I'm from Seattle. Do I need a rain jacket today?

Hourly probability of rainfall
(be a Bayesian for now)

What is the probability of "not raining at all"?

P(no-rain) = 0.78 x 0.73 x 0.45 x 0.45 x 0.53 x 0.40 x 0.51 x 0.40 = $\boxed{0.5\%\ ???}$

(this is a common mistake I see in several "modern" geoneutrino flux calculations)

I'm from Seattle. Do I need a rain jacket today?

Daily forecast: 60% rain

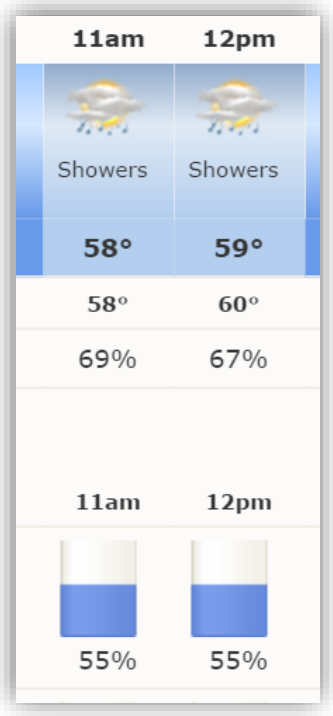
Hourly probability of rainfall
(be a Bayesian for now)

What is the probability of "not raining at all"?

P(no-rain) = 0.78 x 0.73 x 0.45 x 0.45 x 0.53 x 0.40 x 0.51 x 0.40 = $\boxed{\text{0.5\% ???}}$

Of course not. These are not independent from each other.

Weather forecasts must provide correlation matrix



Scenario 1: full correlation

"The overcasting cloud might be a rain cloud, 55% chance"

⇒ 55% chance of rain in this two hours

Scenario 2: anti-correlation

"An intensive thunderstorm is coming, not sure when it arrives"

⇒ 100% chance of rain in this two hours

(be a Bayesian for now)

Depending on correlations, combinations of PDFs can be anything

Stochastic Crustal Model around KamLAND

Earth Science Backgrounds

Nozomu Takeuchi (ERI, U of Tokyo)
Tsuyoshi Iizuka (U of Tokyo)
Kenta Ueki (JAMSTEC)
Akiko Tanaka (GSJ & Tohoku U)
Satoru Haraguchi (ERI, U of Tokyo)

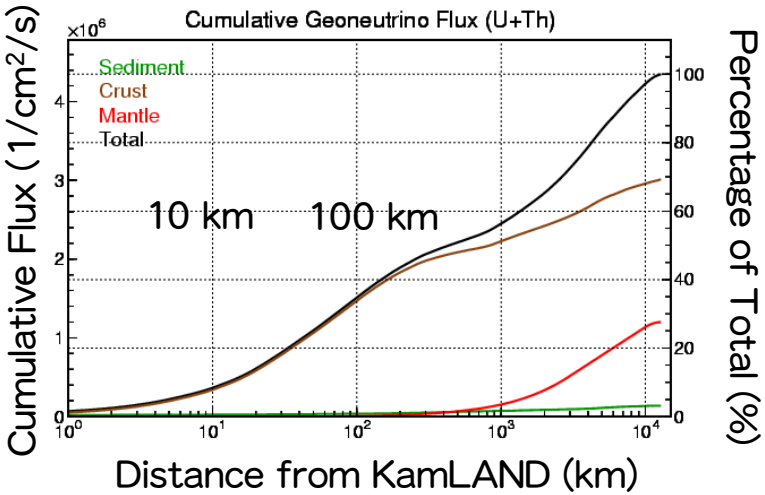
Particle Physics Backgrounds

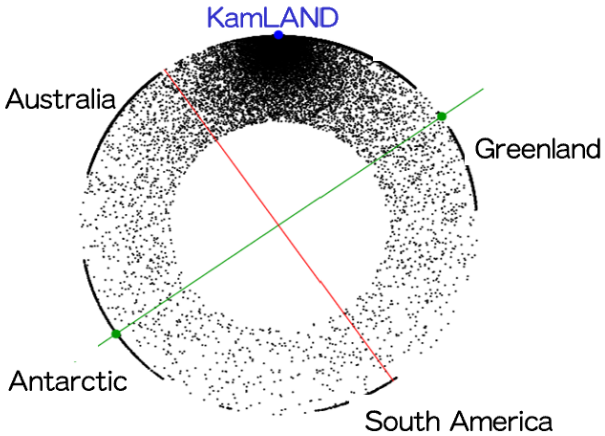
Tsutomu Fukuda (Nagoya U)
Tsukasa Nishikiori (Nagoya U)
Hiroko Watanabe (Tohoku U)
Sanshiro Enomoto (U of Washington)

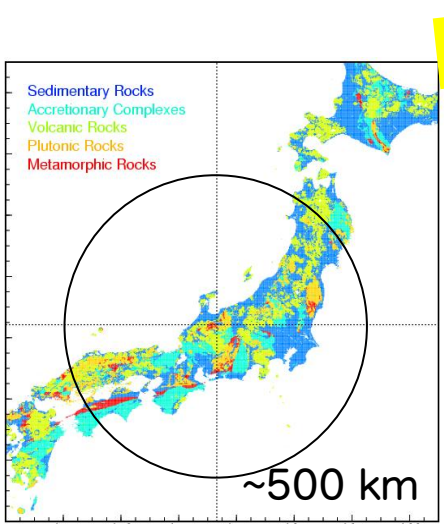
Neutrino Geoscience 2025, Kingston, 28 October 2025

An Inconvenient Truth

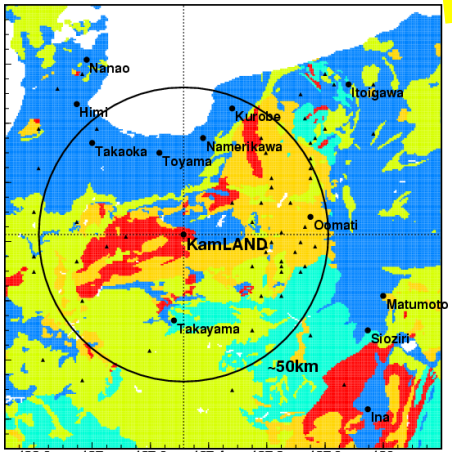
Near-field crust contributes ~half of the total flux at Kamioka



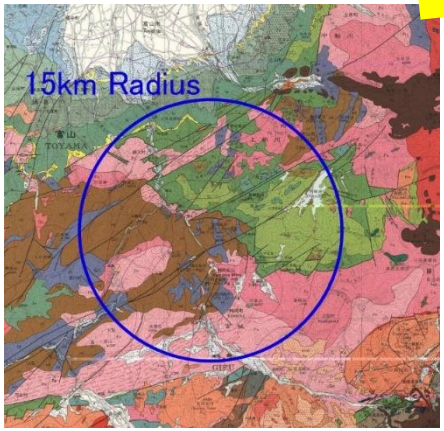




50%



25%

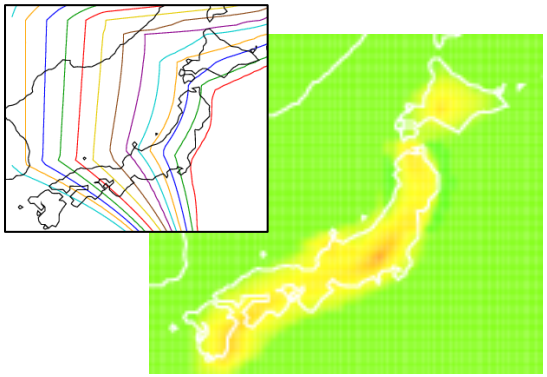


10%

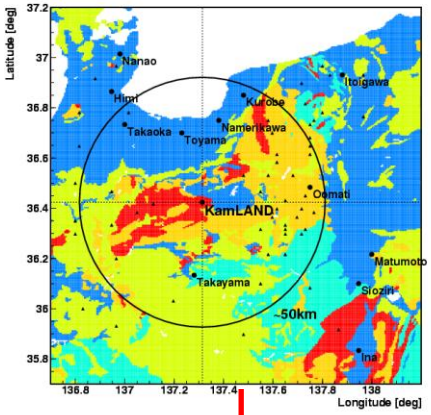
Especially, the Upper Continental Crust is critically important
(largest contribution, complicated 3D structure, global composition model not applicable)

What we did 20 years ago

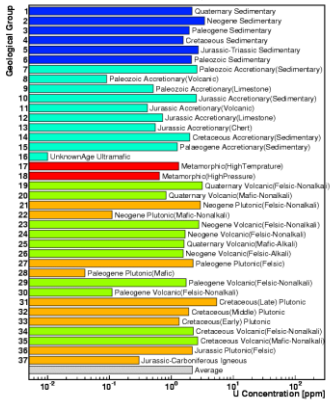
Seismic Crustal Boundaries



Surface Geology



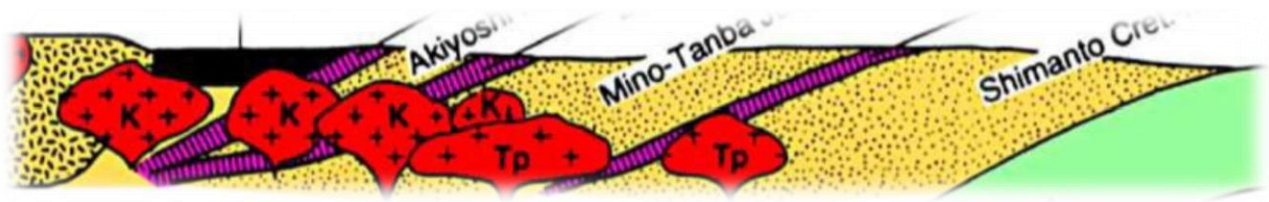
Rock Compositions



Arbitrary
Vertical
Extrapolation

$$F_{U/Th} = A \cdot \int_{Earth} \frac{\rho_{U/Th}(\vec{r})}{4\pi \,|\, \vec{r} - \vec{r}_{KamLAND} \,|^2} P(\bar{\nu}_e \rightarrow \bar{\nu}_e)\, dV$$

With Pretending Not Having Seen This



Seismic boundaries are relatively ok, 3D lithology is the problem

Outline

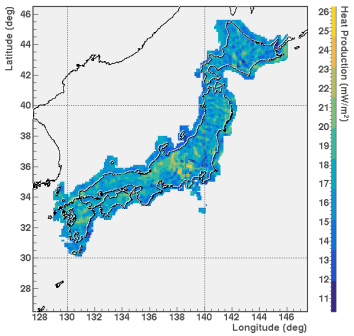
Part I: Building a Fully Stochastic Crustal Model

(presented at Neutrino Geoscience 2019 Prague, published in PEPI 288 (2019))

- Describe everything as Probability Density Function (PDF)
- Avoid arbitrary lines such as crustal boundaries, geological provinces, middle crust, …
- No model-dependent bias, no underestimation of errors

⇩

3D map of U/Th concentration PDFs in Japan



Problem: Multiple PDFs cannot be integrated without correlation information

→ worst case was assumed (max correlation) → very large uncertainties

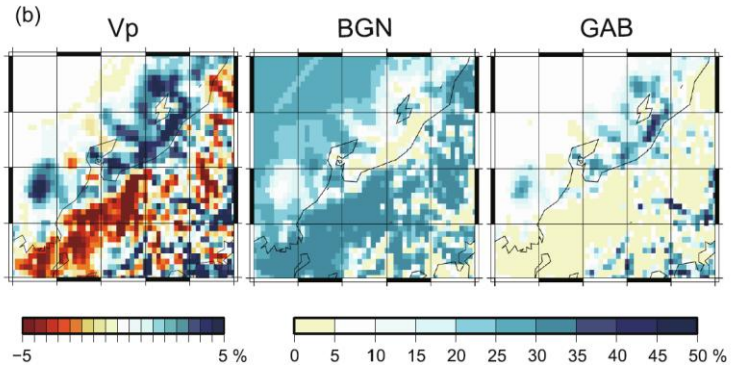
Part II: Reducing the Uncertainties (Correlation Modeling)

(on-going project)

Scale-aware Geochemical Dataset/Model

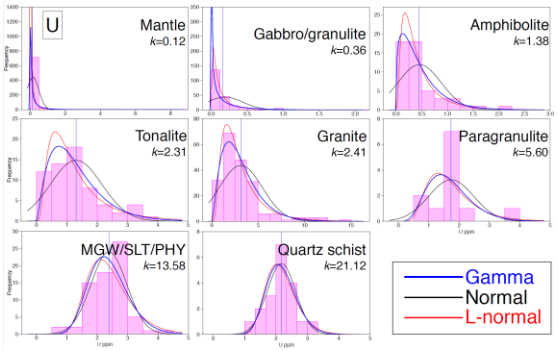
Fully Stochastic Crustal Model - Overview

3D Lithology PDF Map



- Seismic speed tomography
- Vp data for each rock type
- Bayesian inference

Rock Composition PDF

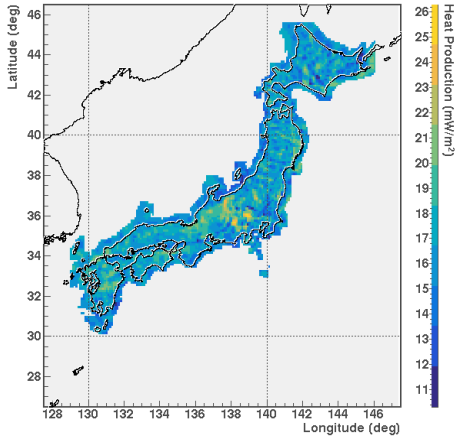


- Rock composition from database
- Mathematically-bias-free modeling
- Bias-free detection limit handling

(separately published in AGU monograph)

Convolution

3D U/Th Distribution PDF Map

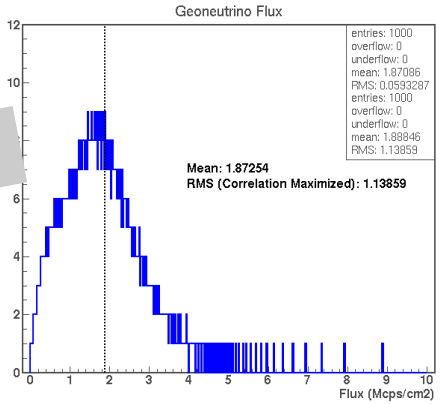


- U/Th concentration as PDF for each point (3D)

Monte-Carlo Integration

Weakest Link

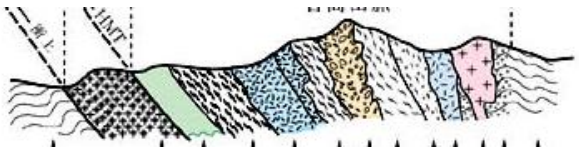
Flux at KamLAND as PDF



Bayesian Lithology Inference

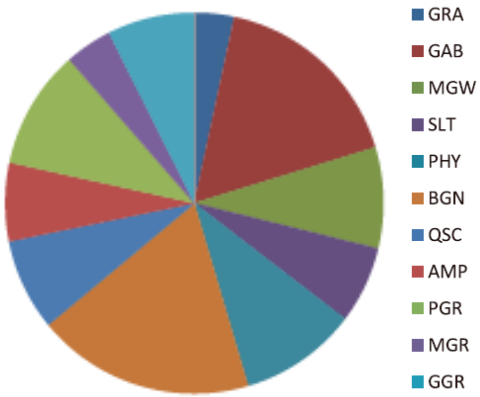
Prior PDF
from Hidaka Exposure

Crustal cross-section at Hidaka



P(i) prior model

i: rock type index

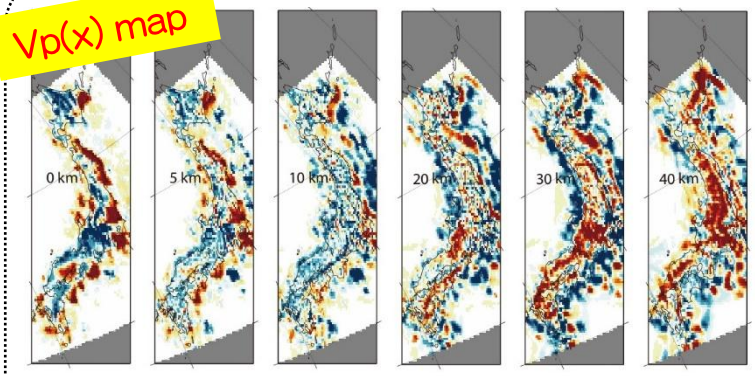


Bulk composition model
(no vertical info here)

×

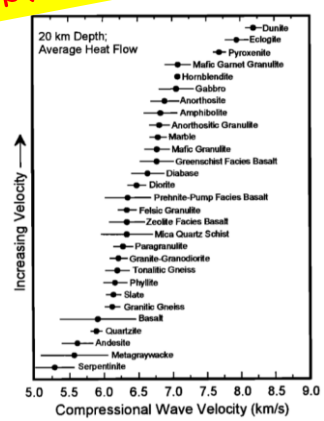
Likelihood
from Seismic Tomography & Petrology

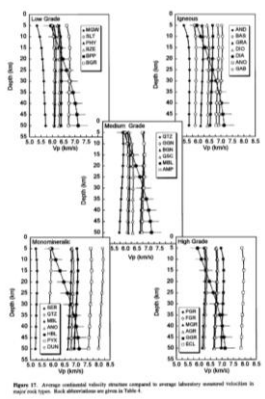
Vp(x) map



Vp(i) model

Christensen&Mooney 1995





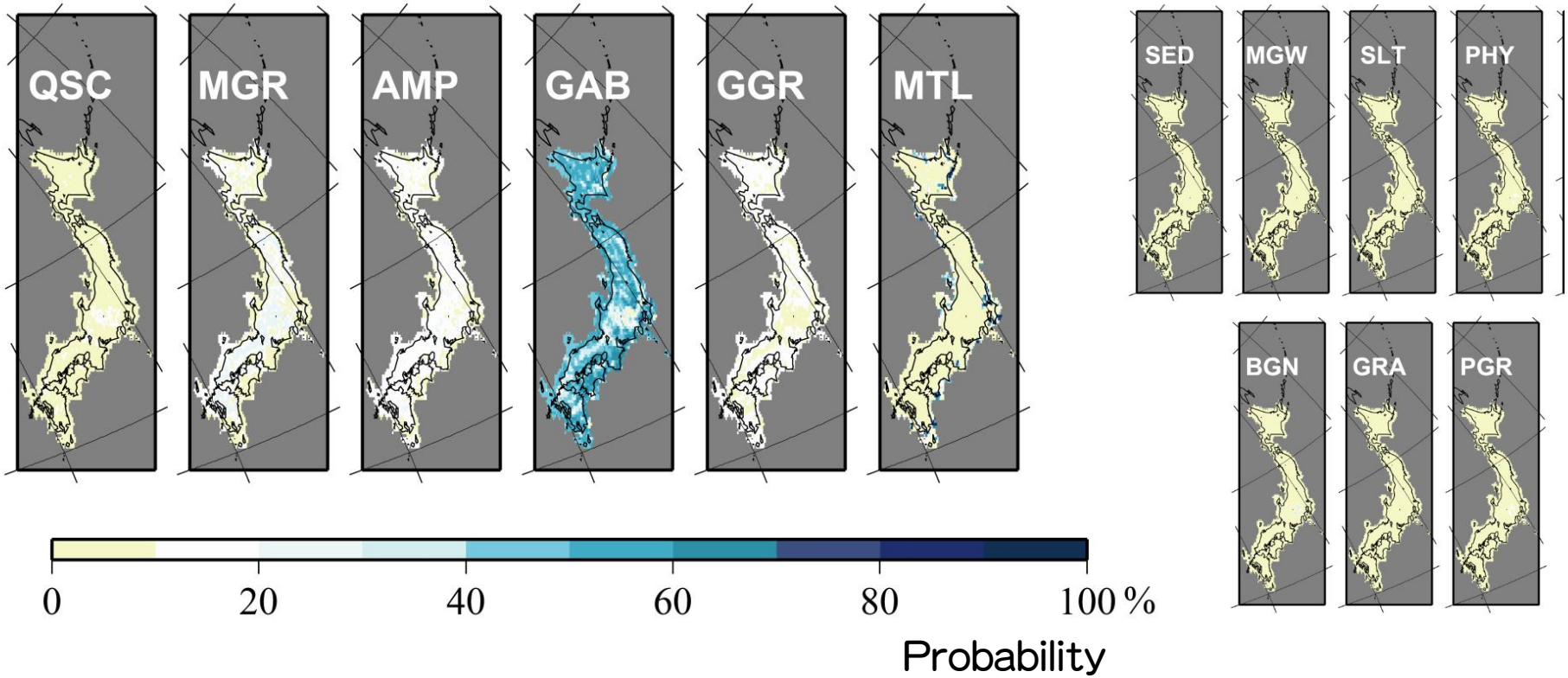
Bayes' Theorem: For rock type i,

$$P(i \mid V_p^{obs}) = \frac{P\left(V_p^{obs} \middle| i\right) P(i)}{\sum_j P\left(V_p^{obs} \middle| j\right) P(j)}$$

Inferred Lithology

Example: 30 km depth

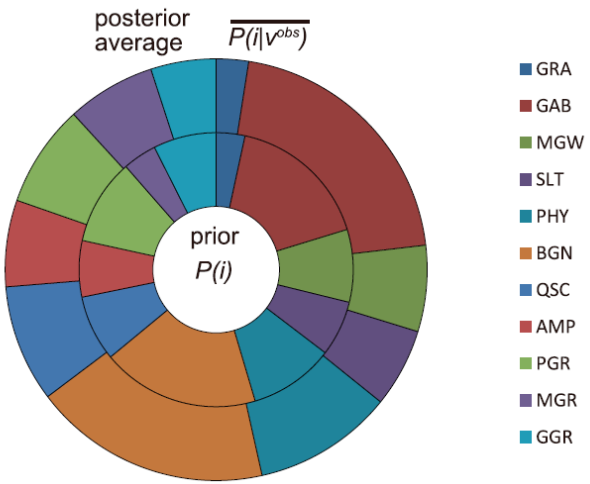
- For each location, the maps show the probabilities over all rock types



