



**sck cen**

Belgian Nuclear Research Centre

**ULB** UNIVERSITÉ  
LIBRE  
DE BRUXELLES

**EPFL**

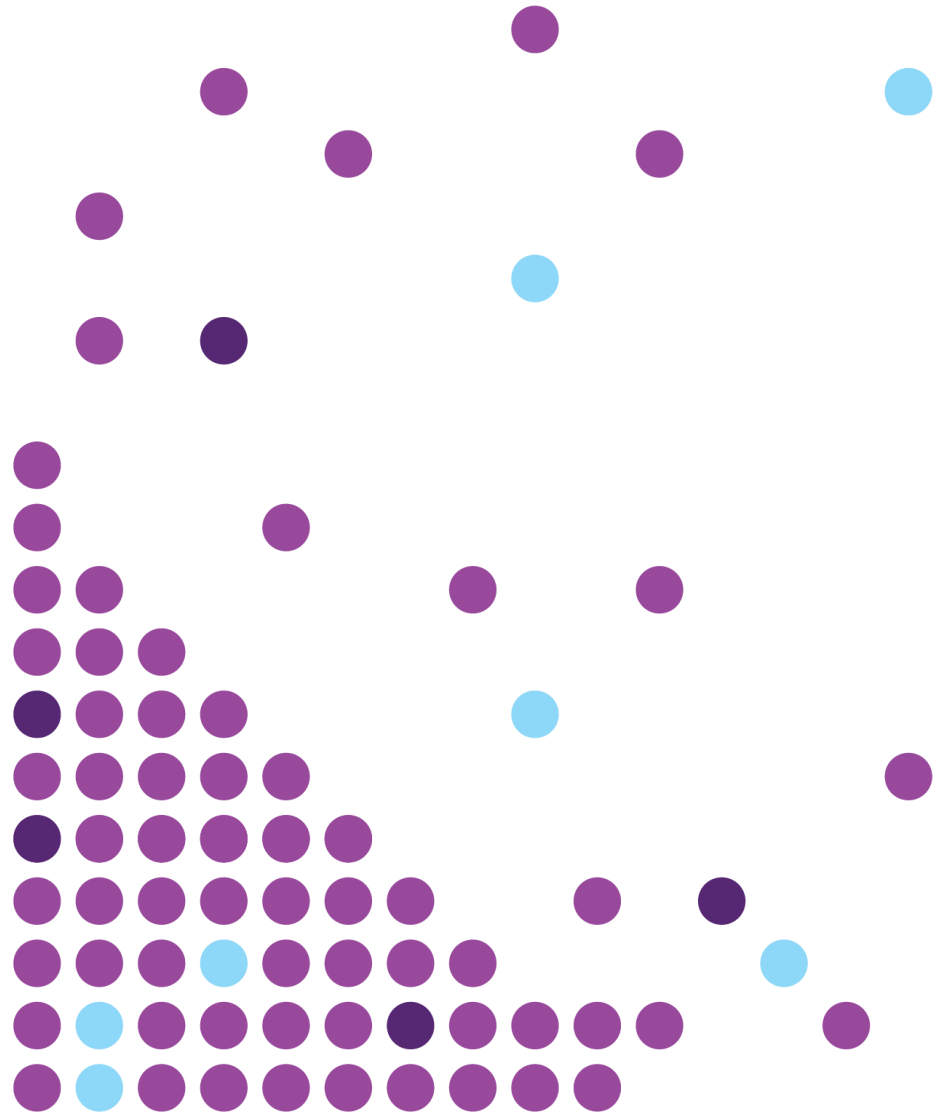
# Bayesian Inference using microscopic and integral measurements to infer nuclear data parameters

*International Workshop On Nuclear Data Evaluation  
for Reactor Applications (WONDER-2026)*

Presenter: Daan Houben

Co-authors: M. Hursin, L. Fiorito, P. Schillebeeckx,  
G. Van den Eynde and P.E. Labeau

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# Introduction

- SCK CEN is committed to building LFR SMR
  - EAGLES-300
  - Cooled with liquid lead
  - MOX fuel to close fuel cycle

