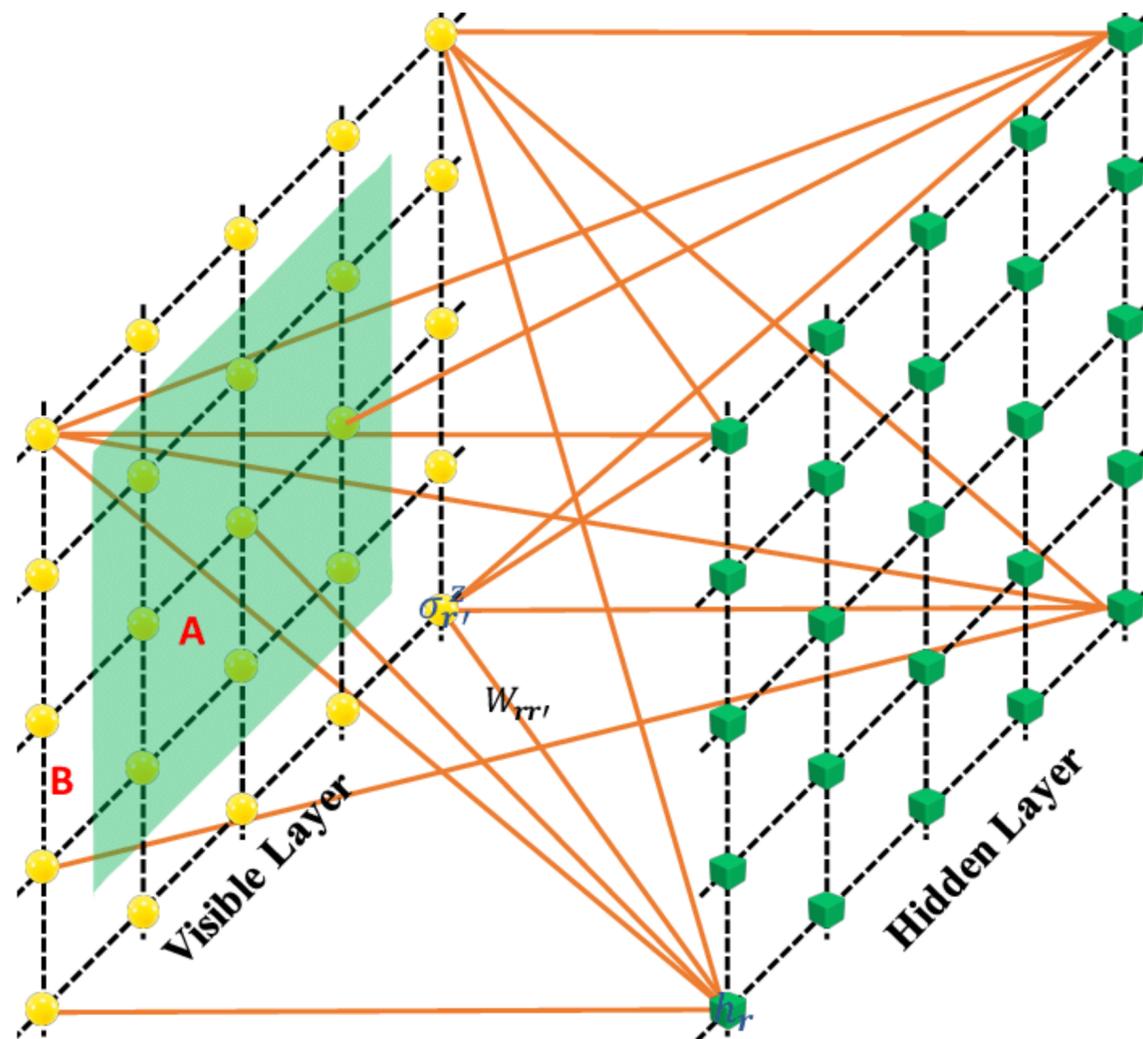


Probing Quantum Gravity Using Neural Quantum States



Name: Júlio Oliveira

Supervisor: Prof. João Viana Lopes

Co-Supervisor: Prof. João Penedones

U. PORTO

FC FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

M | A | P JOINT DOCTORAL
PROGRAMMES

Quantum Gravity

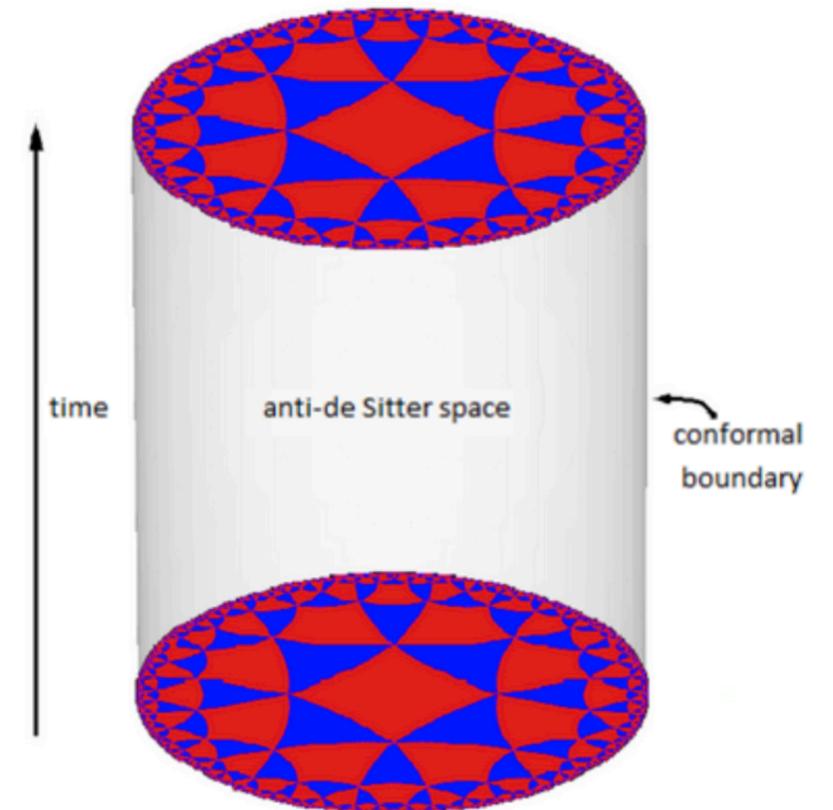
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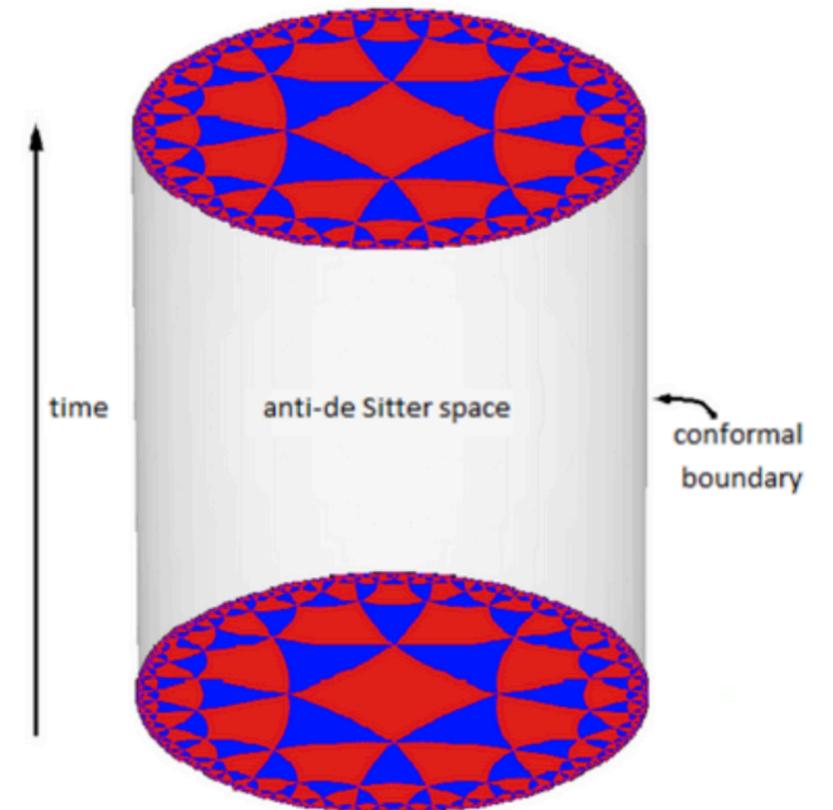
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Gravity
d+1 dimensions