Details in PEPI 288 (2019) 37

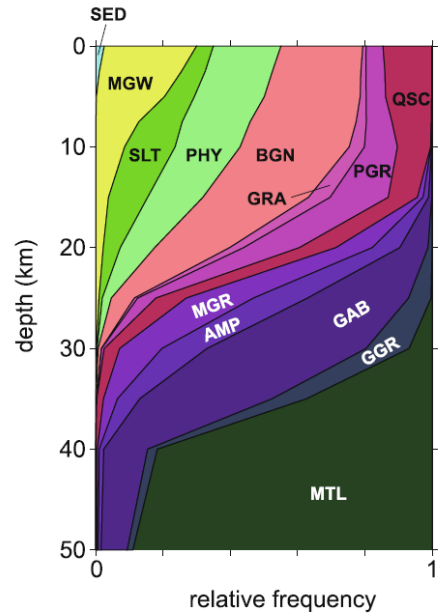
Review: Inferred Lithology

Small update in bulk lithology



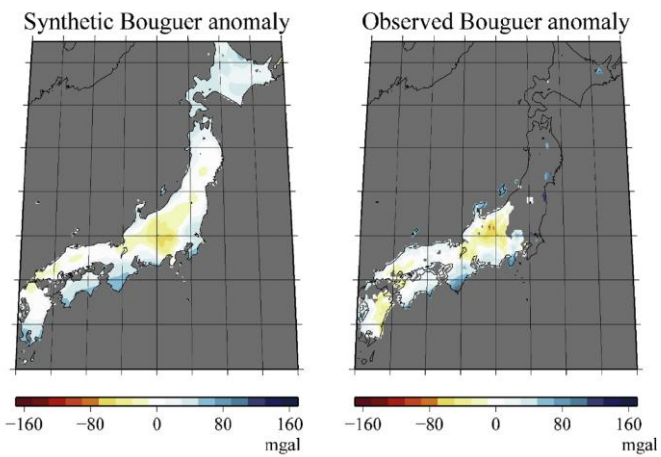
Kullback-Leibler divergence: 0.02
(non-similarity of a PDF from a PDF)

Posterior has vertical resolution

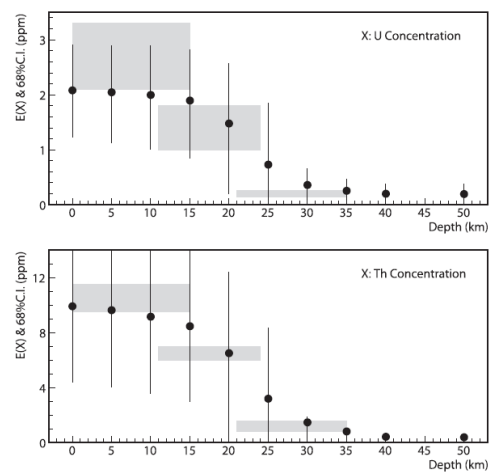


- Basically same structure as in geology textbooks
- Constructed purely from data

Inferred density profile is consistent with gravity anomaly



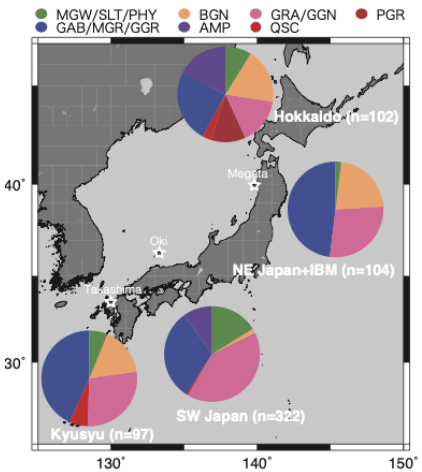
Inferred U/Th distribution has same structure as traditional picture

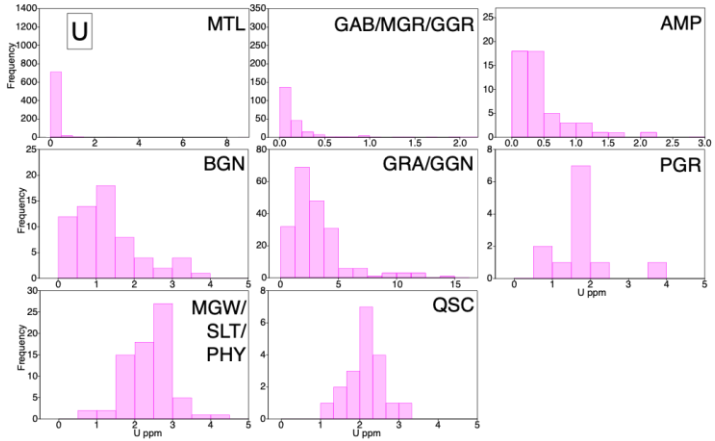


Shade:
CRUST 2.0
Rudnick & Gao 2003

Bias-Free Rock Composition Model - Overview

Composition Data from Database





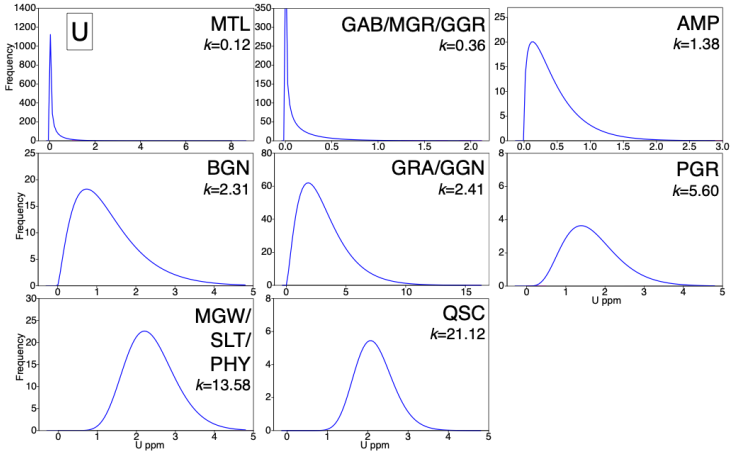
Direct Samples contain Biases

- Detection limits, analysis accuracy
- Contamination of externals
- Human-biased sample selection
- Arbitrary removal of outliers
- Skewed distribution (no negatives)

Common Practices causes Biases

- Use only the mean and SD
- Use the median
- Use the geometric mean
- Use log-normal model

Composition Model with Gamma Distribution



- Conservation of the mean is mathematically guaranteed
  ⇒ No modeling bias (bias by using inaccurate model)

- Proper handling of sample cuts
  (unavoidable for detection limits and outliers)

- Connections to underlying geology are suggested
  - Reproducibility for rock mixing, like normal
  - Logarithmic behavior, like log-normal
    (conservation of geometric mean)

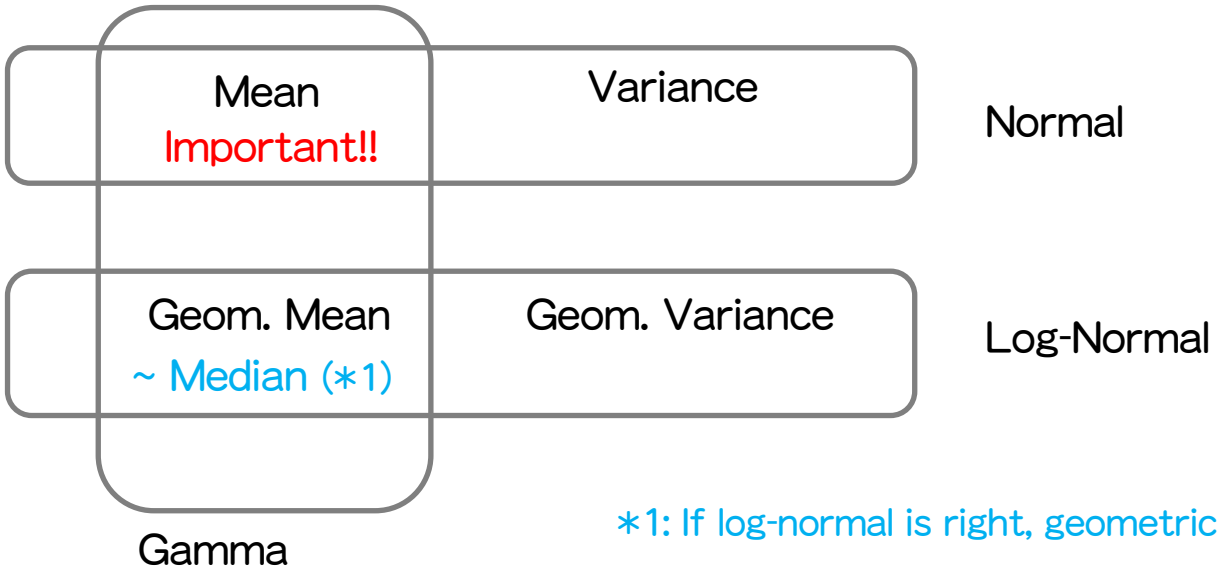
Details in AGU monograph (2023)

Bias-Free Composition Model: Gamma Distribution

Step 1: Consider two parameter distributions ⇒ two controllable quantities

Step 2: Choose two that we think important (i.e., quantities we want to conserve)

Max Entropy (= Min Assumption) Distributions under Two Constraints



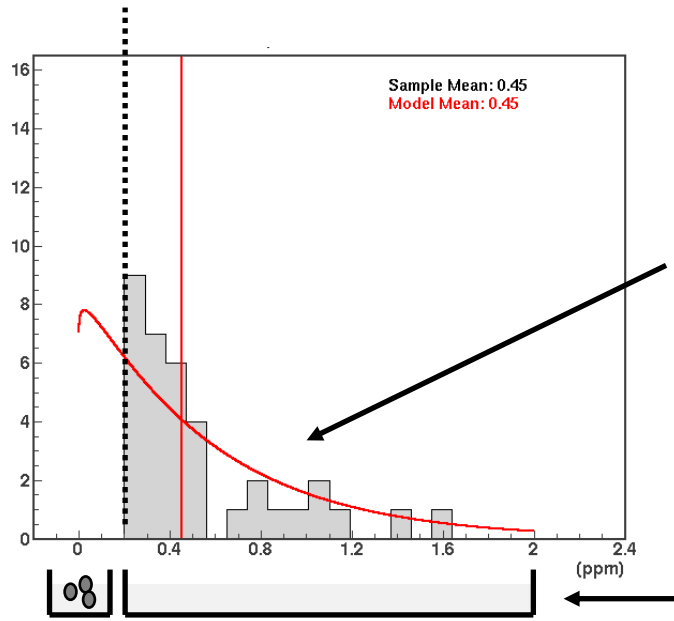
*1: If log-normal is right, geometric mean is equal to median

No bias on mean value, mathematically guaranteed!!

Detection Limits and Outliers

Likelihood is extended to account for cut-offs

Data cut-off at 0.2 ppm



Take into account the probability of Not-Detected

$$L = L_{\text{shape}} + L_{\text{cutoff}}$$

Usual likelihood for samples above cutoff
(with re-normalization to account for cutoff)

$$-2\ \log \mathcal{L}_{\text{shape}}(\boldsymbol{p};\boldsymbol{a}) = -2\sum_{j=1}^{N} \Theta(a_j - B)\ \log\left\{\frac{f(a_j;\boldsymbol{p})}{\int_B^\infty f(\alpha;\boldsymbol{p})\,d\alpha}\right\}$$

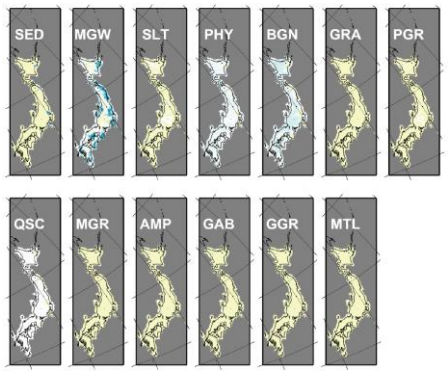
Binominal is used for numbers-below-cutoff

$$-2\ \log \mathcal{L}_{\text{count}}(\boldsymbol{p};\boldsymbol{a}) = -2\log\left\{\binom{N}{N_0}\left(\int_0^B f(\alpha;\boldsymbol{p})d\alpha\right)^{N_0}\left(\int_B^\infty f(\alpha;\boldsymbol{p})d\alpha\right)^{N-N_0}\right\}$$

- Polynomial can be used for both lower/upper cutoffs to remove outliers. (No need for median anymore??)
- The conventional *Quantile Cut* without this treatment will cause bias.

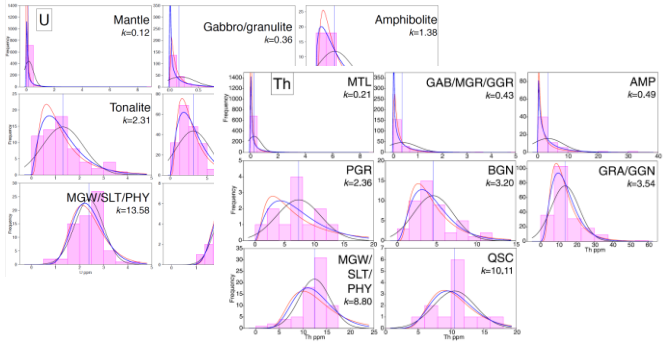
Stochastic Radioactive Heat Production Map

3D Lithology PDF Map

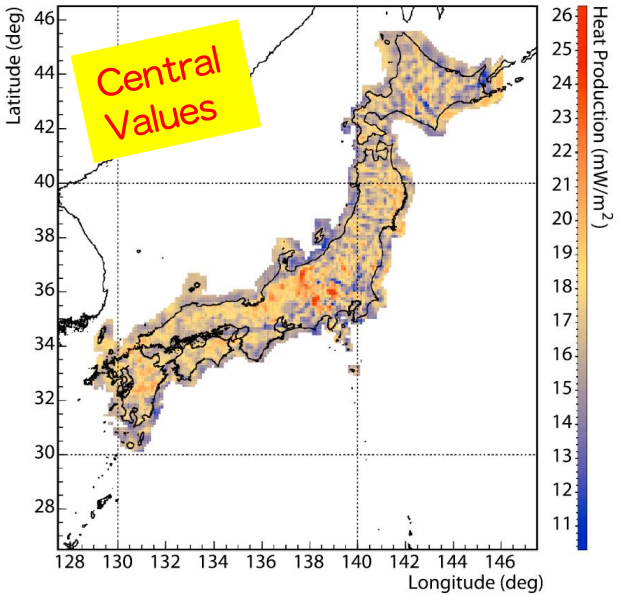


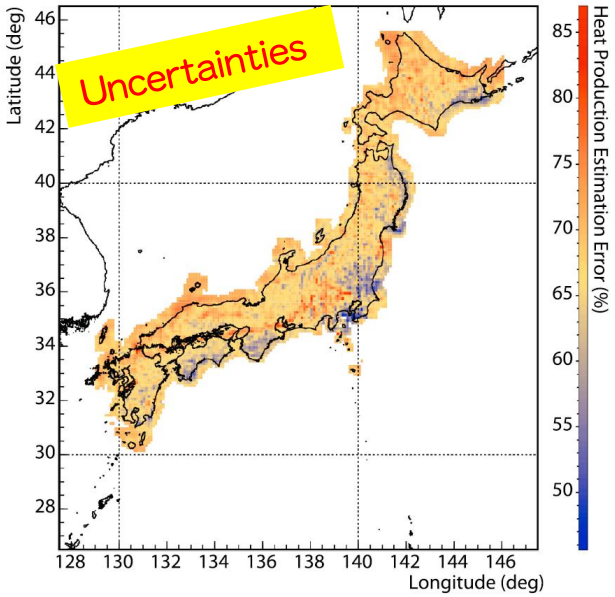
Convolution

Rock Composition PDF



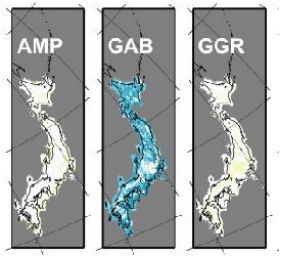
Stochastic Heat Producing Element Distribution Map (vertically integrated)



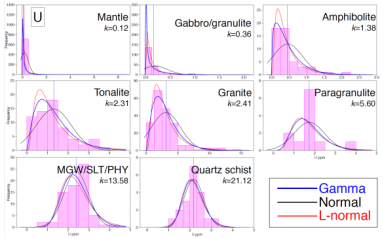


Flux Integration with Monte-Carlo (MC)

Lithology PDF



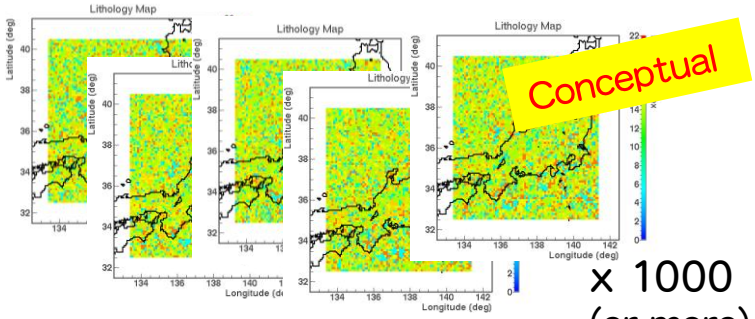
Composition PDF



Convolve

Generate Monte-Carlo Instances

Randomly Generated Distribution Maps



x 1000
(or more)

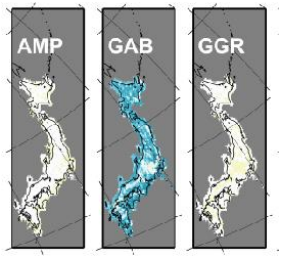
for each, calculate flux

Neutrino Flux PDF

Histogram of
1000 MC flux values

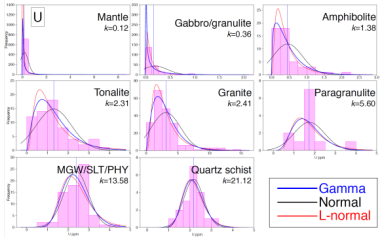
Flux Integration with Monte-Carlo (MC)

Lithology PDF



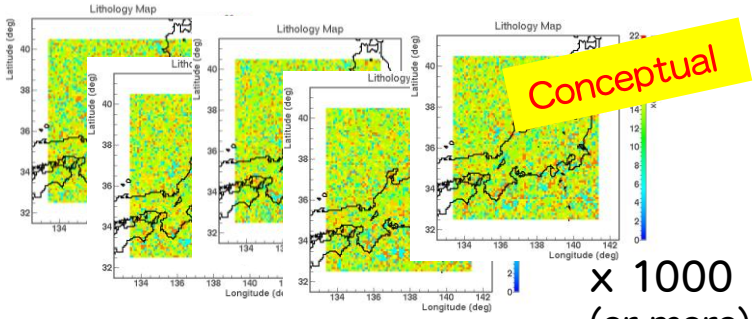
Convolve

Composition PDF



Generate Monte-Carlo Instances

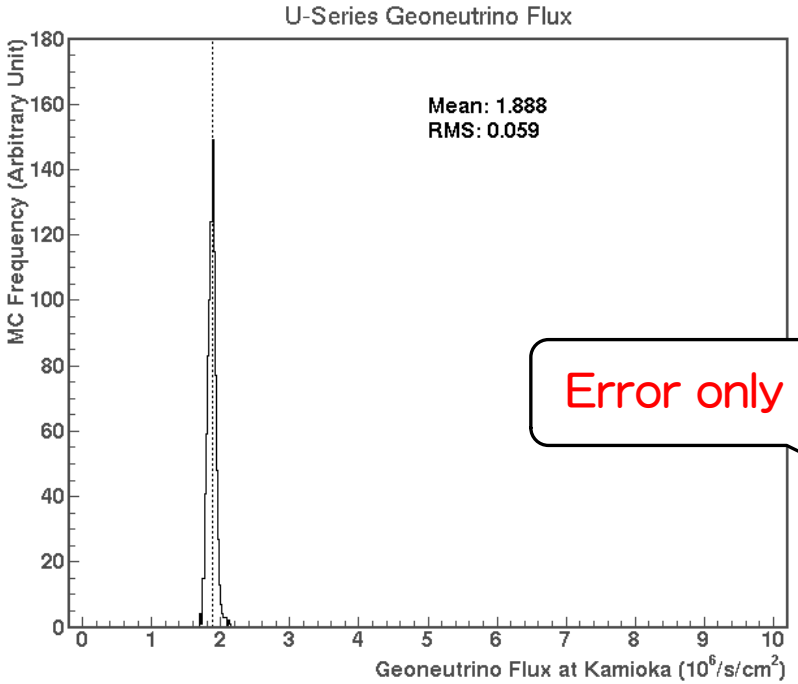
Randomly Generated Distribution Maps



x 1000
(or more)

for each, calculate flux

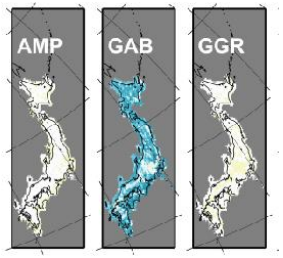
Neutrino Flux PDF



Error only 3% !!!

