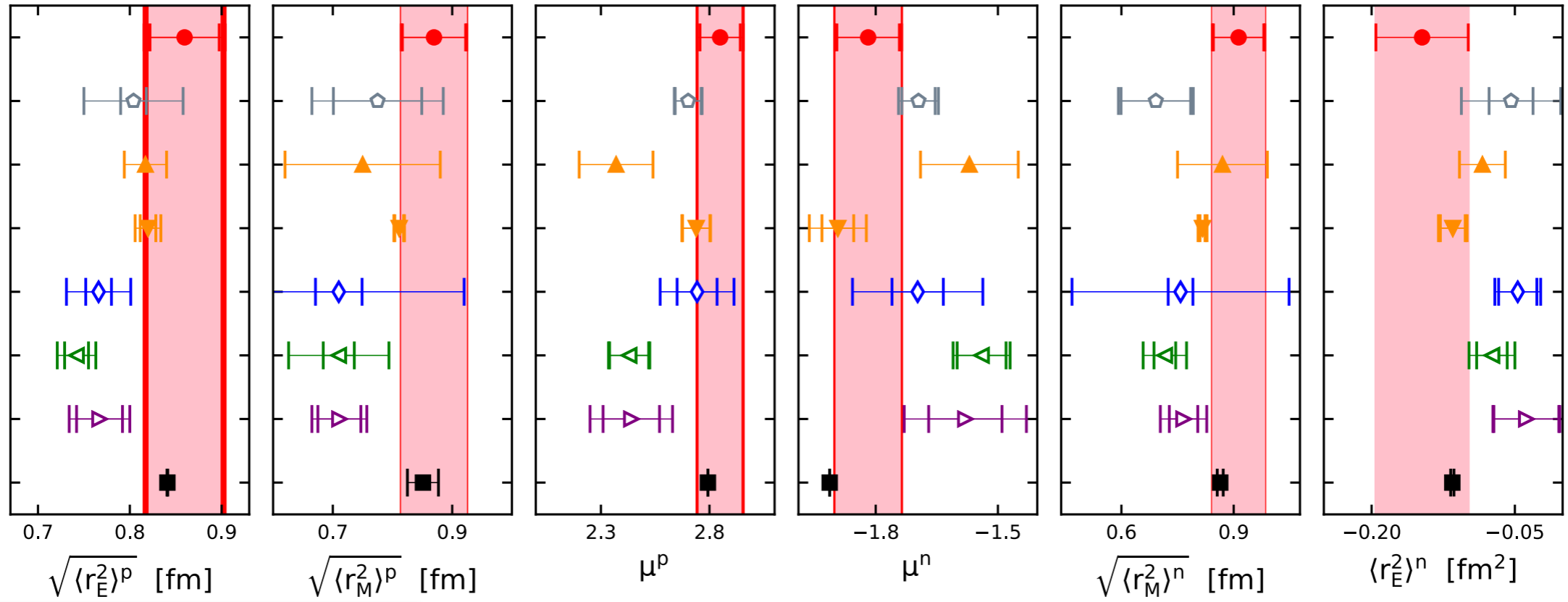
ETMC: C.A., S. Bacchio, G. Koutsou, B. Prasad, G. Spanoudes, arXiv:2507.20910



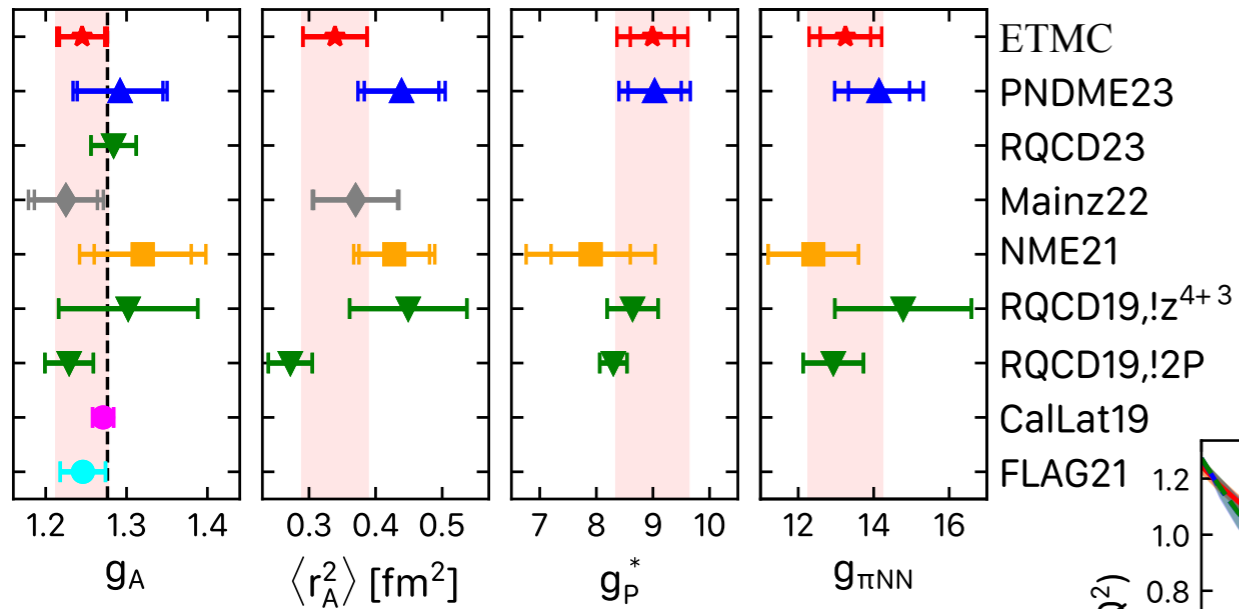
ETMC: C.A. *et al.*, Phys. Rev. D 109(2024)3, 034503

Nucleon EM and axial form factors - comparison

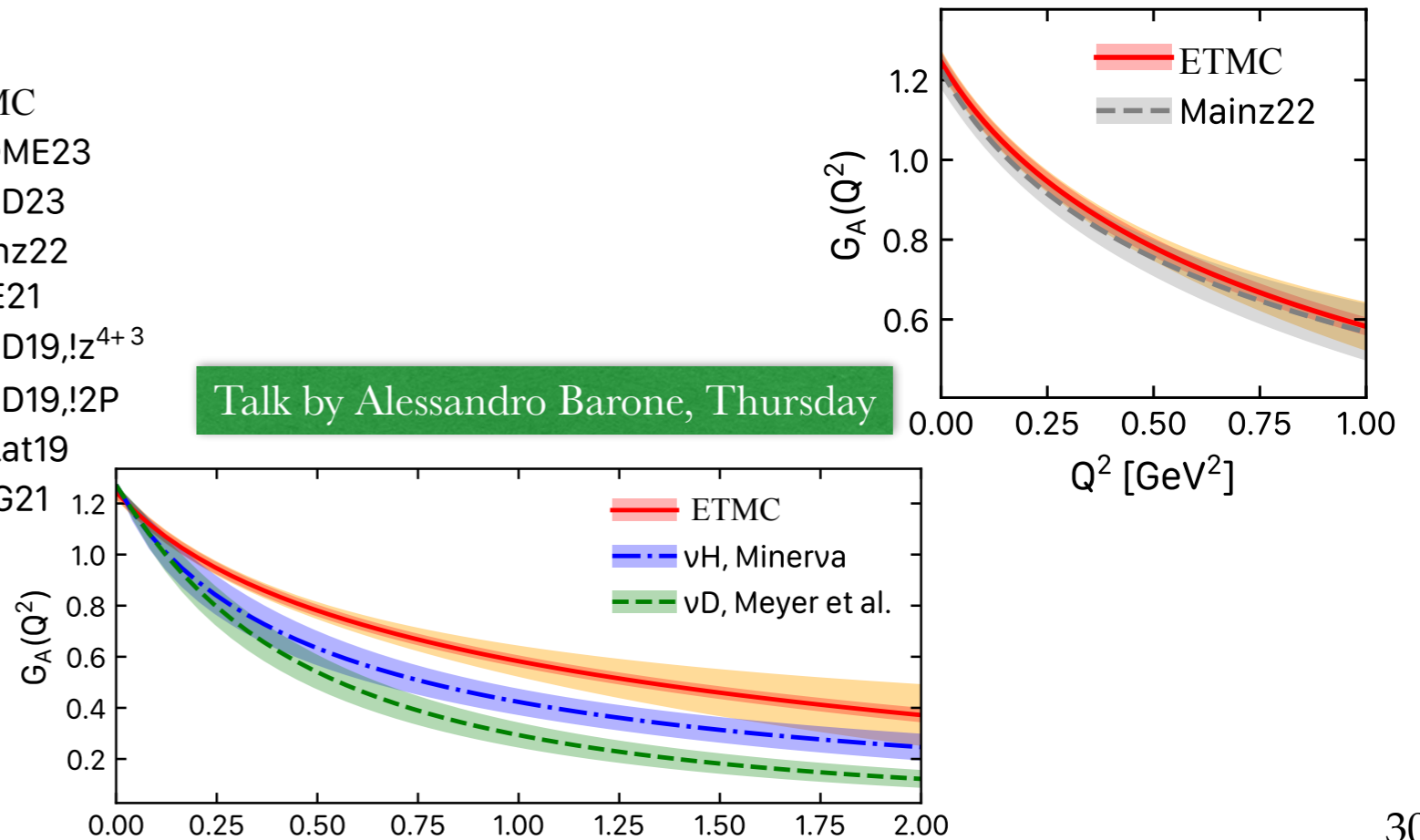
■ PDG ▷ ETMC'17 ◁ ETMC'19 ◊ PACS'19 ▽ Mainz'23 ▲ Mainz'23 z-exp ◻ PACS'23 ● ETMC



Talk by Marcel Rodekam, Thursday



Talk by Alessandro Barone, Thursday



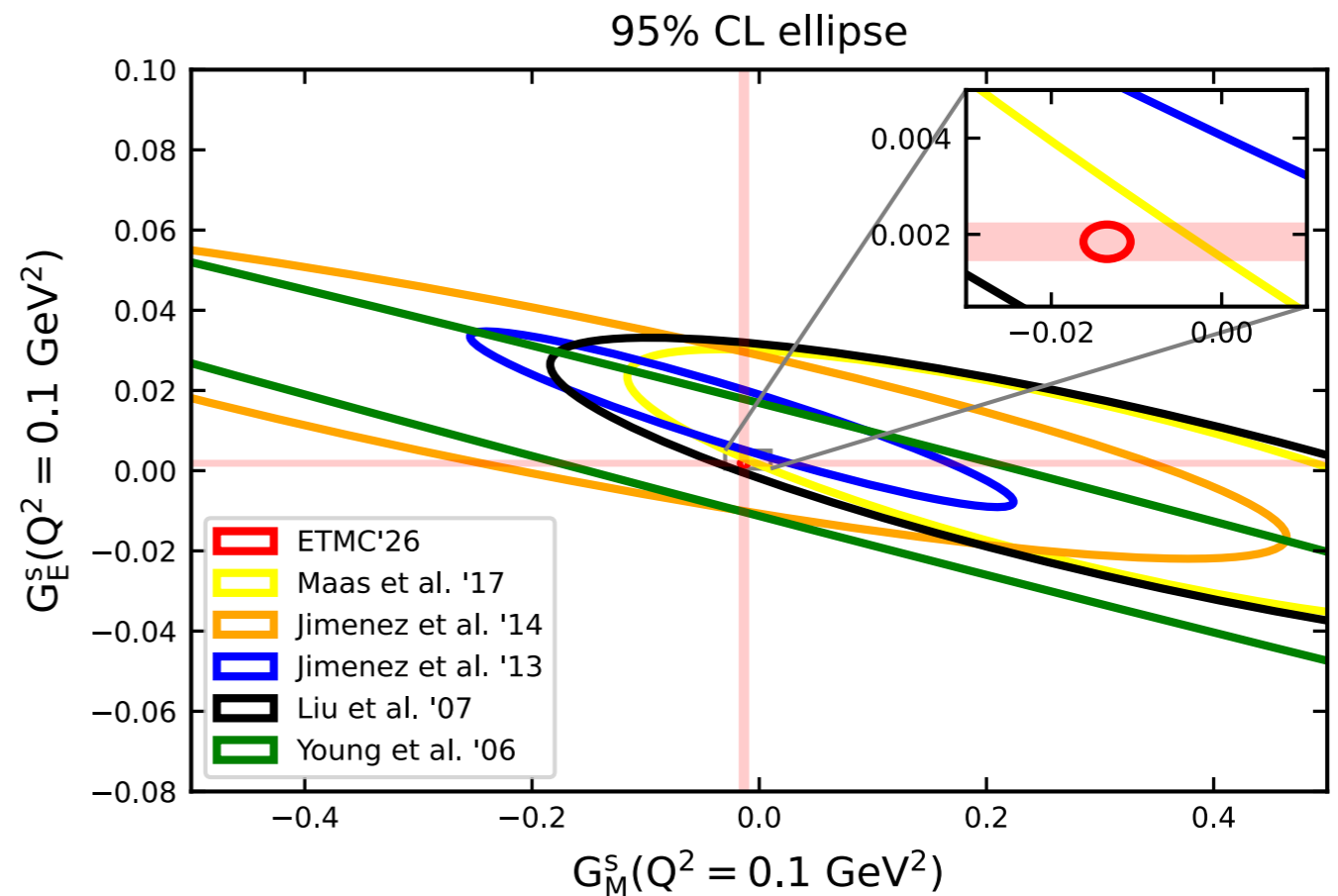
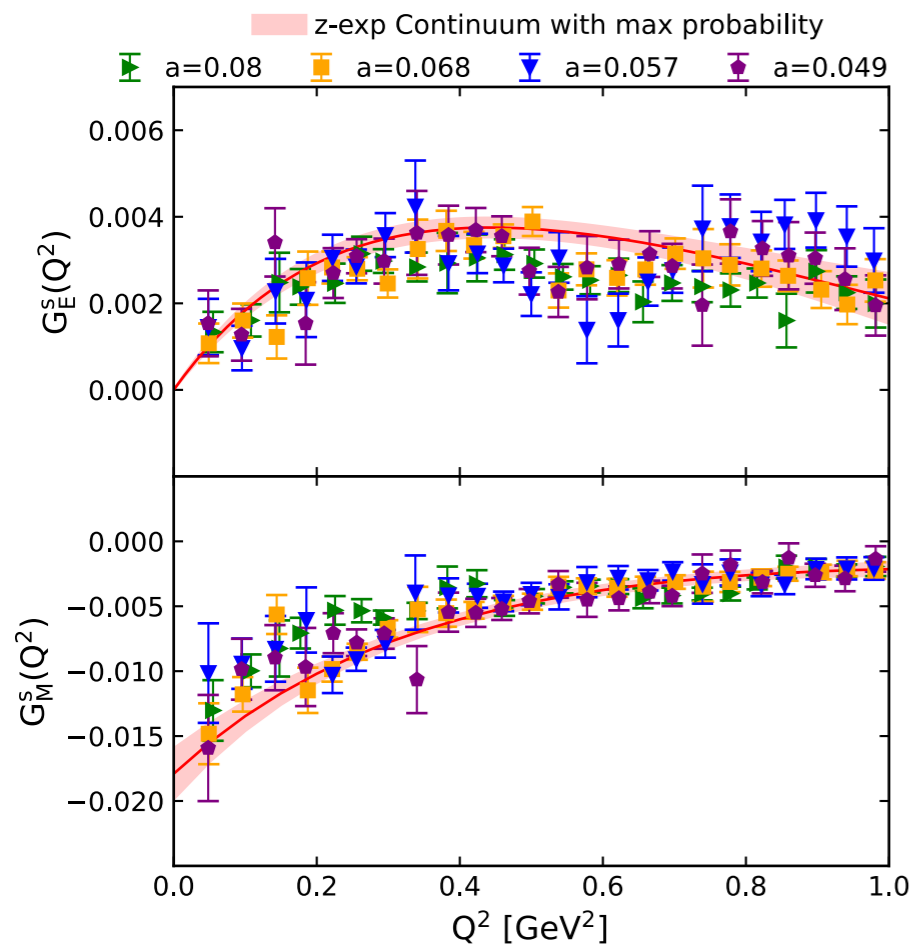
Strangeness in the nucleon from ETMC

$N_f = 2 + 1 + 1$, Twisted – mass fermions : $64^3 \times 128, a = 0.080$ fm, $m_\pi = 140$ MeV

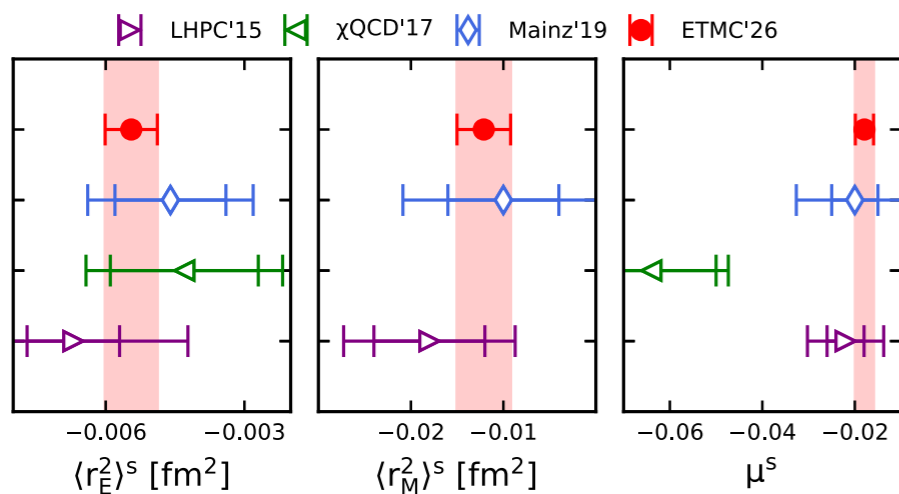
$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV

$96^3 \times 192, a = 0.057$ fm, $m_\pi = 141$ MeV

$112^2 \times 224, a = 0.049$ fm, $m_\pi = 136$ MeV



Significant input to PV experiments e.g. for Q-weak, G0 & HAPPEX @JLab

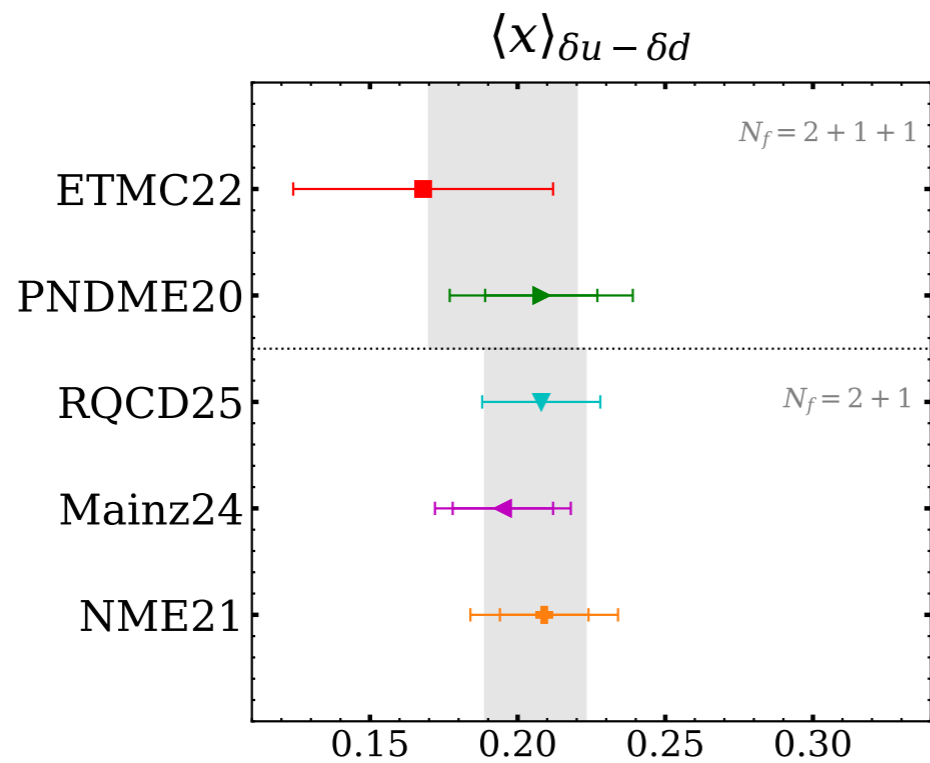
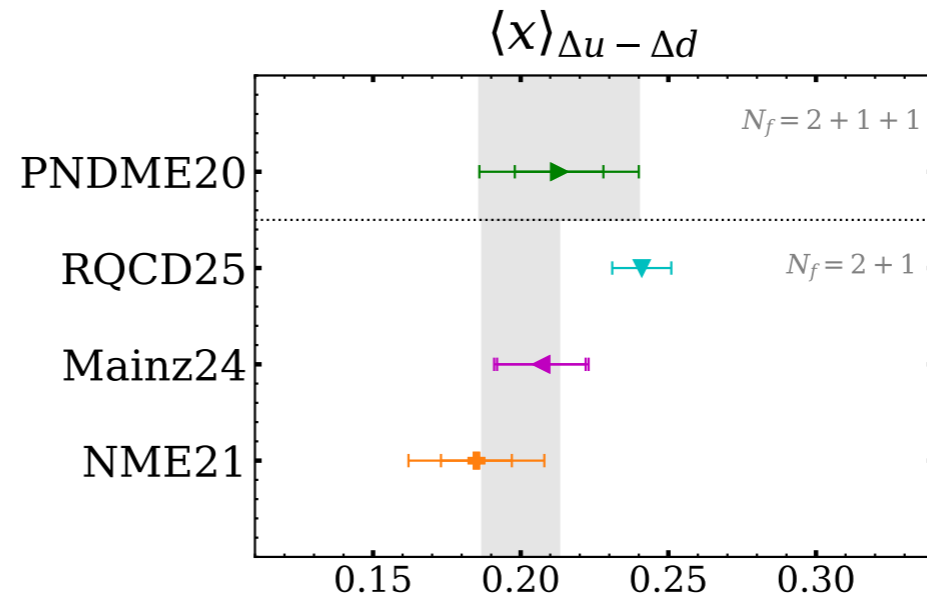
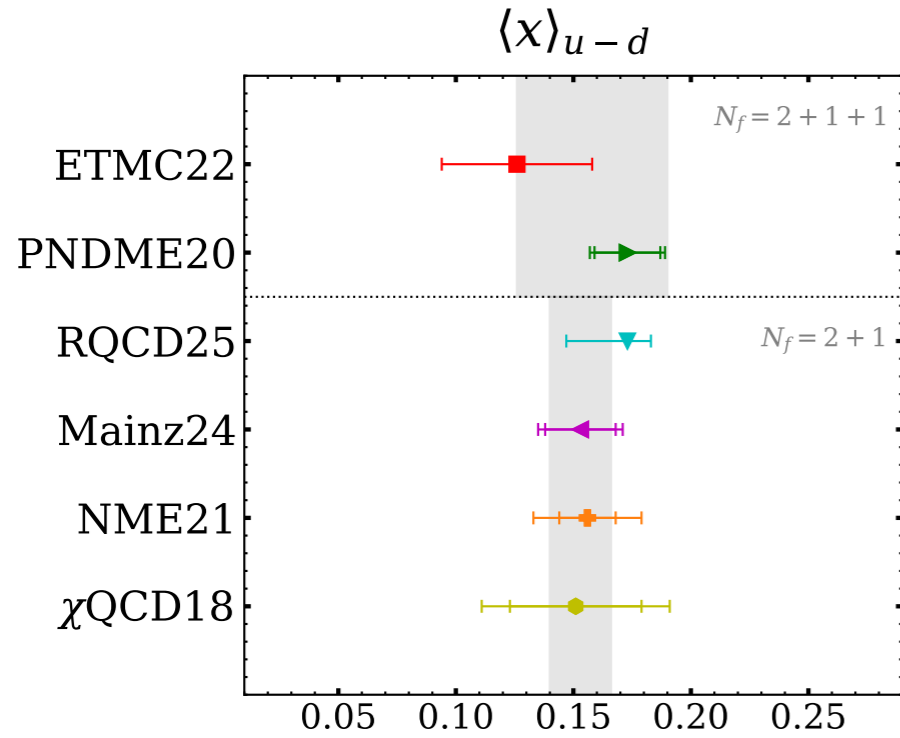