Quantum Field Theory
d dimensions



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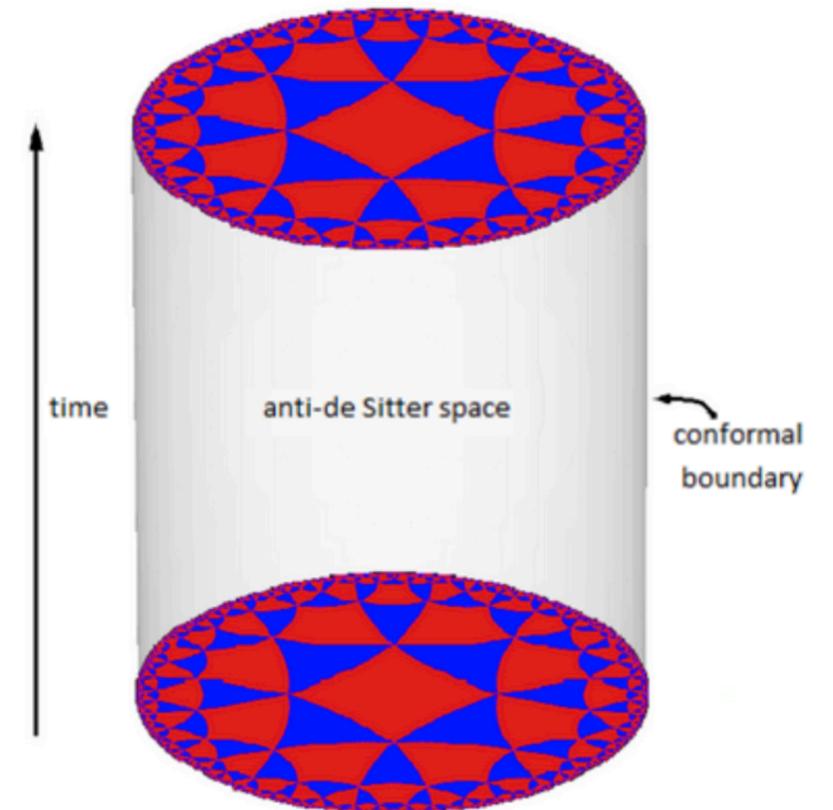
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Notable Case: Type IIB String Theory and $N=4$ SYM ($3+1$ d)

Matrix Quantum Mechanics (MQM)