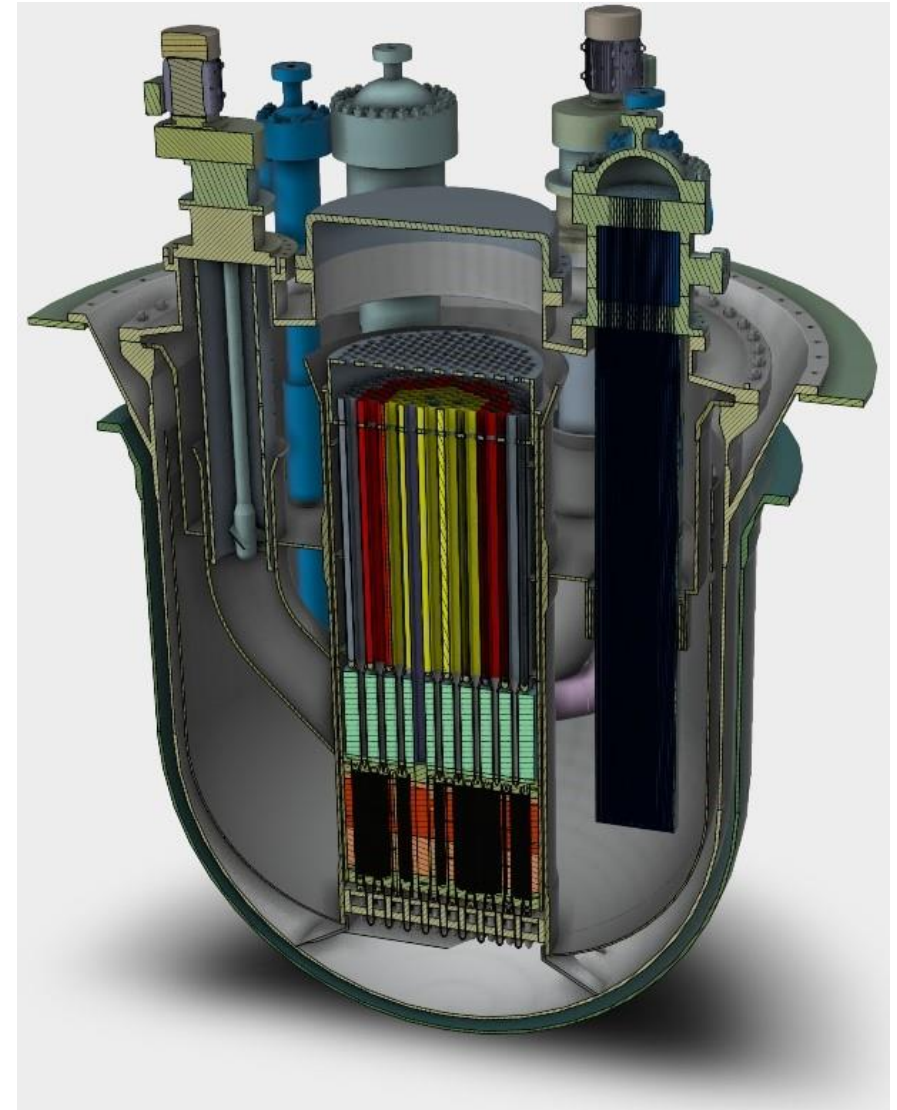


Enhanced safety  
Efficient fuel use  
HL waste reduction  
Passive cooling

Corrosion  
Weight  
Freezing risk  
Limited operating  
experience



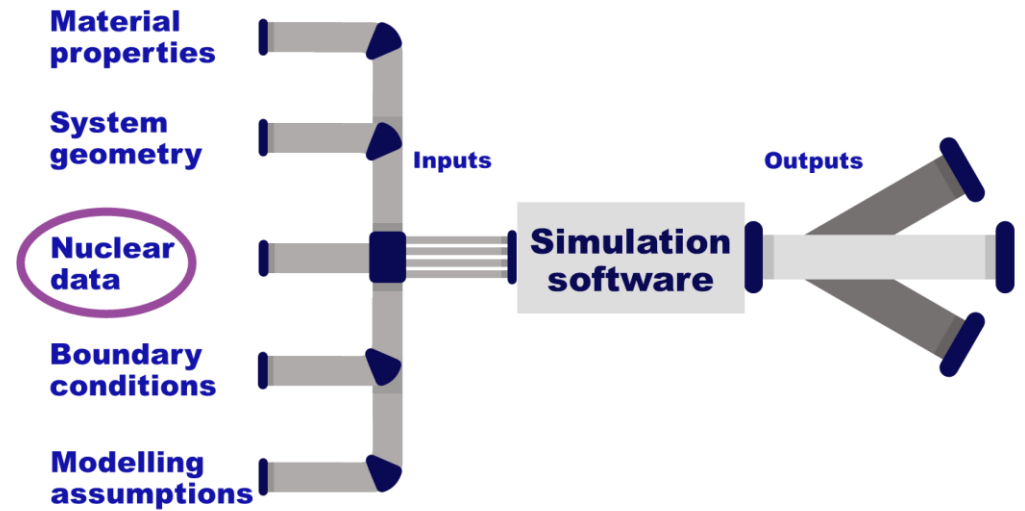
**FOAK**



*EAGLES-300 conceptual design*

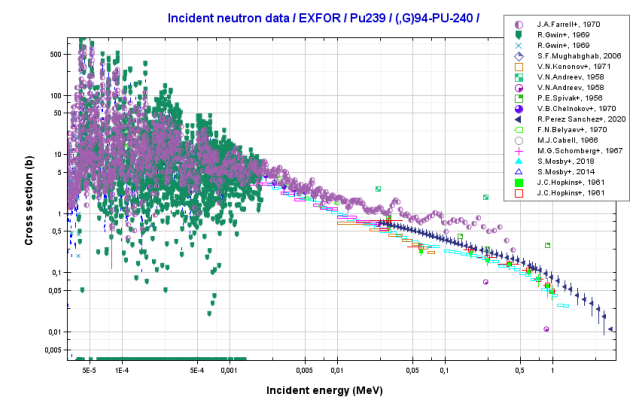
# Introduction

- Design and safety studies of a FOAK
- Use of predictive simulations
- Algorithms well known
- Large uncertainty margins attributed to:
  - Input model parameters
  - **Nuclear data**
- Increased costs for design and safety studies