Talk by B. Prasad, Thursday

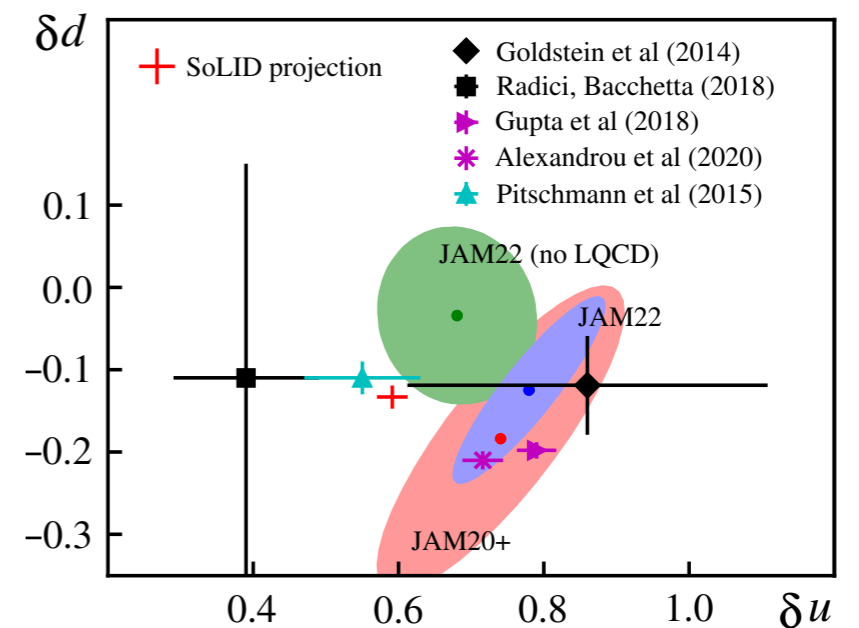
✳ First computation directly at physical pion mass with continuum extrapolation - clearly shows the accuracy that can be reached by lattice QCD on sea quark effects

Isvector nucleon moments

✱ RQCD computed isovector moments for the nucleon, Σ and X using 47 ensembles



SoLID@JLab R. Milner EINN 2025



Nucleon second moments

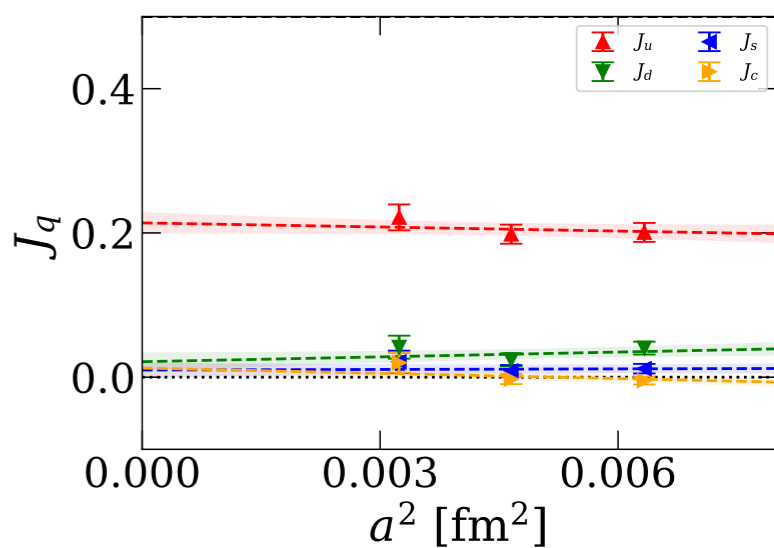
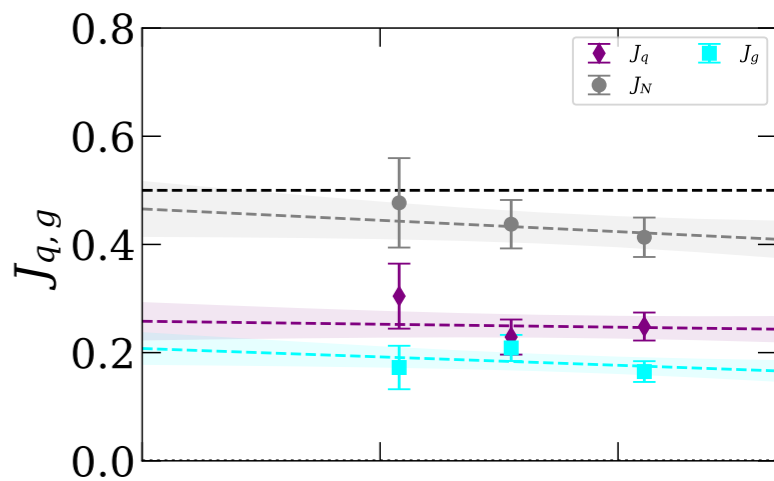
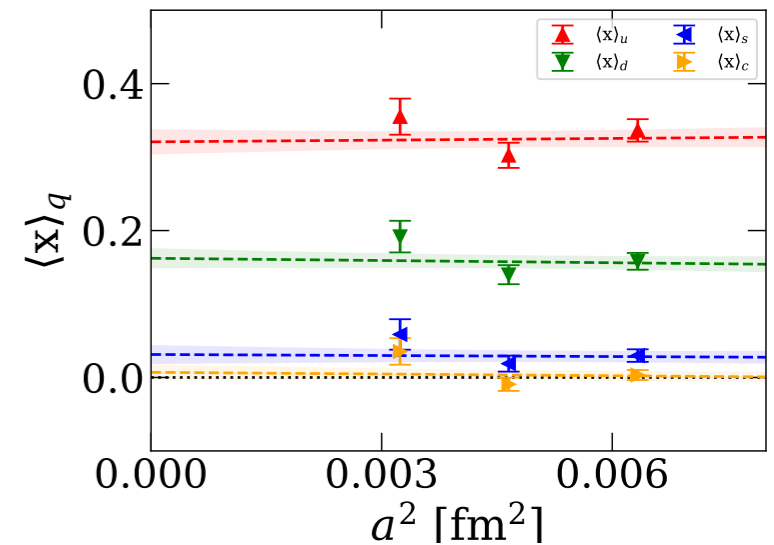
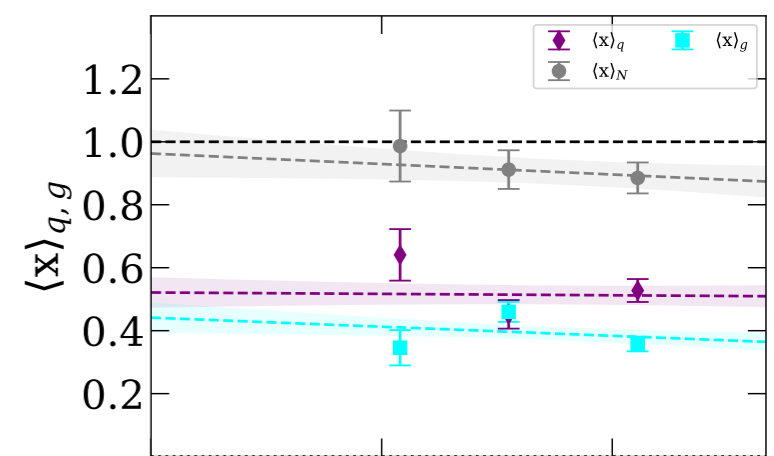
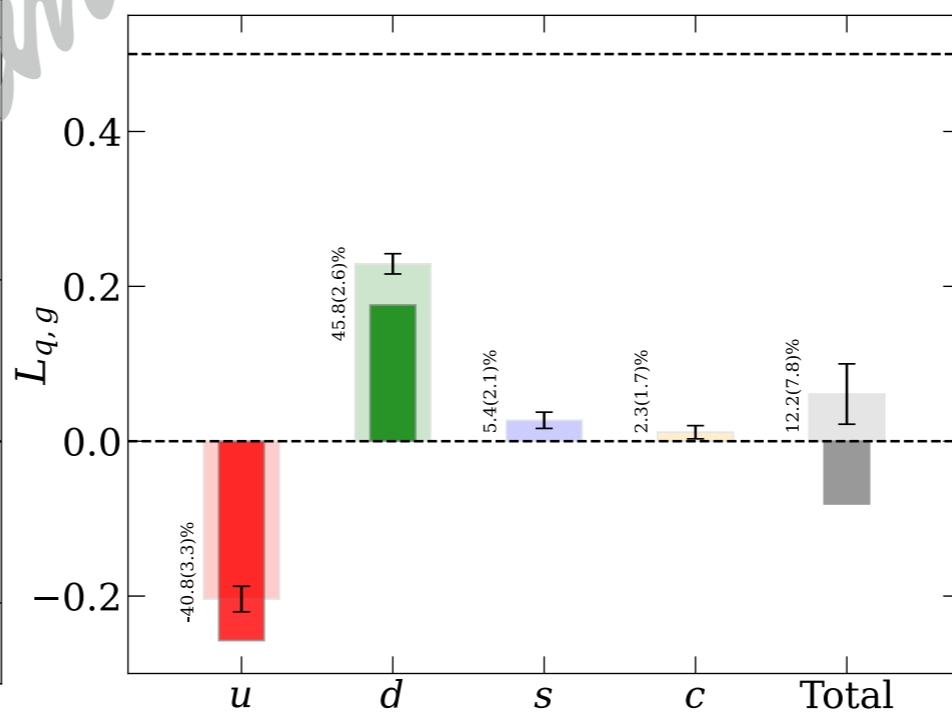
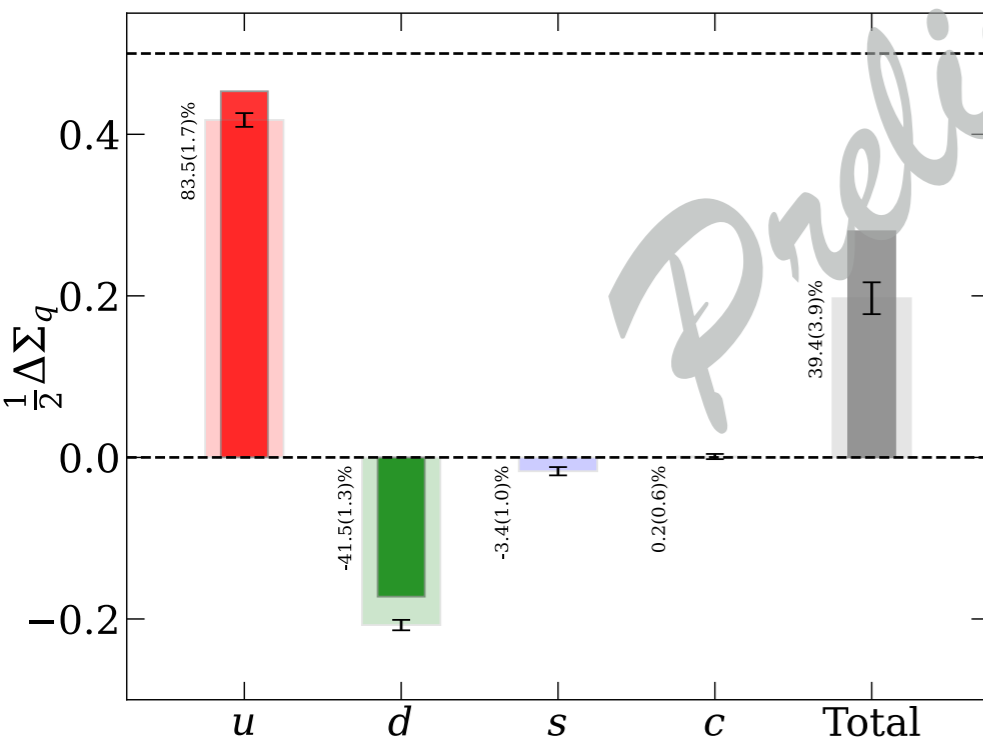
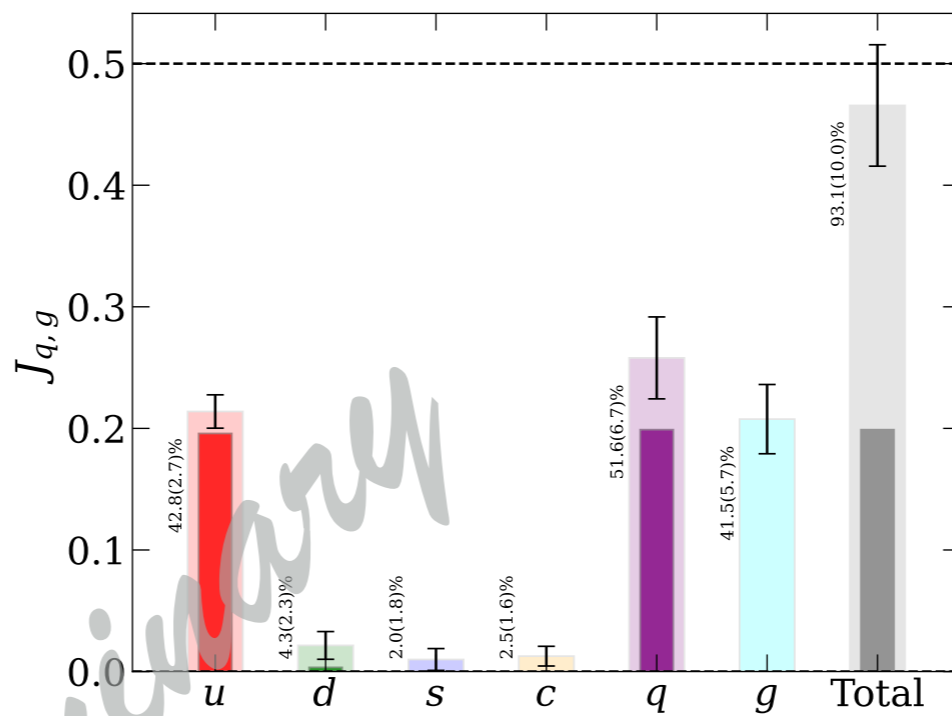
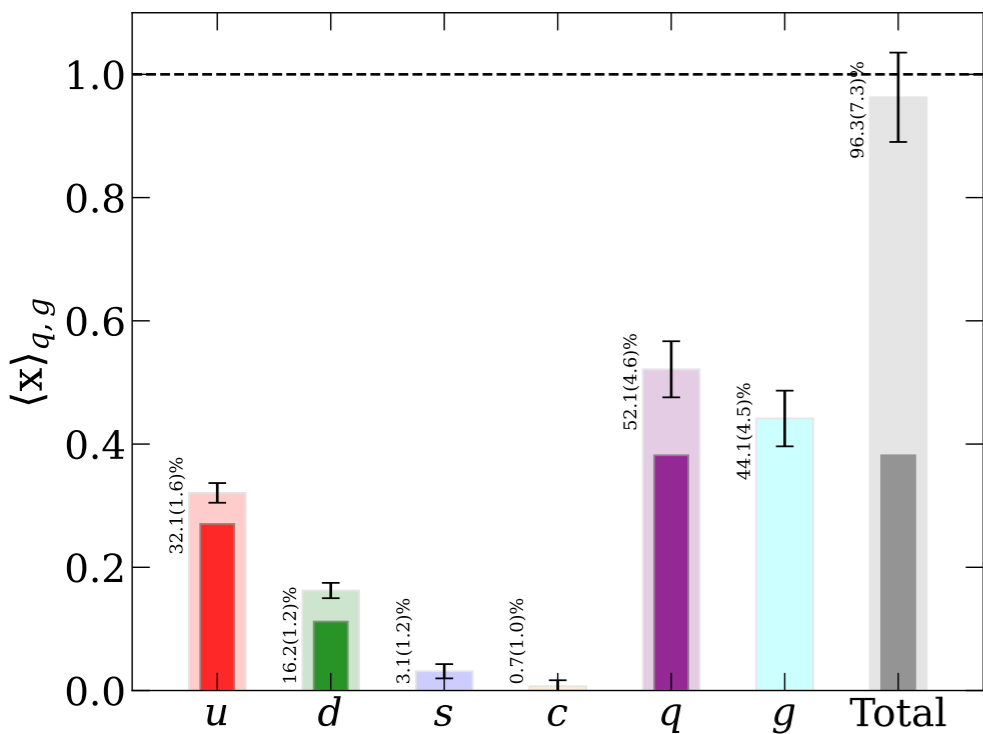
✿ Nucleon momentum and spin sums

$N_f = 2 + 1 + 1$, Twisted – mass fermions : $64^3 \times 128$, $a = 0.080$ fm, $m_\pi = 140$ MeV

Analysis of 4th ensemble is ongoing

$80^3 \times 160$, $a = 0.069$ fm, $m_\pi = 137$ MeV

$96^3 \times 192$, $a = 0.057$ fm, $m_\pi = 141$ MeV

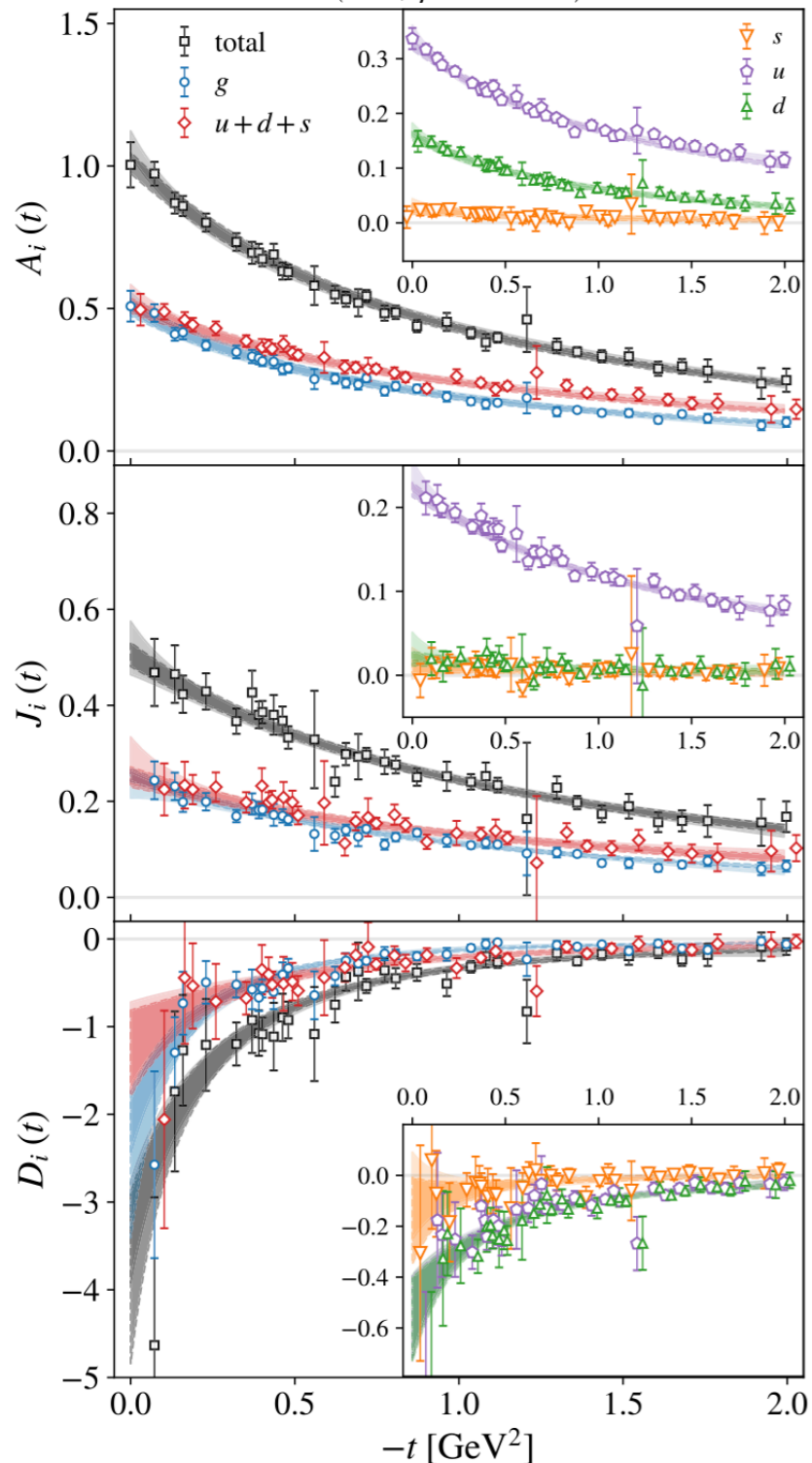


Generalised form factors by MIT group

✳ $N_f = 2 + 1$, Clover fermions : $48^3 \times 96$, $a = 0.091(1)\text{fm}$

Nucleon at $m_\pi \sim 170 \text{ MeV}$

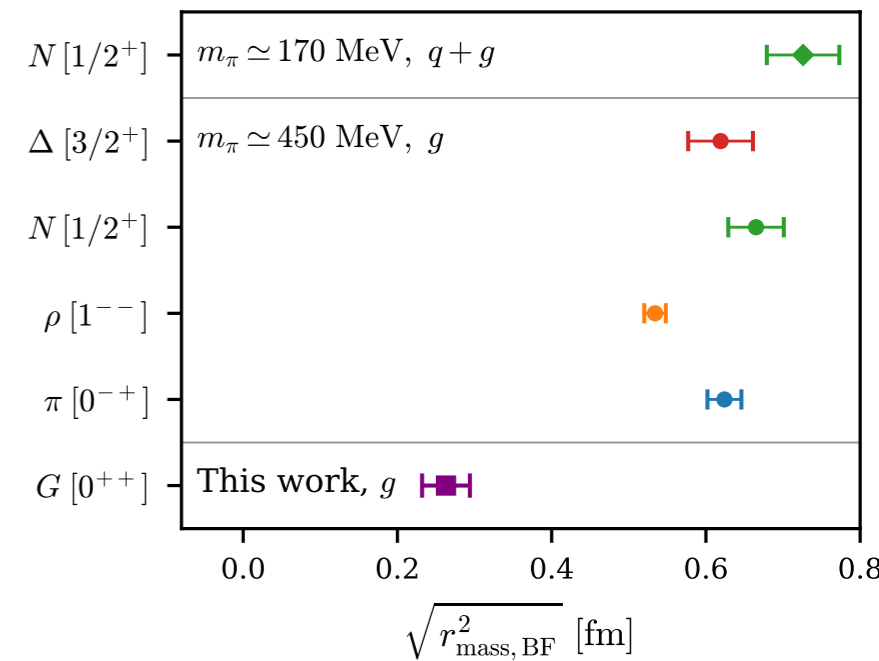
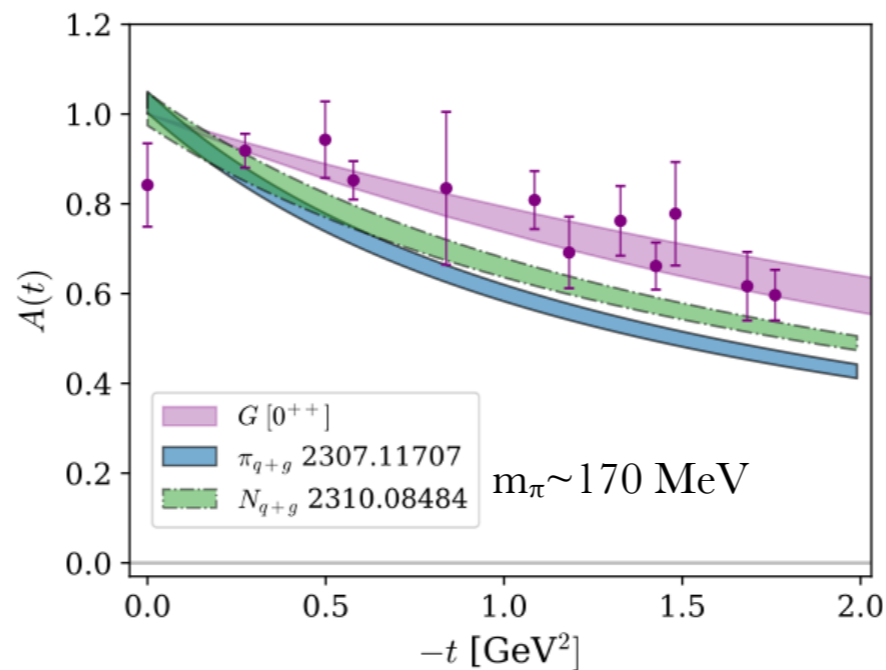
($\overline{\text{MS}}$, $\mu = 2 \text{ GeV}$)