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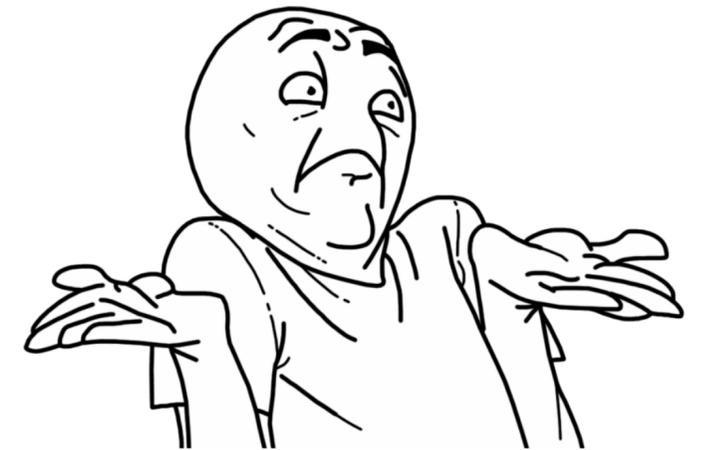
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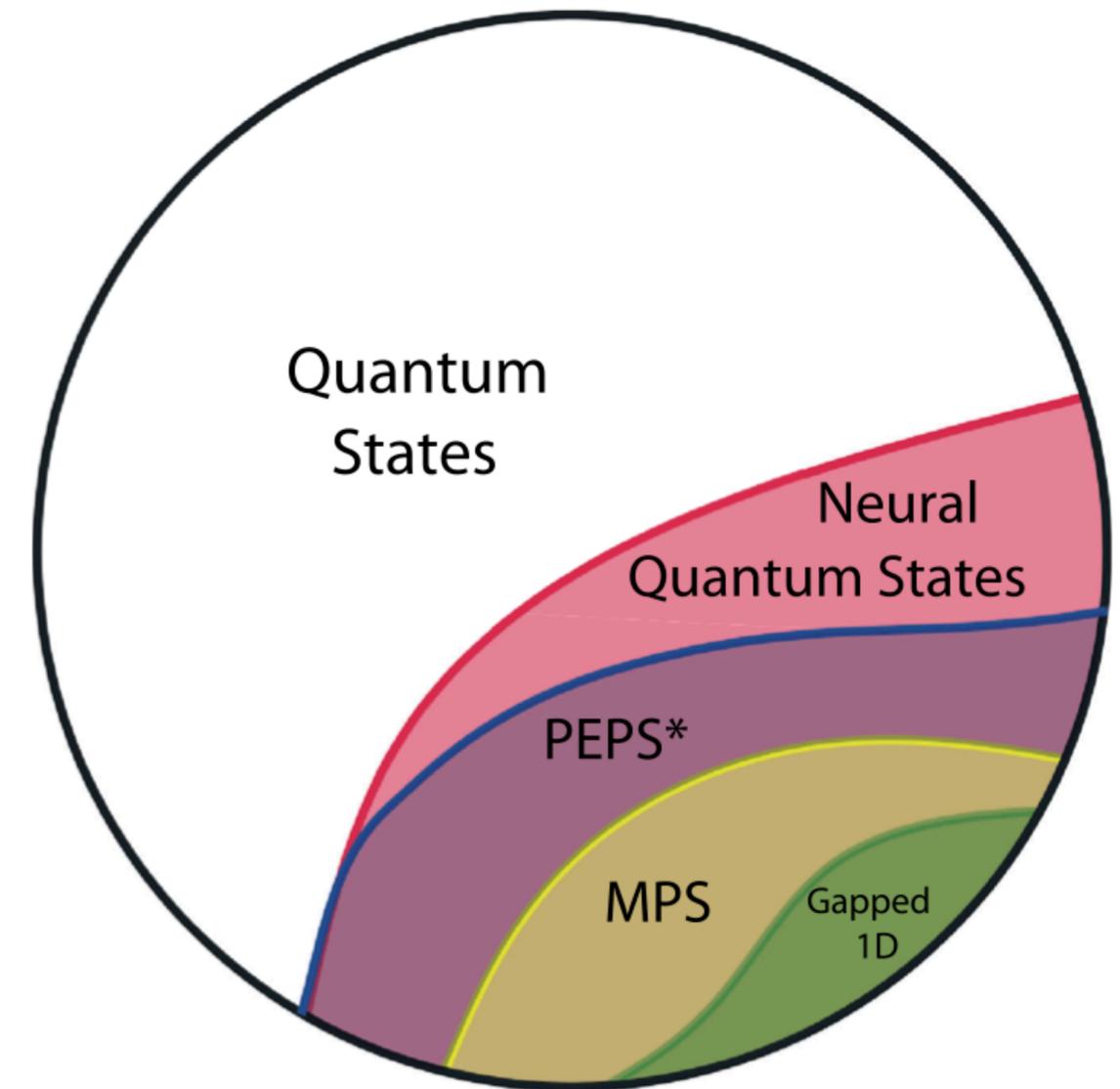
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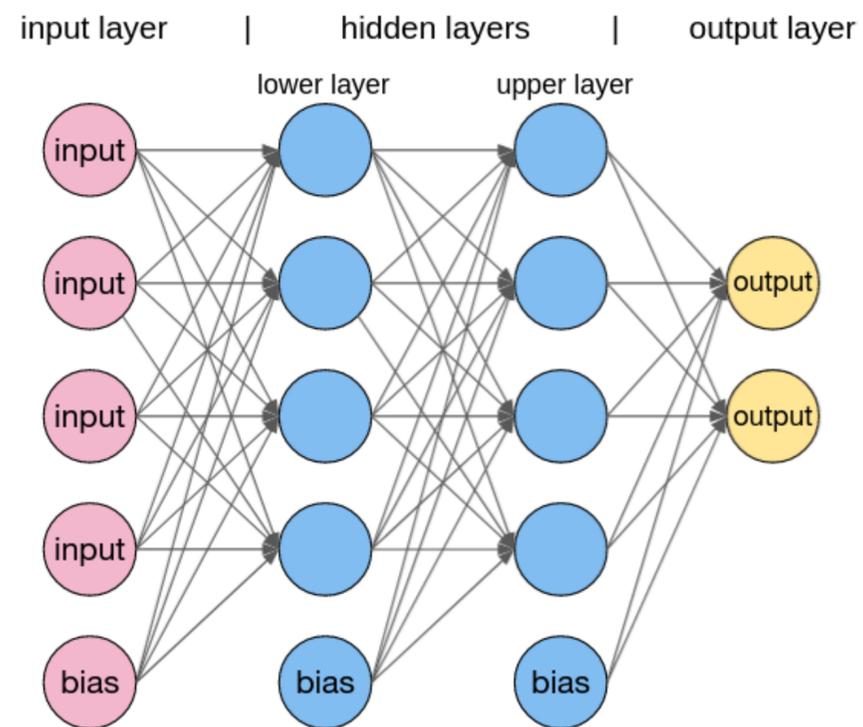
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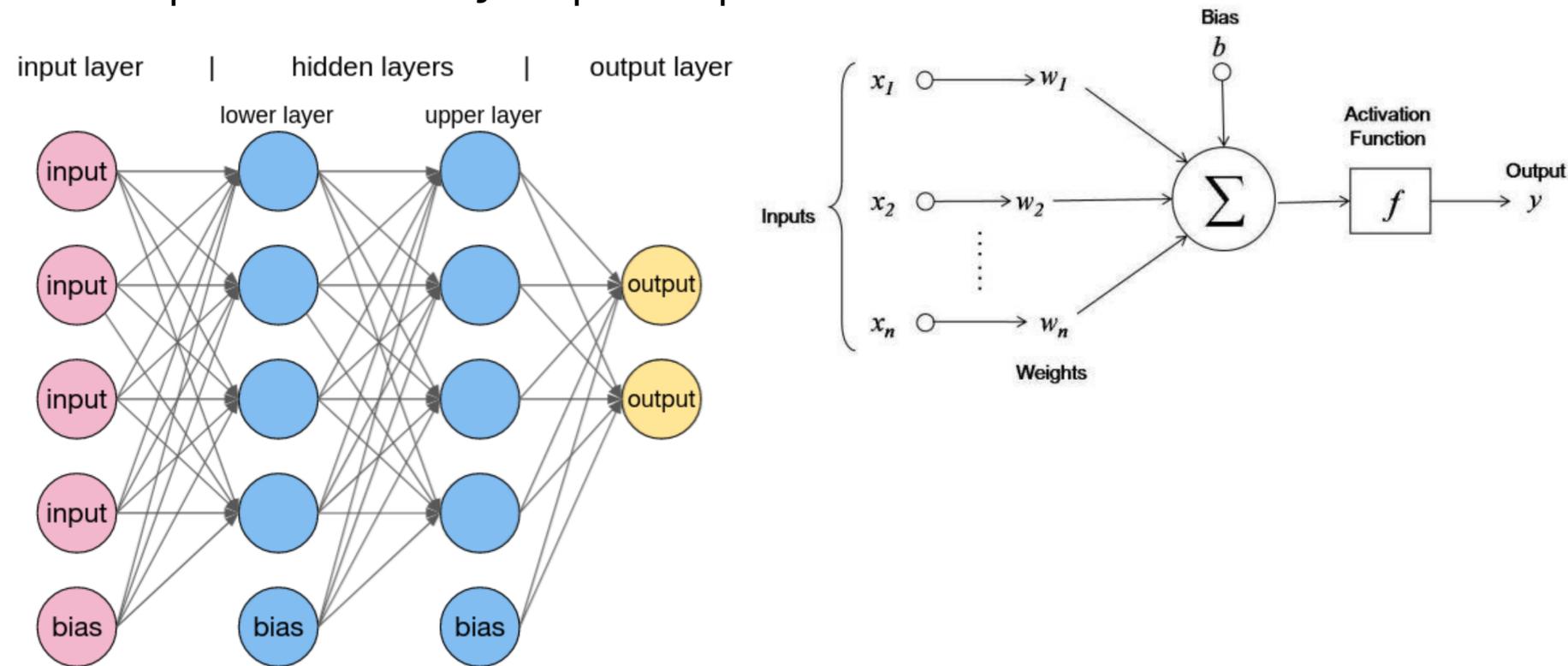
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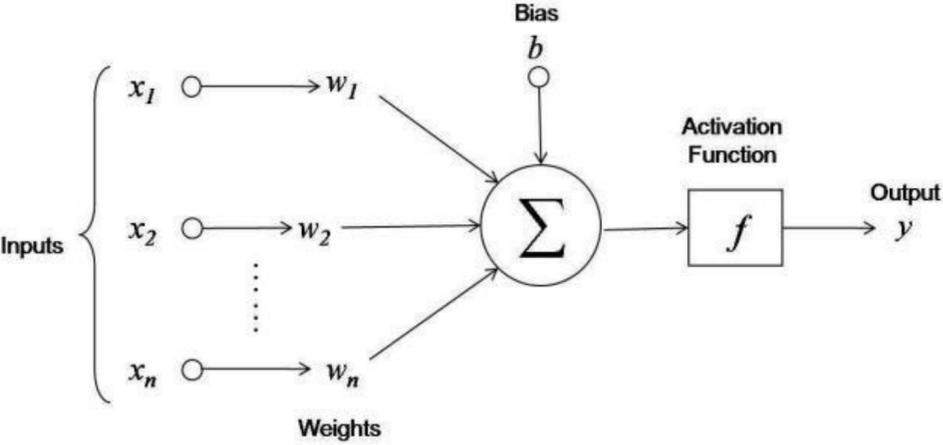
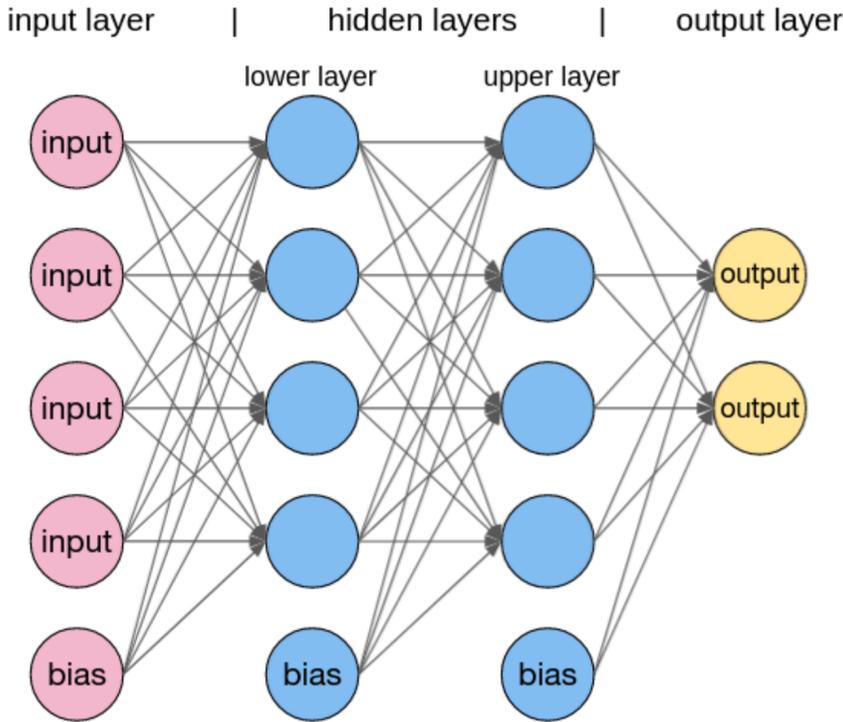
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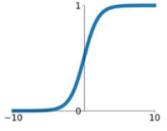
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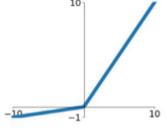