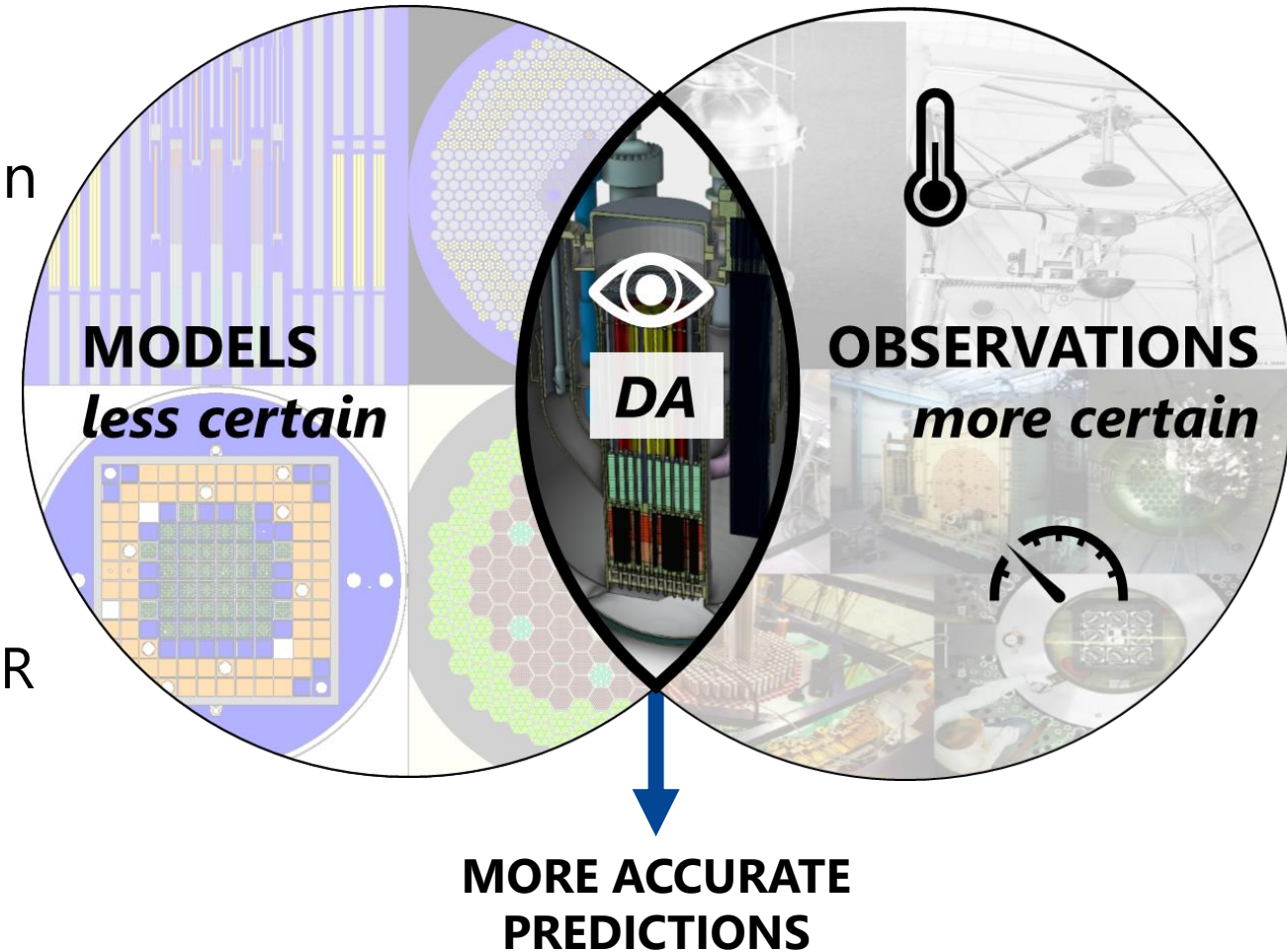
Major source of common uncertainty

→ **Nuclear Data**



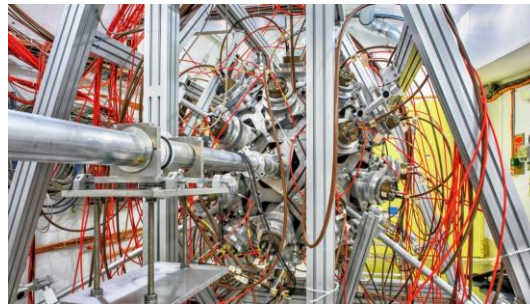
# Introduction

- Possible route to decrease prediction uncertainty:
  - Data assimilation leveraging past experiments
- For LFR highly similar experiments scarce