Scalar glueball

Compute root mean square radius of the energy density of the glueball in the Breit frame

$$r_{\text{mass,BF}}^2 = \frac{1}{A(0)} \left[6 \frac{dA(t)}{dt} \Big|_{t=0} - \frac{3}{4m^2} (A(0) + 2D(0)) \right]$$



Glueball has a very small mass radius

Momentum + spin sums fulfilled

R. Abbot *et al.*, arXiv:2508.21821

Nucleon higher Mellin moments

- ✳ Third Mellin moments of the twist-2 and twist-3 helicity structure functions $g_1(x)$ and $g_2(x)$
- ✳ The twist-3 d_2 is connected to the Sivers function for $b_\perp \rightarrow 0$ and probes quark-gluon correlations
- ✳ First computed by QCDSF QCDSF: M.Goeckeler *et al.*, Phys. Rev. D 72, (2005) 054507
- ✳ Recent calculation by RQCD using clover fermions with six different spacings from 0.039 fm to 0.098 fm and $220 < m_\pi < 420$ MeV to perform chiral and continuum extrapolation

RQCD: M.Buerger *et al.*, Phys.Rev.D 105 (2022) 054504

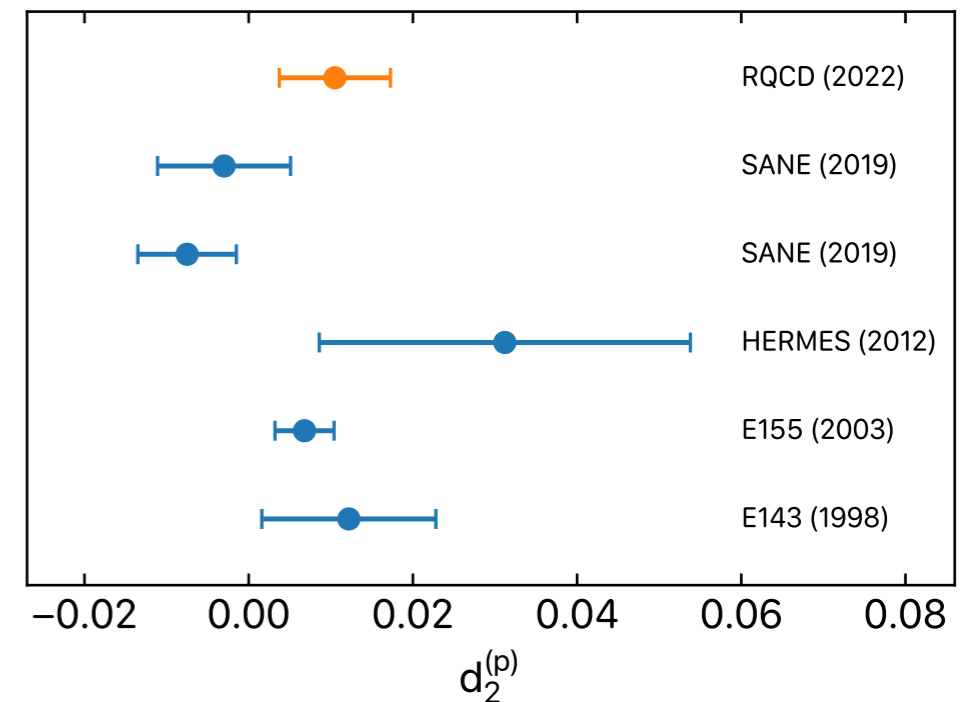
$$d_2(\mu) = 4 \int_0^1 dx x^2 \left[g_1(x, Q^2) + \frac{3}{2} g_2(x, Q) \right]$$

$$\mu^2 = Q^2 = 4 \text{ GeV}^2$$

$$d_2^p = 0.0105(19)(65), \quad d_2^n = -0.0009(14)(69)$$

Helicity third moment in $\overline{\text{MS}}$

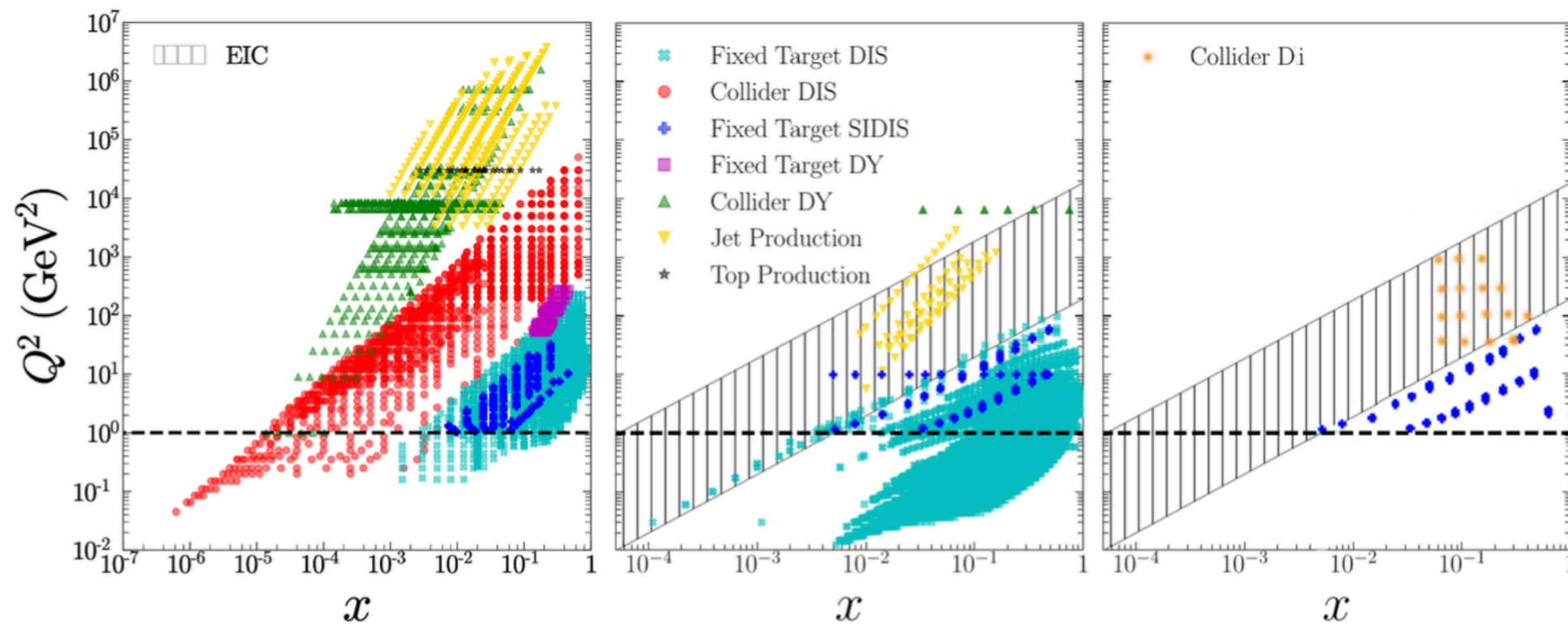
$$\langle x^2 \rangle_{\Delta p} = 0.035(3)(8), \quad \langle x^2 \rangle_{\Delta n} = 0.0034(17)(41)$$



- ✳ Third Mellin moments of unpolarized, helicity and transversity

Talk by Emilio Taggi, Tuesday

Direct computation of GPDs and TMDs



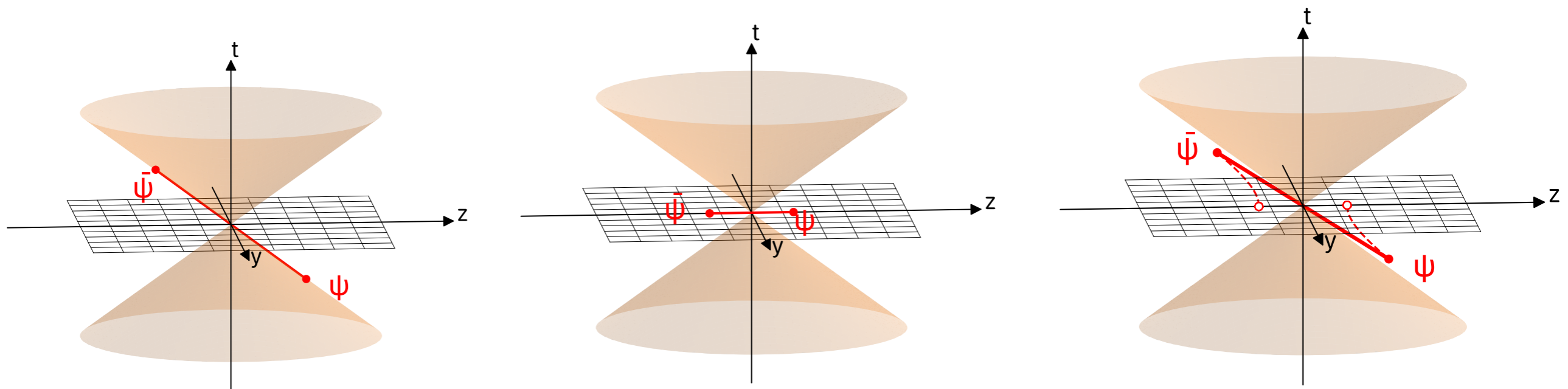
EIC will probe the region of low x \rightarrow non trivial flavor and quark-anti-quark differences

Direct computation of GPDs

- Quasi-approach - X. Ji 2013
- Pseudo-approach - A. V. Radyushkin 2017
- Hadronic tensor - K.-F. Liu 2016
- Forward Compton amplitude - A. Chambers *et al.* 2017
- Auxiliary heavy quark - W. Detmold and C. D. Lin 2006
- Good lattice cross sections - Y.-Q. Ma and J.-W. Qiu 2018
- ...

✳ Large momentum effective theory (LaMET) X. Ji, *Phys. Rev. Lett.* 110 (2013) 262002

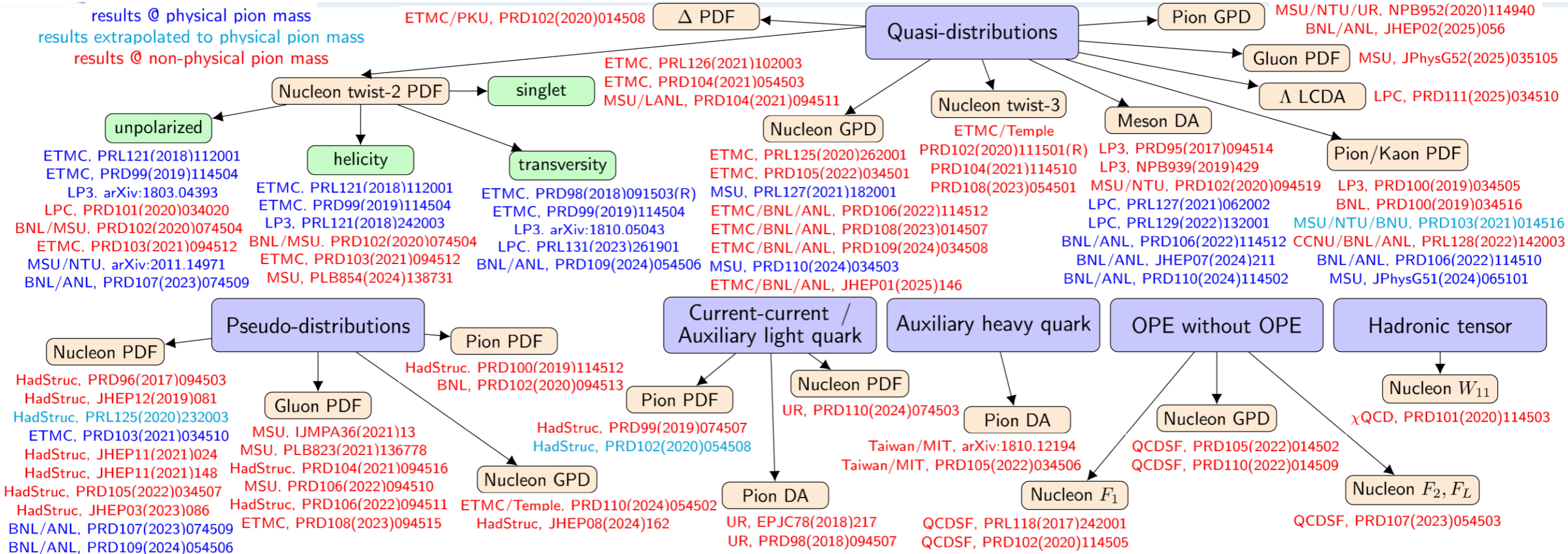
- Define spatial correlators and boost nucleon state to large momentum \rightarrow quasi PDFs (have same IR behavior)
- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (possible due to asymptotic freedom of QCD)
- Allow momentum transfer \rightarrow generalised parton distributions



✳ Short distance factorization (SDF) \rightarrow Pseudo- distributions V. Radyushkin, *Phys. Rev. D* 96 (2017) 034025

✳ Quasi- and Pseudo- distributions are extracted from the same spatial corrector

Many studies on PDFs/GPDs



K. Cichy, EINN 2025

Computation of quasi- and pseudo-PDFs

✱ Compute space-like matrix elements for boosted states $M_\Gamma(z, P_3) = \langle P_3 | \bar{\psi}(0) \Gamma W(0, z) \psi(z) | P_3 \rangle$

- **Quasi-distributions:** Take Fourier transform and the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3z} M_\Gamma(z, P_3) \Big|_{\mu} \leftarrow \begin{array}{l} \text{Renormalise non-perturbatively, } Z(z, \mu) \\ \text{Need to eliminate both UV and exponential divergences} \end{array}$$

Match using LaMET

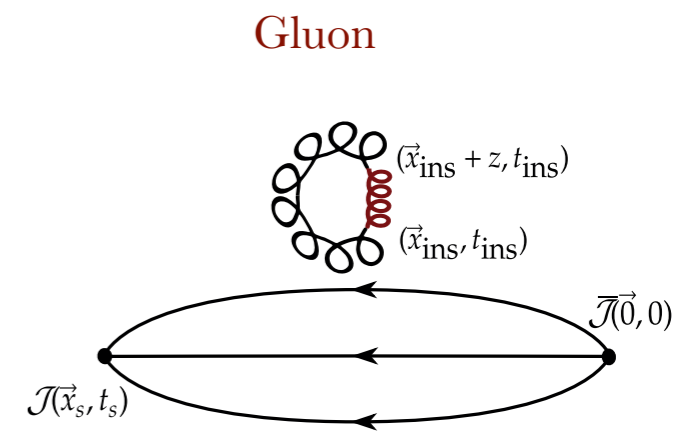
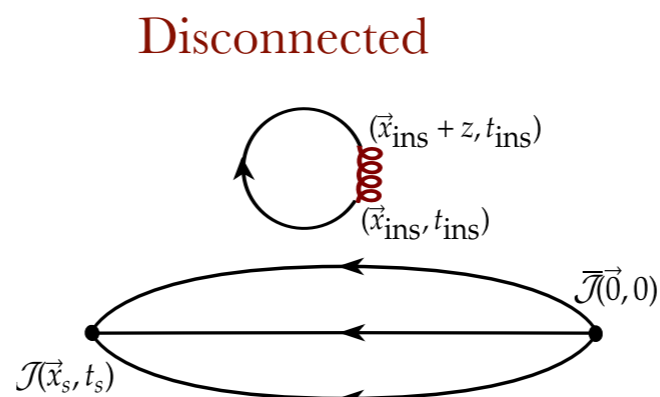
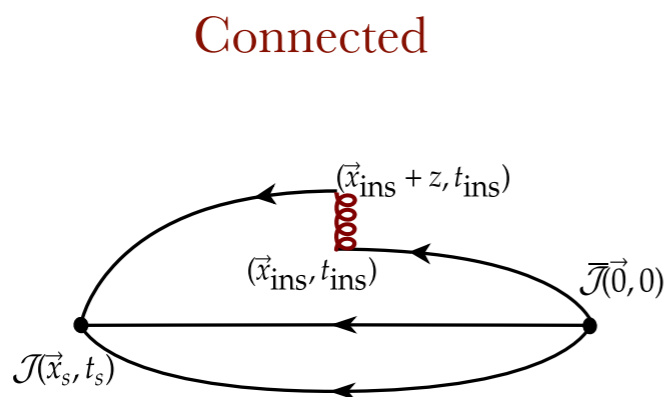
$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^1 \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_3^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_3^2}\right)$$

↙ Perturbative kernel ↘ Higher twist

Validity range:
 $x \in (x_{\min} - x_{\max}) \approx (0.2 - 0.8)$

$\Gamma =$	γ_0	unpolarised
	$\gamma_5 \gamma_3$	helicity
	$\sigma_{3i}, i = 1, 2$	transversity

- **Nucleon**



Computation of quasi- and pseudo-PDFs

✱ Compute space-like matrix elements for boosted states $M_\Gamma(z, P_3) = \langle P_3 | \bar{\psi}(0) \Gamma W(0, z) \psi(z) | P_3 \rangle$

• **Quasi-distributions:** Take Fourier transform and the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3z} M(z, P_3) \Big|_{\mu} \leftarrow \begin{array}{l} \text{Renormalise non-perturbatively, } Z(z, \mu) \\ \text{Need to eliminate both UV and exponential divergences} \end{array}$$

Match using LaMET

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^1 \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_3^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_3^2}\right) \quad \text{Validity range: } x \in (x_{\min} - x_{\max}) \approx (0.2 - 0.8)$$

↙ Perturbative kernel
↘ Higher twist

• **Pseudo-distributions:** use same matrix elements as quasi-distributions and write in terms of $\nu = z.P$ Ioffe time

Renormalize matrix element e.g. $\tilde{\mathcal{M}}_\Gamma(\nu, z^2) = \frac{\bar{M}_\Gamma(\nu, z^2)}{\bar{M}_\Gamma(0, z^2)}$

Match in coordinate space via short distance factorization (SDF)

$$\tilde{\mathcal{M}}_\Gamma(z^2, \nu) = \int_{-1}^1 dy C(y) \mathcal{M}_\Gamma(y\nu, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

↖ Ioffe time distribution
↖ Higher twist

limits max. ν_{\max} value \longrightarrow max. moments $x^{n_{\nu_{\max}}}$

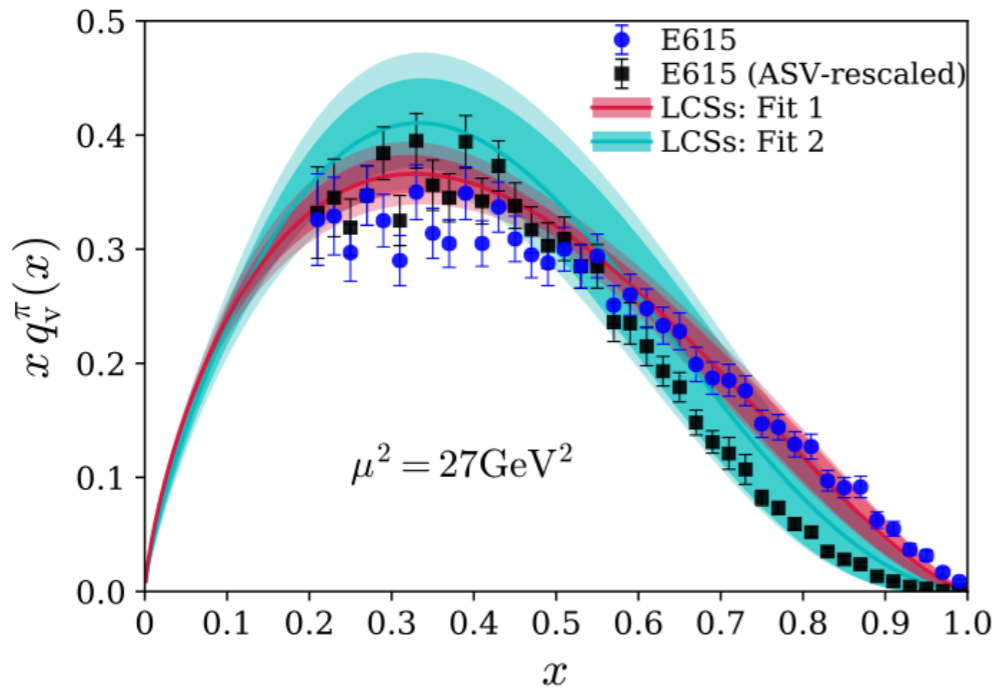
$$\mathcal{M}_\Gamma(\nu, \mu) = \int_{-1}^1 dxe^{ix\nu} F_\Gamma(x, \mu)$$

✱ Quasi- and pseudo- are complementary X. Ji, Research 8 (2025) 0695

Pion and kaon unpolarized PDF

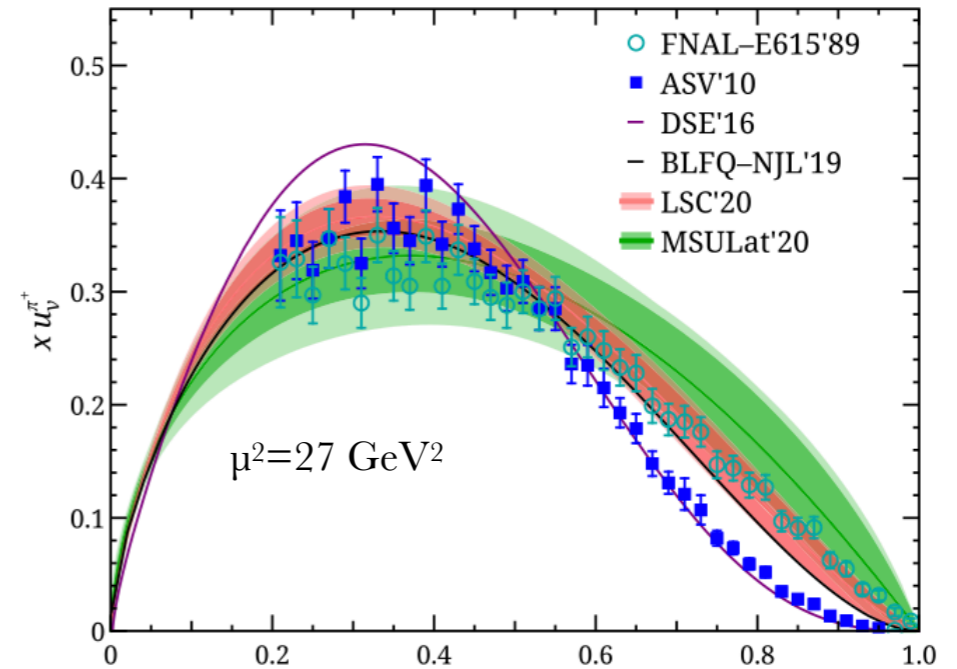
✳ Pion

R. S. Sufian *et al.*, Phys. Rev. D 102 (2020) 054508



“Good lattice cross-sections” approach with $N_f=2+1$ clover, $m_\pi \sim 410-280 \text{ MeV}$ and $a=0.127$ and 0.094