Activation Functions

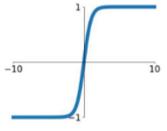
Sigmoid
 $\sigma(x) = \frac{1}{1+e^{-x}}$



Leaky ReLU
 $\max(0.1x, x)$

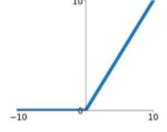


tanh
 $\tanh(x)$

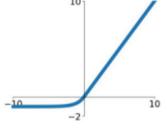


Maxout
 $\max(w_1^T x + b_1, w_2^T x + b_2)$

ReLU
 $\max(0, x)$



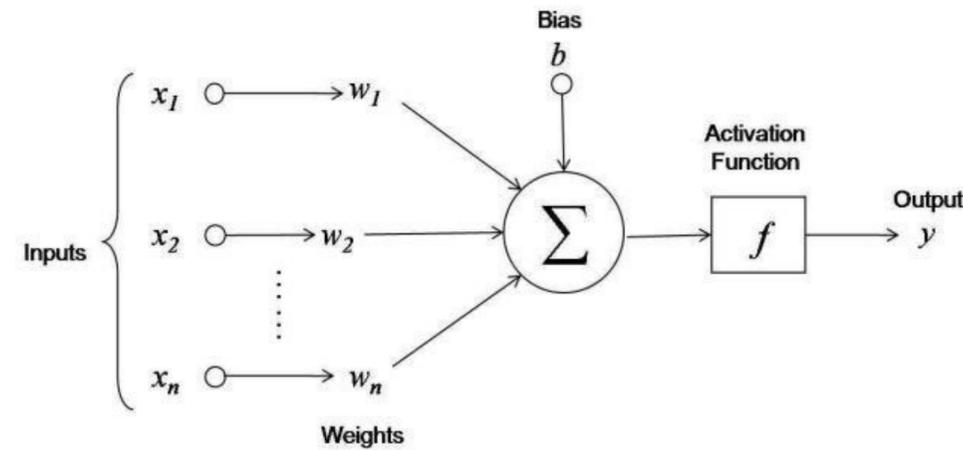
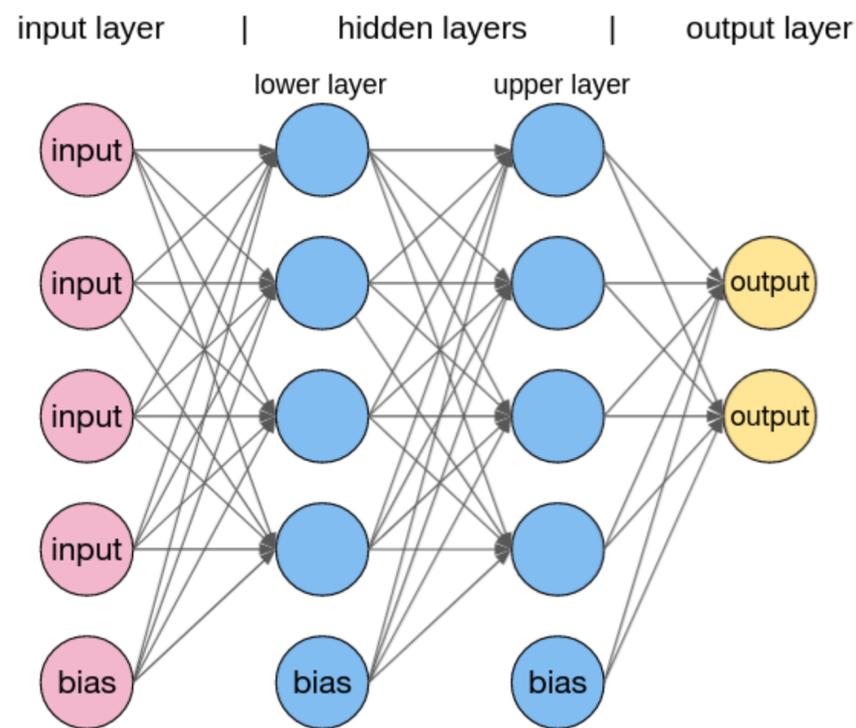
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 $\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$