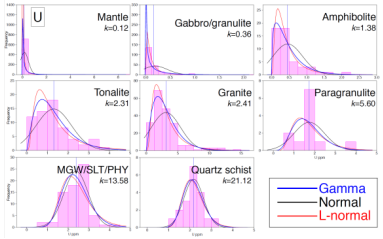
Flux Integration with Monte-Carlo (MC)

Lithology PDF



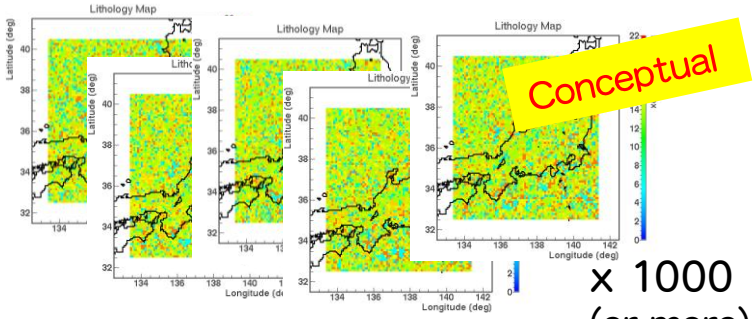
Convolve

Composition PDF



Generate Monte-Carlo Instances

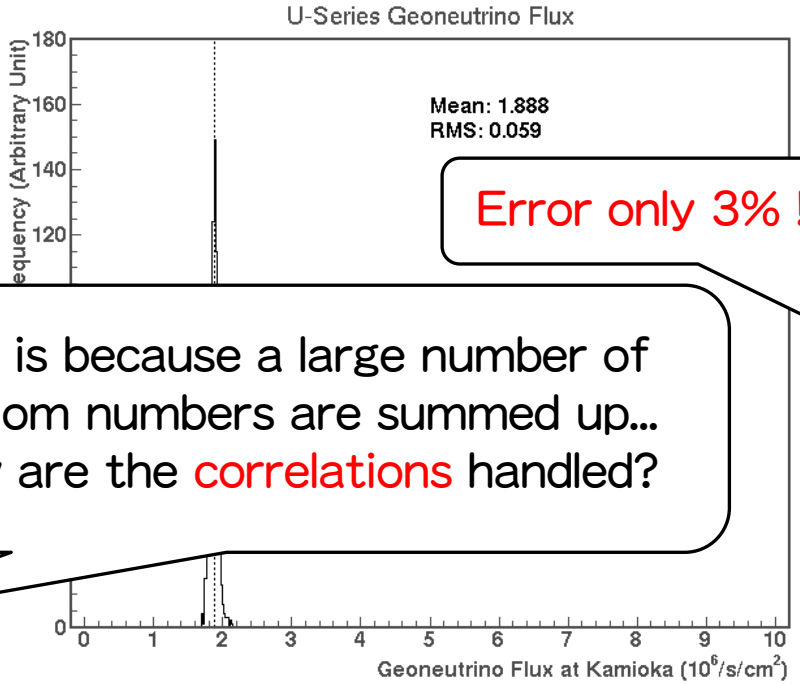
Randomly Generated Distribution Maps



x 1000
(or more)

for each, calculate flux

Neutrino Flux PDF

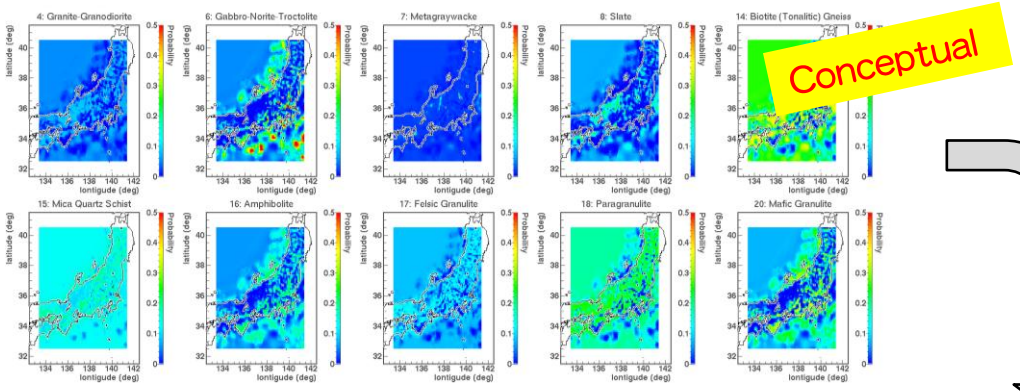


Error only 3% !!!

This is because a large number of random numbers are summed up...
How are the correlations handled?

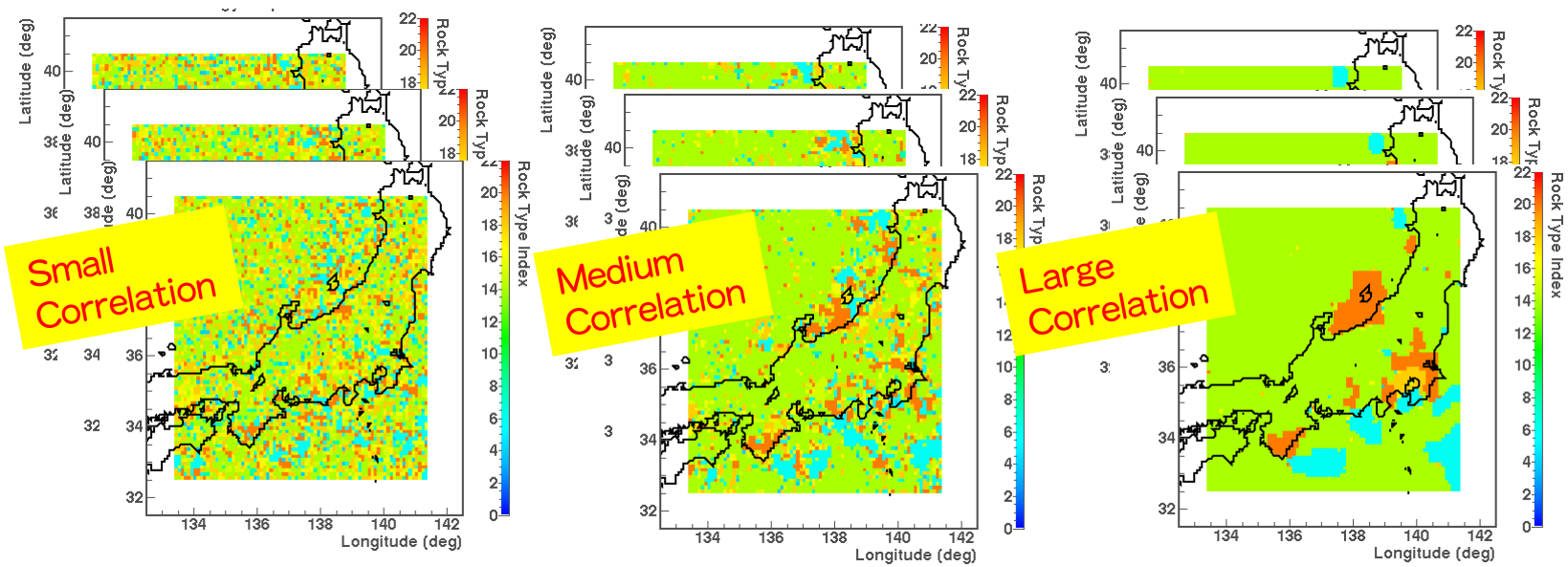
MC Flux Integration and Correlation Problem

Lithology Probability Map



Generate random Monte-Carlo instances according to probability

Randomly Generated Distribution Maps



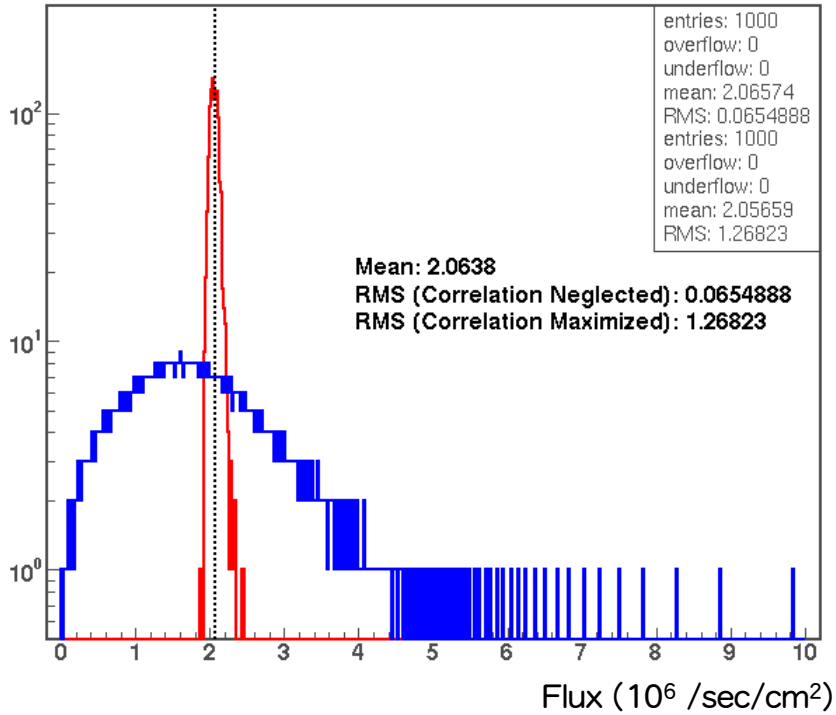
Small Flux-Estimation Error

Large Flux-Estimation Error

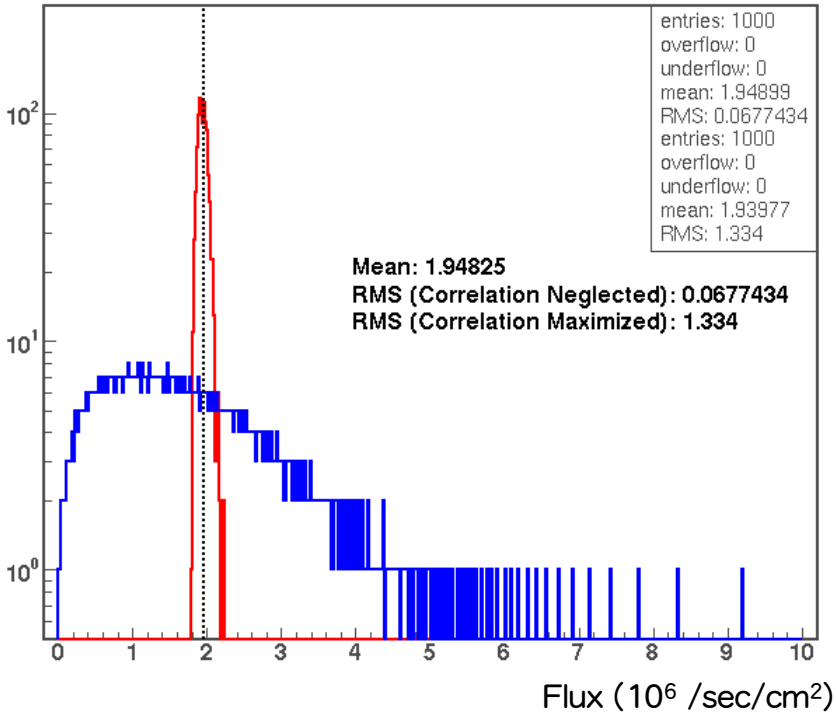
Completing the Chain Anyway…

- Covariance is unknown → assumed the worst case (max correlation)
- Errors are too large
- A fully stochastic model has been constructed anyway

U-Series Geoneutrino Flux PDF



Th-Series Geoneutrino Flux PDF

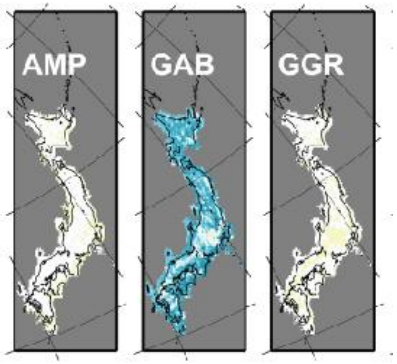


Error of flux estimation

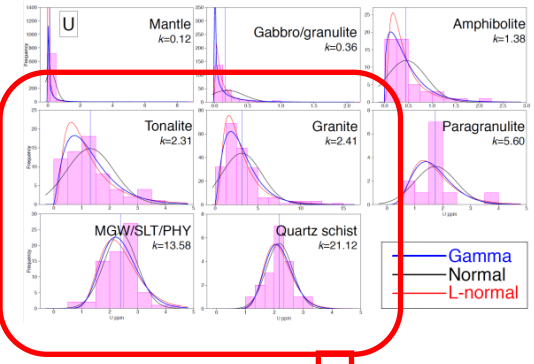
Simply summing MC ensembles (common mistake): 3% error
Maximum Correlation Assumed (conservative): 62~68% error

The Problem

3D Lithology Map (PDF)

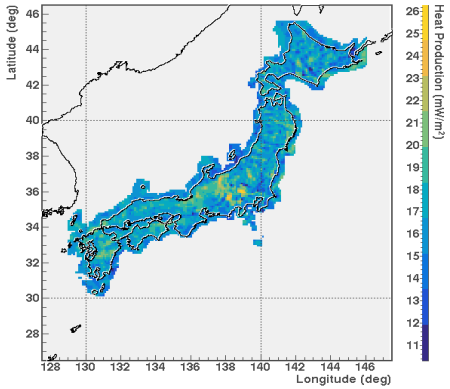


Rock Composition (PDF)



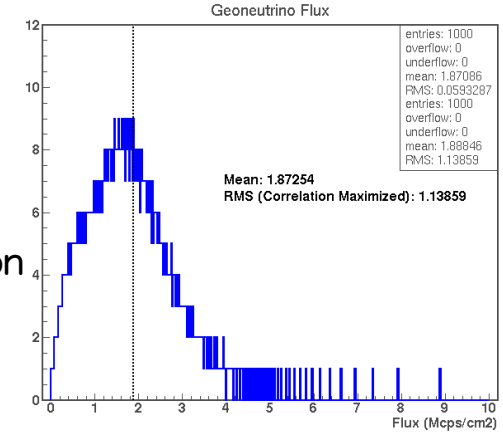
Convolution

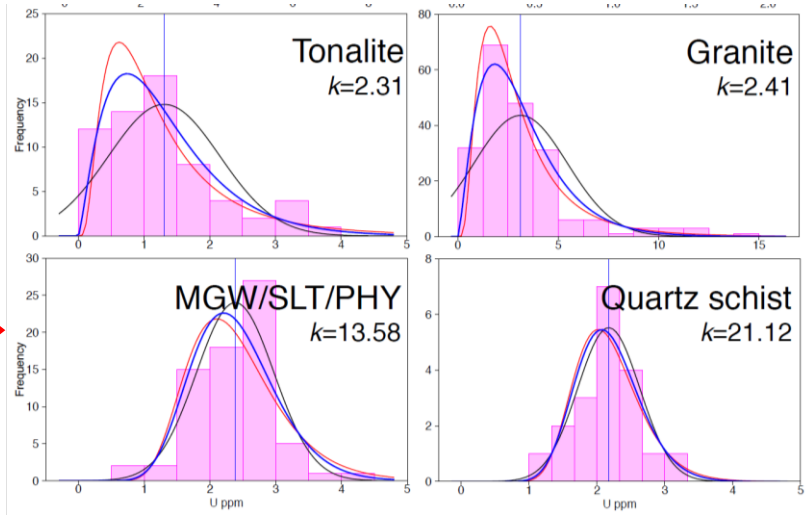
3D U/Th Distribution Map (PDF)



Integration

Geoneutrino Flux at Kamioka (PDF)



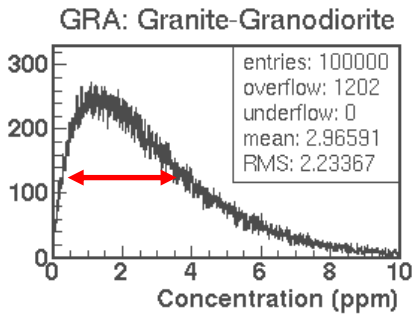


Rock samples show large variations of U/Th concentrations

Conservative assumption (maximum correlation) projects the variations to final flux model

The Problem

Our rock composition model is very broad



Example: Granite (花崗岩)
76% compositional spread

The maximum correlation assumption directly transfers this to the broad flux PDFs

Rock composition often shows a large variation

U/Th Contents of Kamioka Mine Rock Samples (Enomoto thesis, 2005)

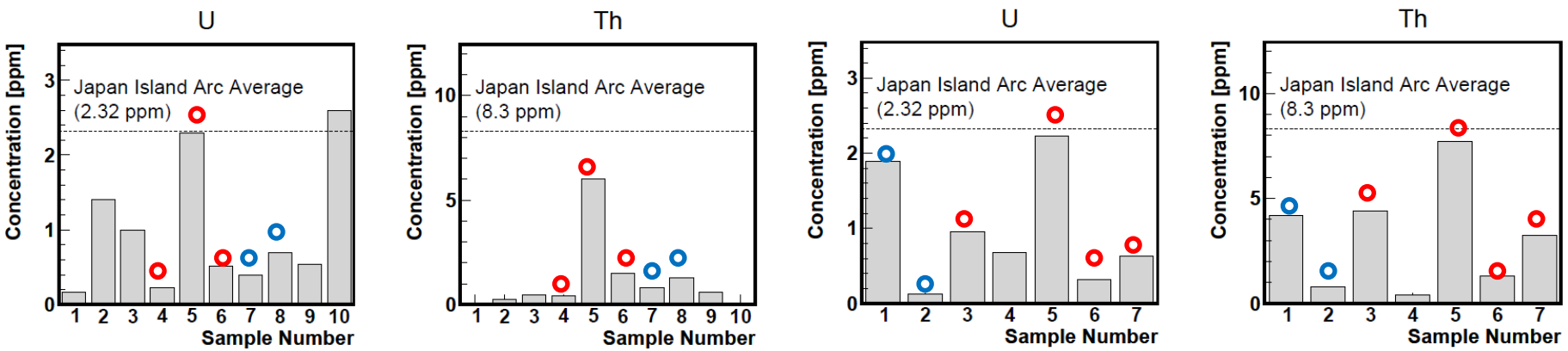


Figure 2.31: samples collected by us

Figure 2.32: samples from another study ples (M.Yamamoto *et al.*)

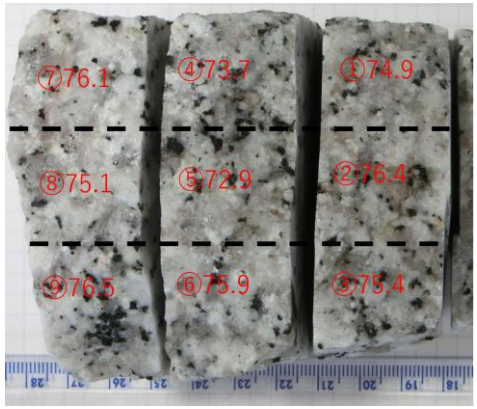
Gneiss

Inishi Rock (common in Kamioka)

Extreme Case Test: Comparing parts of a sample

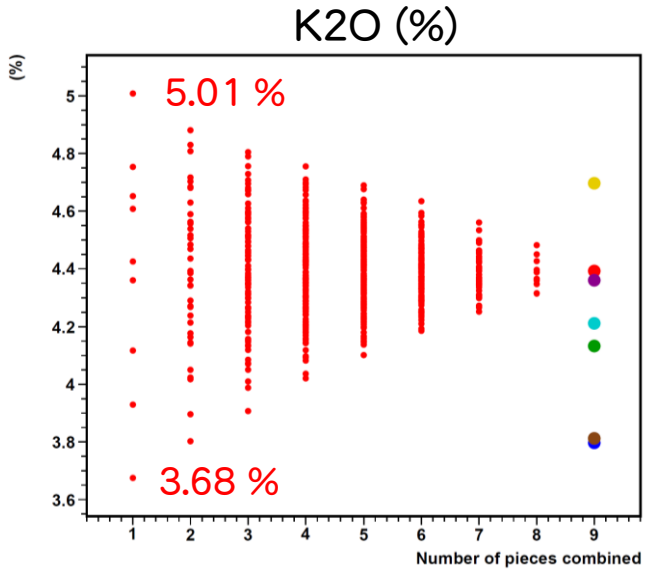
"Typical-size" Samples from One Large Chunk

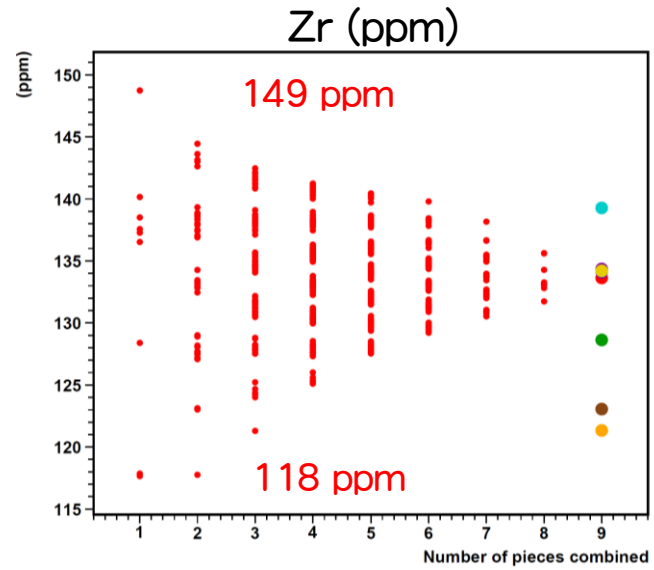
~10 cm



~3 cm

each piece is roughly 30 g





pieces combined

Smallest divided

Whole chunk
(7 chunks shown)

- Even in a super small scale (~10 cm), the compositional variations are very large
- These can be integrated out. But how much can we safely remove?