H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516

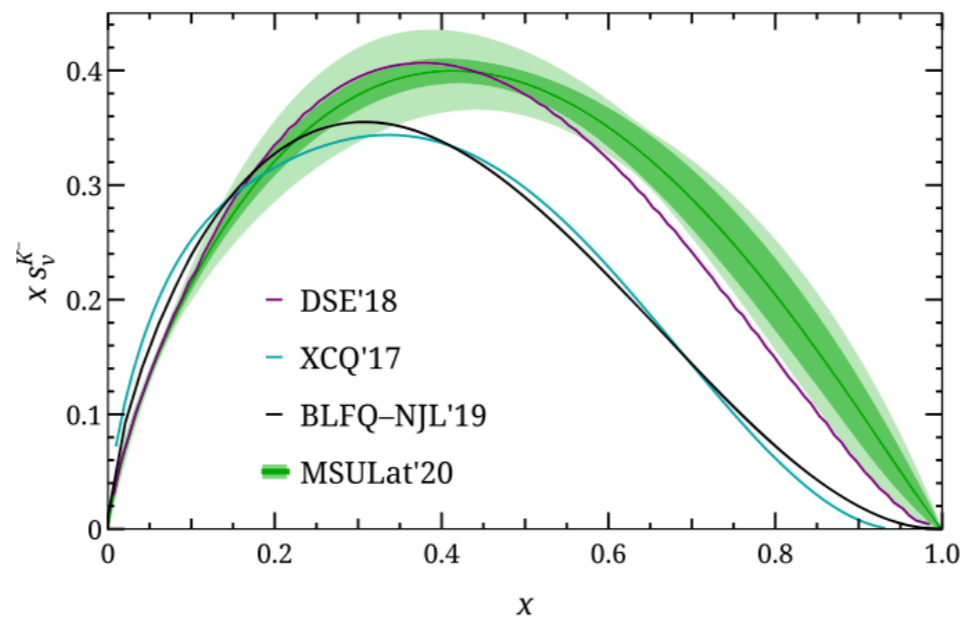


Quasi-PDF approach with clover valence on staggered sea with $m_\pi = 220, 310, 690 \text{ MeV}$ and $a = 0.12, 0.06 \text{ fm}$ with extrapolation to the continuum and physical m_π via

$$c_0 + c_1 m_\pi^2 + c_3 a^2$$

✳ Kaon

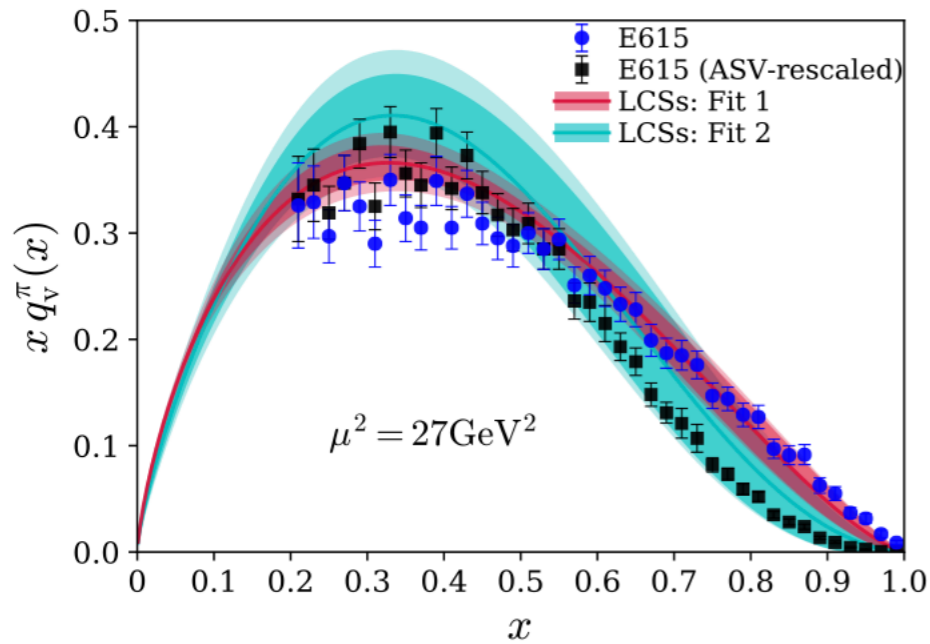
H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516



Pion and kaon unpolarized PDF

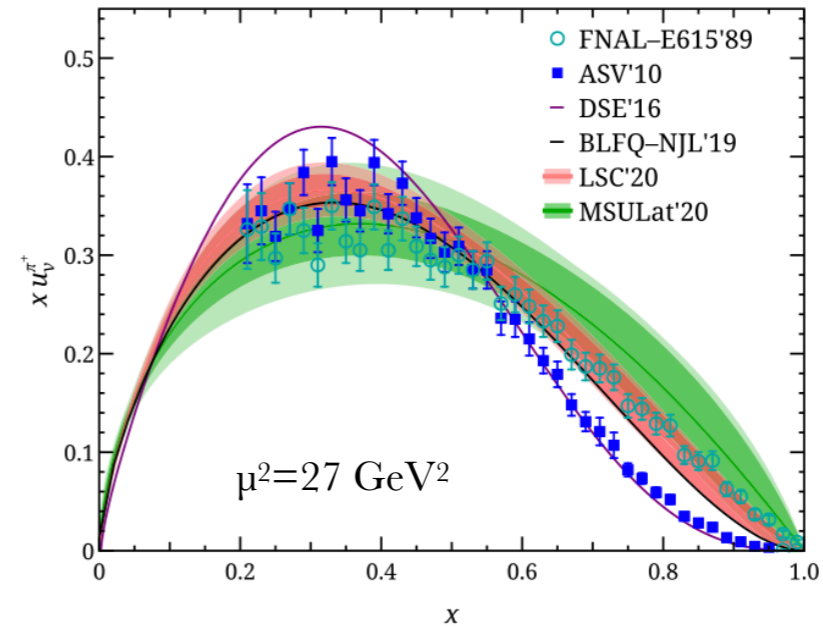
✿ Pion

R. S. Sufian *et al.*, Phys. Rev. D 102 (2020) 054508



“Good lattice cross-sections” approach with $N_f=2+1$ clover, $m_\pi \sim 410-280$ MeV and $a=0.127$ and 0.094

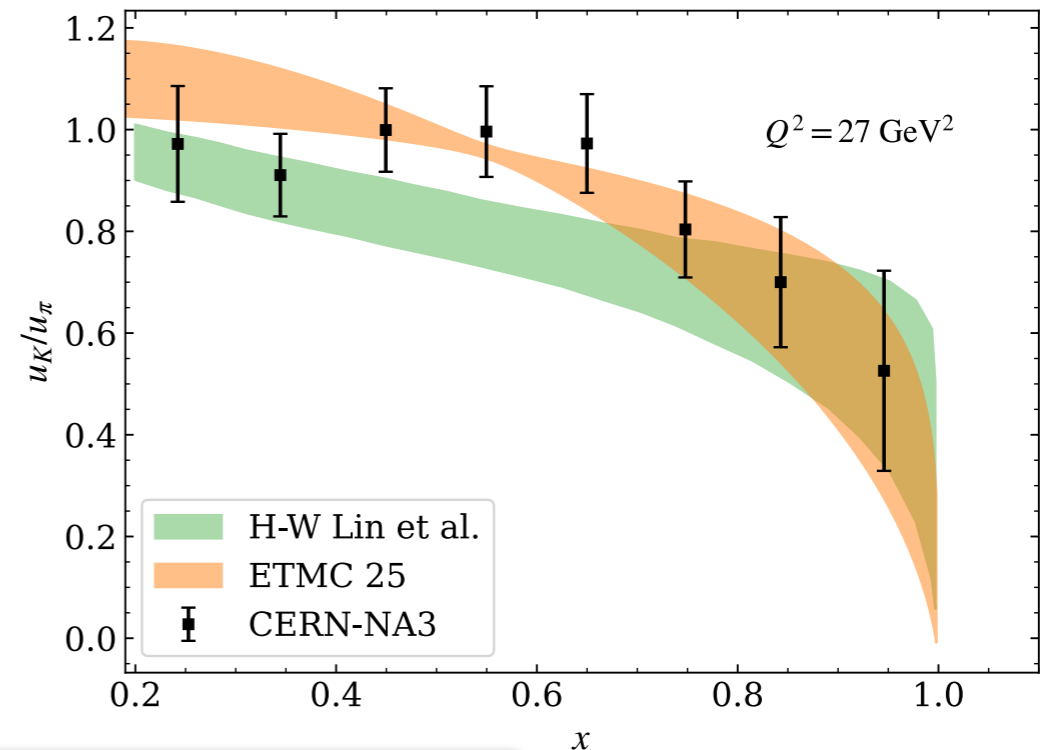
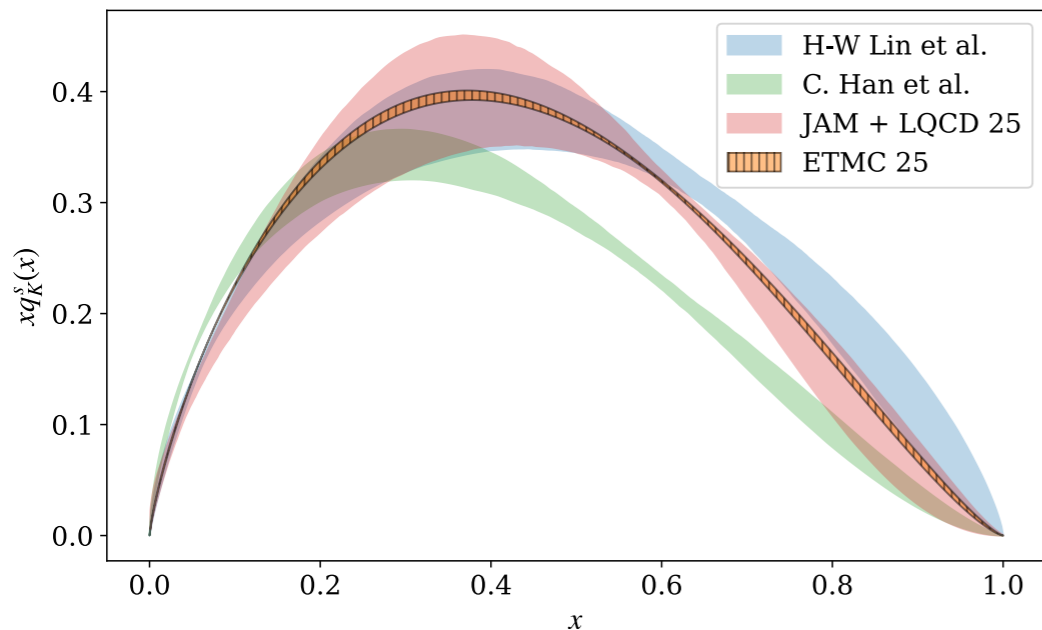
H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516



Quasi-PDF approach with clover valence on staggered sea with $m_\pi=220, 310, 690$ MeV and $a=0.12, 0.06$ fm with extrapolation to the continuum and physical m_π via $c_0 + c_1 m_\pi^2 + c_3 a^2$

✿ Kaon

H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516



To be measured by the AMBER experiment at CERN

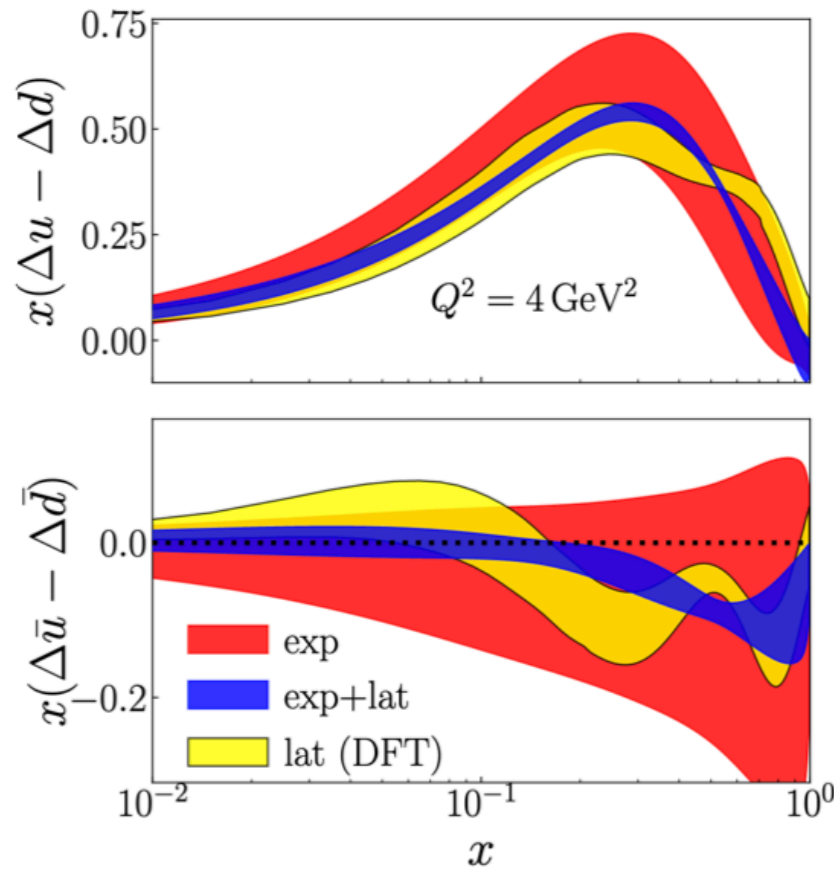
Nucleon helicity and strange quark PDFs

✱ Studies in the quasi-PDF approach helicity have had an impact on phenomenology

✱ Isovector at physical pion mass

• LP³: $N_f=2+1+1$, $m_\pi \sim 135$ MeV, $a=0.09$ fm
 H.-W. Lin *et al.* Phys. Rev. Lett. **121**, 242003 (2018)

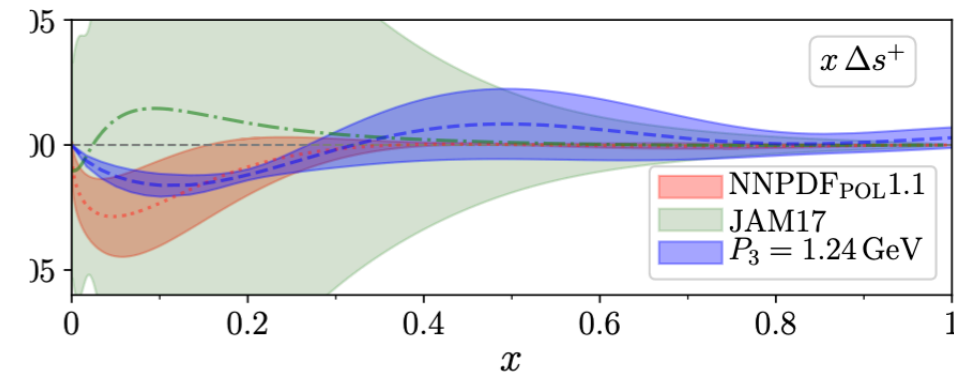
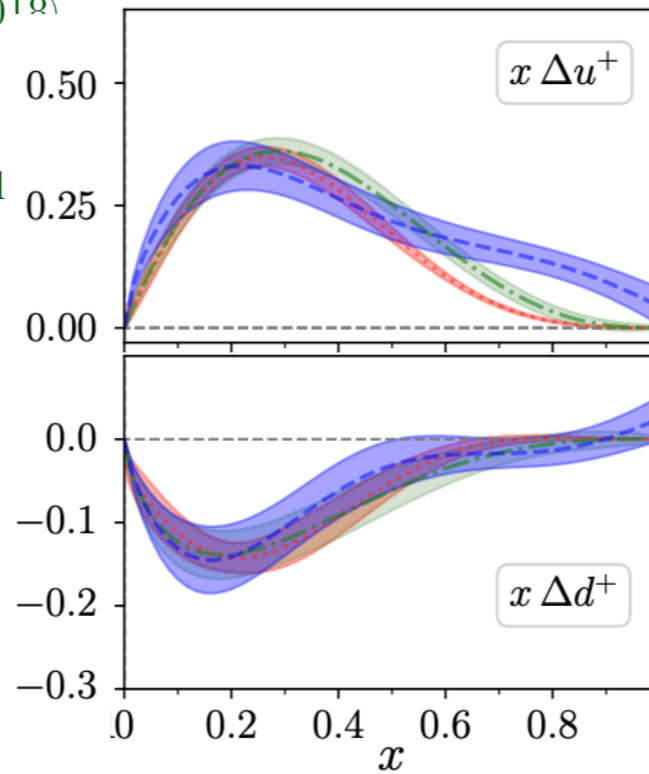
• ETMC: $N_f=2$, $m_\pi=135$ MeV, $a=0.094$ fm
 C.A. *et al.* (ETMC) Phys. Rev. Lett. **121** (2018) 112001



J. Bringewatt *et al.* (JAM) Phys.Rev.D **103** (2021) 016003

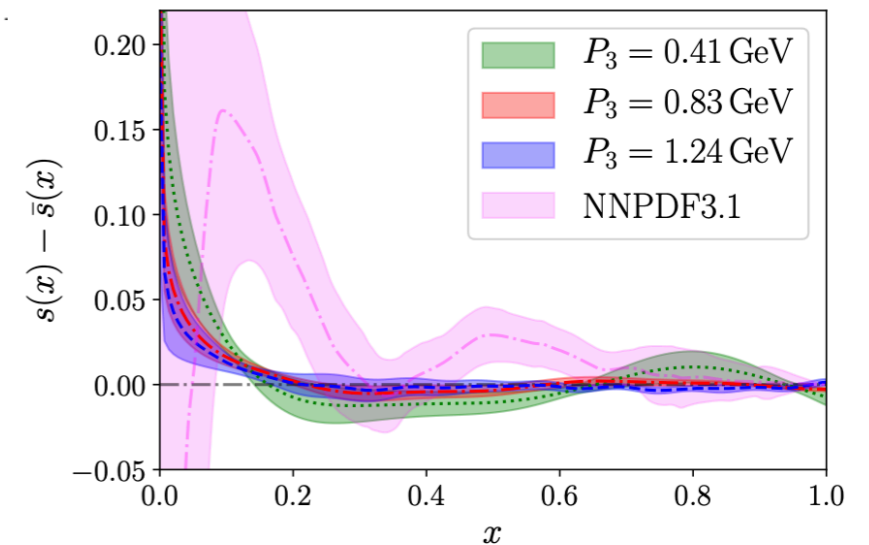
✱ Disconnected at heavier than physical pion mass

• ETMC: $N_f=2+1+1$, $m_\pi=260$ MeV, $a=0.094$ fm



C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. **126** (2021) 102003

C.A. *et al.*, Phys.Rev.D **104** (2021) 5, 054503



• Unpolarized s- and c-quark PDFs with mixed action (clover on staggered sea) $m_\pi=310$ MeV and $m_\pi=690$ MeV, $a=0.12$ fm

R. Zhang, H.W. Lin, B. Yoon (2020), 2005.011

• Ongoing effort by the HadStruc Collaboration

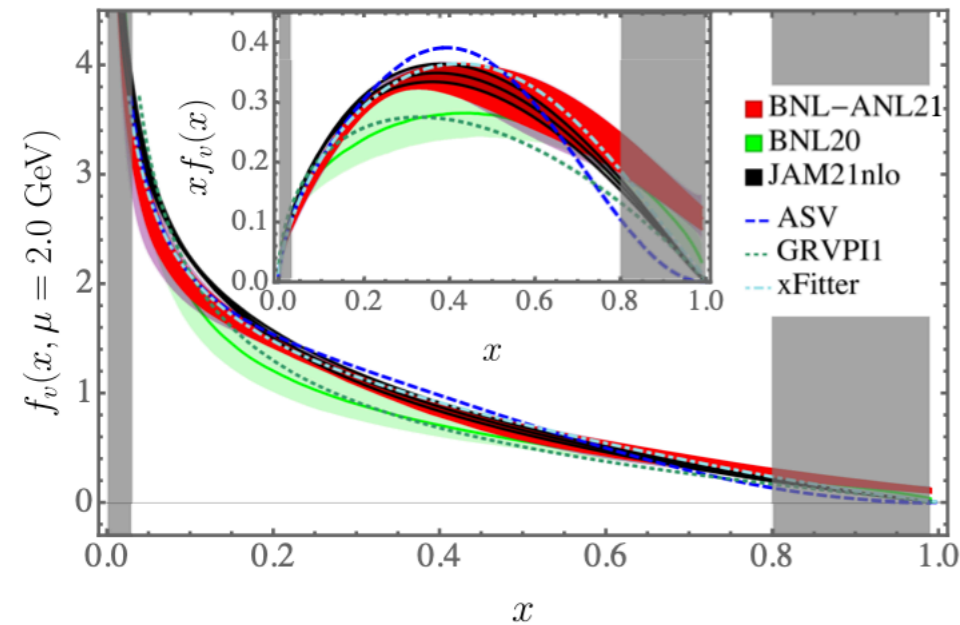
Pion and isovector nucleon unpolarized PDFs

✿ Improvements in renormalization and matching by e.g. LPC, BNL, ETMC, MSULAT, ...