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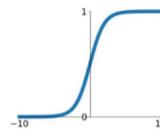
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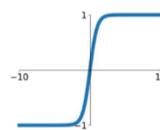
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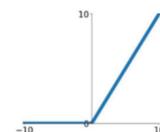
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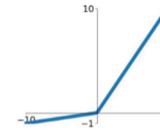
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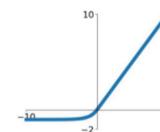


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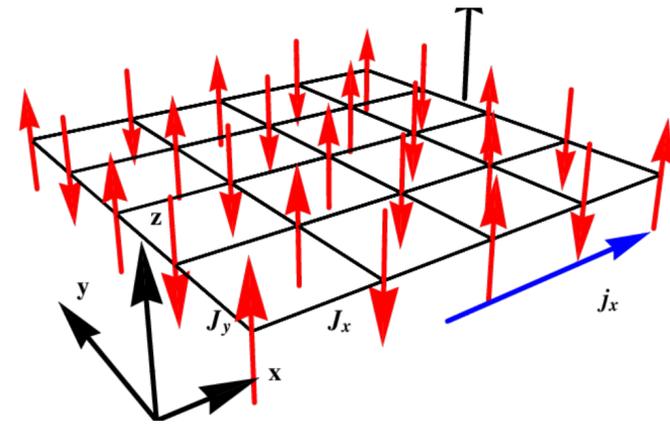


ADVANTAGES

- No Sign-Problem
- No Exponential Growth of the Hilbert space
- More expressive than other ansätze
- Direct access to the wavefunction

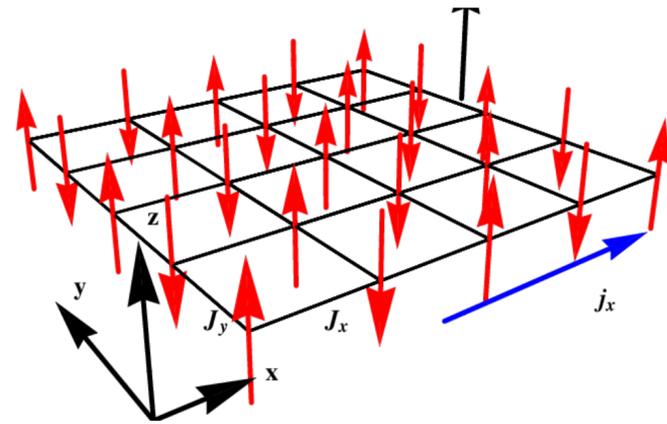
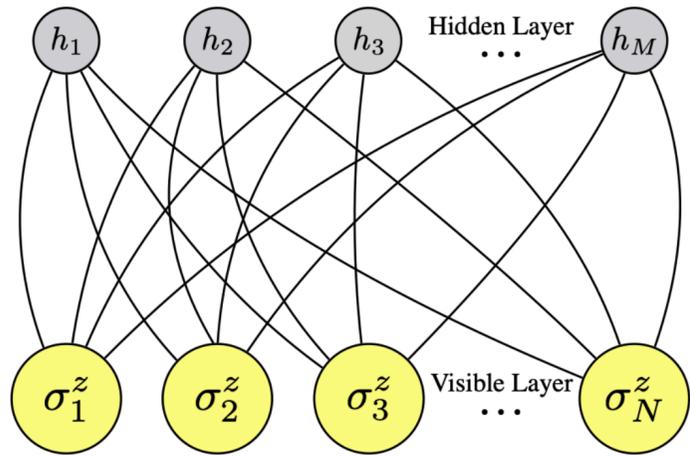
Applications

Applications



- Antiferromagnetic Heisenberg Model 2D
(100 spins)

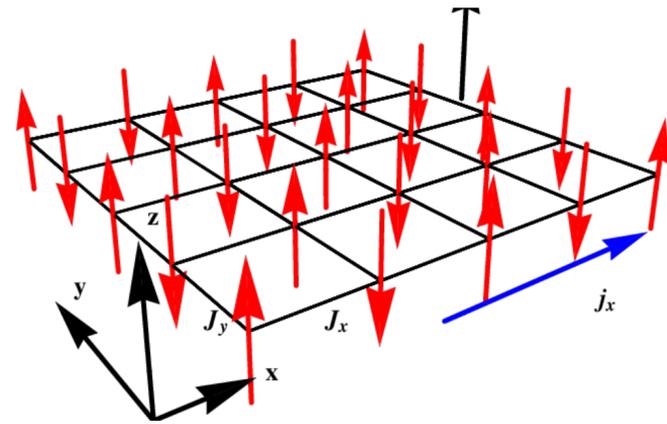
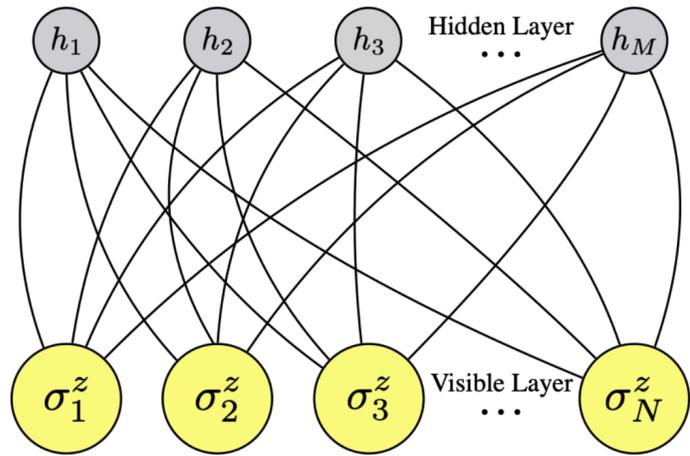
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$$- \psi(\sigma_i^z, \{a_j, b, j, W_{ij}\}) = \sum_{\{h_i\}} e^{\sum_j a_j \sigma_j^z + \sum_j b_j h_j + \sum_{ij} W_{ij} h_i \sigma_j^z}$$