Maybe Too Conservative; What if some averaging assumed?

Lithology Scale

100 km^2 X 2.5~10 km

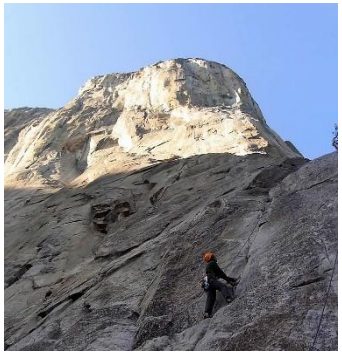
Samples Scale

Geochemists' samples

1 cm^3
X 1000

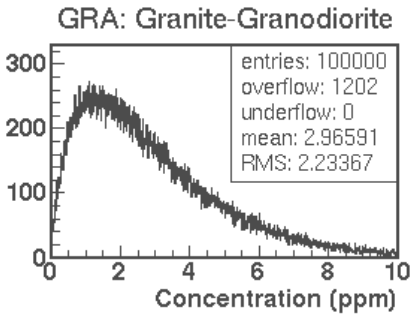
A physicist collecting samples

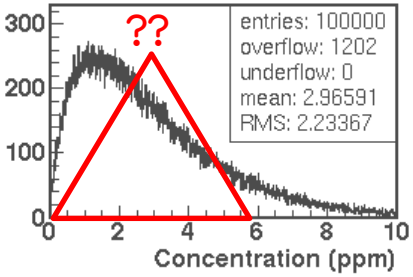
1 m^3
X 1000

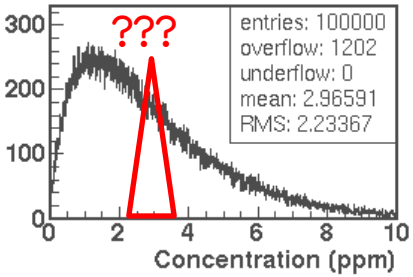


100 m^3
X 1000

Samples Distributions







- Many geoneutrino models use mean & error-of-mean for composition
- This assumes complete averaging (no local variation) ... Can this be justified ???

Compositional Variation Modeling: Approaches

Start: Physicists´ Naive Thoughts

- mm-scale structure due to mineralogical processes ⇒ Negligible
- km-scale structure due to rock body formation (differentiation in solidification) ⇒ Mostly Negligible might need Kamioka Model
- >10 km-scale structure due to geological processes ⇒ Need to be included

Brainless (aka Data Driven) Approach

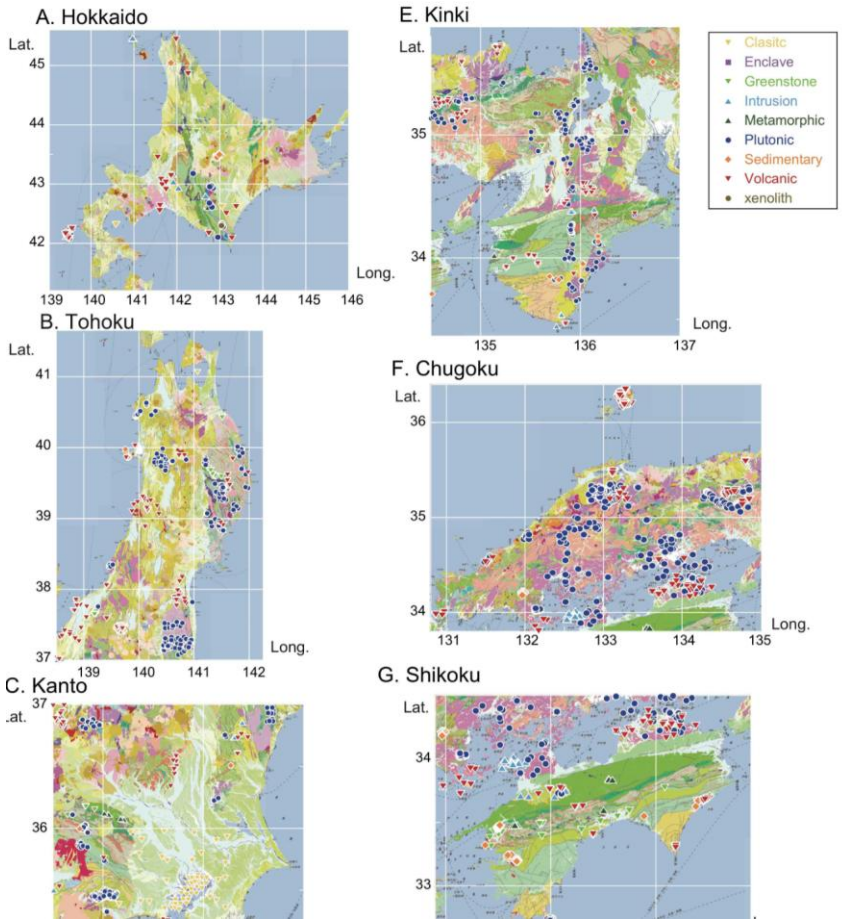
- Collect many samples with location information
- Extract special structure

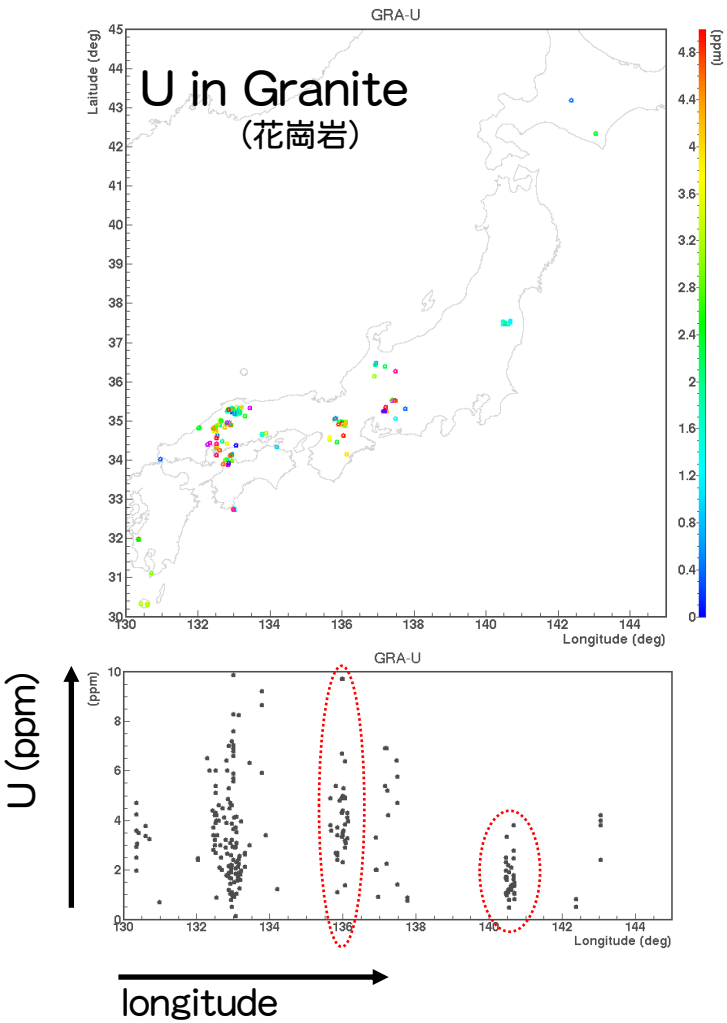
Bottom-up (Model Based) Approach

- Analyze and describe mineralogical spatial variations
- Investigate rock body structures with strategic sampling

Data Driven Approach: Variations and Spreads

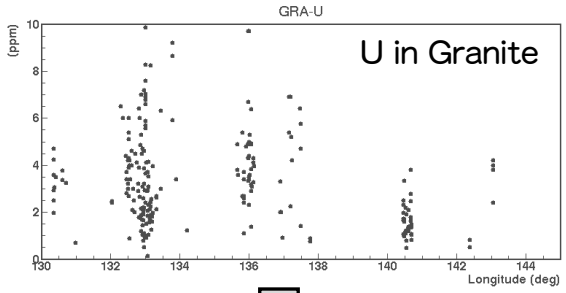
DODAI database
(rock DB with detailed location information)





- Dispersions within a small region is large; can be averaged out
- Differences among regions are negligible ???

Data Driven: Extracting Regional Variations

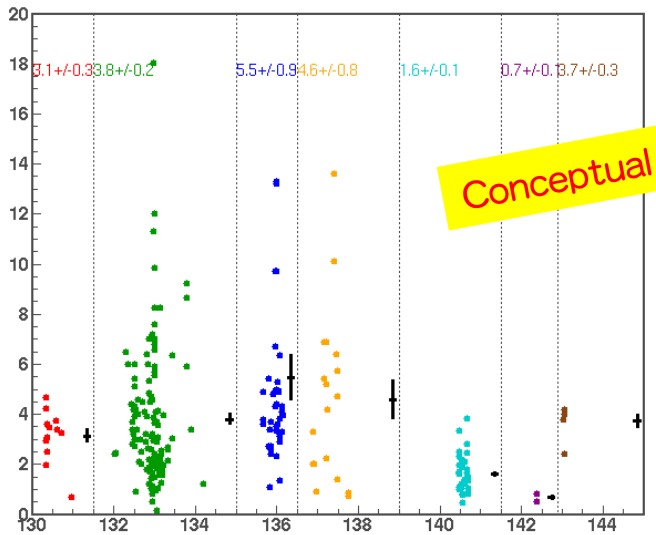


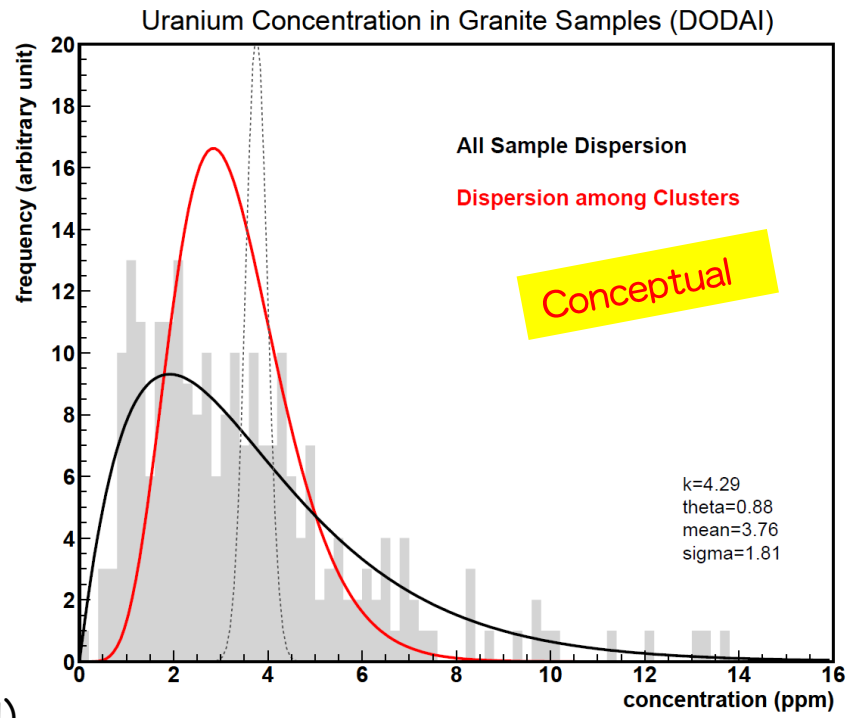
Hierarchical Bayesian Inference

- a_k: variations within the k-th cluster
- θ: variations of a_k

$$\prod_i P_i(a_i, \theta \mid x_i^{obs}) \propto \prod_i P_i(x_i^{obs} \mid a_i) \times P(a_0, a_1, a_2, \ldots \mid \theta) \times P(\theta)$$

Clustering (location & age)





For this particular example (conceptual!),

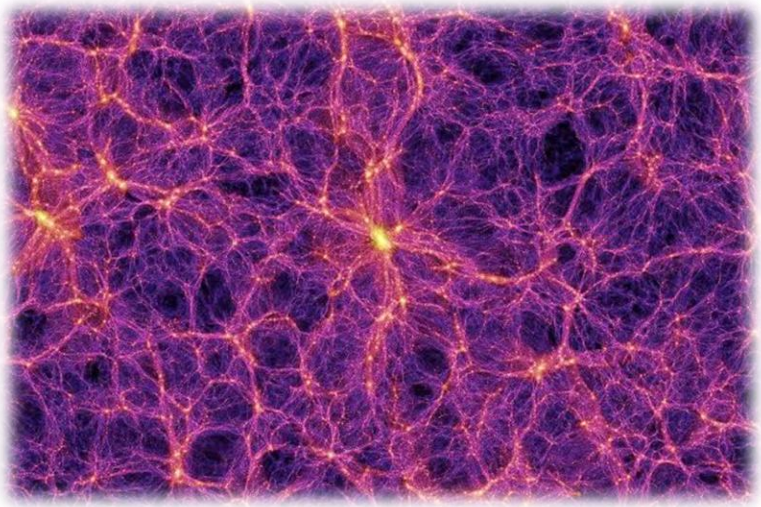
- All-sample dispersion: 48%
- Dispersion among clusters (cannot be neglected): 32%
- Remaining (~35%): dispersion within a region ⇒ sample size? rock-body structure?

Bottom up: What makes the sample variations?

Physicists Simplification on Overdispersion

Attracting force
creates overdispersion

Ex) Cosmic large-scale-structure



Depending on the location,
the local density varies a lot
⇒ inhomogeneous composition

Repulsive force
creates underdispersion

Ex) Young couples in Kyoto (Kamogawa)

Density is constant everywhere
⇒ homogeneous composition

Geological processes are often element-selective → behaves like attractive force
(gradients in chemical potential...)

Model Driven: Understand the Rock Structure

Initial Guess

Overdispersions (micro-variations) are caused by mineral grain structures

- Rocks composed of minerals
- Rock samples must be much larger than the size of mineral grains

Example: Granite

Consists of:
- Quartz
- Feldspar
- Plagioclase
- Biotite

← This sample might not contain a sufficient number of mineral grains

To show this:
- characterize the size variations of the mineral grains
- model the U and Th concentrations (PDF!) of the minerals

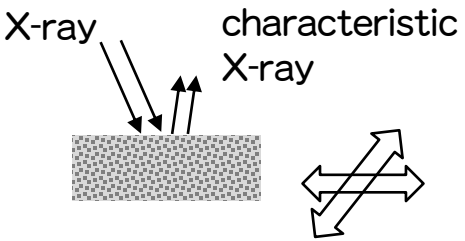
Goal: show the overdispersion factors for various sample sizes

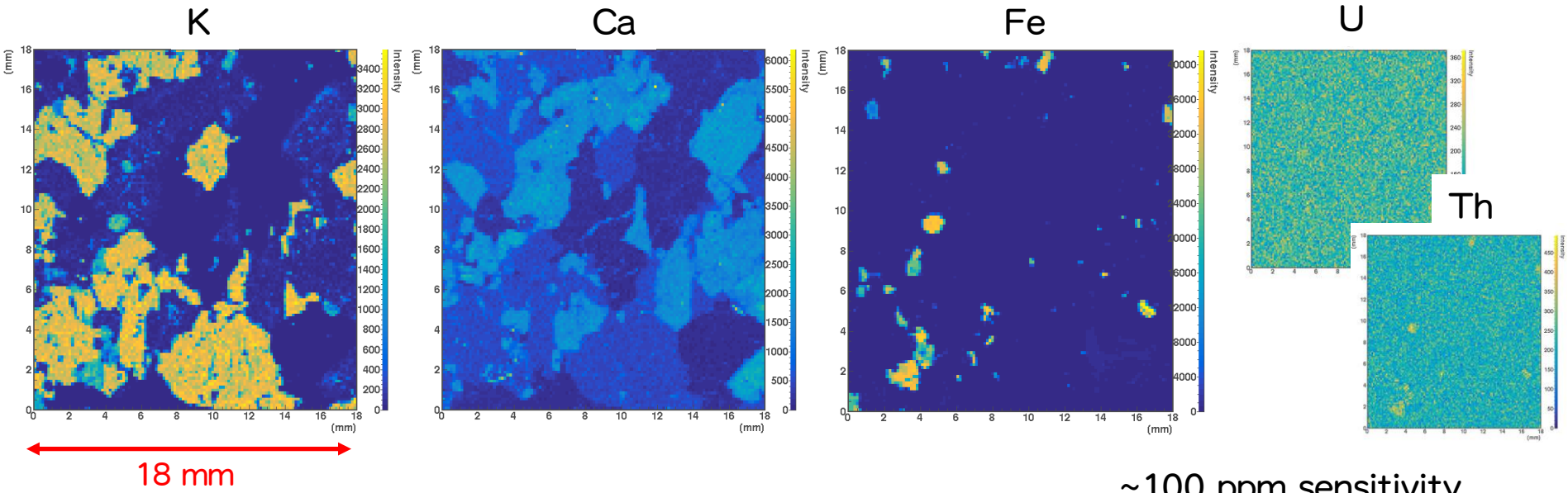
Surface Mineral/Element Mapping using X-ray

Weapon 1: Surface Scanning X-ray Fluorescence (XRF)

- Large area coverage (2 dim mapping)
- Resolution ~25 μm

- Not sensitive to U and Th
- Good enough to identify mineral structures



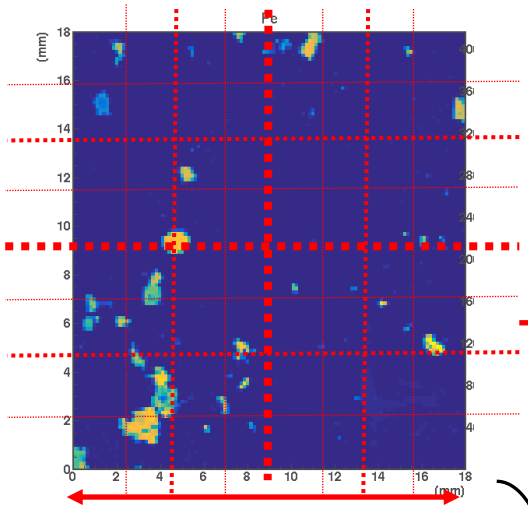


~100 ppm sensitivity
(not sufficient for U / Th)

Internal Structures and Overdispersion

Surface Element Distribution (Fe on Granite, XRF scan)

(true correlation model)

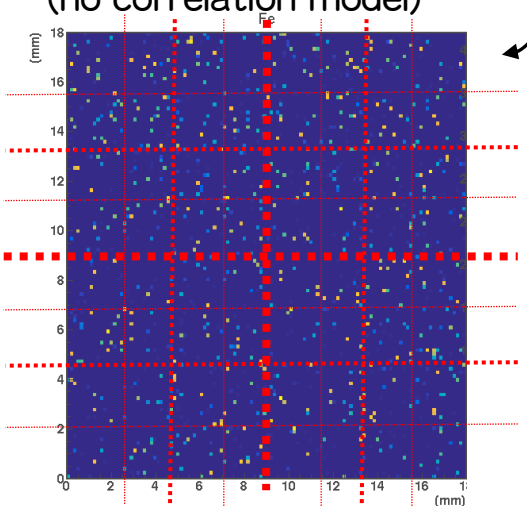


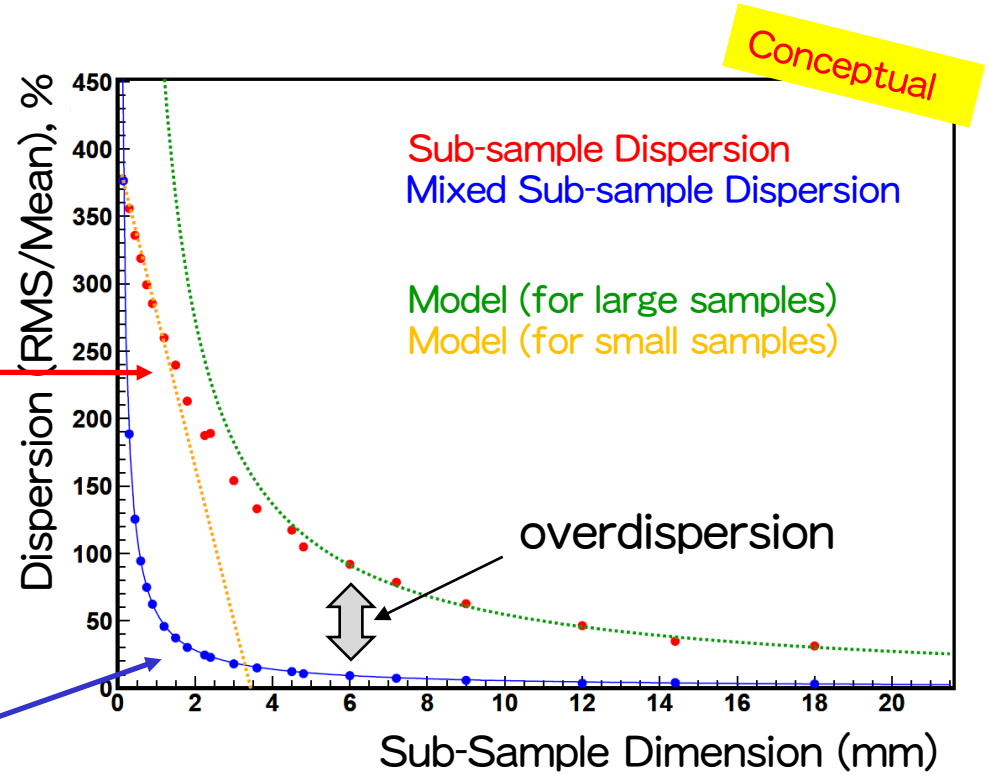
numerically sub-divide the sample

numerically mix the grains

Pixels are mixed

(no correlation model)





Observed Sample Size Dependence

- ~50% dispersion for 1cm x 1cm samples
- ~25% dispersion for 2cm x 2cm samples

Simple overdispersion model works!