- PDFs using NNLO matching and the hybrid renormalization scheme by BNL

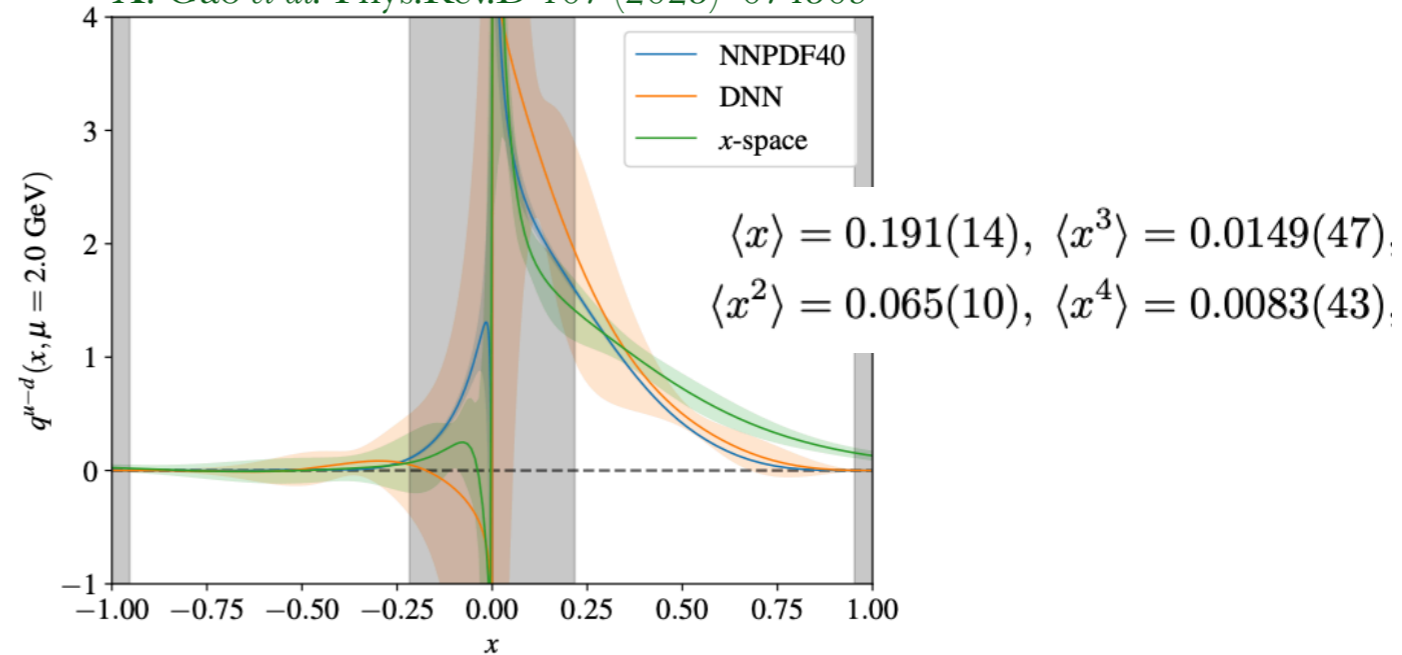
Pion

X. Gao *et al.* Phys. Rev. Lett. 128 (2022) 142003



Nucleon isovector

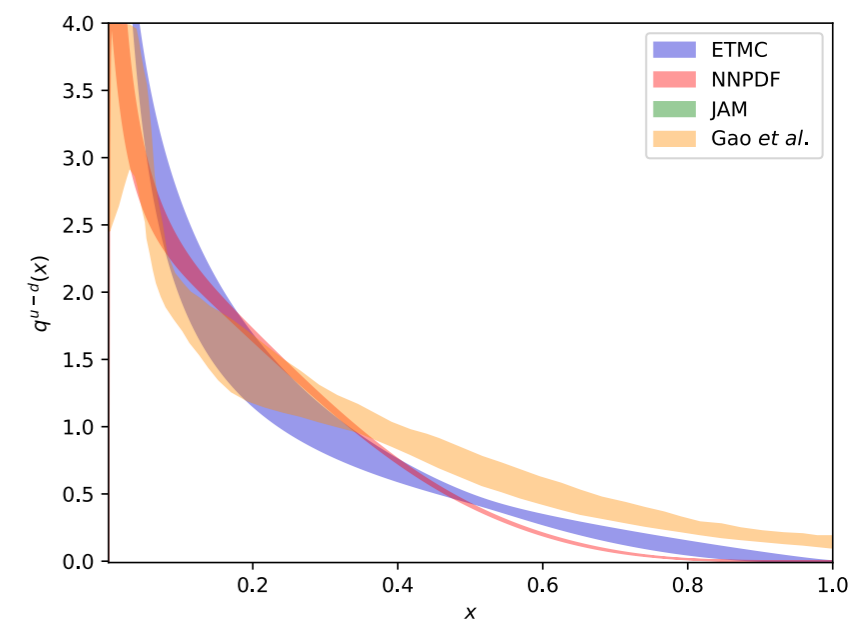
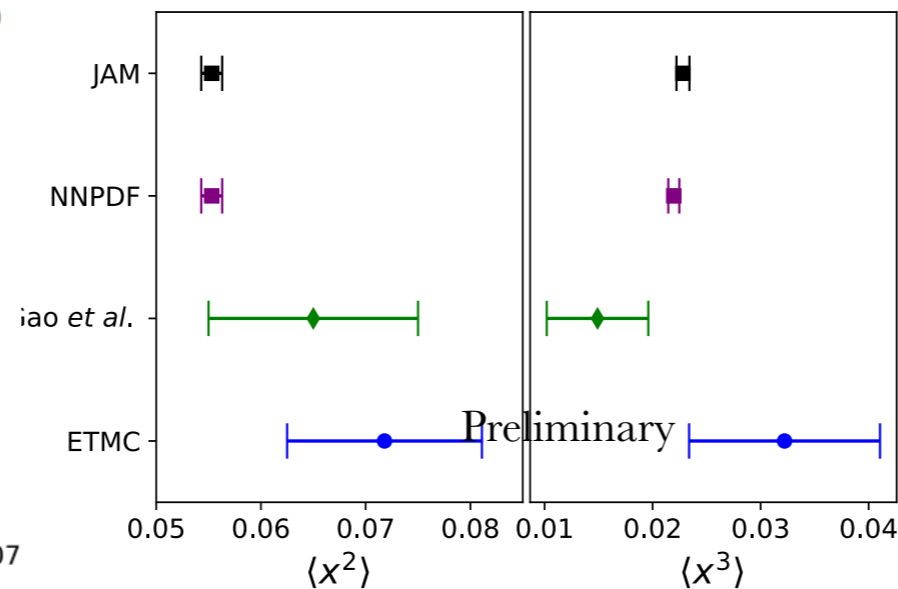
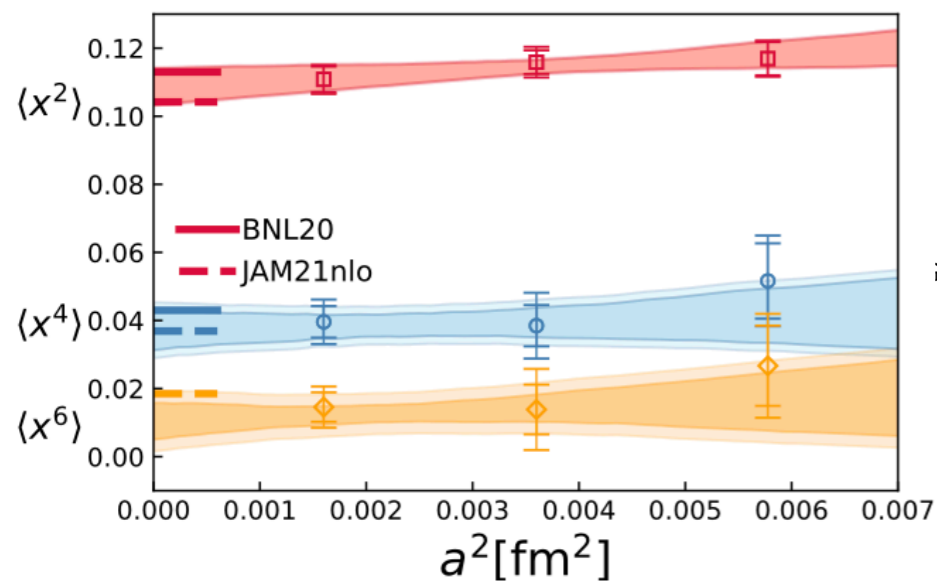
X. Gao *et al.* Phys.Rev.D 107 (2023) 074509



Quasi-approach with NNLO matching using hybrid action:
clover on $N_f=2+1$ staggered $m_\pi=300$ MeV, $a=0.04$ fm

$m_\pi=140$ MeV, 64^4 , $a=0.076$ fm in same setup as pion

X. Gao *et al.* Phys.Rev.D 106 (2022) 11, 114510



Pseudo-approach to extract moments with $m_\pi=140$ MeV,
 $a=0.076$ fm and $m_\pi=300$ MeV, $a=0.06, 0.04$ fm

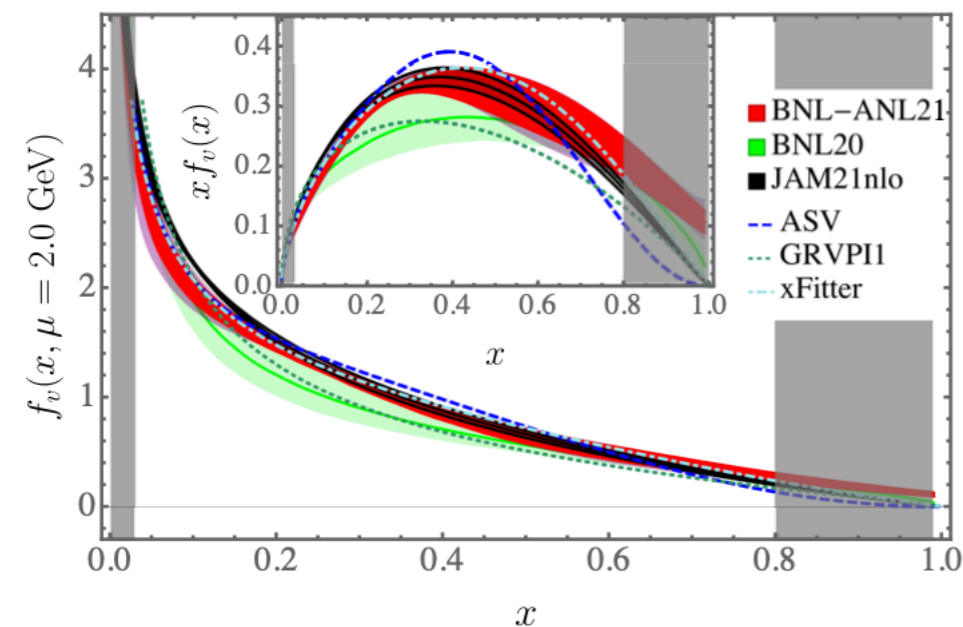
Pion and isovector nucleon unpolarized PDFs

✿ Improvements in renormalization and matching in LaMET approach by e.g. LPC, BNL, ETMC, MSULAT, ...

- PDFs using NNLO matching and the hybrid renormalization scheme mixed action by BNL

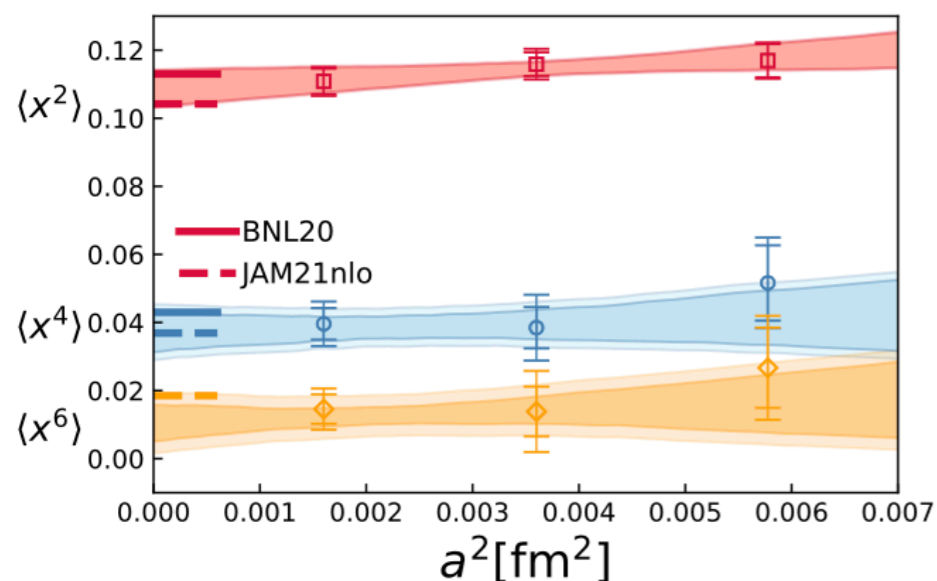
Pion

X. Gao *et al.* Phys. Rev. Lett. 128 (2022) 142003



Quasi-approach with NNLO matching using hybrid action: clover on $N_f=2+1$ staggered $m_\pi=300$ MeV, $a=0.04$ fm

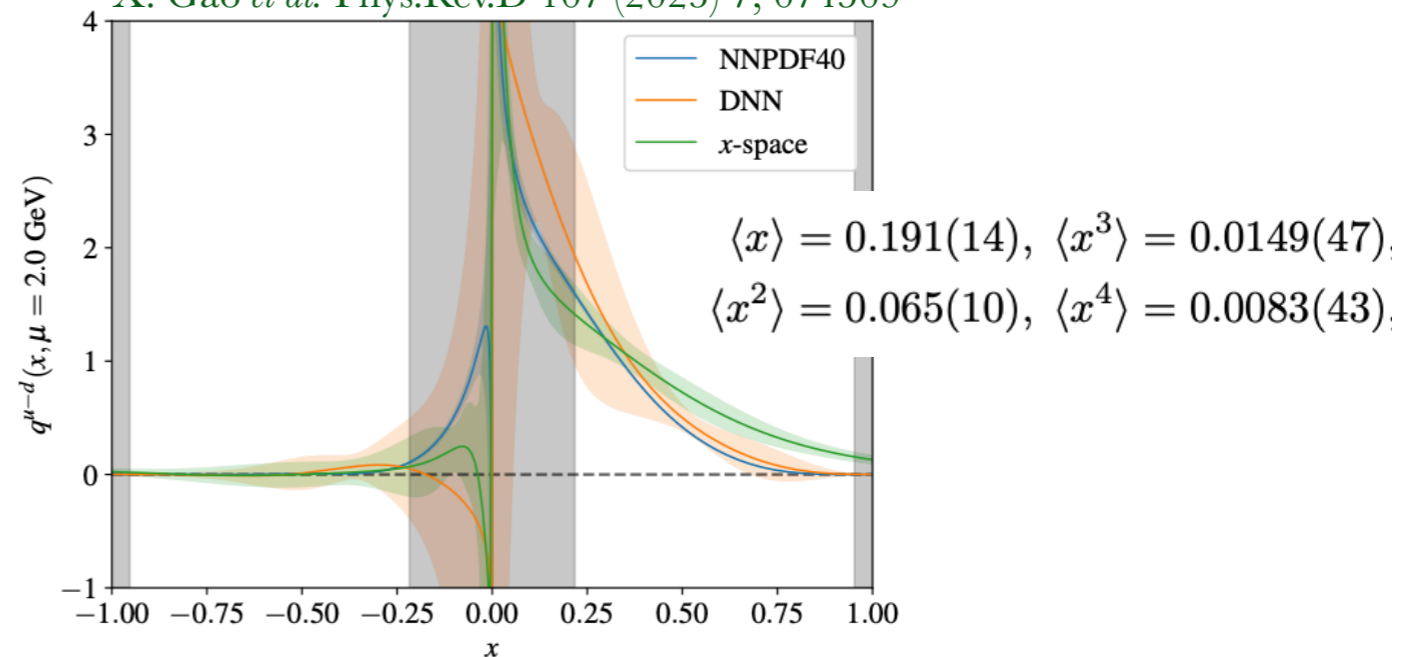
X. Gao *et al.* Phys. Rev. D 106 (2022) 11, 114510



Pseudo-approach to extract moments with $m_\pi=140$ MeV, $a=0.076$ fm and $m_\pi=300$ MeV, $a=0.06, 0.04$ fm

Nucleon isovector

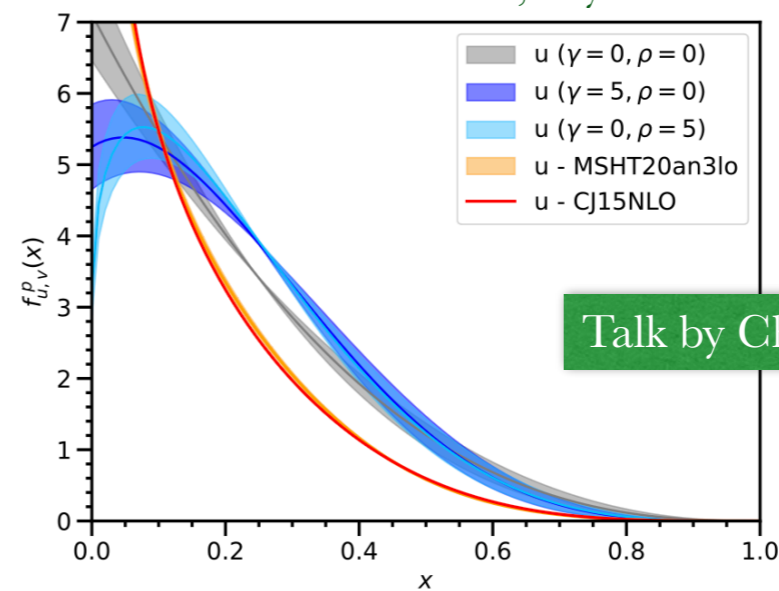
X. Gao *et al.* Phys. Rev. D 107 (2023) 7, 074509



$m_\pi=140$ MeV, 64^4 , $a=0.076$ fm in same setup as pion

- Auxiliary heavy quark approach

C. Zimmerman and A. Schäfer, Phys. Rev. D 110 (2024) 074503



Talk by Christian Zimmerman, Tuesday

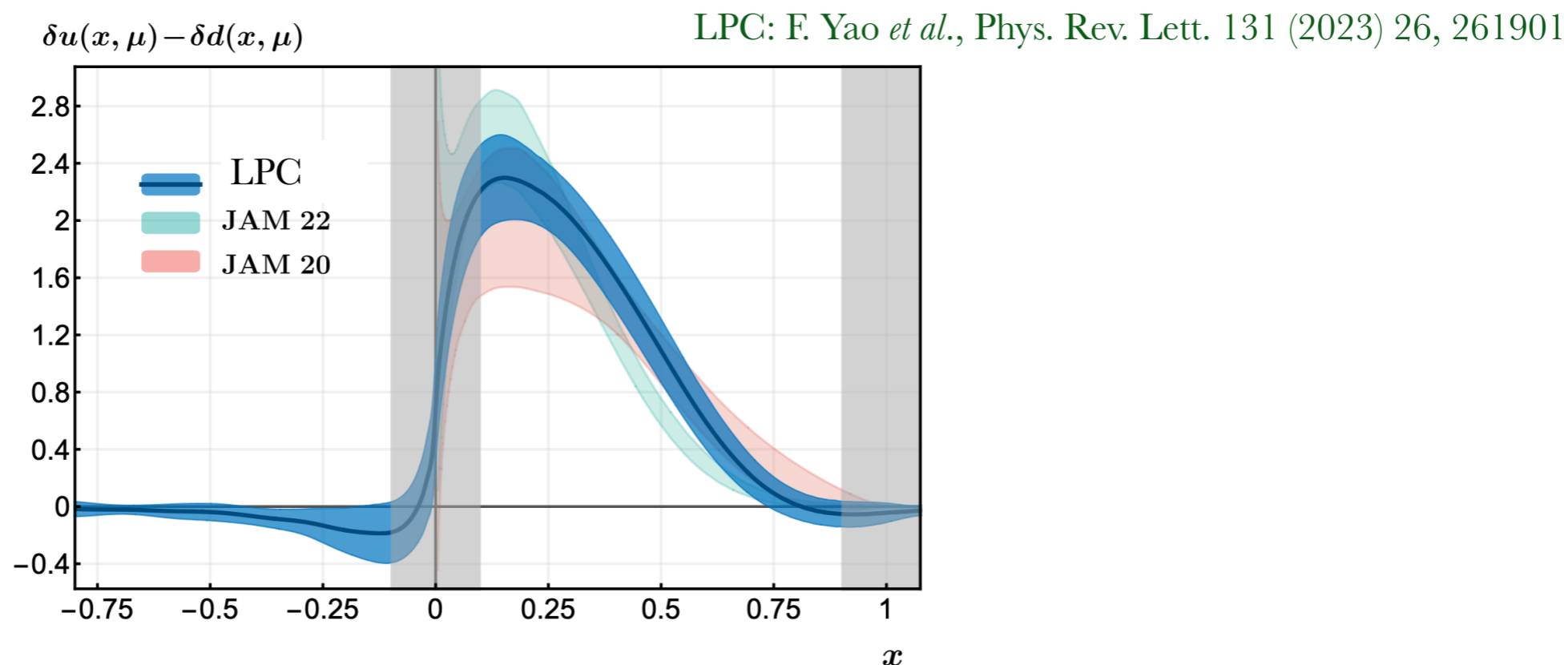
CLS ensemble with $m_\pi=335$ MeV, 64^4 , $a=0.086$ fm

Transversity PDF

✳ Lattice Parton Collaboration (LPC) analyzed CLS ensembles with 4 lattice spacings (0.098-0.049 fm), pion masses ranging from 220 to 350 MeV and momentum boosts from 1.6 GeV to 2.8 GeV

- Continuum and chiral extrapolations are performed as well as the large momentum limit
- Renormalization done using a hybrid scheme separating the short and long distances

$$\tilde{h}_R(z, P^z) = \frac{1}{\tilde{h}(0, P^z, 1/a)} \left[\frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z, 0, 1/a)} \theta(z_s - |z|) + \frac{Z_R(z_s, 1/a)}{Z_R(z, 1/a)} \frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z_s, 0, 1/a)} \theta(|z| - z_s) \right]$$



✳ BNL also computed the transversity isovector and isoscalar PDFs using clover on staggered and one ensemble at physical pion mass for both the quasi- and pseudo-approaches

X. Gao *et al.*, Phys. Rev. D 109 (2025) 054506

✳ HadStruc Collaboration used "window observables" to compare lattice and phenomenology for isovector transversity

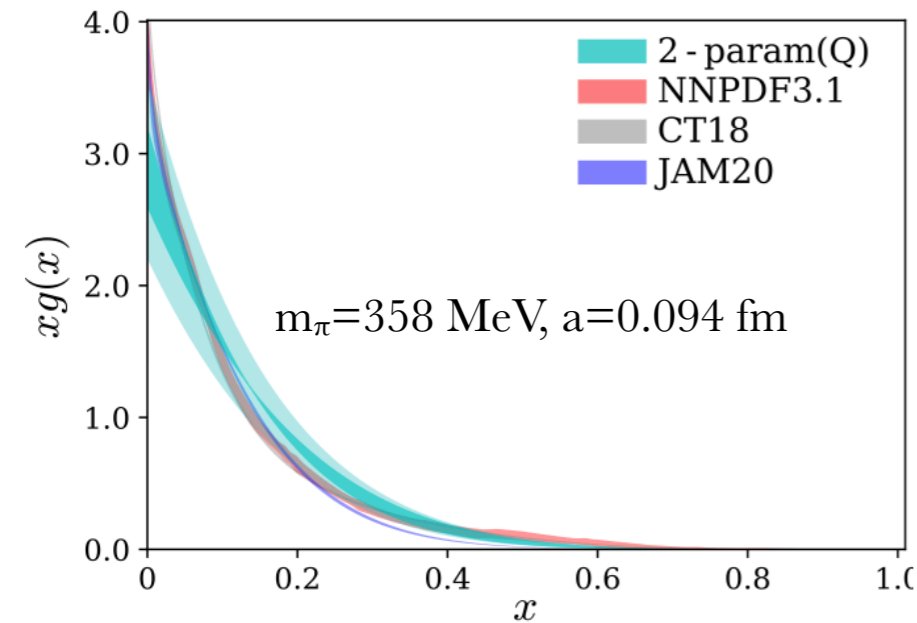
$$a_n(x_- - x_+) = \int_{x_-}^{x_+} dx x^n f_{j/H}(x)$$