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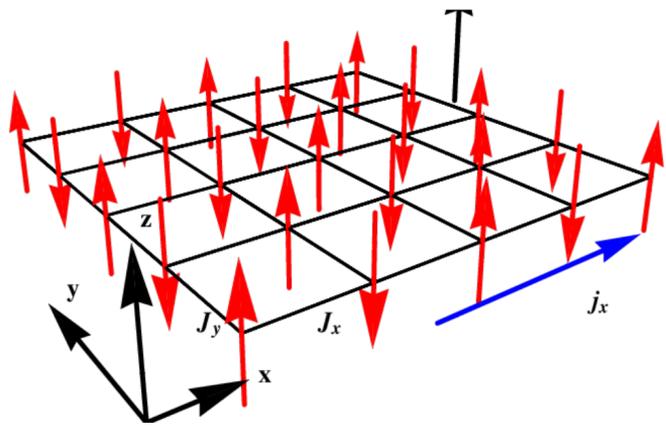
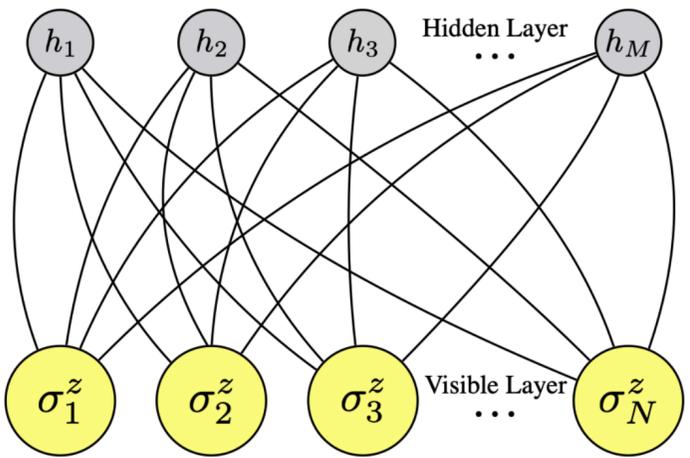
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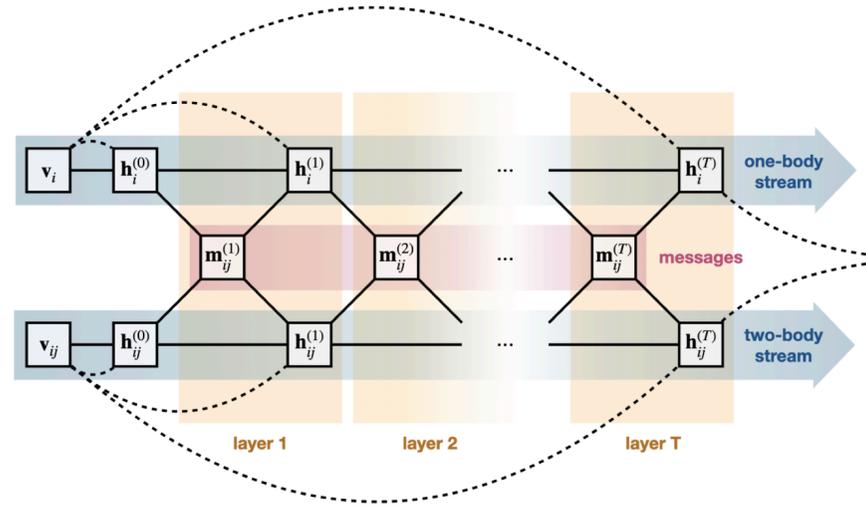
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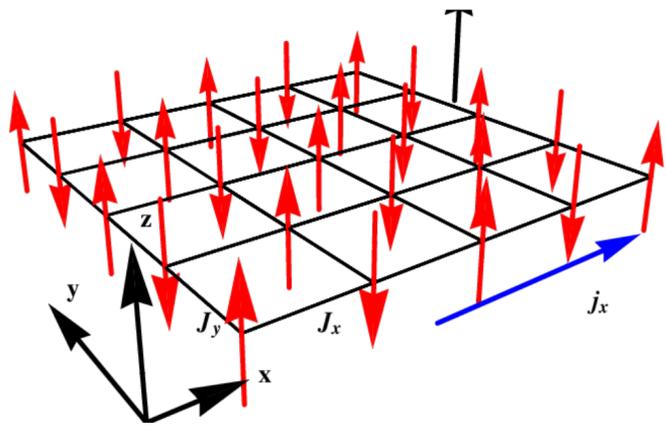
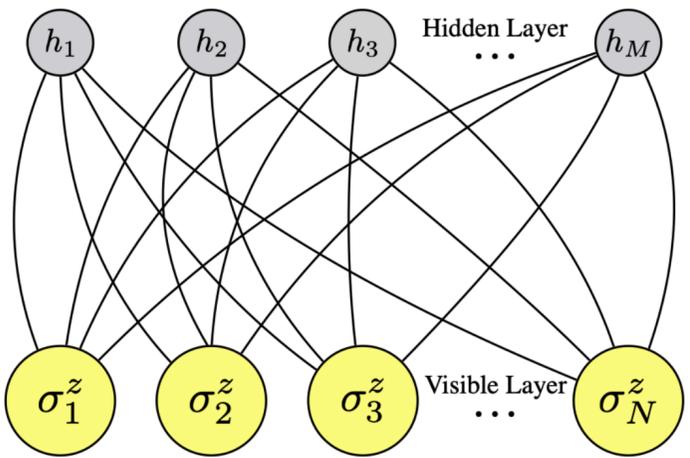
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Message Passing Neural Network

- Encode Symmetries
- Used in Electron Gas

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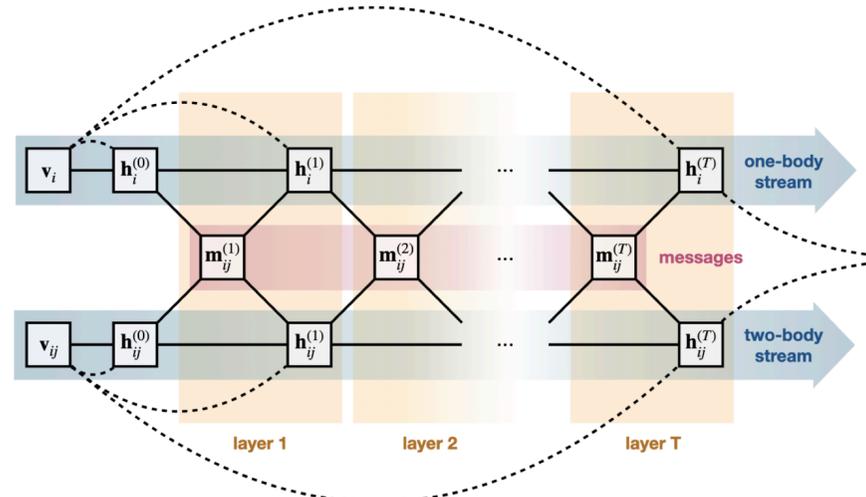


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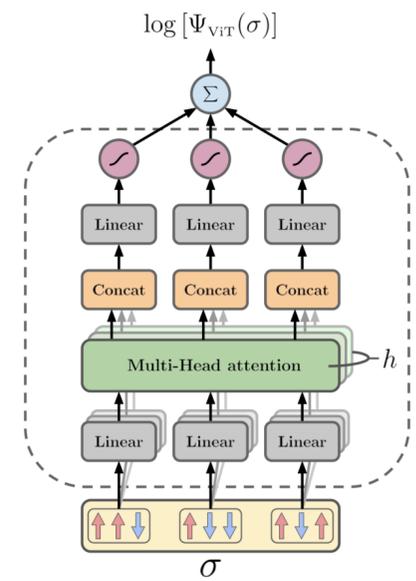
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iv:2305.07240 & arXiv:2305.07240