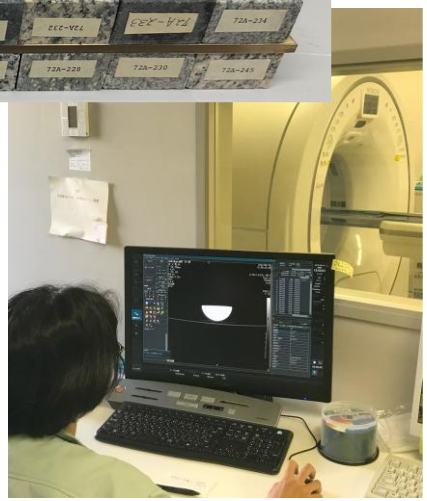
2-D surface model to 3-D structure

Weapon 2: Medical CT

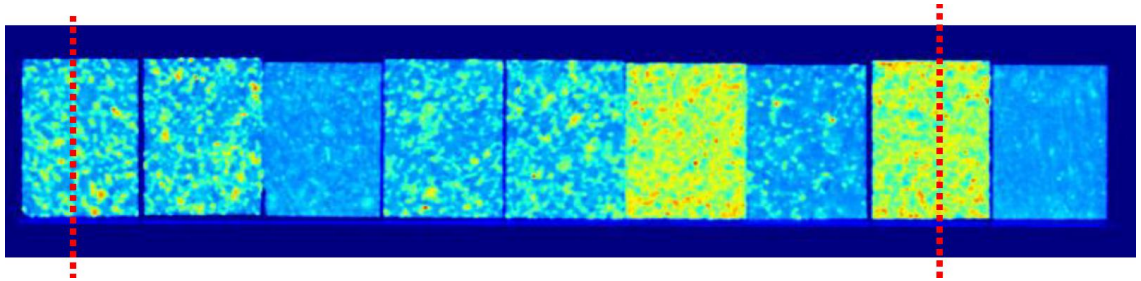
- 3D!!!
- Basically sensitive to density, possibly for element mapping (dual energy mode)
- ~100 μm or better resolution possible

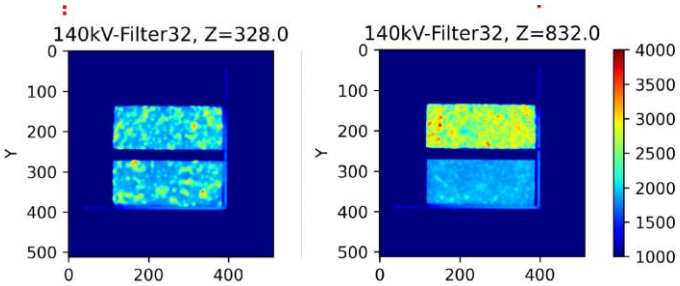
CT Operator

Granite samples from GSJ (we have >1000!!)



3D Internal Structures can be pictured



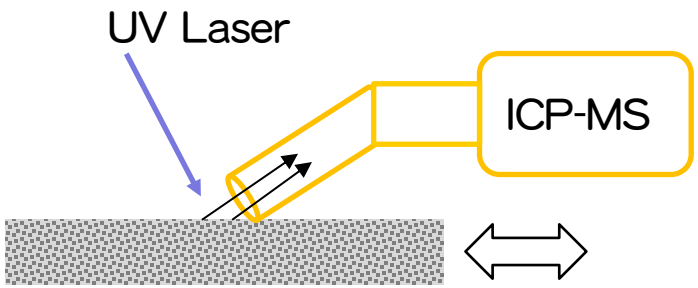


Quantitative analysis (e.g., comparison with XRF) is ongoing

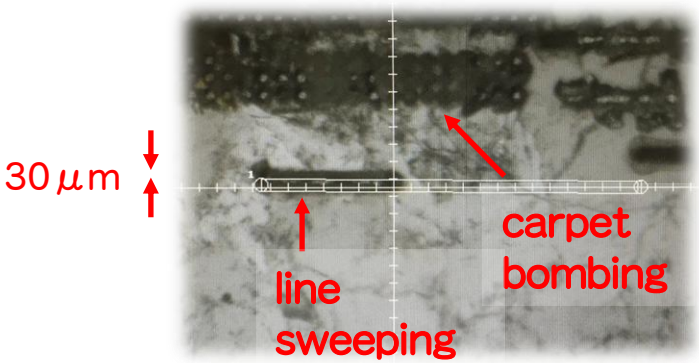
Searching for Uranium and Thorium

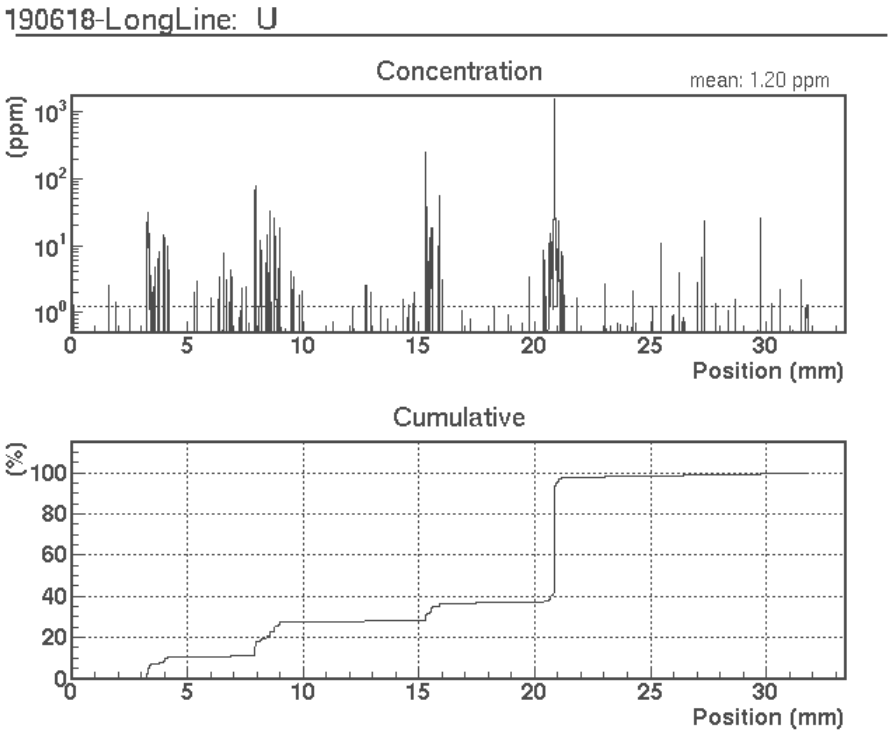
Weapon 3: Laser Ablation ICP-MS

- Sensitive to U and Th, destructive
- ~30 μm resolution
- Line or pin-point bombing



First Test





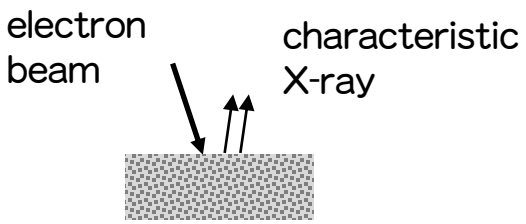
Observation

- Uranium is NOT contained in major minerals (not in "area")
- Maybe in "accessory" minerals?

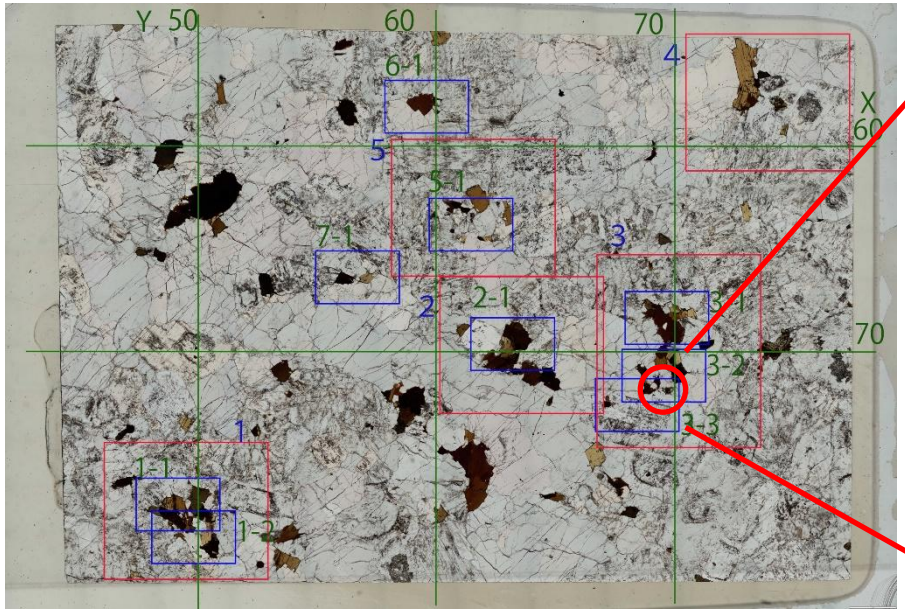
Uranium / Thorium Exploration

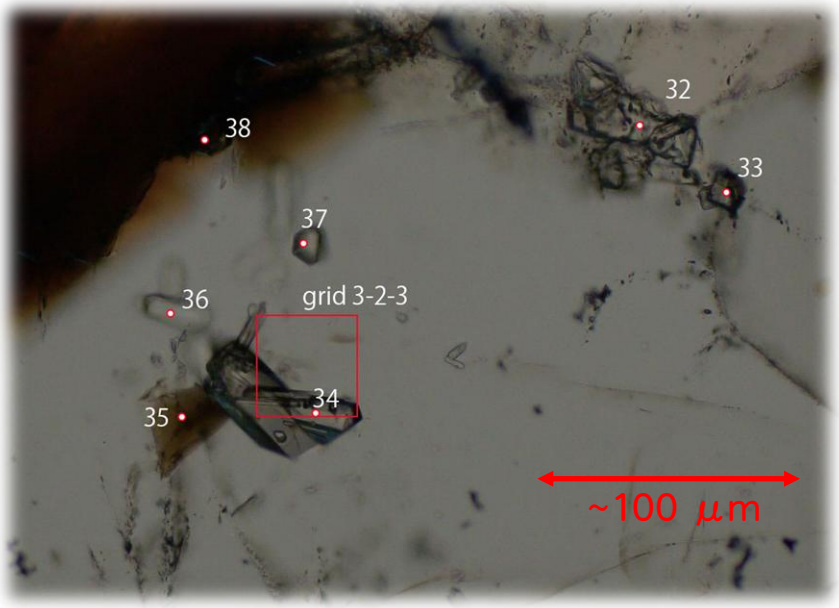
Weapon 4: EPMA (Electron Probe Micro Analyzer)

- High resolution (~1 μm or better)
- Not sensitive to U, limited sensitivity to Th



Electron microscope + X-ray spectroscopy





Observation: Th (and likely U) are contained in small "accessory" minerals

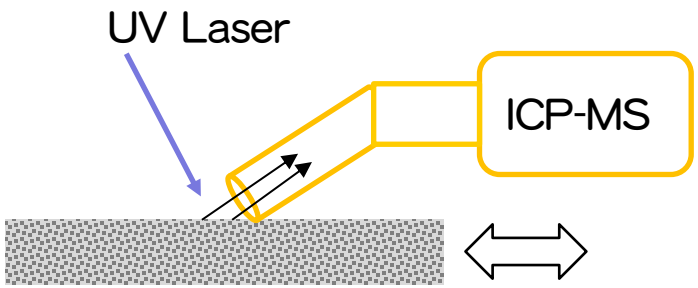
Such U/Th-rich minerals are much more localized than the major minerals

⇒ Overdispersion due to mineralogy can be larger than it appears

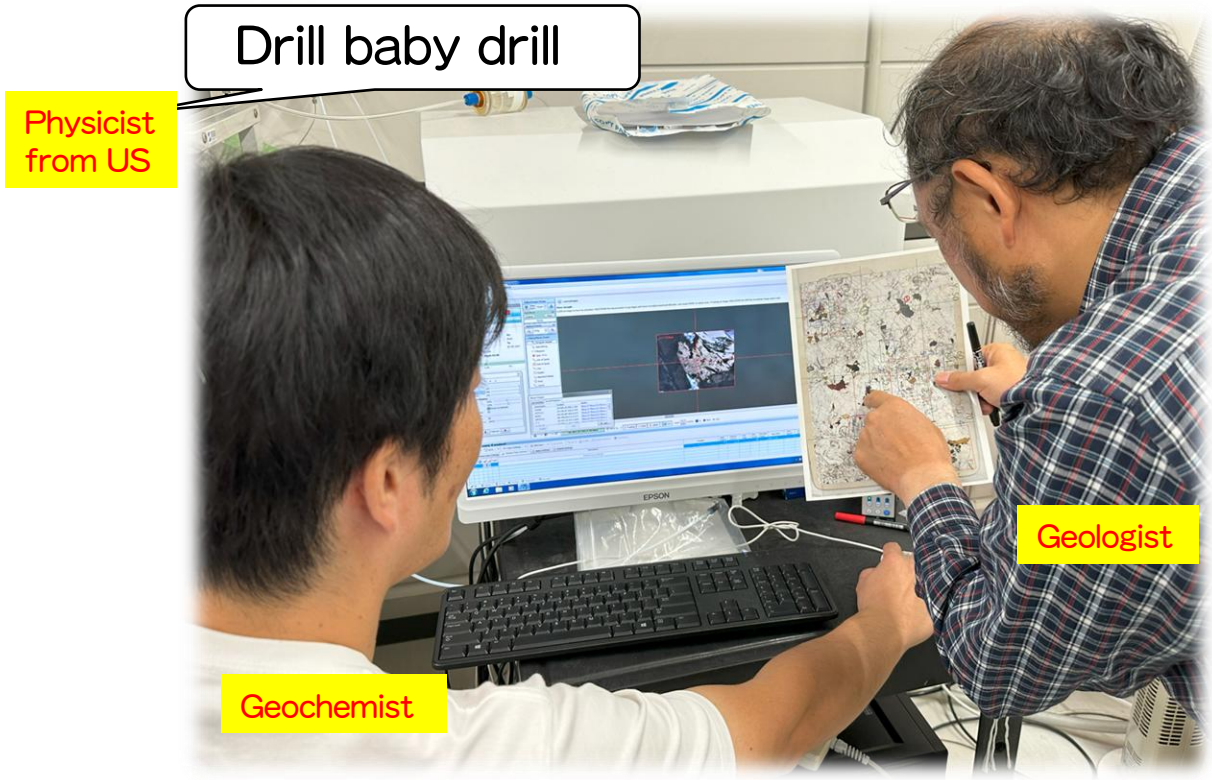
Uranium / Thorium Digging

Weapon 3: Laser Ablation ICP-MS

- Sensitive to U and Th, destructive
- ~30 μm resolution
- Line or pin-point bombing



Second Round Measurement



Precision Guided Bombing
based on the mineralogy map from EPMA

Hope:
accessory minerals contain majority of U and Th

If so, the next step would be mapping the accessory minerals (easier than looking at U/Th)

New Findings and New Problems

New Observation (maybe known to some people...)

U and Th distribute not only in accessory minerals,
but also along the boundaries between mineral grains.

This is understandable, because

- U and Th are large elements, difficult to fit into compact crystals (major minerals)
- During magma solidification, they remain in the residual melt
- Their space becomes smaller and smaller. Two choices:
  - Get gathered, make their own crystal → accessory minerals
  - Cannot find a way out → frozen in between crystals

This explains the large overdispersion well, but

- Quantitative estimation became very difficult (ICP-MS is 1-dim line)
- Needs:
  - high spatial resolution (~μm) AND large size (~10 cm),
  - 2-dim (or 3-dim),
  - sensitive to ppm-level U and Th
- Alpha radiography? Rates are too low for Si detectors (e.g. CCD)...

Breakthrough: Nuclear Emulsion!!

Enhanced photo-films, "early 20th century technology"

WIKIPEDIA
The Free Encyclopedia

Nuclear emulsion

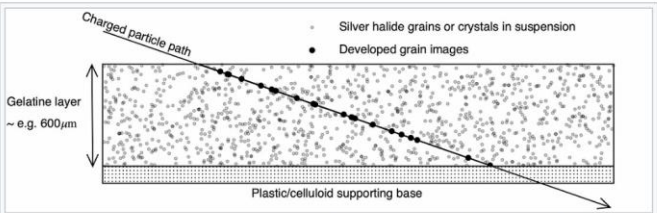
8 languages

Article Talk Tools

From Wikipedia, the free encyclopedia

A **nuclear emulsion** plate is a type of particle detector first used in nuclear and particle physics experiments in the early decades of the 20th century.[1][2][3] It is a modified form of photographic plate that can be used to record and investigate fast charged particles like alpha-particles, nucleons, leptons or mesons. After exposing and developing the emulsion, single particle tracks can be observed and measured using a microscope.

Description [edit]

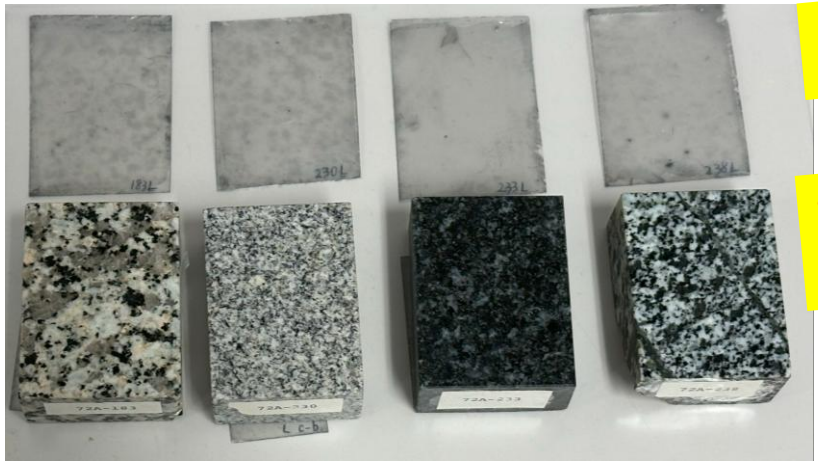


Schematic edge-view cross section of nuclear emulsion, not to scale.

The nuclear emulsion plate is a modified form of photographic plate, coated with a thicker photographic emulsion of gelatine

Survived at Nagoya University
(with lots of unique applications)

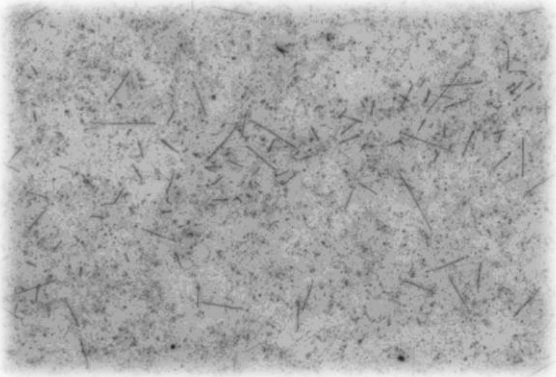
First Measurement in 2024



Developed films

Granite samples from GSJ

Aligned Alpha Tracks Pictured!!!