J. Karpie *et al.* arXiv: 2505.22795

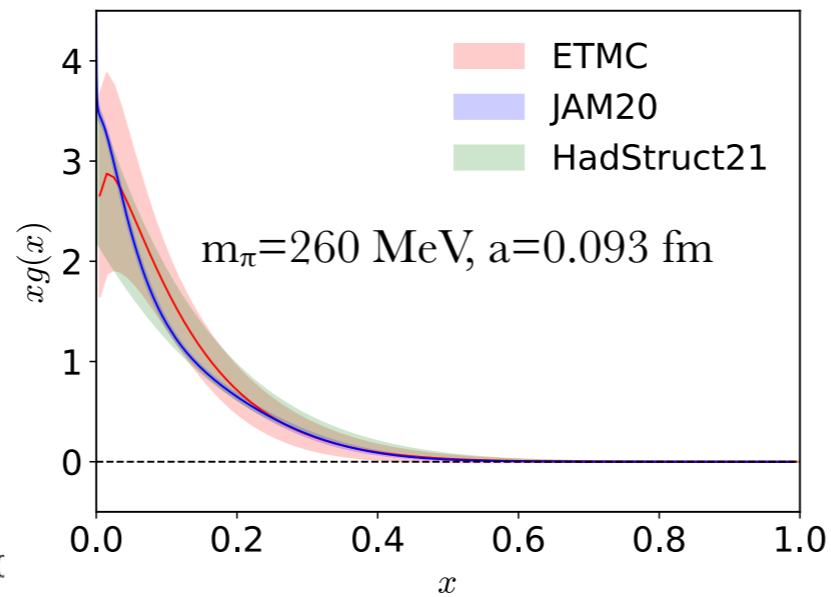
Gluon PDFs

✿ Most studies in the pseudo-pdf approach

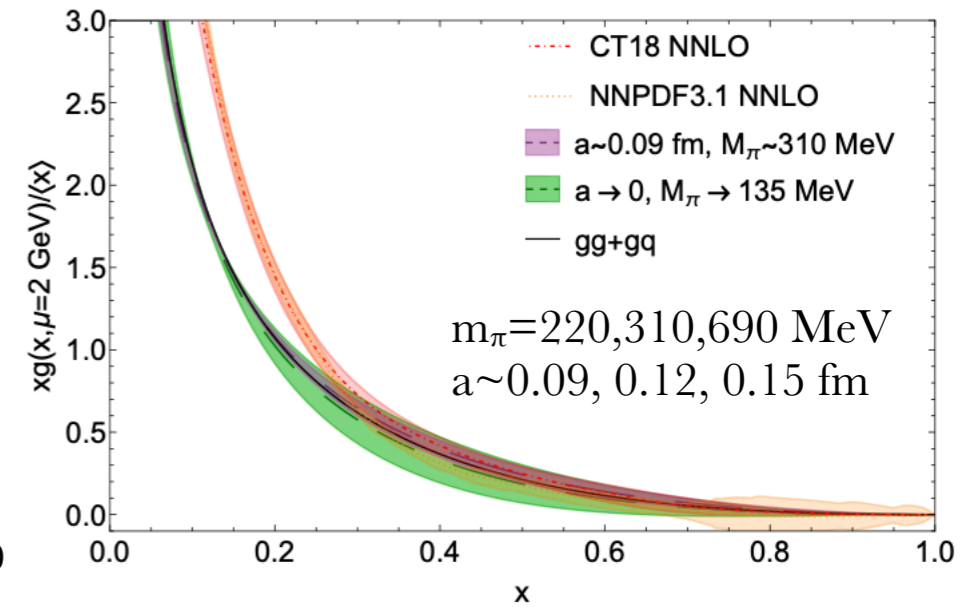
- Nucleon unpolarized PDF



HadStruc Collaboration: T. Khan *et al.*, Phys. Rev. D 104, (2021) 094516

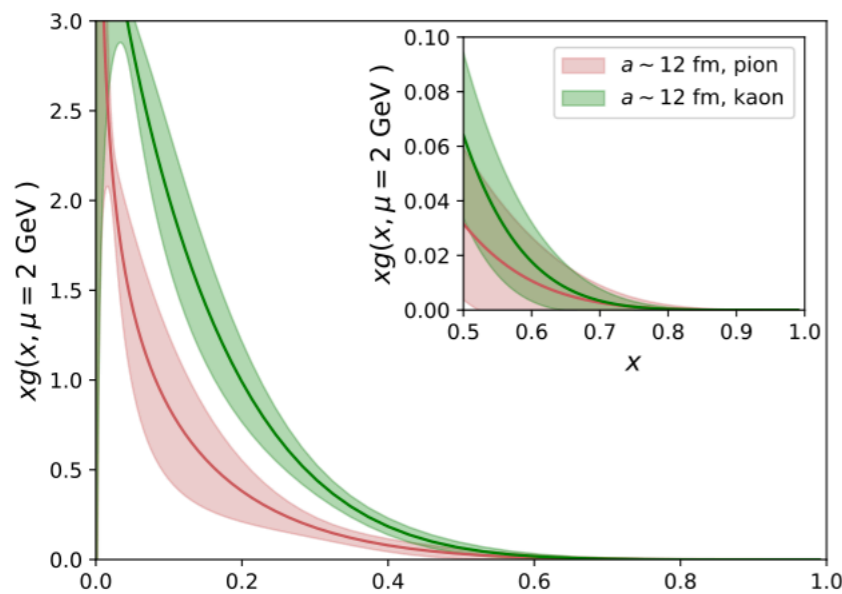


ETMC: J. Delmar *et al.*, Phys. Rev. D 108 (2023) 094515

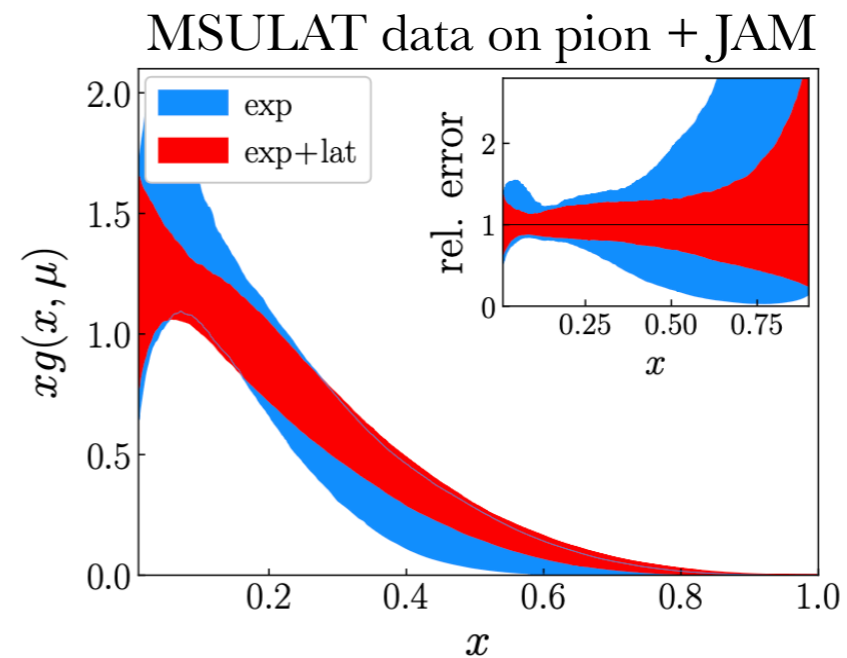


MSULAT: Z. Fan, W. Good, H.-W. Lin, Phys. Rev. D 108 (2023) 014508

- Pion and kaon gluon PDFs by MSULAT



A. NieMiera, W. Good, H.-W. Lin, Phys.Rev.D 112 (2025) 074504



W. Good *et al.* arXiv: 2507.22730

- Nucleon gluon helicity

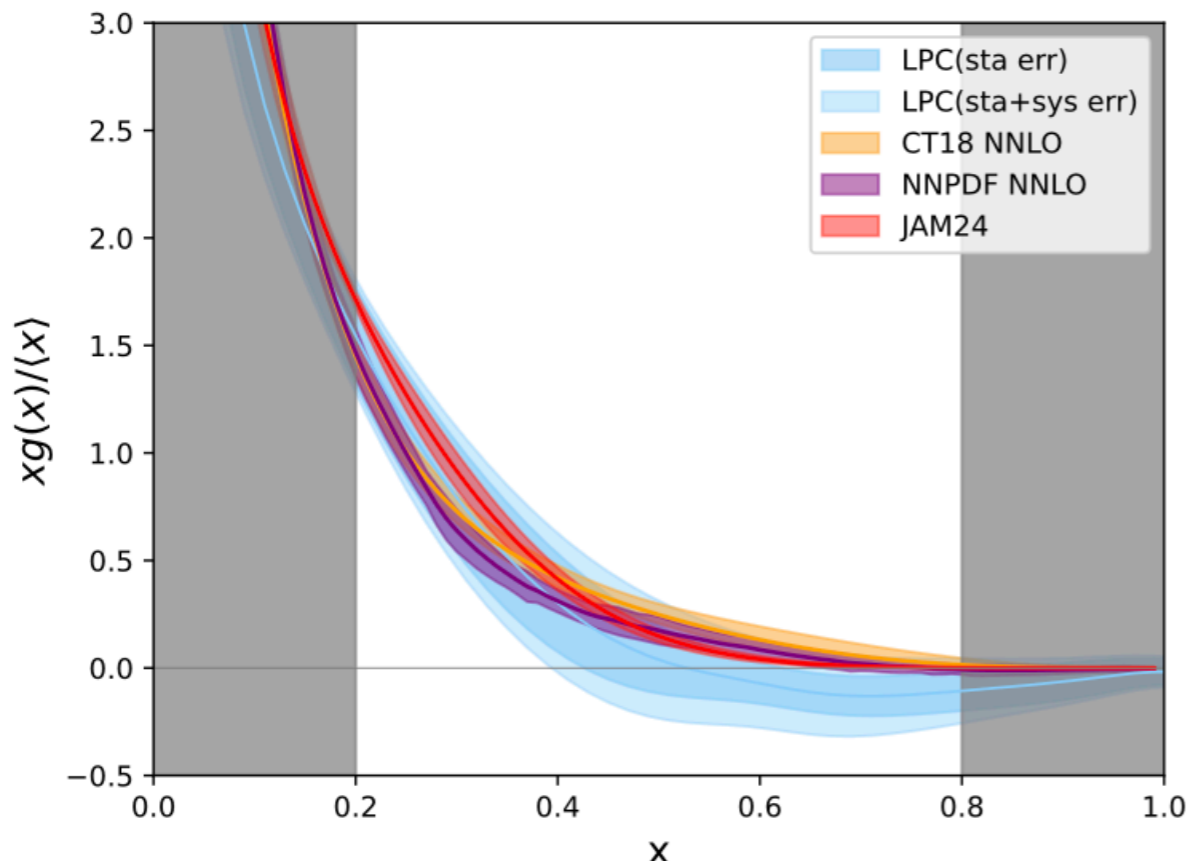
HadStruc Collaboration: C. Egerer *et al.*, Phys.Rev.D 106 (2022) 094511
T. Khan, T. Liu, R. S. Sufian, Phys. Rev. D 108, (2023) 074502

Unpolarized gluon PDF

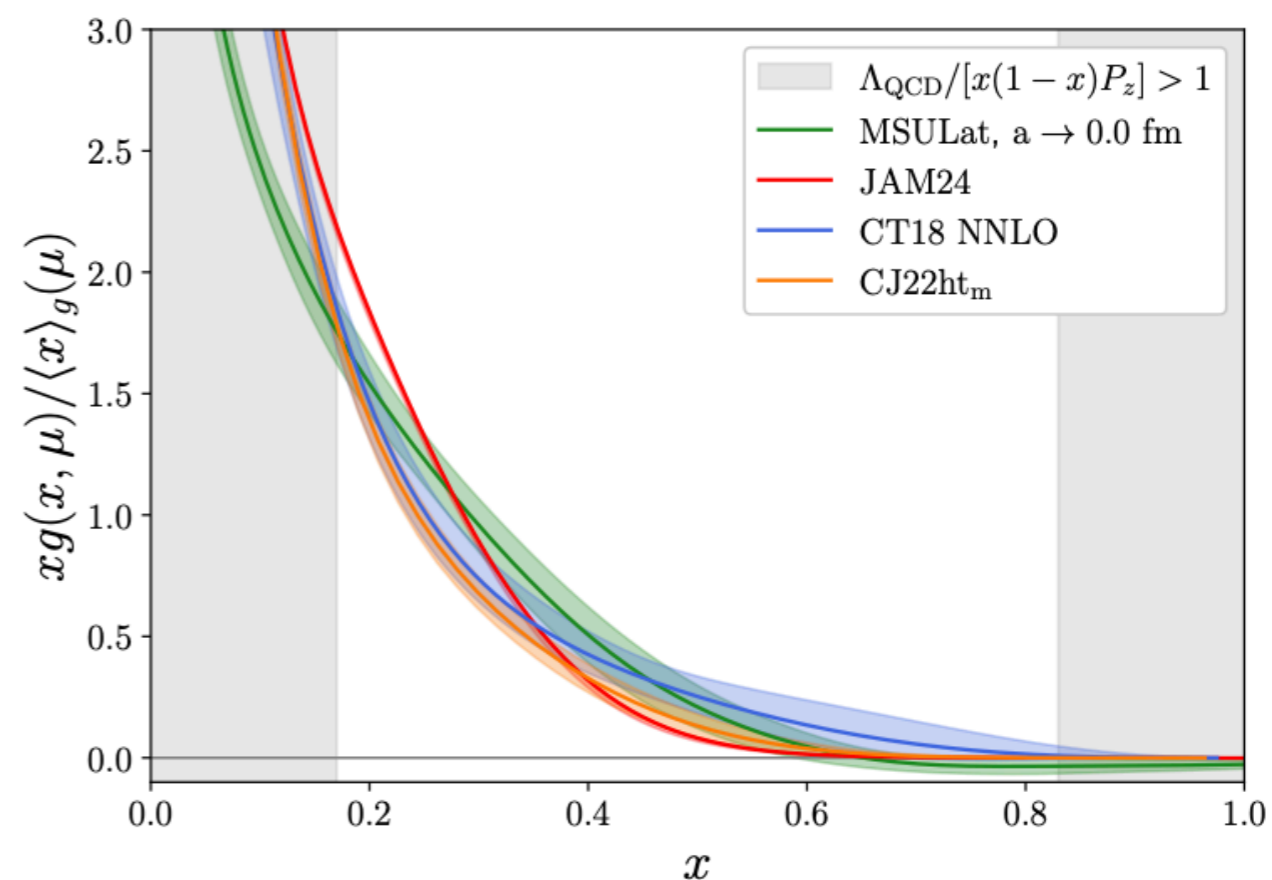
- ✳️ LPC analyzed 2+1 clover ensembles at 3 lattice spacings (0.105, 0.0897, 0.0775 fm) and $m_\pi \sim 300$ MeV with HYP smeared gluons
- ✳️ Continuum and large momentum limit are performed
- ✳️ Renormalization done using a hybrid scheme separating the short and long distances

$$\tilde{h}_R(z, P^z) = \frac{1}{\tilde{h}(0, P^z, 1/a)} \left[\frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z, 0, 1/a)} \theta(z_s - |z|) + \frac{Z_R(z_s, 1/a)}{Z_R(z, 1/a)} \frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z_s, 0, 1/a)} \theta(|z| - z_s) \right]$$

LPC: Ch. Chen *et al.*, arXiv:2510.26425



MSULAT: A. NieMiera, W. Good, H.-W. Lin, F. Yao, arXiv:2510.17758



- ✳️ MSULAT analyzed 2+1+1 staggered ensembles with clover valence at 3 lattice spacings (0.15, 0.12, 0.09 fm) and $m_\pi \sim 310$ MeV performing the continuum limit. Used Wilson flow for the gluon
- ✳️ Renormalization done using a hybrid scheme separating the short and long distances

Twist-3 PDFs

✱ PDFs are classified in terms of twists:

$$f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} + \dots$$

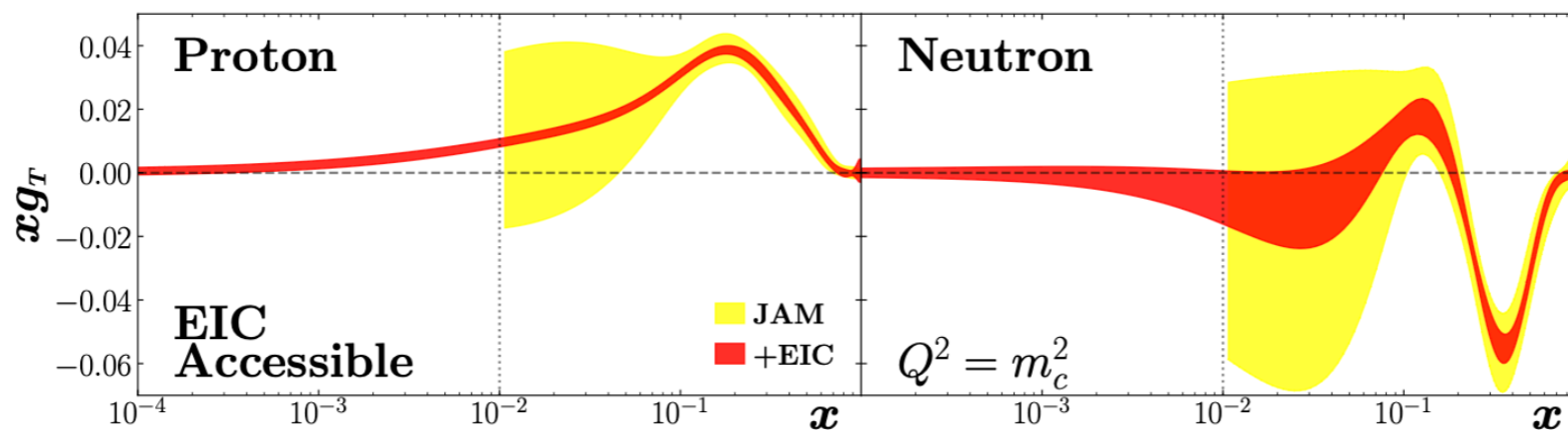
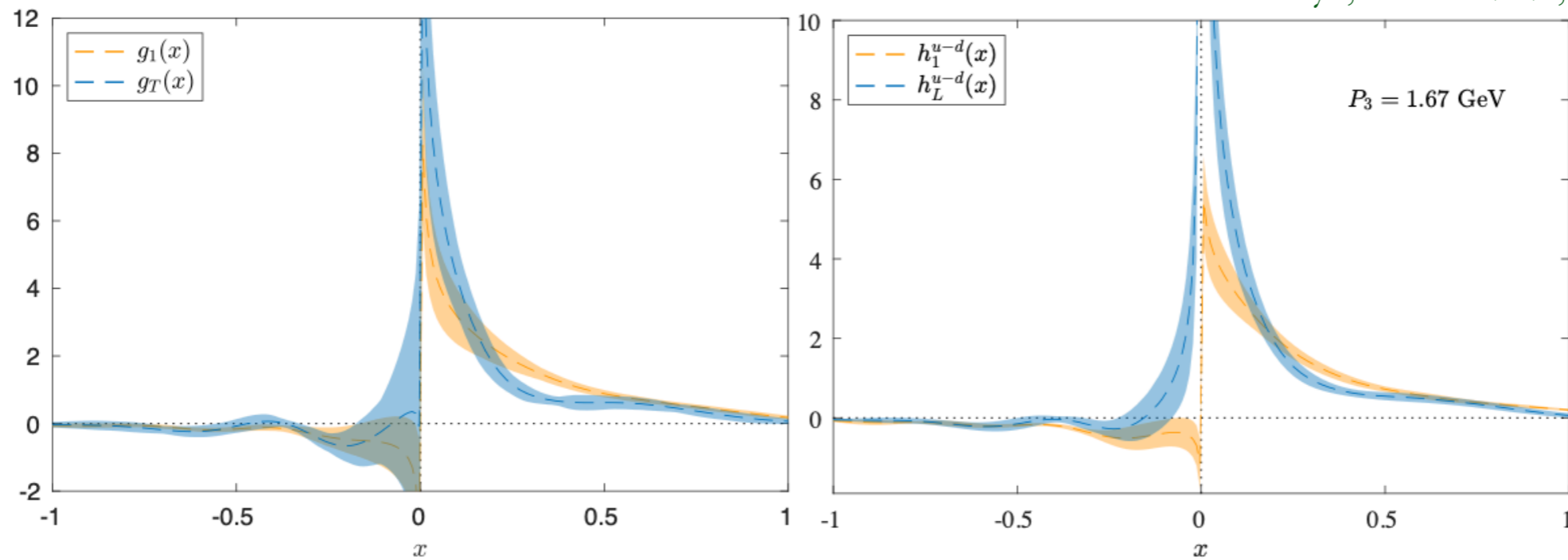
\downarrow twist-2 \downarrow twist-3

✱ Three types: $e(x)$, $g_2(x)$ and $h_2(x)$. No density interpretation but e.g. the x^2 - moments of $e(x)$ and $g_2(x)$ can yield information on the transverse force and encode information on quark-gluon correlations

✱ $g_2(x)$ ($g_T(x)$) and $h_2(x)$ ($h_L(x)$) computed by ETMC using the quasi-distributions approach at a pion mass of 260 MeV

✱ $g_T(x)$ extracted from $\langle P | \bar{\psi}(0, z) \gamma_j \gamma_5 W(z) \psi(0, 0) | P \rangle$ with $j, k \perp P$ S. Bhattacharya, *et al.*, PRD 102, 111501 (2020)

✱ $h_L(x)$ extracted from $\langle P | \bar{\psi}(0, \vec{0}) \sigma_{kj} W(0, \vec{z}) \psi(0, \vec{z}) | P \rangle$ with $j, k \perp P$ S. Bhattacharya, *et al.*, PRD 104 (2021) 114510
 S. Bhattacharya, Lattice 2024, arXiv: 2502.00481



Generalised parton distributions

✱ Quasi-approach: Compute space-like matrix element with different initial and final nucleon boosts e.g. in the Breit frame

$$h_{\Gamma}(z, \tilde{\xi}, Q^2, P_3) = \langle N(P_3 \hat{e}_z + \vec{Q}/2) | \bar{\psi}(z) \Gamma W(z, 0) \psi(0) | N(P_3 \hat{e}_z - \vec{Q}/2) \rangle$$

$$\tilde{\xi} = -\frac{Q_3}{2P_3} : \text{quasi-skewness} \quad \tilde{\xi} = \xi + \mathcal{O}\left(\frac{1}{P_3^2}\right)$$

Renormalise, take the Fourier transform and match and final nucleon boosts

$$\tilde{F}_{\Gamma}(z, \tilde{\xi}, Q^2, P_3, \mu^0, \mu_3^0) = \int_{-1}^1 \frac{dy}{y} C_{\Gamma} \left(\frac{x}{y}, \frac{\mu}{yP_3}, \frac{\mu_3^0}{yP_3}, \frac{(\mu^0)^2}{(\mu_3^0)^2} \right) F_{\Gamma}(y, Q^2, \xi, \mu) + \mathcal{O} \left(\frac{m^2}{P_3^2}, \frac{Q^2}{P_3^2}, \frac{\Lambda_{\text{QCD}}^2}{x^2 P_3^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_3^2} \right)$$

Reduces to the matching kernel for $\xi=0$
Does not depend on Q^2

RI-scale

$\overline{\text{MS}}$ - scale

X.Ji *et al.*, Phys.Rev. D92 (2015) 014039

X.Xiong, J-H. Zhang, Phys.Rev. D92 (2015) 054037

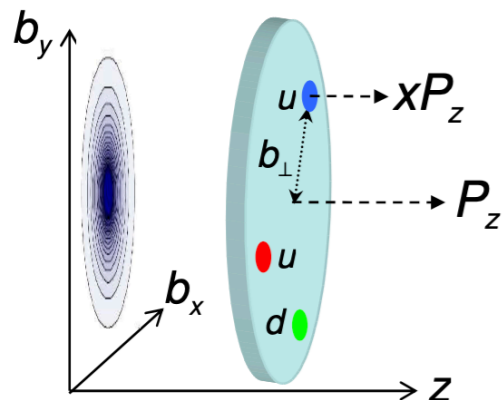
Y-S. Liu *et al.*, Phys.Rev. D100 (2019), 034006

✱ Pseudo-approach: Compute space-like matrix element with different initial and final nucleon boosts e.g. in the Breit frame

Use e.g. the double ratio $\mathcal{M}(\nu, z) = \frac{M(\nu, z) M(0, 0)}{M(0, z) M(\nu, 0)}$, involve, match and take FT in ν to extract the GPDs

A. Radyushkin, Int. J. Mod. Phys. A 35, 2030002 (2020)

✱ Geometrical interpretation for $\xi=0$

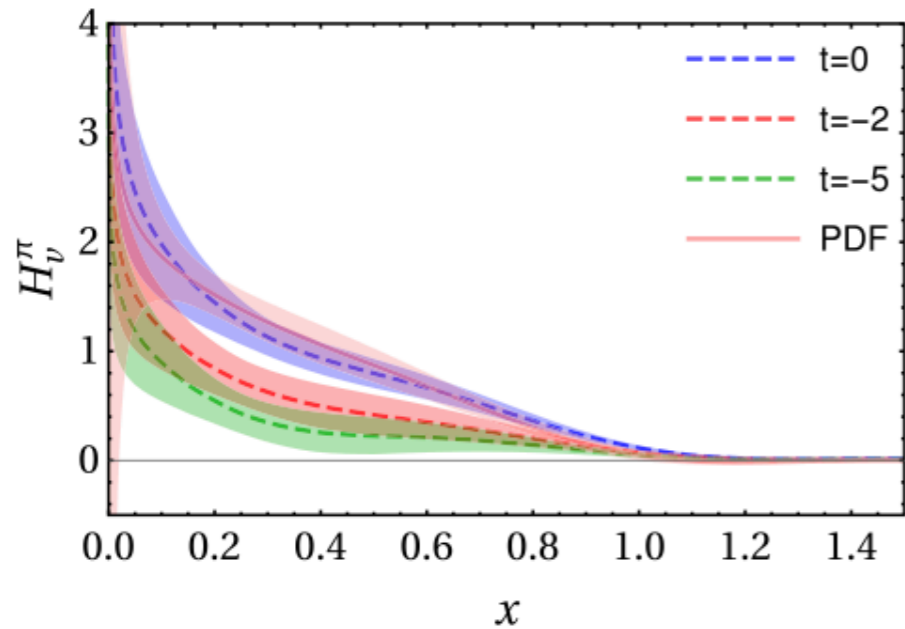


$$\mathcal{H}(x, b_{\perp}) = \int_{-\infty}^{\infty} \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\Delta_{\perp} \cdot b_{\perp}} H(x, \xi = 0, \tau)$$