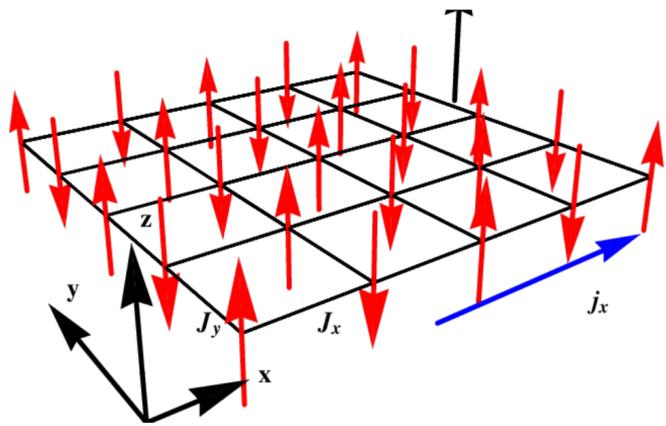
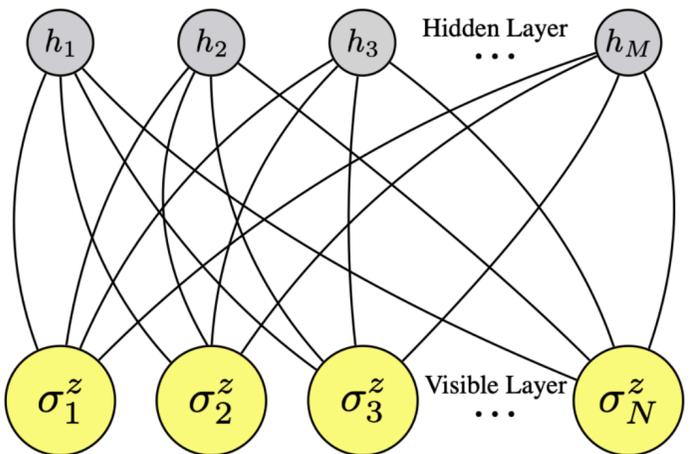


Transformers

- More Expressive
- Learn the dependence on other parameters, like the coupling
- Used in spin chains

arXiv:2211.05504 & arXiv:2502.09488

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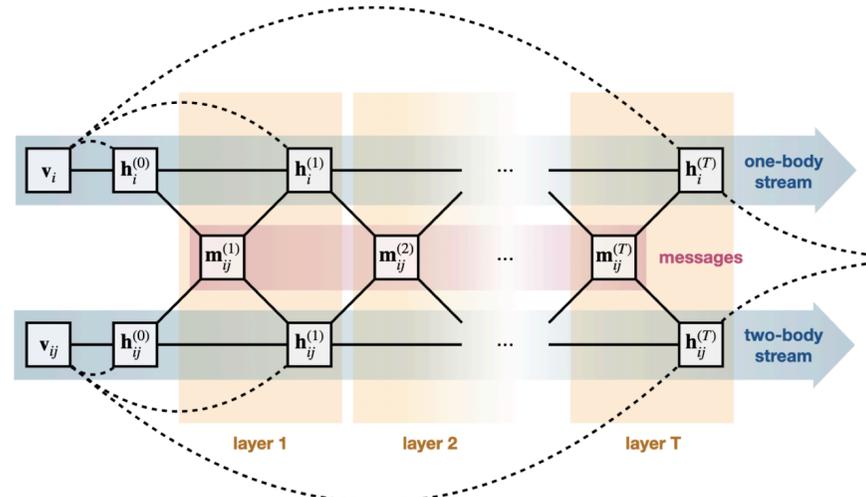


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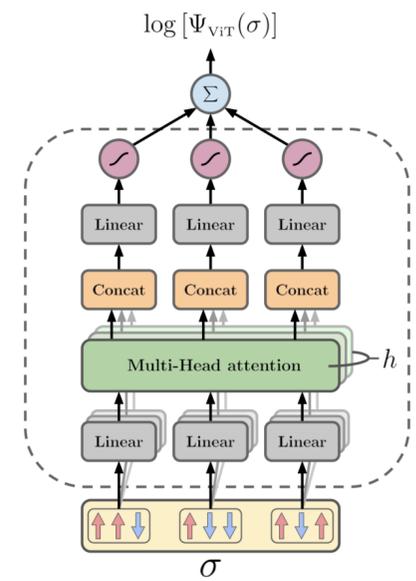
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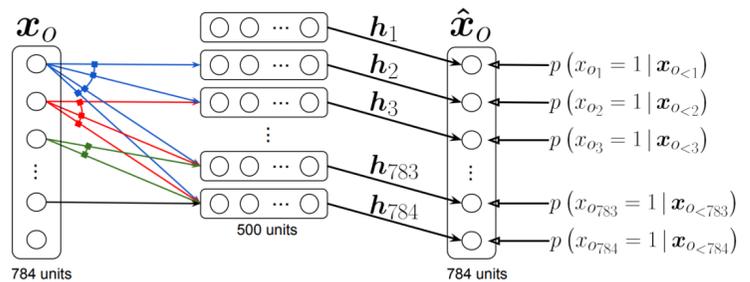
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Autoregressive Networks

- Can directly sample from a given distribution
- Used in MQM

Goals & Objectives

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GOAL: Study the emergent gravity in matrix quantum mechanics using techniques from artificial intelligence.

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Objectives:

- 1) Develop a new efficient machinery for computing quantum states of strongly interactive many-body;
- 2) Test gauge-gravity duality by measuring observables in the BMN/BFSS models;
- 3) Use similar machine learning techniques to study the polarised IKKT;
- 4) Investigate which properties of MQM are relevant for the emergence of a the emergence of a gravitational description, e.g. "Is supersymmetry essential?";
- 5) Create a public database with trained NQS as open-source models.

Research Plan

Development

Apply

Research Plan

Development

Research Plan - Development

Challenges - Development

1) Imposing symmetries and Pauli principle

2) Improve training efficiency

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Bosonic Matrix Model (Only Bosons)

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4) Degeneracy and Excited States

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Bosonic Matrix Model (Only Bosons)