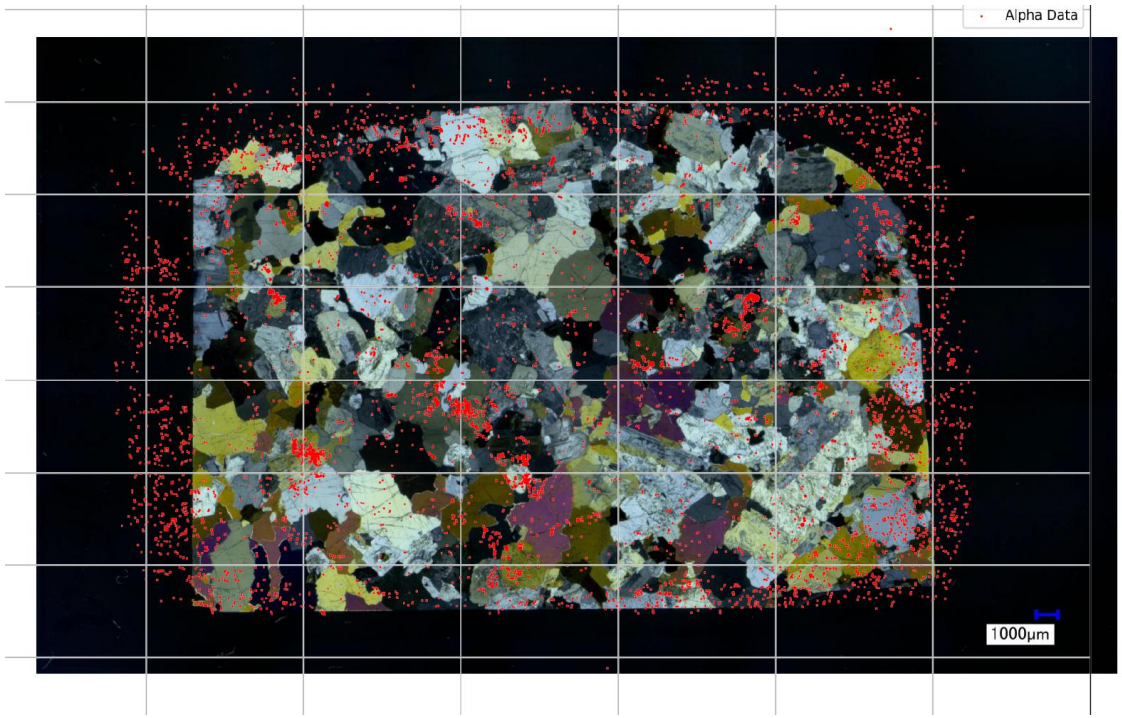


alpha tracks
(thick straight lines)
typ. ~ 10 μm long

Nagoya University Technology Pool: Automated Film Scanning, AI Track Recognition, …

See Next Talk by Nishikiori

Spoiler: Alpha tracks overlaid on mineral structures

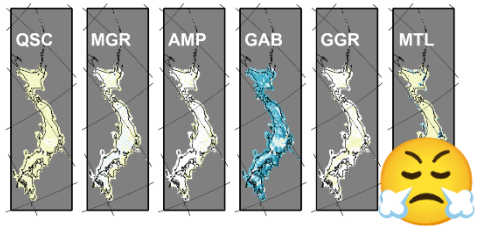


The non-uniformity of U/Th distribution is much larger than you would think from the appearance (mineral structures)

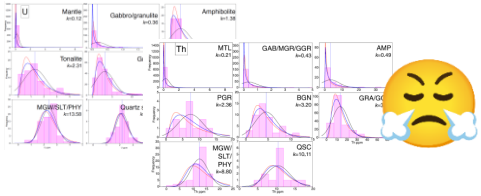
Summary and Outlook

Part I: Building a Fully Stochastic Crustal Model

3D Lithology Inference (PDF)

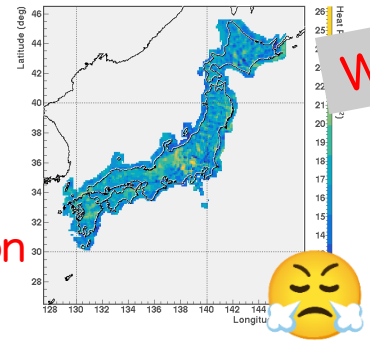


Bias-Free Composition Model (PDF)



Convolution

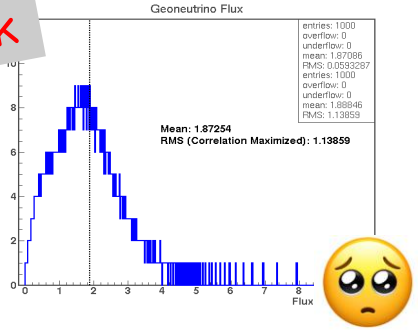
3D U&Th Map (PDF)



Weakest Link

MC Integration

Neutrino Flux at KamLAND (PDF)



Problem: Multiple PDFs cannot be integrated without correlation information

Part II: Understanding the Overdispersion in Rock Samples (to subtract)

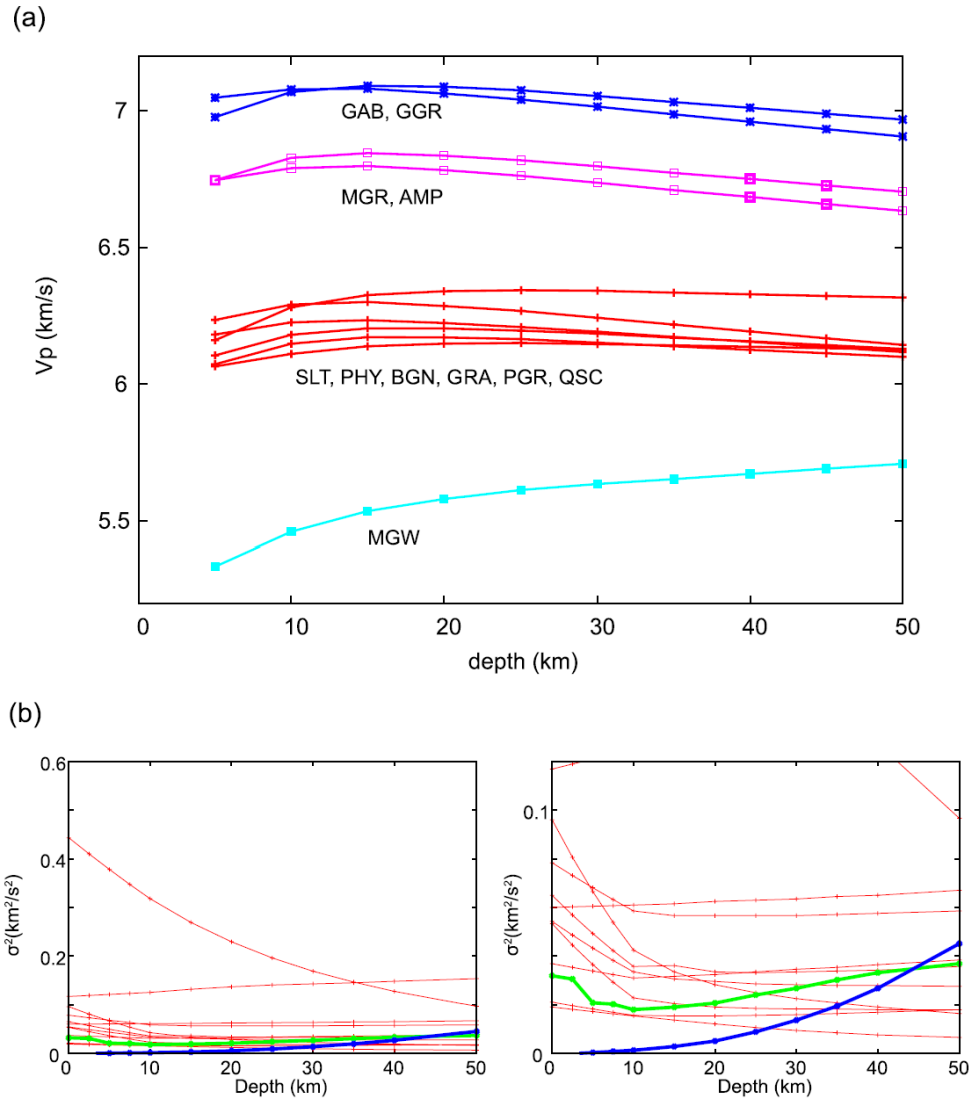
Data Driven Approach: Separation of local and regional variations (prototype working)

Model Driven Approach: U/Th non-uniformity structure is larger than you would think

- Simple overdispersion model seems effective
- New method to picture U/Th distribution structures → next talk by Nishikiori

Appendix

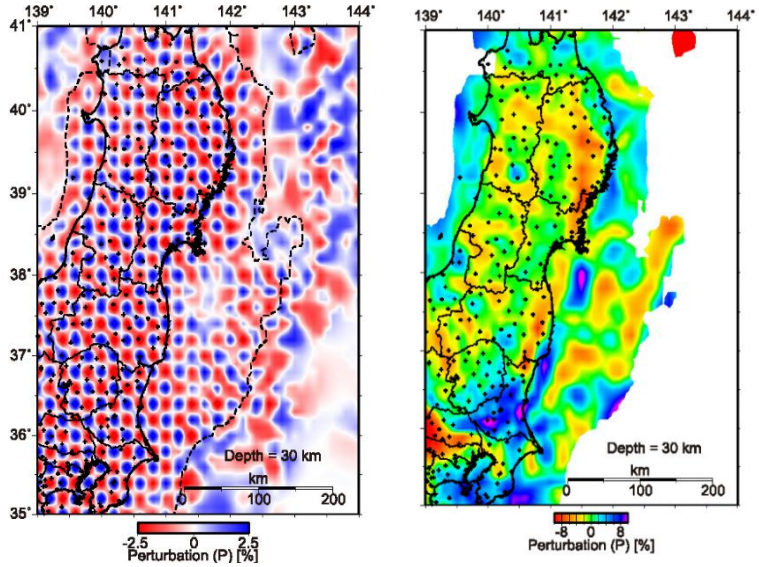
Lithology Vp Model



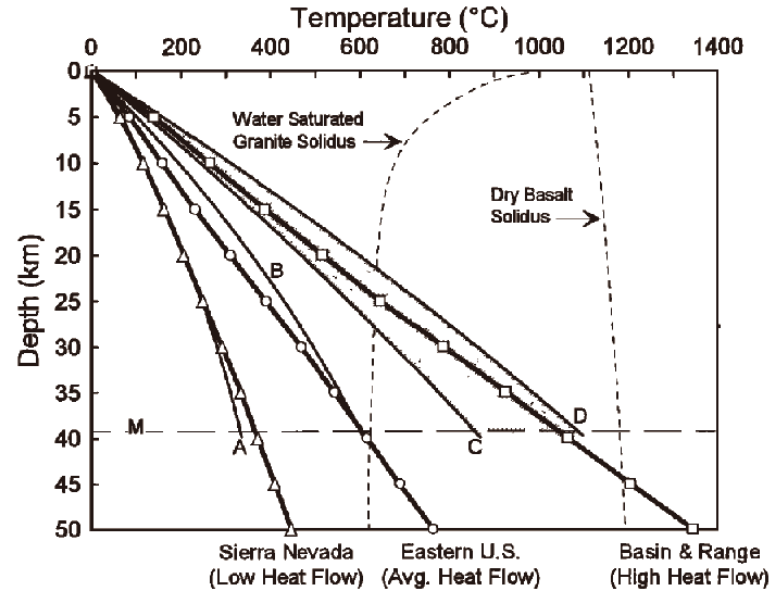
Tomography Model Errors

Tomography Resolution and Errors

Evaluated by pattern recovery test



Temperature Variations



$$P(i \mid V_p^{obs}) = \frac{P\left(V_p^{obs} \middle| i\right) P(i)}{\sum_j P\left(V_p^{obs} \middle| j\right) P(j)}$$

$$P(V^{obs} \mid i) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma^{obs\,2} + \sigma^{temp\,2} + \sigma^{(i)\,2}}} \exp\left(-\frac{\left(V^{obs} - V^{(i)}\right)^2}{2\left(\sigma^{obs\,2} + \sigma^{temp\,2} + \sigma^{(i)\,2}\right)}\right)$$

$V^{(i)}$: Velocity in the laboratory experiment data

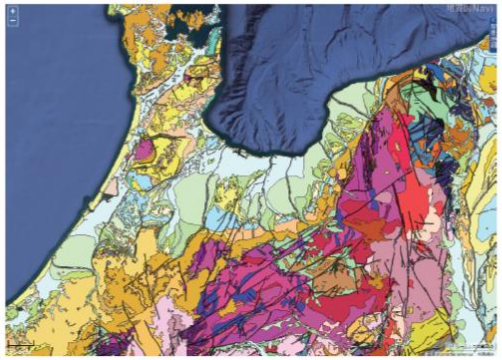
$\sigma^{(i)}$: Velocity fluctuations in laboratory experiment data

σ^{obs}: Error of the tomography data

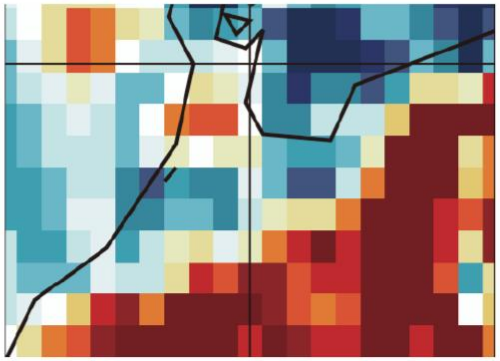
σ^{temp}: Error of the standard temperature profile

Another Scale Factor: Seismology Resolution

Geological Map



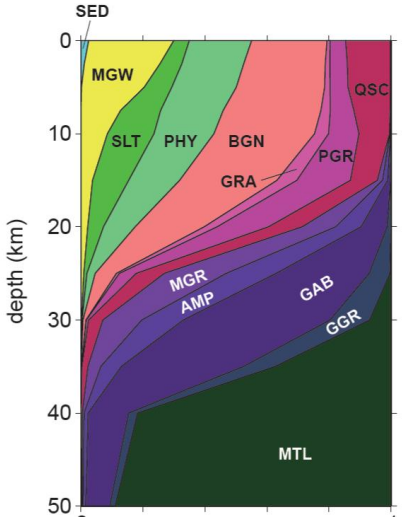
Tomography Model



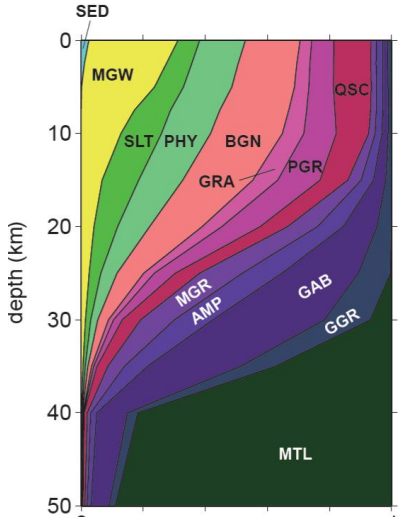
Bayesian Translator was extended to include Two Rock Mixture

$$P(ij,q) \propto (1-q)P(i) + qP(j) \qquad P(ij,q|v^{obs}) = \frac{P(v^{obs}|ij,q)\,P(ij,q)}{\sum_{i',j'(i'\neq j')} \int P(v^{obs}|i'j',q)\,P(i'j',q)\,dq}$$

Mono- Lithology Posterior



Bi- Lithology Posterior



Small differences

⇩

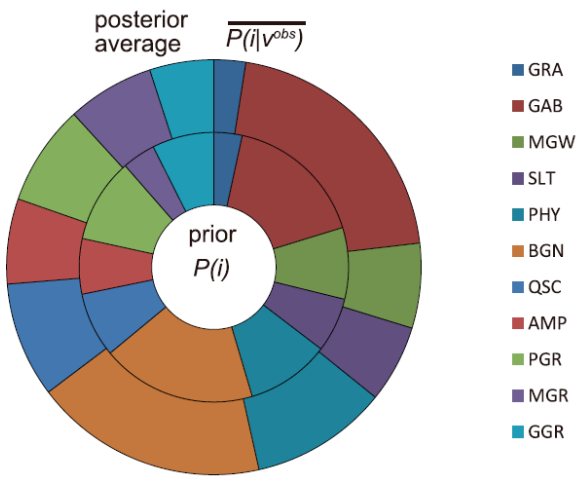
Cells are small enough

⇩

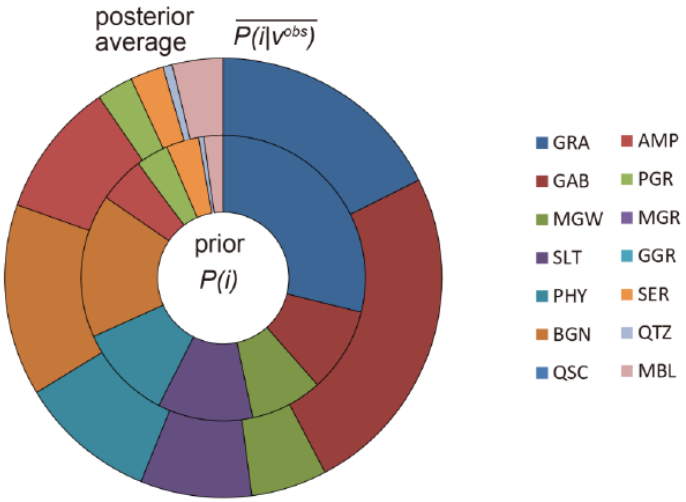
Inter-cell correlation can be significant

Prior / Posterior Comparison

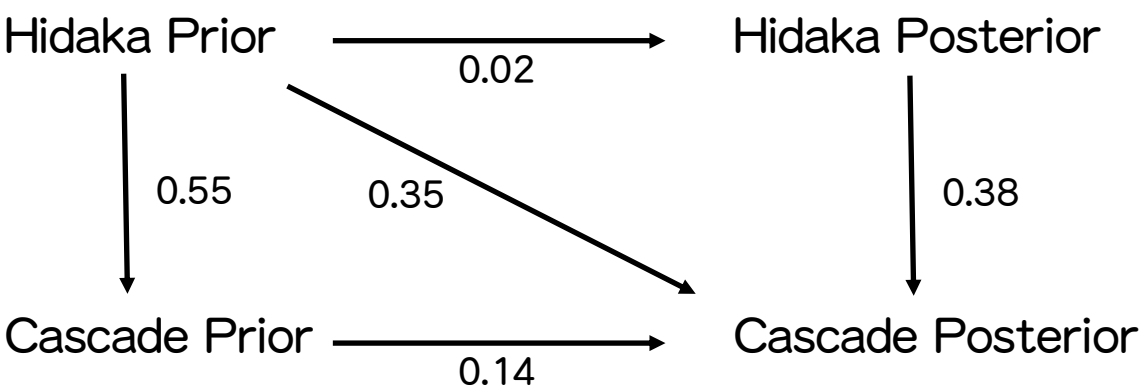
Hidaka (North Japan) Prior



North-Cascade (Seattle Region) Prior



Kullback-Leibler divergences



Zero-Bias Composition Model

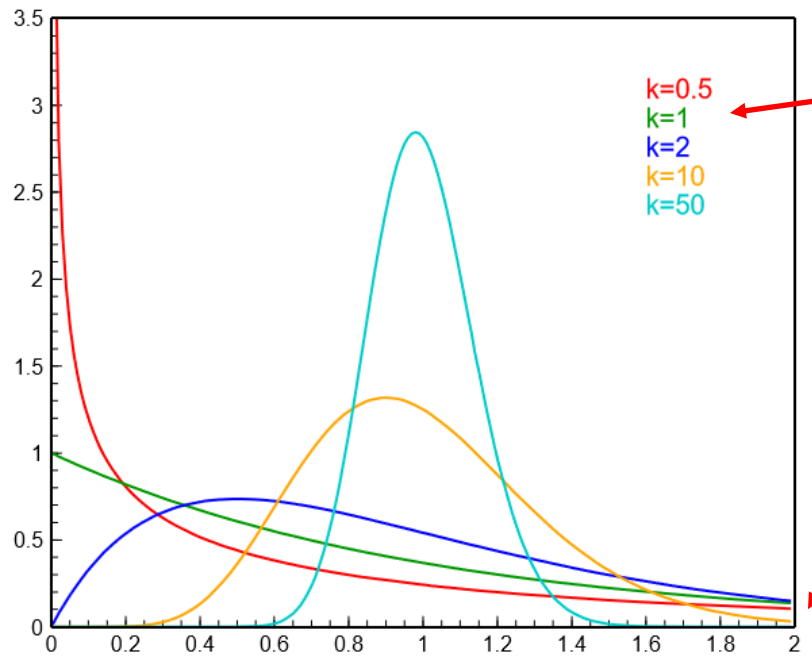
- Normal model conserves mean and sigma
- Log-normal model conserves geometric mean (~median) and log-sigma

- If mean and geometric mean are to be conserved? ⇒ Gamma distribution

$$f(x;k,\theta) = \frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\theta^k}$$

k: shape parameter θ: scaling parameter

mean: $k\theta$ skewness: $1/k$



Parameter k controls skewness

$k \to 0$ ⇒ 1/x distribution
$k = 1$ ⇒ exponential distribution
k large ⇒ gaussian-like

Parameter θ scales horizontal axis

more info here

Maximum Likelihood Fitting of Gamma Distribution

Gamma distribution PDF is:

$$f(x;k,\theta) = \frac{x^{k-1}e^{-x/\theta}}{\Gamma(k)\theta^k}$$

For samples $\{x_1, \dots, x_N\}$, log likelihood is:

$$\log L_{(k,\theta)} = (k-1)\sum_i \log x_i - \frac{1}{\theta}\sum_i x_i - N(\log\Gamma(k) + k\log\theta)$$

To find maximum w.r.t. θ , take the derivative:

$$\frac{\partial \log L}{\partial \theta} = \frac{1}{\theta^2}\sum_i x_i - Nk\frac{1}{\theta}$$

Setting the derivative zero yields:

$$\hat{\mu} = \hat{\theta}\hat{k} = \frac{1}{N}\sum_i x_i$$

Therefore, for any underlying sample distribution,

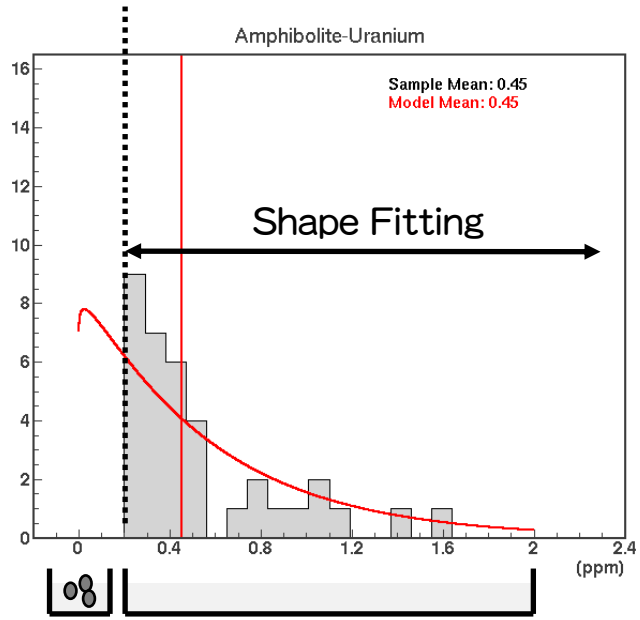
Expected value of fit PDF == Mean value of samples

Detection Limits and Outliers

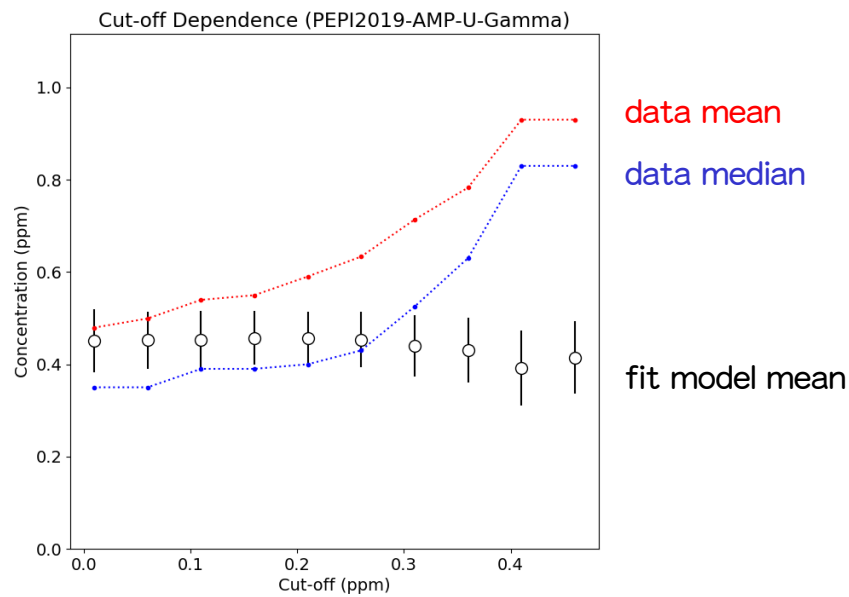
Likelihood is extended to account for cut-offs

Data cut-off at 0.2 ppm ⇨ Take into account the probability of Not-Detected

$$L = L_{\text{shape}} + L_{\text{count-below-cutoff}}$$



Binominal Counts under Cutoff

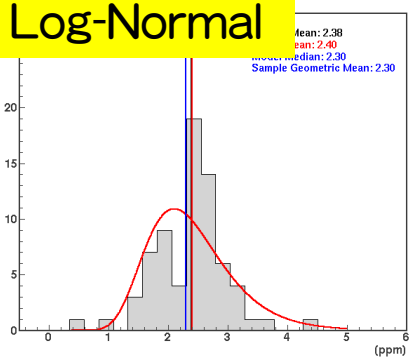


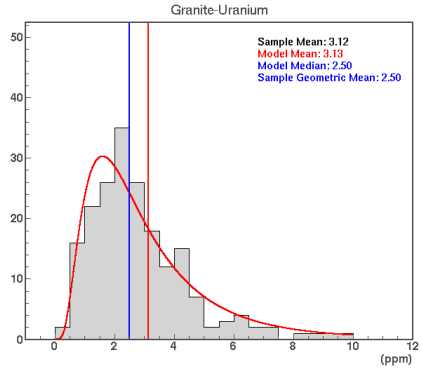
- Polynomial can be used for both lower/upper cutoffs to remove outliers. (No need for median anymore??)
- The conventional *Quantile Cut* without this treatment will cause bias.