M. Burkardt, Phys. Rev., D62:071503, 2000, hep-ph/0005108

Pion and nucleon GPDs

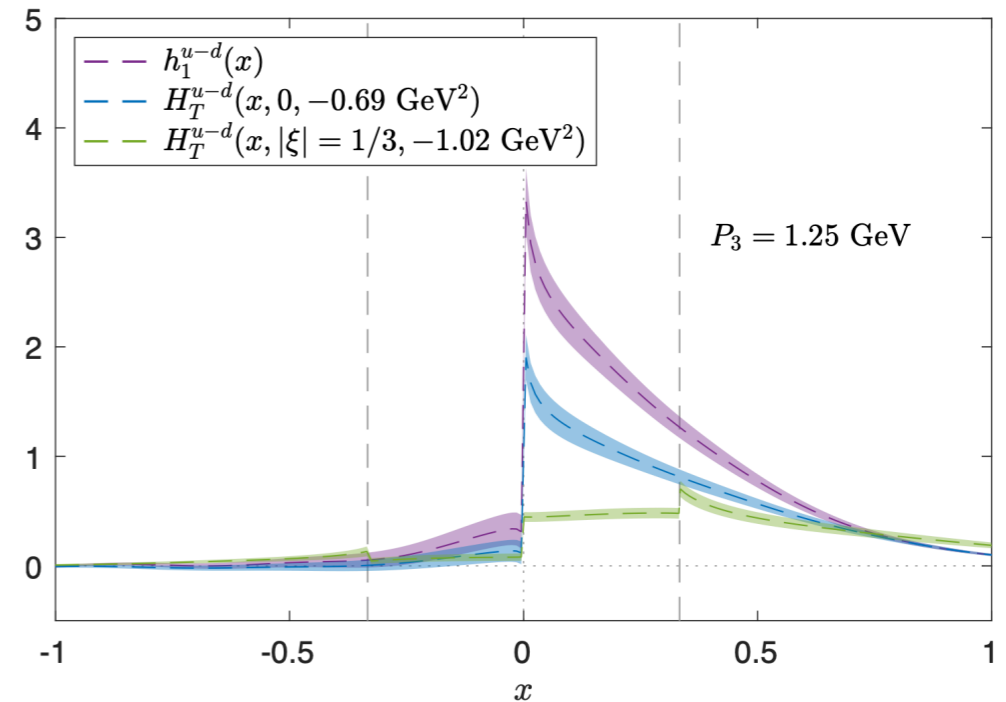
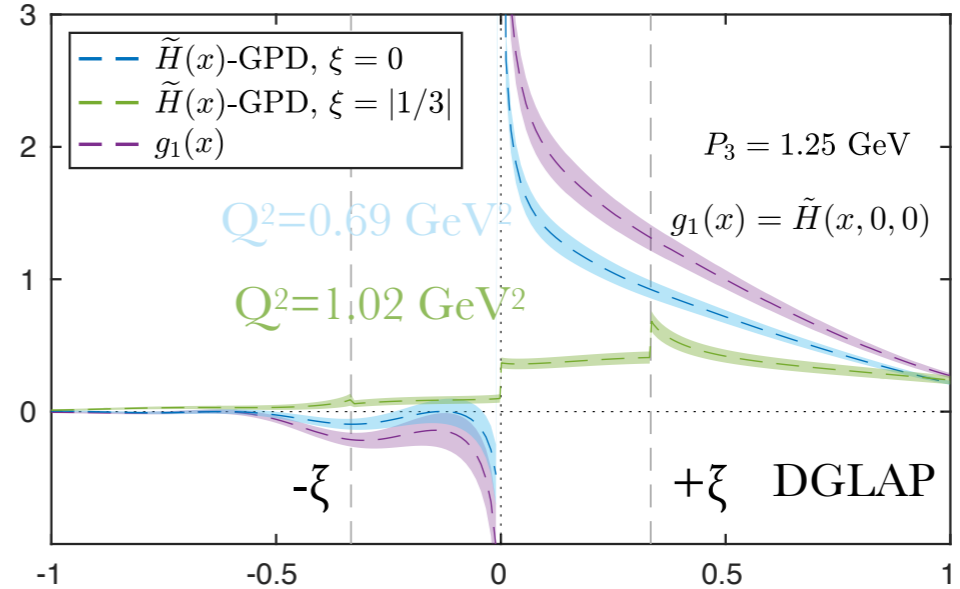
- Pion GPD at $\xi=0$ and $m_\pi=300$ MeV hybrid action



J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376

—> GPDs flatten as Q^2 increases

- Isovector nucleon GPDs, $m_\pi=260$ MeV

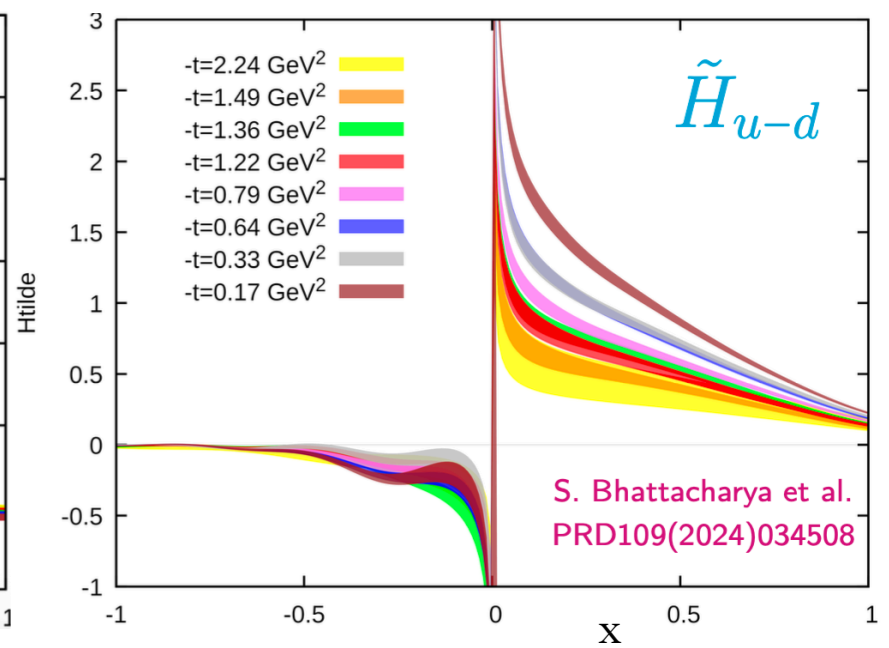
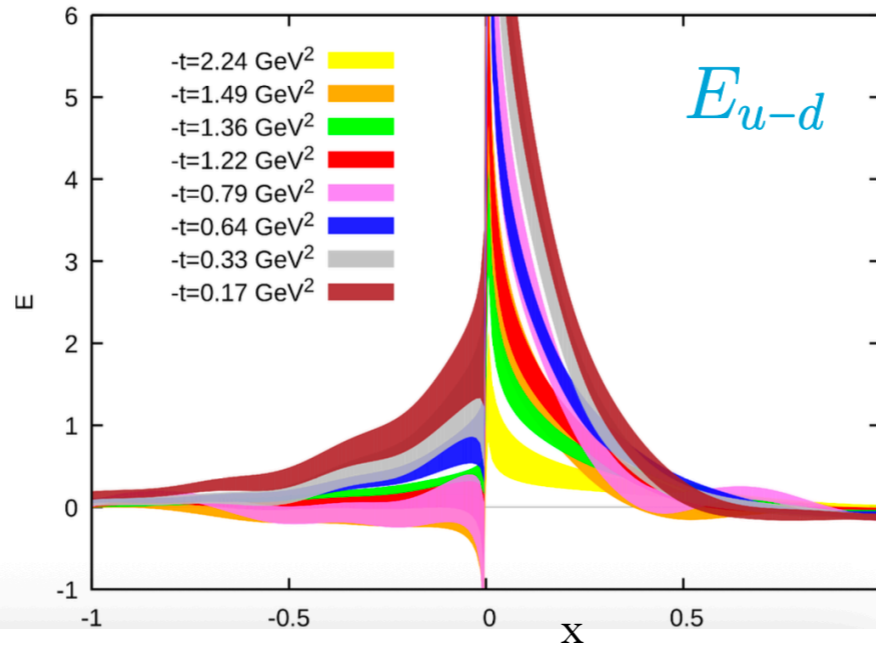
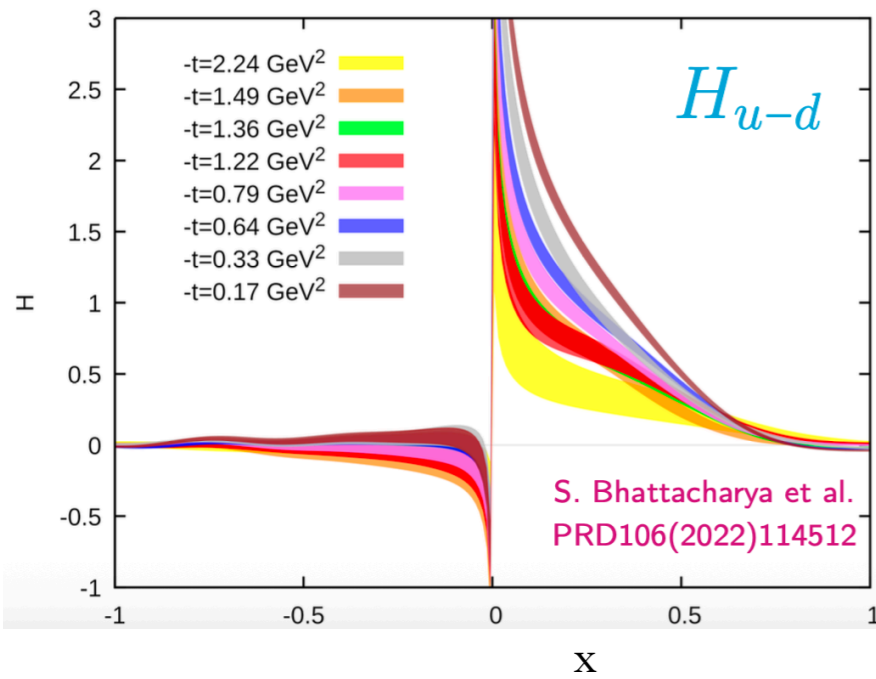


C. A. *et al.* (ETMC), Phys. Rev. Lett. 125 (2020) 262001, 2008.10573
 C.A. *et al.* (ETMC), Phys. Rev. D 105 (2022) 3, 034501, 2108.10789

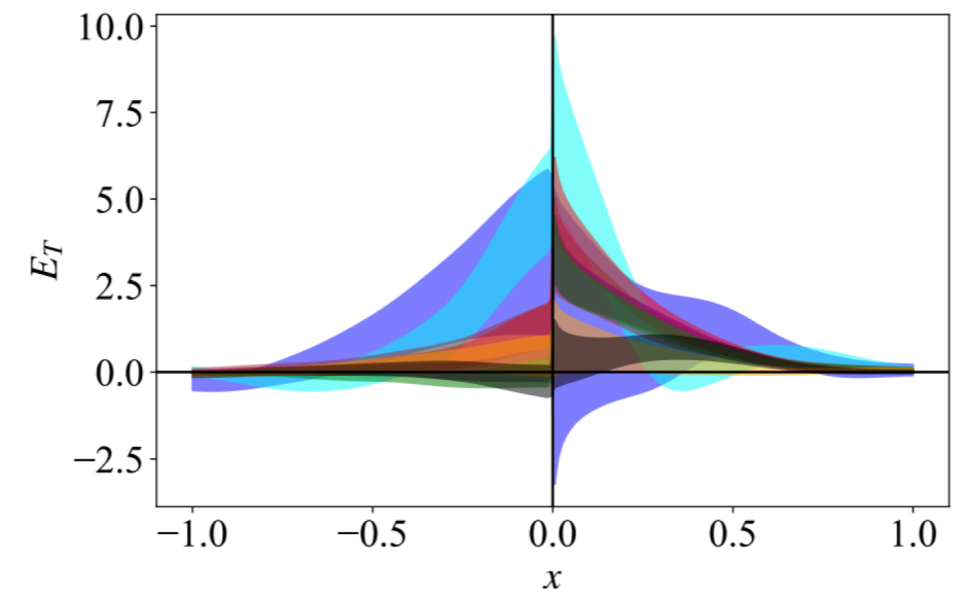
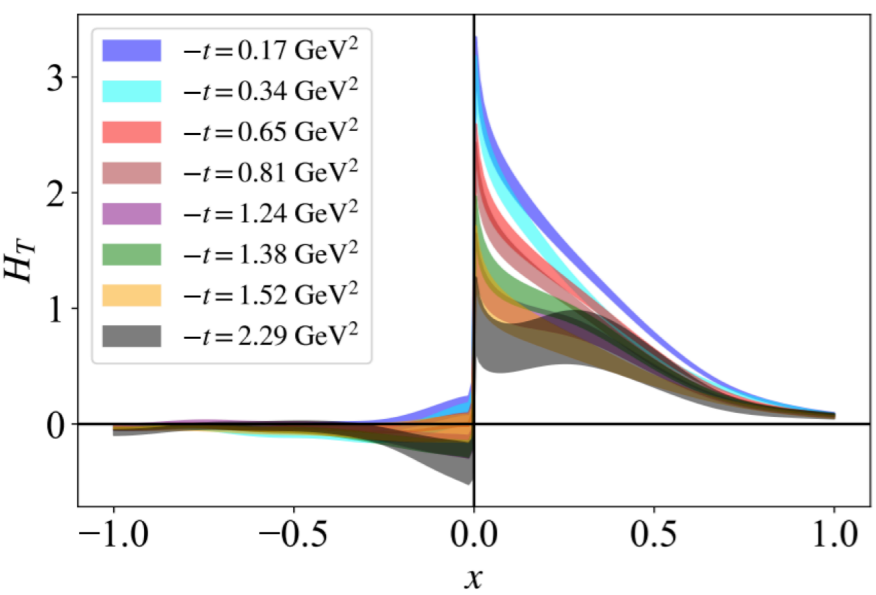
Asymmetric frame

✳ Express GPDs in terms of Lorentz invariant amplitudes that can be computed in any frame allowing easier access to a range of Q^2 similar to FFs

S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 114512; Phys. Rev. D 109 (2024) 034508; Phys. Rev. D 112 (2025) 11, 114504



K. Cichy, EINN 2025



J. Miller, EINN 2025

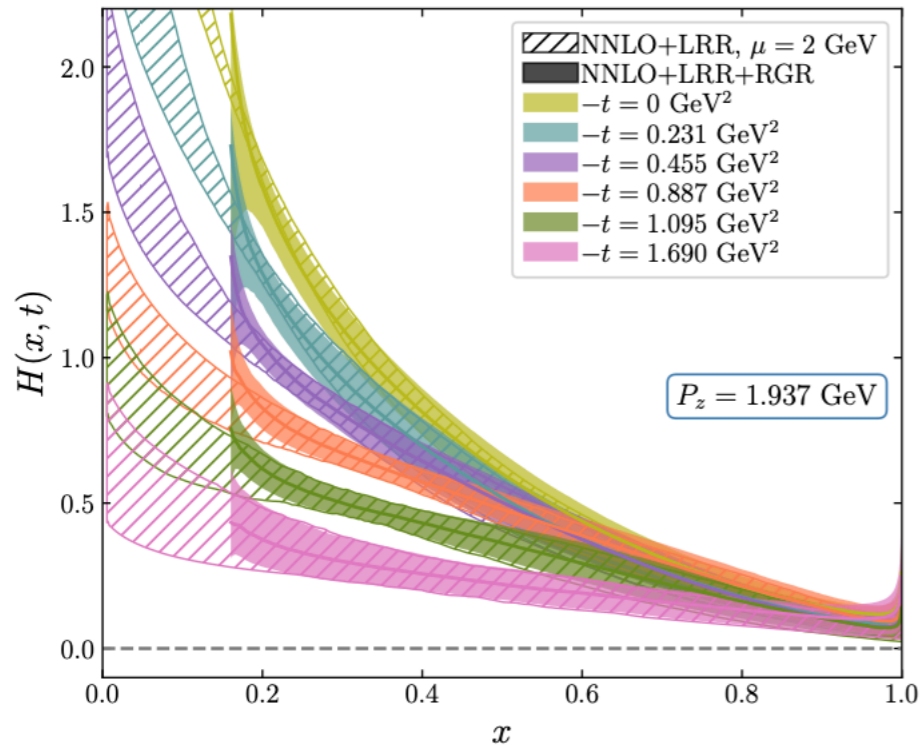
✳ Other advancements is the usage of neural networks

M. Chu, K. Cichy, M. Constantinou, P. Sznajder, J. Wagner, arXiv:2509.15931

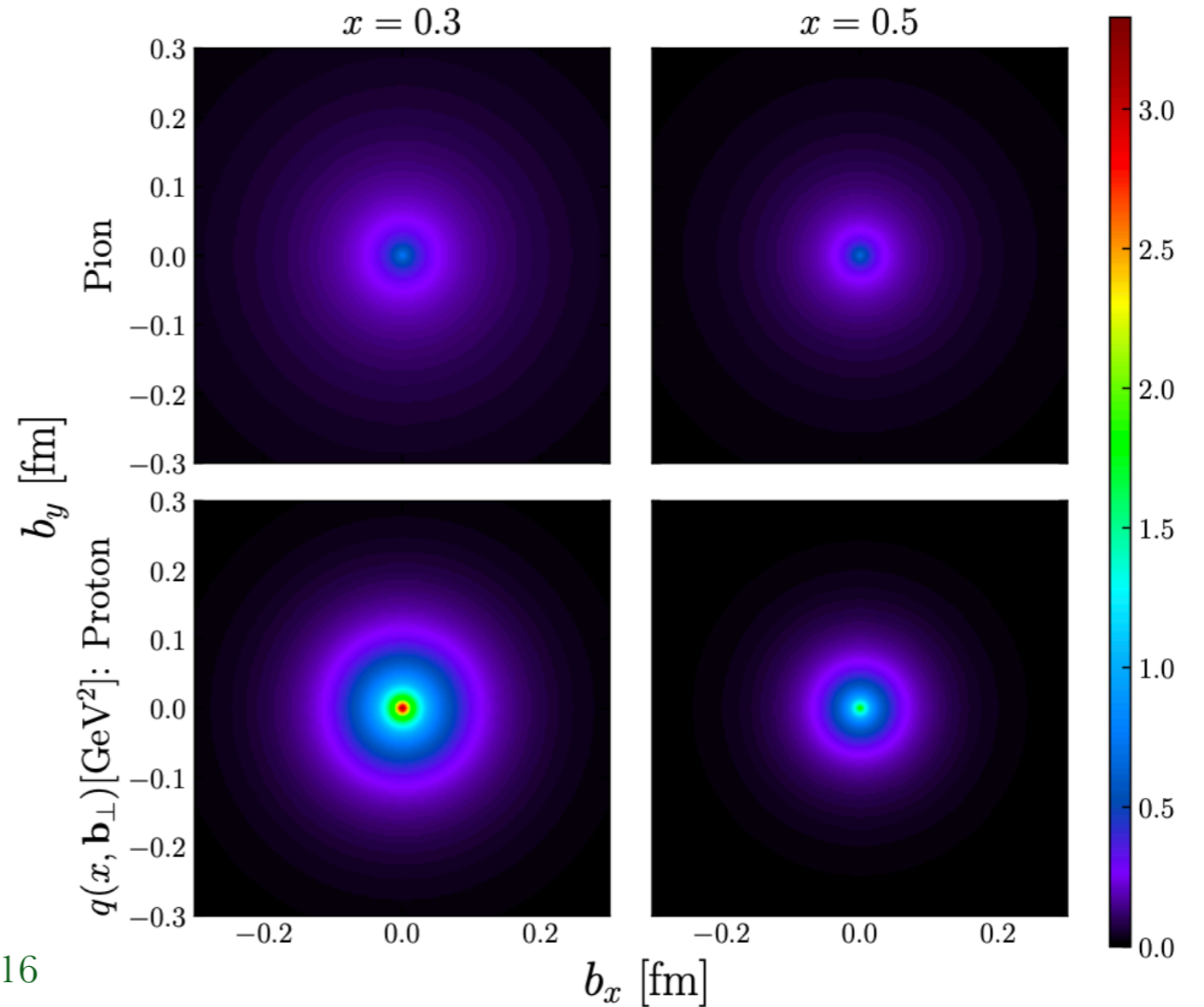
3-D imaging of pion and nucleon

Pion unpolarized GPD for $\xi=0$

Hybrid action, $m_\pi=300$ MeV, $a=0.04$ fm



H.-T. Ding *et al.*, JHEP 02 (2025) 056, arXiv:2407.03516



$$q(x, b_\perp) = \int_{-\infty}^{\infty} \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} H(x, \xi = 0, t)$$

Nucleon data are from ETMC: S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 11, 114512

Nucleon GFFs determined from GPDs

- ✳ Comparison of first moment of isovector unpolarized GDPs from pseudo ($m_\pi=358$ a=0.094 fm MeV) to direct computation of form factors
- ✳ Also computed second and third moments for non-zero skewness

HadStruc Collaboration: H. Dutrieux et al., JHEP 08 (2024) 061

- ✳ Comparison of first and second moments of isovector helicity GDP using quasi and SDF ($m_\pi=260$ MeV and a=0.093 fm)