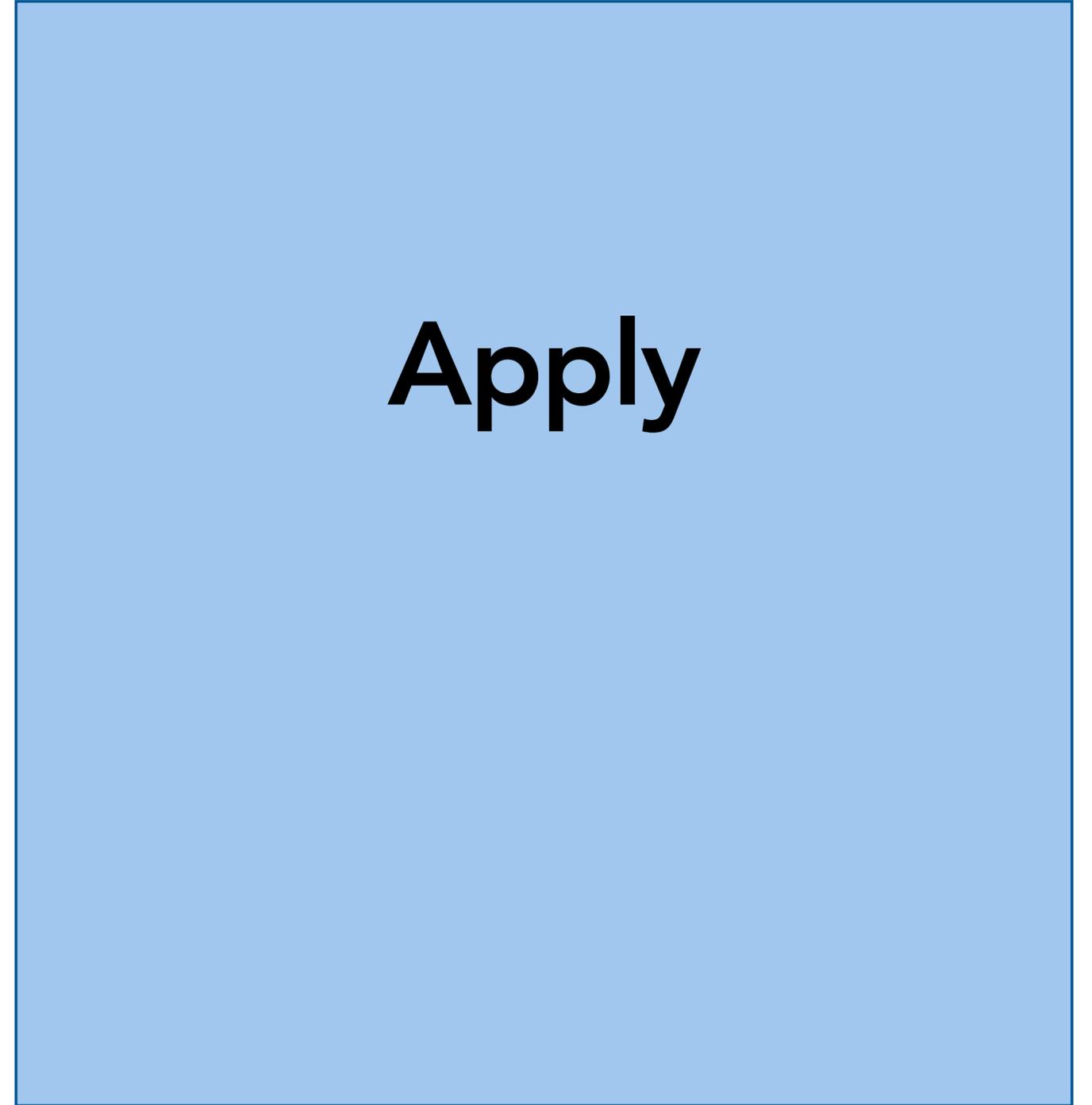
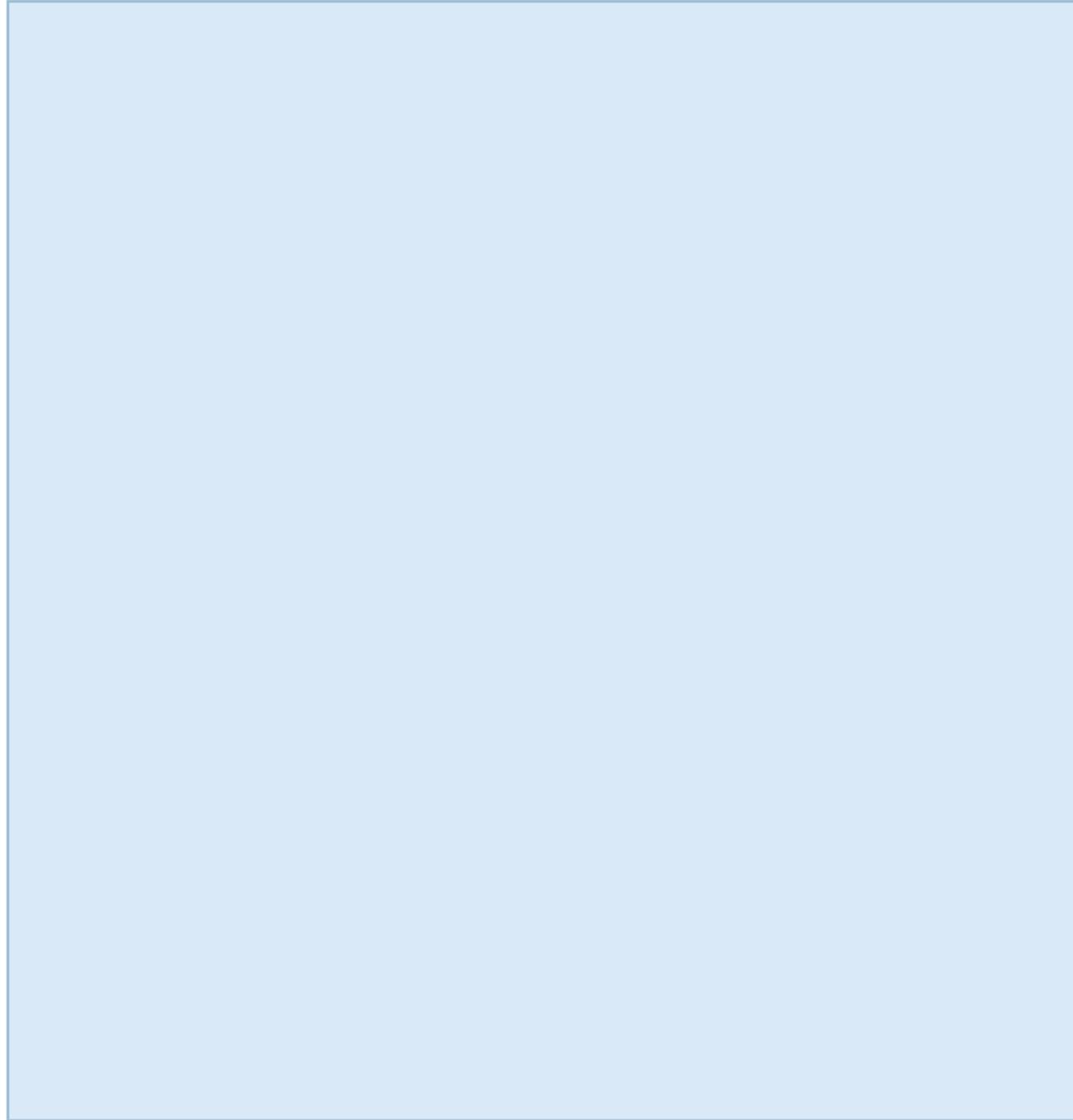
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4) Degeneracy and Excited States

Minimal-BMN (3 bosons, 2 fermion)

Research Plan



Research Plan - Apply

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- Measure observables in the groundstate and compare the results with the predictions from the gravitational dual.
- Investigate the relevance of each microscopic property in the emergence of the gravitational dual.
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1) IKKT

- Compute the distribution of physical observables.
- Understand the mechanism that gives rise to the emergent 10D geometry.

Timetable

YEAR	YEAR 1												YEAR 2												YEAR 3											
Month - Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Month	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
Bosonic Matrix Model	█																																			
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BMN & BFSS																							█													
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Thank you!
Any questions?