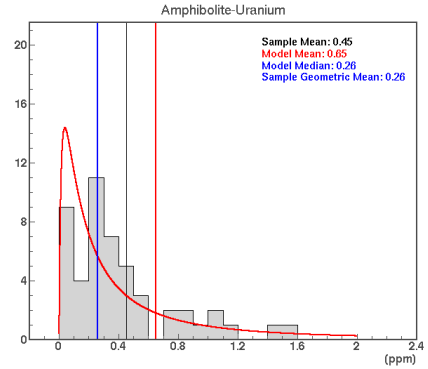
Bias Comparison

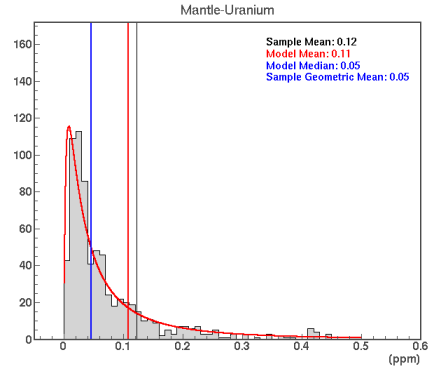
SLT / MGW / PHY | Granite | Amphibolite | Mantle

Log-Normal

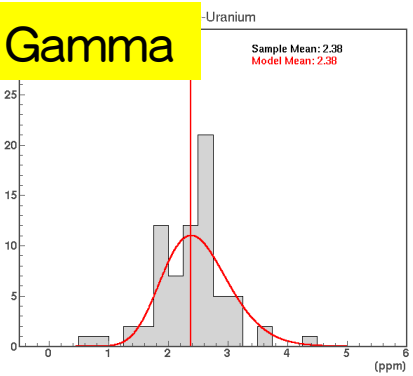


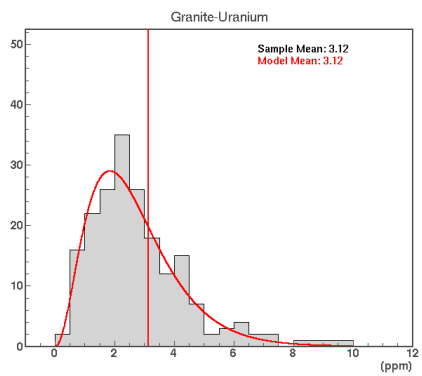


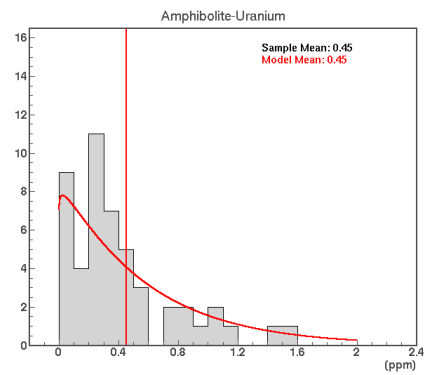


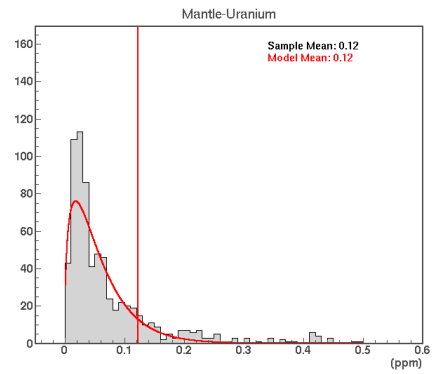


Gamma









Deeper / More Depleted / More Skewed / More Bias in Median

- Bias in Log-Normal model depends on how well data fits to the model
- Bias in Gamma model is free from bias, independent from data distribution
- Median / Geometric-mean is out of question

Goodness-of-fit Comparison

AIC (Akaike Information Criterion): Model Selection Criterion

WIKIPEDIA
The Free Encyclopedia

Main page
Contents
Featured content
Current events

Akaike information criterion

From Wikipedia, the free encyclopedia

The **Akaike information criterion** (**AIC**) is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection.

$$\mathrm{AIC} = 2k - 2\ln(\hat{L})$$

k: number of model parameters

(Caution: this is not the only model selection criterion; there are AIC, BIC, CIC, DIC, EIC, FIC, GIC, HIC, IIC, ⋯)

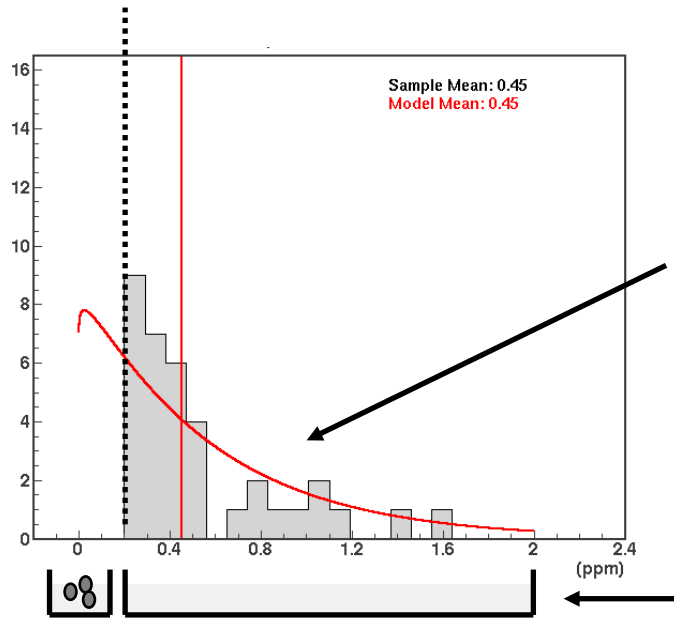
	Rock type	Element	Gamma	Normal	Log-normal
Amphibolite	AMP	U	7.83	43.73	5.83
		Th	231.29	355.48	236.82
Biotite Gneiss	BGN	U	134.86	153.07	138.23
		Th	210.70	219.14	214.50
Gabbro & Granulite	GAB/MGR/GGR	U	-28.91	327.57	-33.52
		Th	53.74	497.42	43.75
Granite	GRA/GGN	U	797.72	918.18	784.62
		Th	1304.02	1374.56	1299.10
Slate etc	MGW/SLT/PHY	U	26.52	126.57	146.34
		Th	399.57	370.09	421.02
Mantle	MTL	U	6.52	2251.65	-9.09
		Th	141.09	2518.97	116.48
Para-granulite	PGR	U	25.15	26.79	25.66
		Th	67.78	65.71	71.90
Mica / Quartz	QSC	U	24.85	24.21	25.60
		Th	67.15	66.91	67.88

: obviously wrong

One Step Further: Detection Limits and Outliers

Likelihood is extended to account for cut-offs

Data cut-off at 0.2 ppm



Take into account the probability of Not-Detected

$$L = L_{\text{shape}} + L_{\text{cutoff}}$$

Usual likelihood for samples above cutoff
(with re-normalization to account for cutoff)

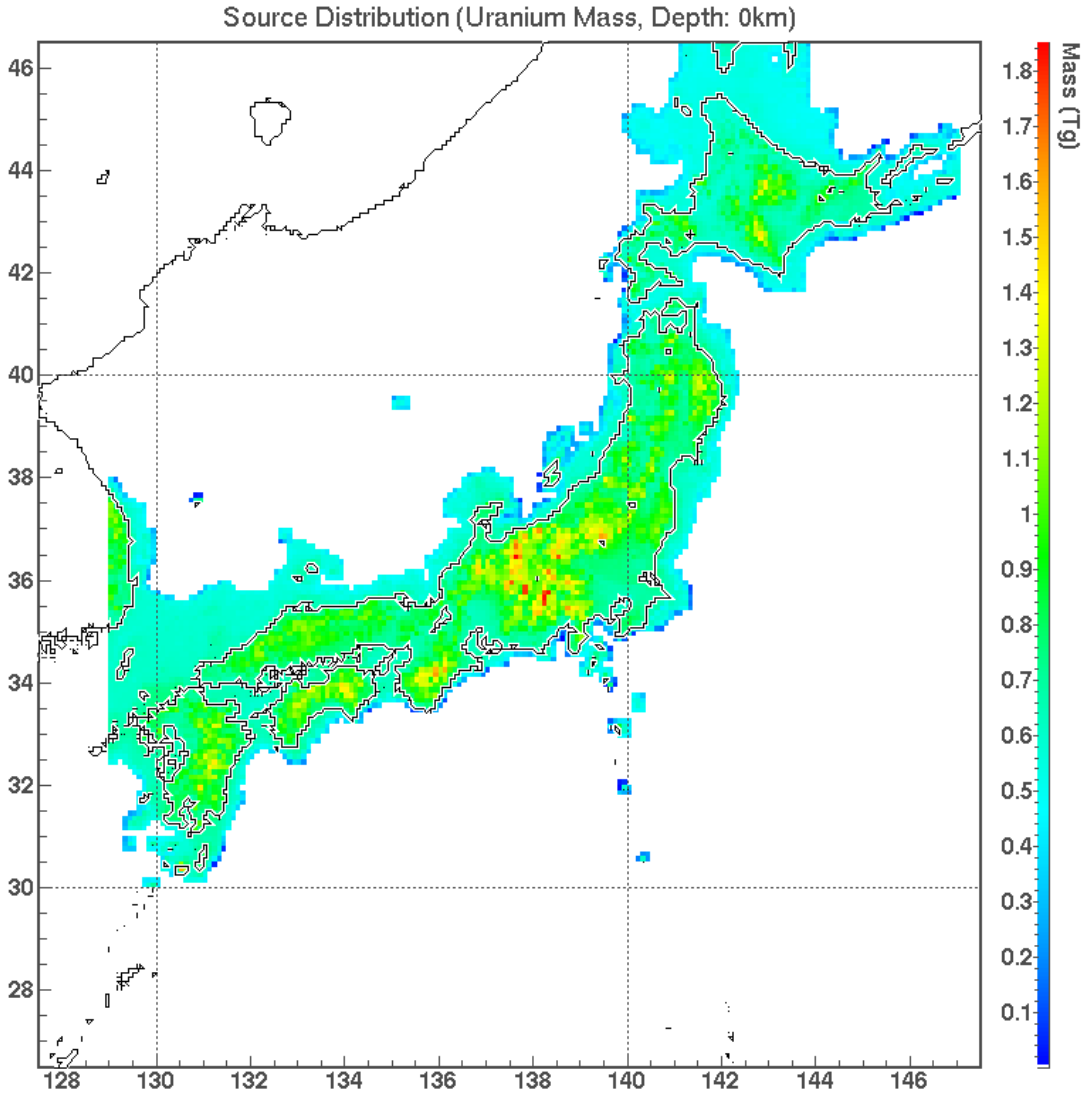
$$-2\ \log \mathcal{L}_{\text{shape}}(\boldsymbol{p};\boldsymbol{a}) = -2\sum_{j=1}^{N} \Theta(a_j - B)\ \log\left\{\frac{f(a_j;\boldsymbol{p})}{\int_B^\infty f(\alpha;\boldsymbol{p})\,d\alpha}\right\}$$

Binominal is used for numbers-below-cutoff

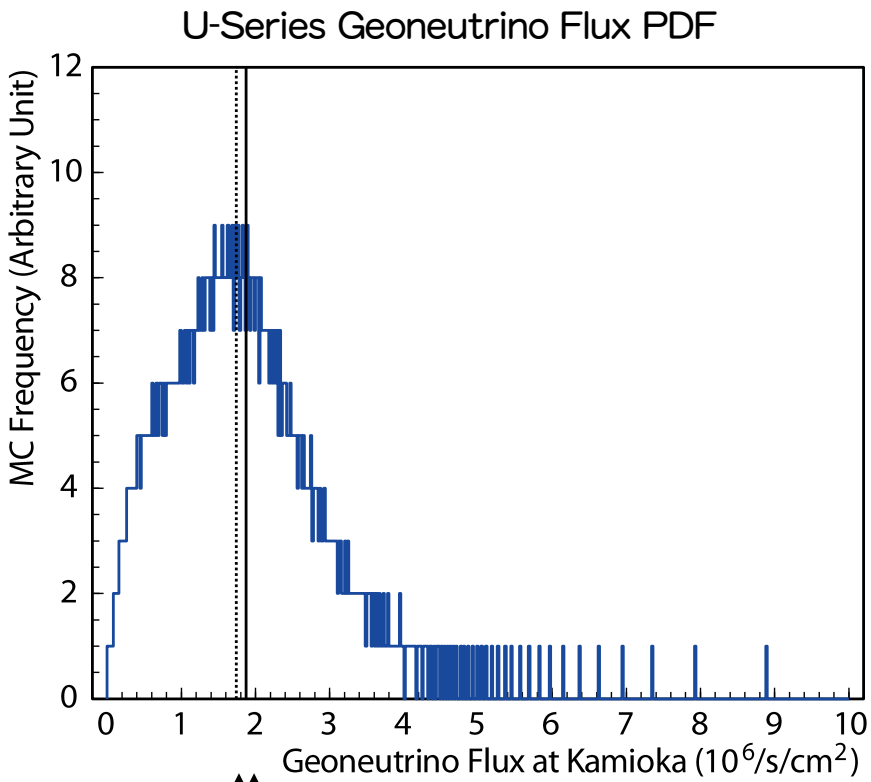
$$-2\ \log \mathcal{L}_{\text{count}}(\boldsymbol{p};\boldsymbol{a}) = -2\log\left\{\binom{N}{N_0}\left(\int_0^B f(\alpha;\boldsymbol{p})d\alpha\right)^{N_0}\left(\int_B^\infty f(\alpha;\boldsymbol{p})d\alpha\right)^{N-N_0}\right\}$$

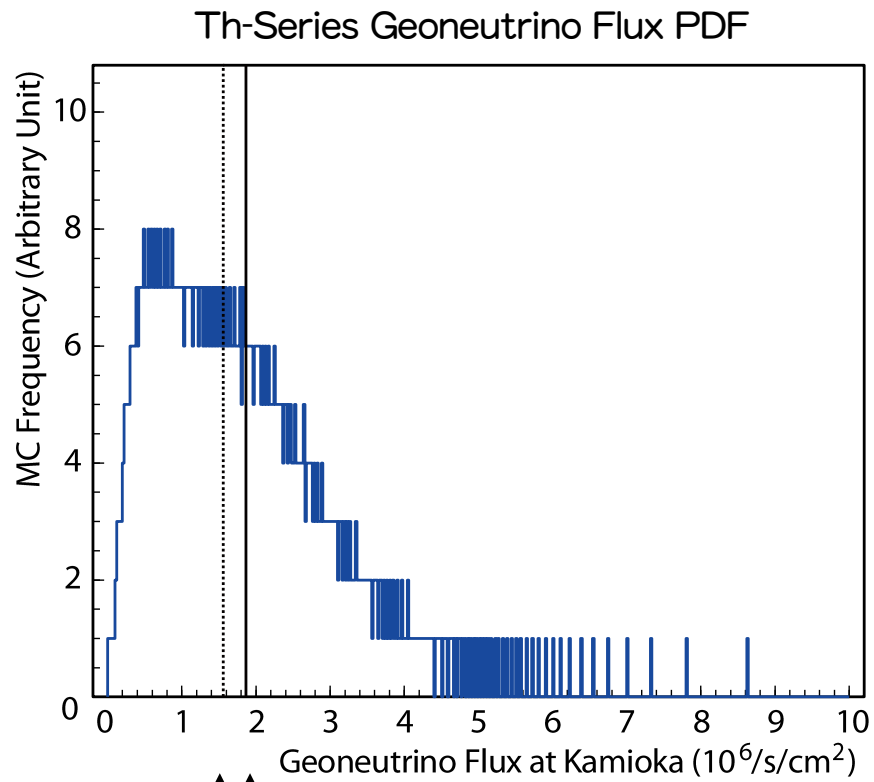
- Polynomial can be used for both lower/upper cutoffs to remove outliers. (No need for median anymore??)
- The conventional *Quantile Cut* without this treatment will cause bias.

Surface Uranium Map (Mean Value)



Comparison with Previous Models





Previous Subjective Model
(Enomoto et al. 2007)

This Study: Bias-Free Mean

- Due to large errors, everything is consistent …

I'm from Seattle. Do I need an umbrella today?

Hourly probability of rainfall
(be a Bayesian for now)

What is the probability of "not raining at all"?

P(no-rain) = 0.78 x 0.73 x 0.45 x 0.45 x 0.53 x 0.40 x 0.51 x 0.40 = $\boxed{0.5\%\ ???}$

I'm from Seattle. Do I need an umbrella today?

Daily forecast: 60% rain

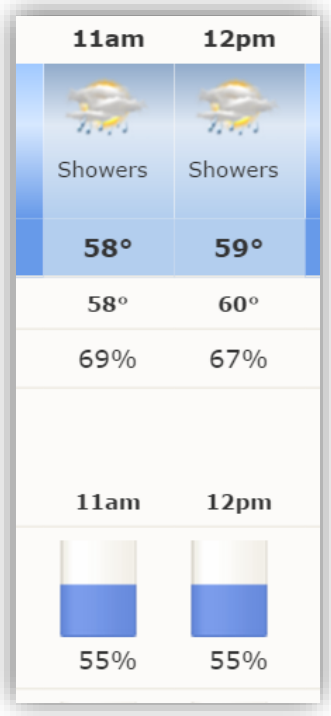
Hourly probability of rainfall
(be a Bayesian for now)

What is the probability of "not raining at all"?

P(no-rain) = 0.78 x 0.73 x 0.45 x 0.45 x 0.53 x 0.40 x 0.51 x 0.40 = $\boxed{0.5\%\ ???}$

Of course not. These are not independent from each other.