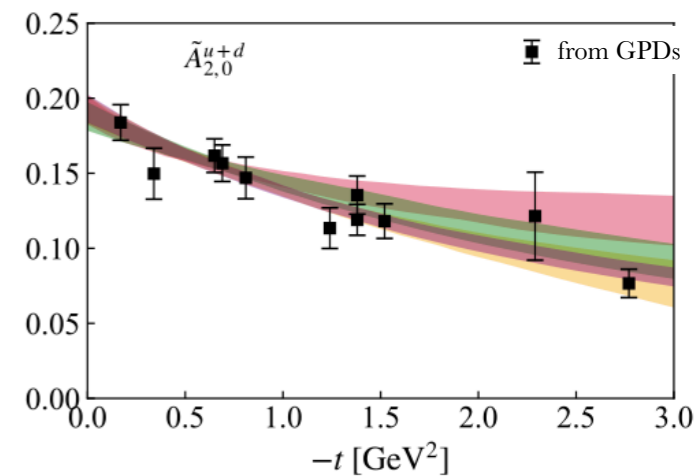
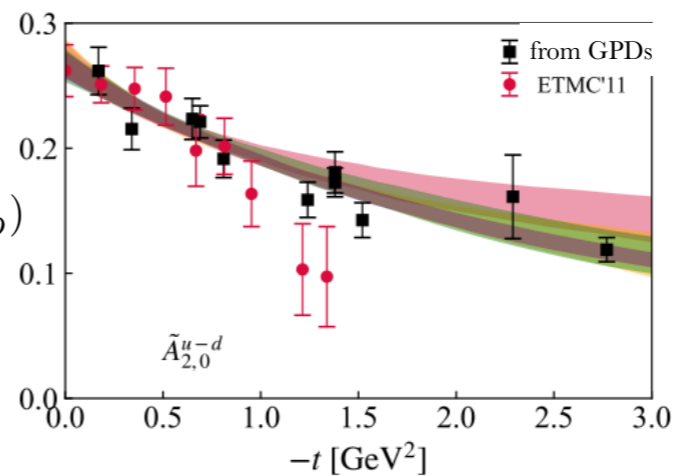
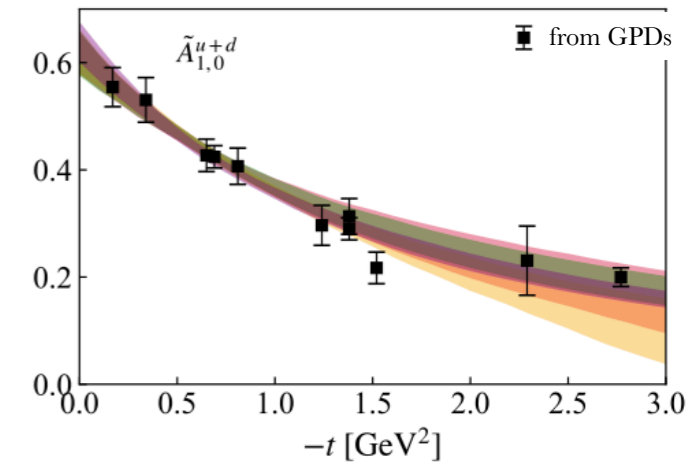
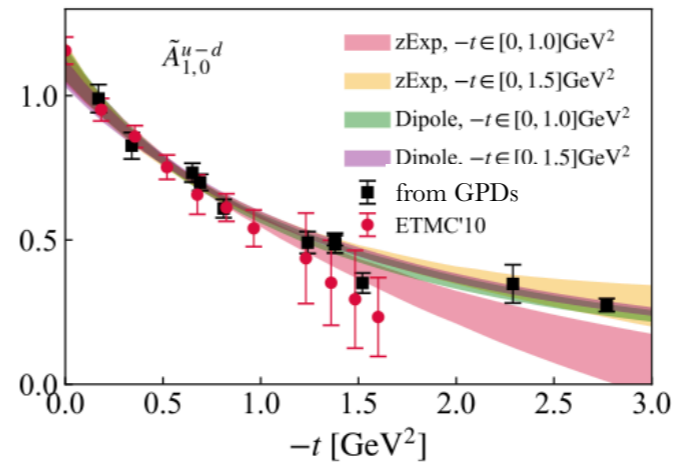
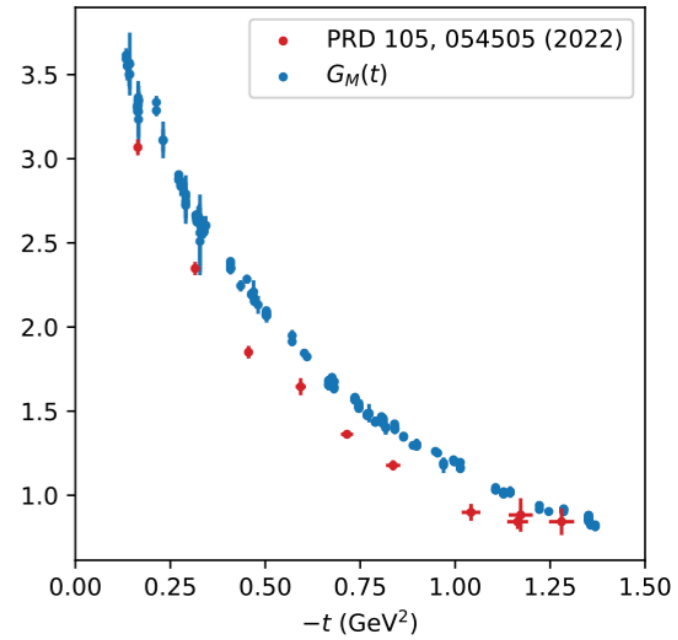
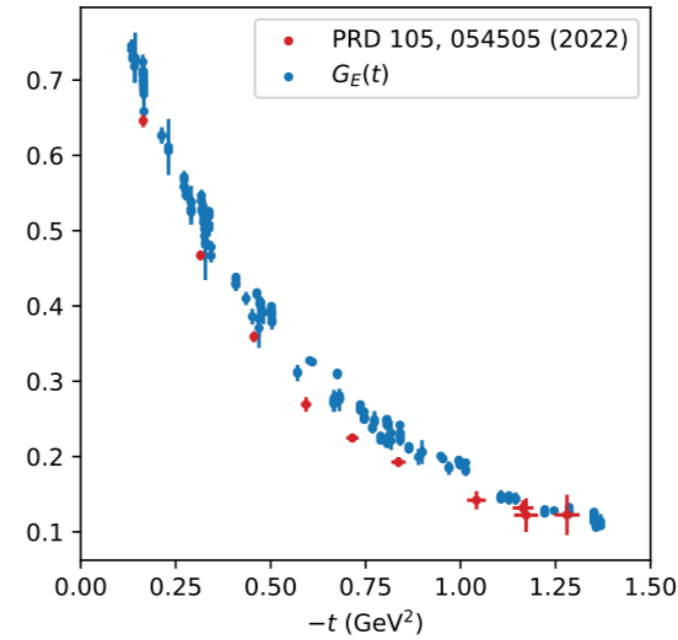
ETMC: S. Bhattacharya et al. JHEP 01 (2025) 146

$$\tilde{H}^R(z_3, P_3, \Delta) = \frac{\tilde{H}(z_3, P_3, \Delta; \mu)}{\tilde{H}(z_3, 0, 0; \mu)} \times g_A$$

to make $\mathcal{M}(0, 0, 0) = g_A$

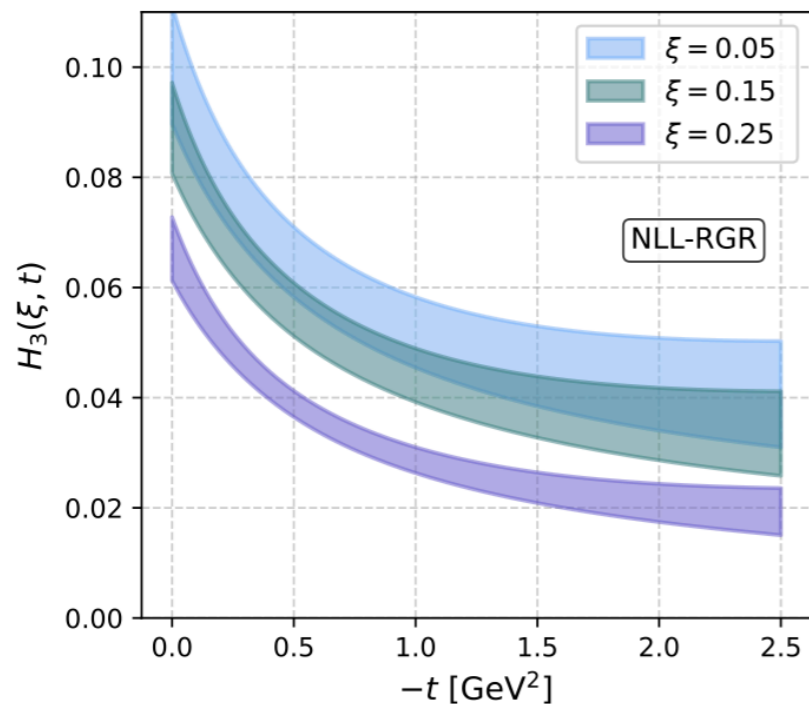
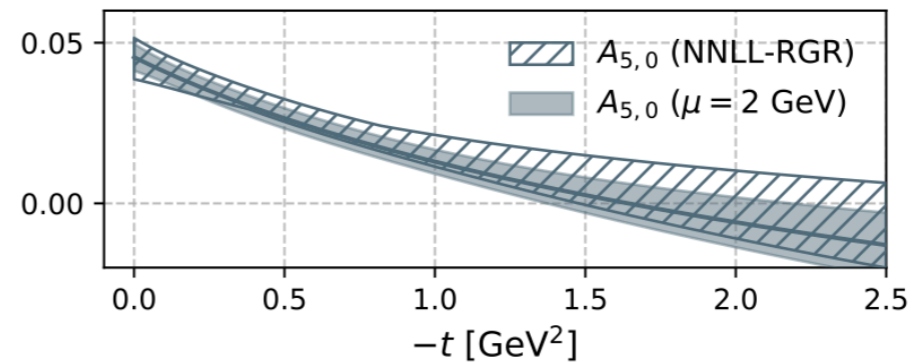
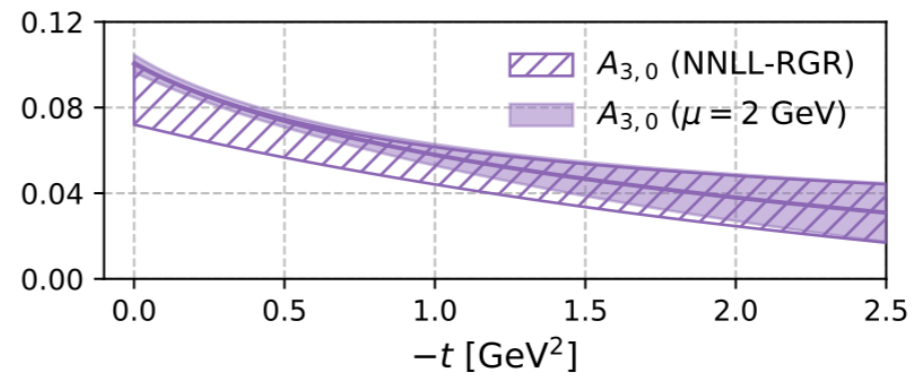
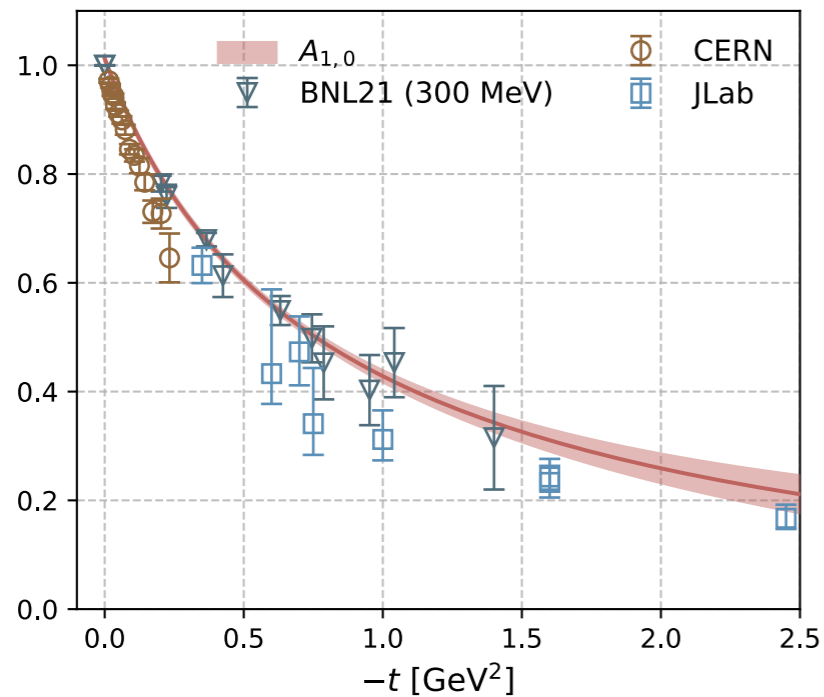
$$\tilde{H}^R(z_3, P_3, \Delta) = \sum_{n=1}^{\infty} C_n^{\overline{\text{MS}}}(\mu^2 z^2) \frac{(-iz_3 P_3)^{n-1}}{(n-1)!} \tilde{A}_{n,0}(t) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

$$A_{n0}(t; \mu) = \int_{-1}^1 x^{n-1} \tilde{H}^R(x, \xi = 0; \mu)$$



Pion unpolarized GFFs

- ✱ An $N_f=2+1$ ensemble of staggered fermions at $a=0.04$ fm 64^4 and clover valence at $m_\pi=300$ MeV
- ✱ Computation is done using the gauge invariant decomposition and ratio method to renormalize
- ✱ Renormalization with ratio and short distance expansion to compute moments for zero and non-zero skewness



n^{th} moment

$$H_n(\xi, t) = \sum_{k=0,2,\dots}^{n-1} A_{n,k}(t) (2\xi)^k \pm \text{mod}(n-1, 2) (2\xi)^2 C_n(t)$$

GPD Mellin moments decrease as ξ increases

X. Gao, S. Mukherjee, Qi Shi, Fei Yao and Yong Zhao, arXiv:2511.01818

Conclusions

✳ EIC is a versatile machine that will bring a wealth of data on pion, kaon and proton structure:

1. Extend measurement of form factors to large momentum transfer
2. Probe the low x -region of PDFs and GPDs improving our knowledge of sea quark and gluon distributions
—> spin structure of proton, mass generation, mechanical properties

—> 3D imaging of pion, kaon and proton
3. Measure helicity and transversity TMD PDFs such as the Sivers structure and the Boer-Mulders functions and higher twist PDFs probing multi-particle correlations

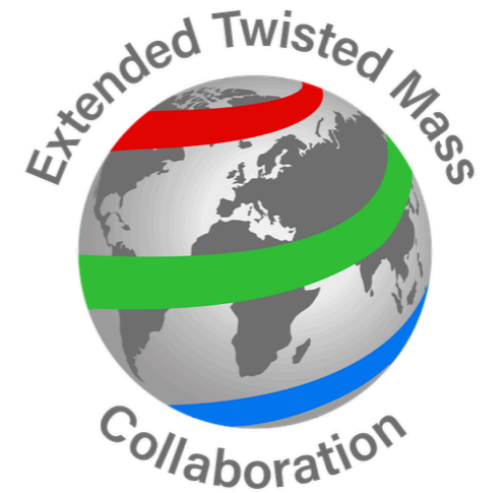
✳ Lattice QCD can greatly impact the scientific program of EIC:

1. Precise computations of charges, form factors and Mellin moments of PDFs and GPDs taking into account all major lattice systematics

As 1% is reached in some of these quantities one needs to include isospin breaking

—> New era of QCD+QED
2. Great progress for direct computations of PDFs, GPDs and TMDs including twist-3 with better matching and renormalisation procedures

Thanks to ETMC members for their crucial contributions



And to all who sent highlights

Andrea Shindler, Artur Avkhadiev, Christian Zimmerman, Christopher Monahan, Dimitra Pefkou, Wayne Morris, Sara Collins, Wei Wang, Zhao Yong



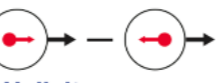

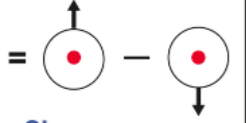
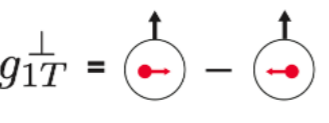


Backup slides

Transverse momentum distributions (TMDs)

* Transverse momentum dependent distributions encode 3D structure in momentum space, e.g. correlations between the direction of parton and hadron spins and intrinsic transverse momenta

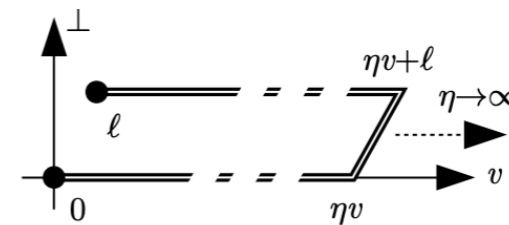
—> EIC with polarized beams will provide results on these quantities

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

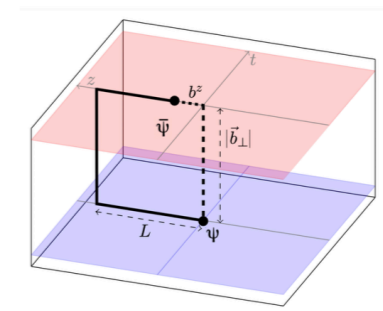
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$ 		$h_1^\perp = \text{Boer-Mulders}$ 
	L		$g_{1L} = \text{Helicity}$ 	$h_{1L}^\perp = \text{Worm-gear}$ 
	T	$f_{1T}^\perp = \text{Sivers}$ 	$g_{1T}^\perp = \text{Worm-gear}$ 	$h_1 = \text{Transversity}$  $h_{1T}^\perp = \text{Pretzelosity}$ 

TMD Handbook: arXiv:2304.03302

- * The unpolarized TMD is the best measured one
- * There are different schemes to define TMDs e.g.
 - Use space-like Wilson lines and Lorentz-invariance to get ratios where the soft function cancels:
B. U. Musch *et al.* (LHC) Phys. Rev. D 83 (2011) 094507



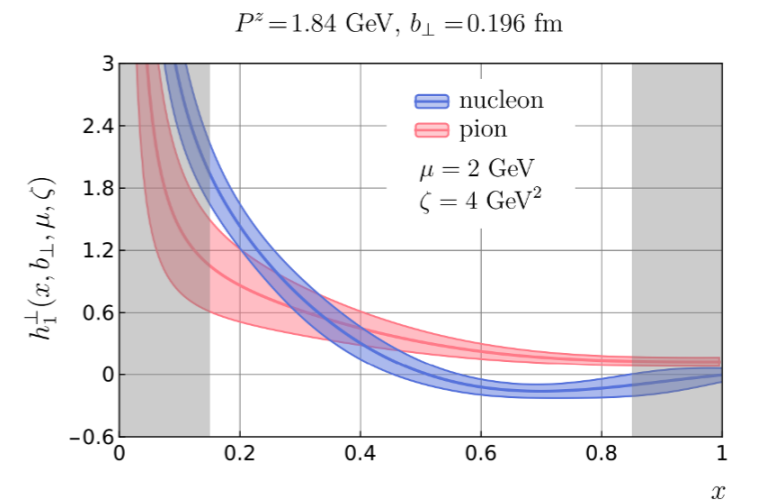
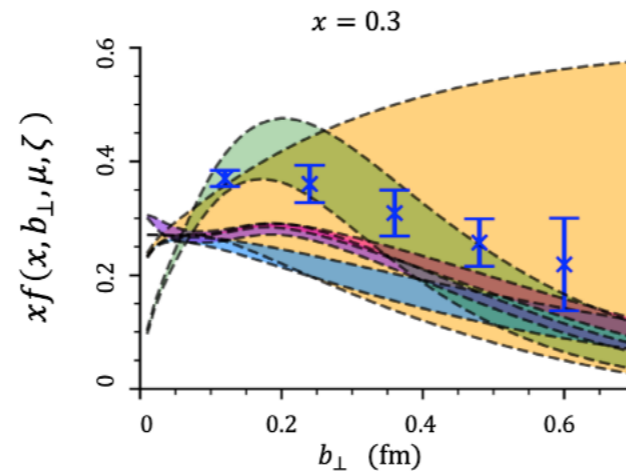
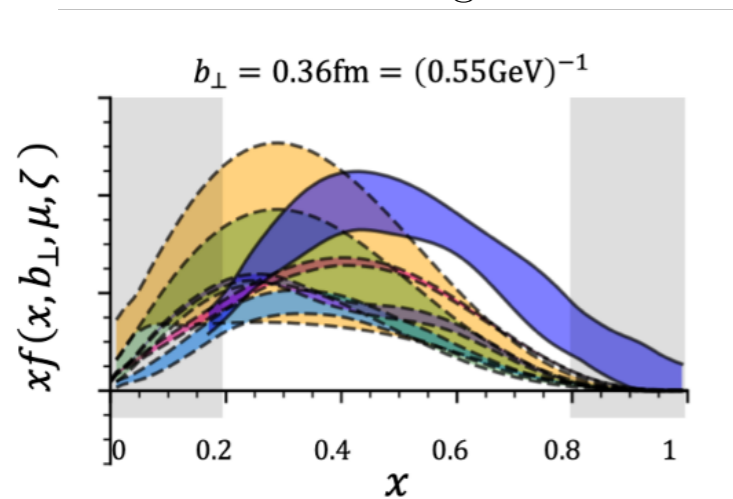
- LaMET: X. Ji, *et al.* Phys. Rev. D 99 (2019) 114006; M. A. Ebert, I. W. Stewart, Y. Zhao, Phys.Rev.D 99 (2019) 034505; JHEP 09 (2019) 037; JHEP 03 (2020) 099



$$\Phi^{[\Gamma]}(x, k_\perp, P, s; \mathcal{C}) = \frac{1}{2} \int dk^- \int \frac{d^4x}{(2\pi)^4} e^{-ik \cdot x} \langle P, s | \bar{\psi}(x) \Gamma \mathcal{U}_{\mathcal{C}}(x; 0) \psi(0) | P, s \rangle |_{k^+ = xP^+}$$

Recent highlights of TMDs

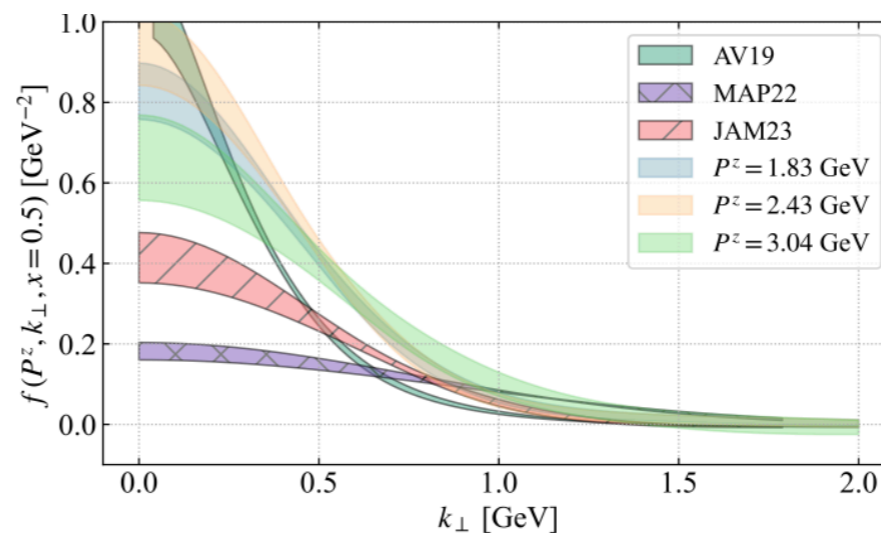
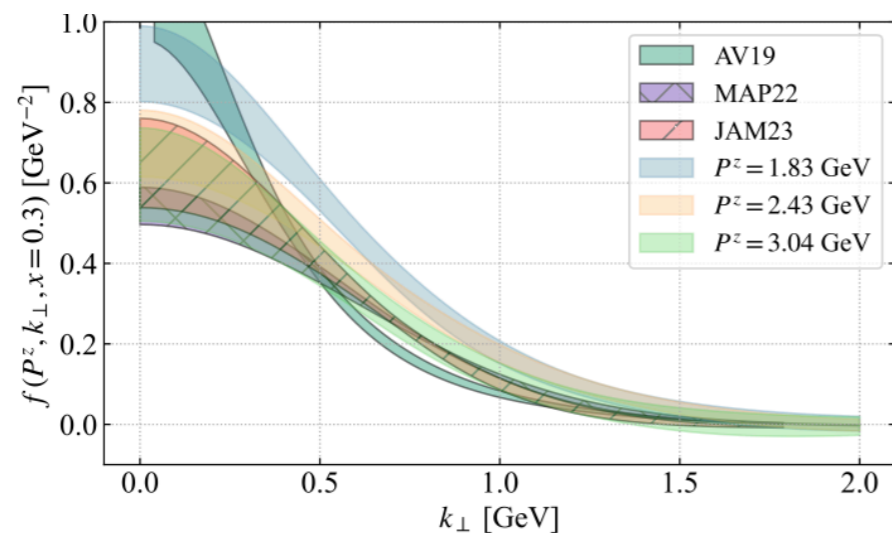
- Lattice Parton Collaboration (LPC) within the quasi- approach computed the unpolarized nucleon TMD-PDF using a hybrid action, $m_\pi=220$ and 310 MeV, $a=0.12$ fm and up to $P^z=2.58$ GeV
- LPC also computed the Boer-Mulders function: Pion using Clover fermions, $m_\pi\sim 340$ MeV and $a=0.098, 0.085, 0.064$; nucleon using $m_\pi\sim 340$ MeV, $a=0.098$ fm



J.-C. He *et al.*, Phys. Rev. D 109 (2024) 114513

L. Walter *et al.*, Phys. Rev. D 109 (2024) 114513; JHEP 08 (2025) 086

- ANL-BNL used gauge fixing to compute the pion TMD-PDF with a hybrid action (Clover valence on staggered), $m_\pi\sim 300$ MeV $a=0.06$ fm and up to $P^z=3$ GeV; proton helicity TMD-PDF using an ensemble of domain-wall fermions with physical pion and $a=0.084$ fm



D. Bollweg *et al.* Phys.Rev.D 112 (2025) 3, 034501