Weather forecasts must provide correlation matrix



Scenario 1: full correlation

"The overcasting cloud might be a rain cloud, 55% chance"

⇒ 55% chance of rain in this two hours

Scenario 2: anti-correlation

"An intensive thunderstorm is coming, not sure when it arrives"

⇒ 100% chance of rain in this two hours

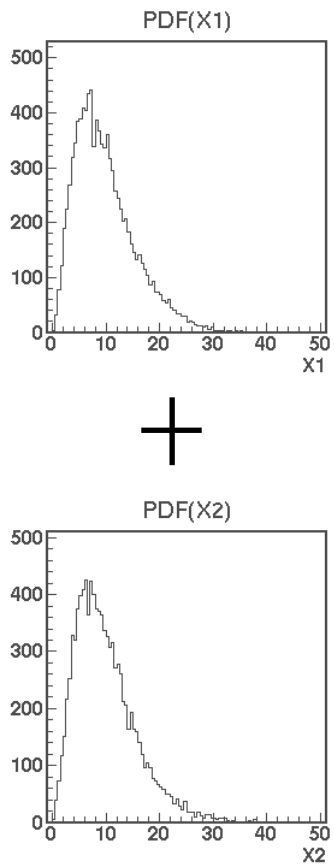
(be a Bayesian for now)

Depending on correlations, a sum of two PDFs can be a any number

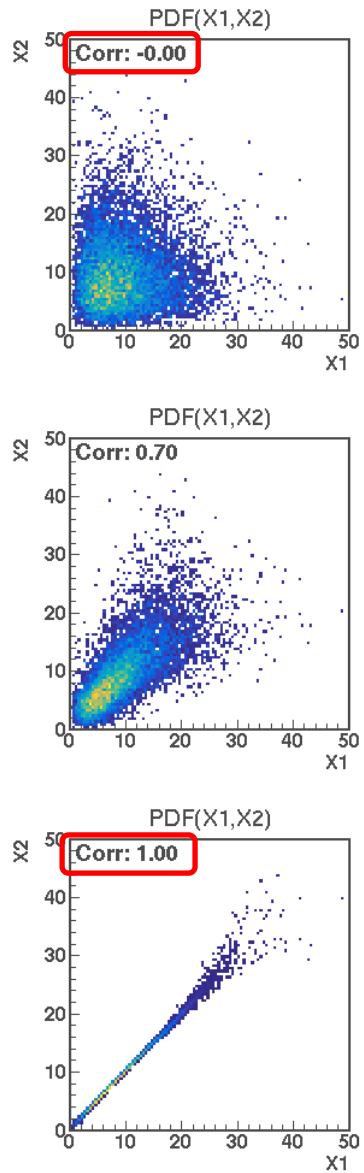
Sum over PDFs and Correlations

Example: Summing probabilistic values in two locations / elements / reservoirs / whatever

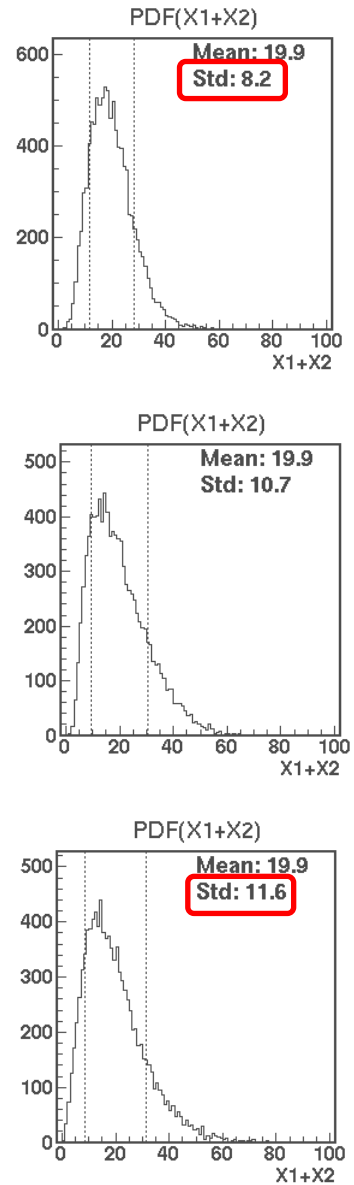
N=2 System



Combine



Sum



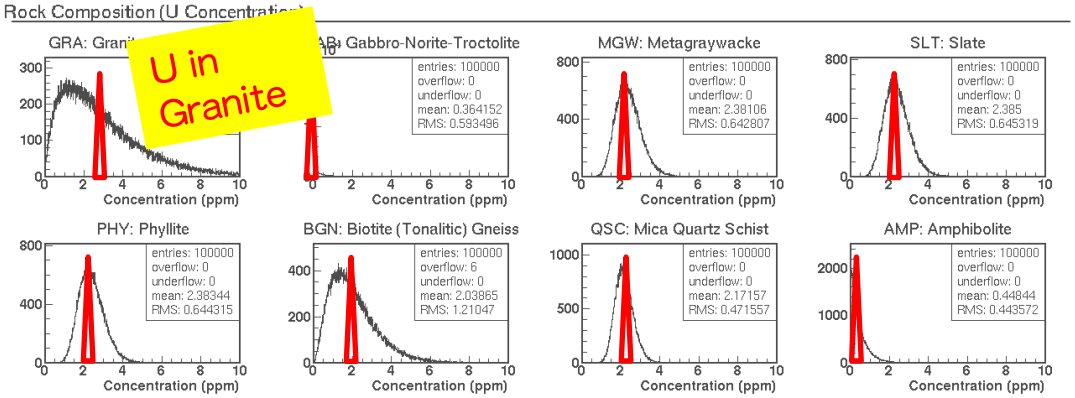
Small Correlation

Small Errors
(~1/√N)

Large Correlation
Large Errors

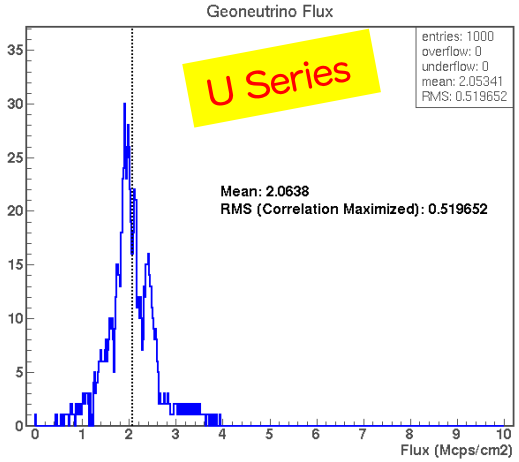
If Compositional Correlations were Neglected

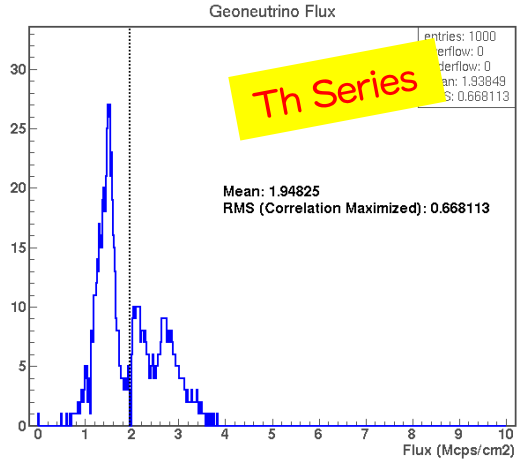
Rock Composition Models



- Many other studies already use mean & error-of-mean

Crustal Geoneutrino Flux





Errors 25~30%
(12~15% of total flux)

- This neglects regional variations. Isn´t it too aggressive?

- Micro-variations can be neglected
- Regional variations are not

Correlations: What other people are doing?

Past and Present Approaches

Correlation Unknown

Model it

- Variogram
- Surface map
- By comparison
- *Insights*

So far none is stochastic

Assume maximum

Too large errors to be usable

Use global average

e.g., quoting Rudnick & Gao (2014)

Underestimation of errors, if error-of-mean is used (common)

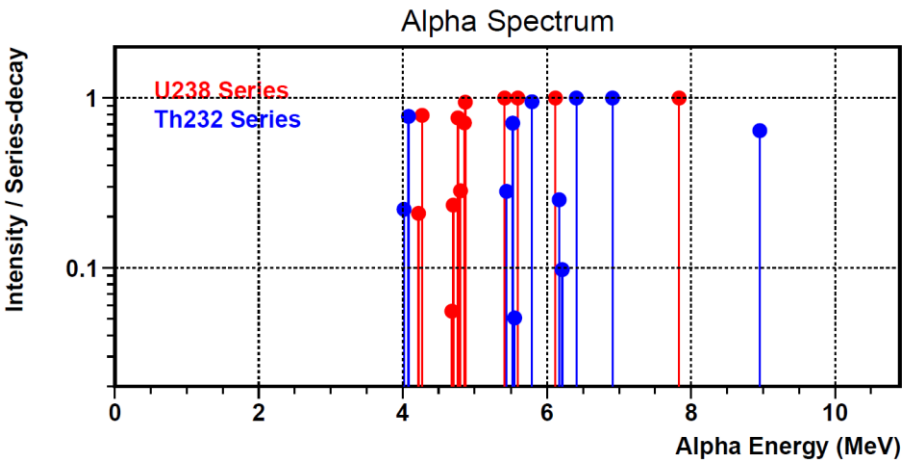
Possibly biased if median is used

Neglect

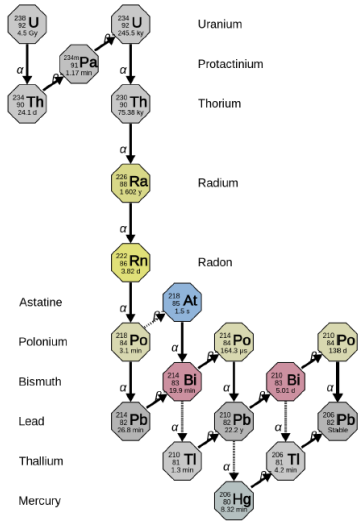
sometimes without recognizing it

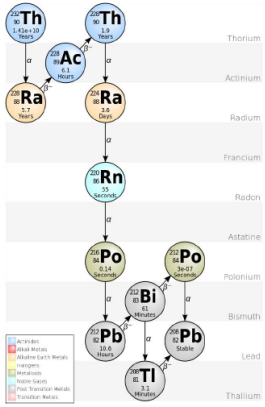
Totally wrong (error depends on the number of cells; ⇒ arbitrary error!)

Alphas in Rocks and Emulsion Film

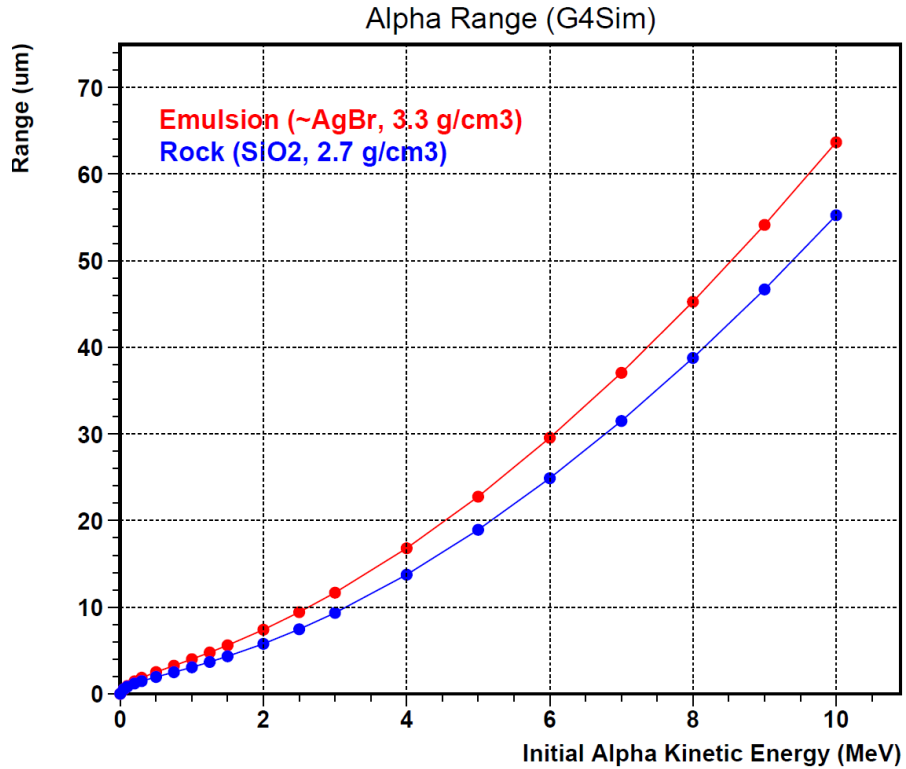


Alphas from U and Th decay chains





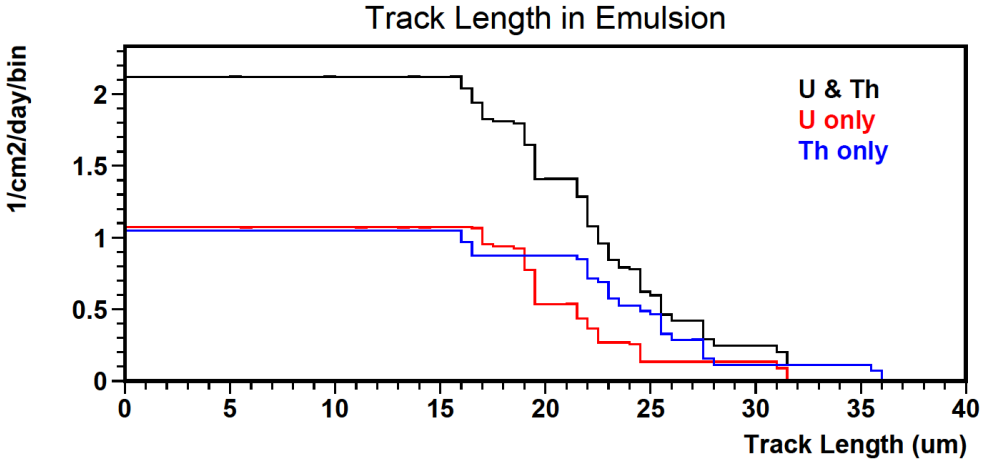
(Rn could be a breaking point)

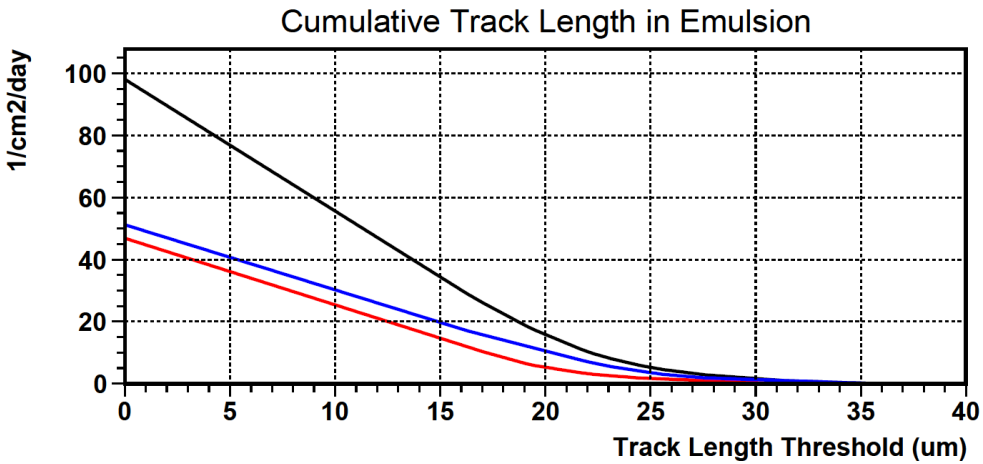


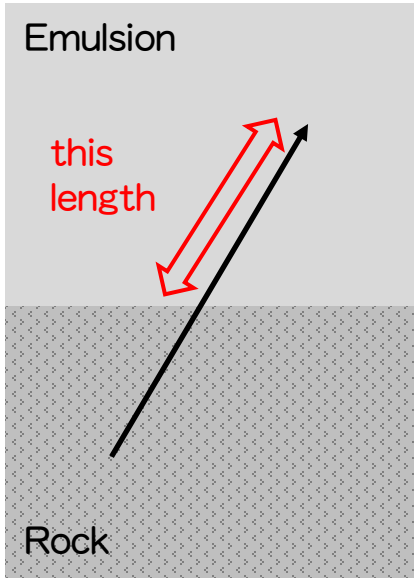
10~50 μm range:
Perfect match to film thickness!

Alphas in Rocks and Emulsion Film

Rates and Track Lengths Simulation



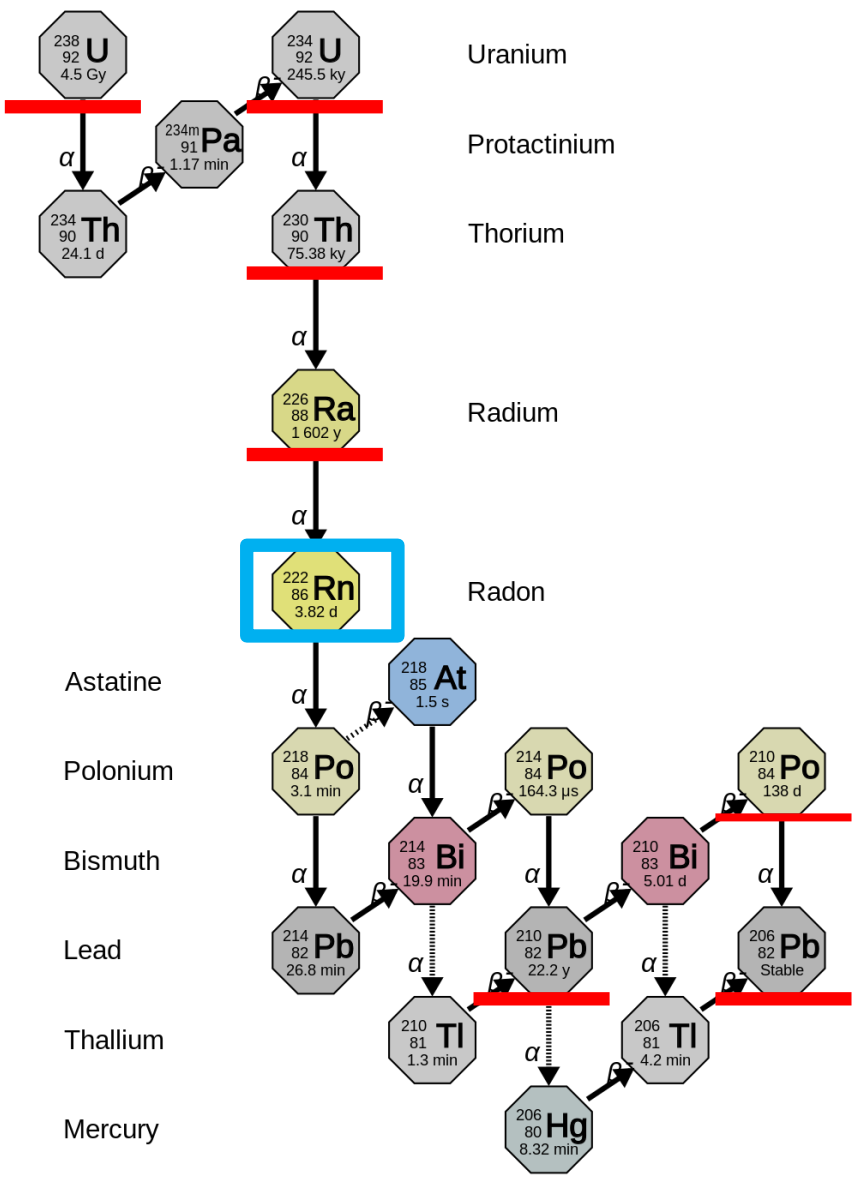


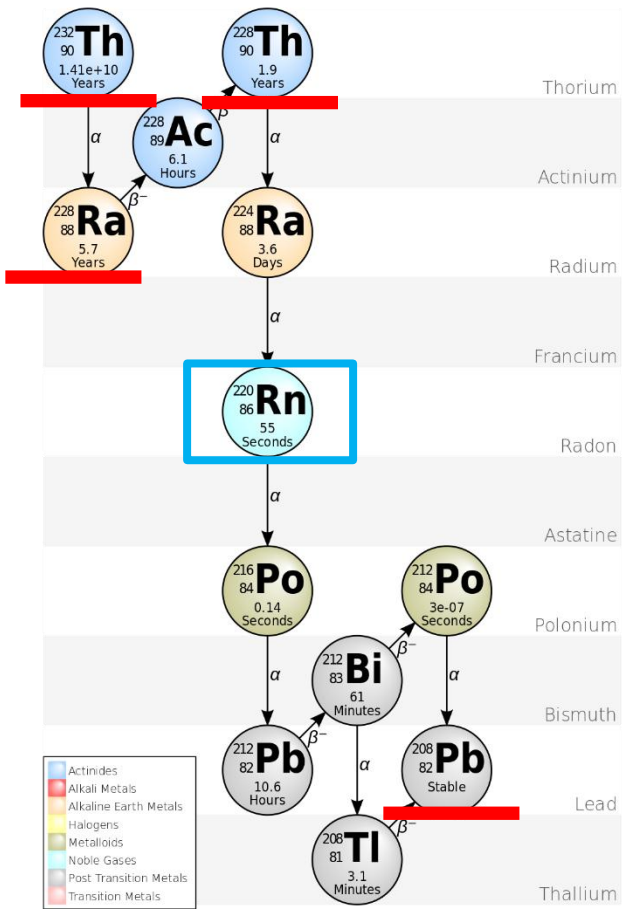


Model:
- Decay equilibrium (incl. Rn)
- U and Th uniform in rock
- Isotropic emission

- Expected: ~50 events/cm^2 /day (for tracks longer than 10 um)
- Promising !!!

Expected Multiplicity Distribution (100 days)





U-Series 1 + 1 + 1 + (1+3) + 1

Th-Series 1 + (2+3)

After Radon

FYI: Rock Types and Rock Code (taken from C&M (1995))

	AND	"Andesite"
	BAS	"Basalt"
	DIA	"Diabase"
C H	GRA	"Granite-Granodiorite"
	DIO	"Diorite"
C H	GAB	"Gabbro-Norite-Troctolite"
C H	MGW	"Metagraywacke"
C H	SLT	"Slate"
C	PHY	"Phyllite"
	BZE	"Zeolite Facies Basalt"
	BPP	"Prehnite-Pumpellyite Facies Basalt"
	BGR	"Greenschist Facies Basalt"
	GGN	"Granitic Gneiss"
C H	BGN	"Biotite (Tonalitic) Gneiss"
H	QSC	"Mica Quartz Schist"
C H	AMP	"Amphibolite"
H	FGR	"Felsic Granulite"
C H	PGR	"Paragranulite"
	AGR	"Anorthositic Granulite"
H	MGR	"Mafic Granulite"
	GGR	"Mafic Garnet Granulite"
	ECL	"Mafic Eclogite"
C	SER	"Serpentinite"
C	QTZ	"Quartzite"
C	MBL	"Calcite Marble"
	ANO	"Anorthosite"
	HBL	"Hornblendite"
	PYX	"Pyroxenite"
	DUN	"Dunite"
C H	SED	"Sediment Layer"
C H	MTL	"Mantle"

H: rocks in the Hidaka prior
C: rocks in the Cascade prior