  - Use experiments not designed for LFR

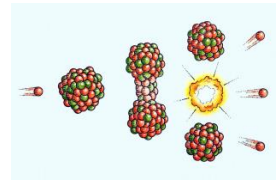


# Introduction

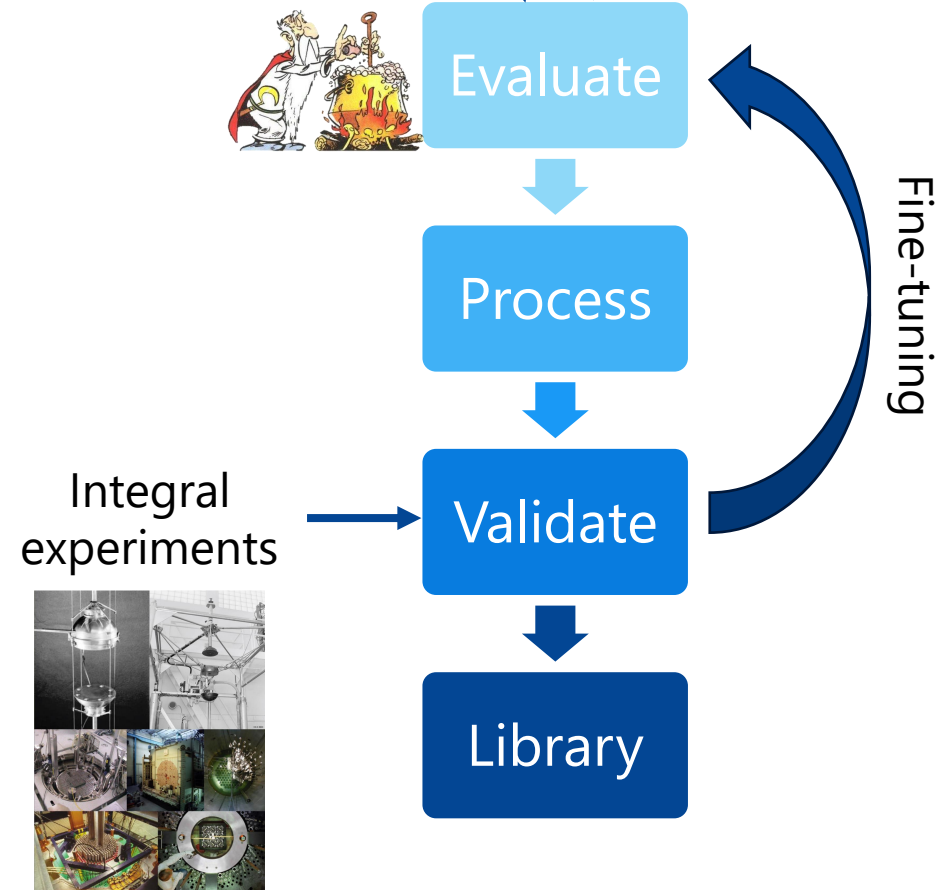
- Nuclear data evaluation process
  - Multi-step process
  - Evaluation → Bayesian approach
  - Integral experiments used for validation



Microscopic experiments

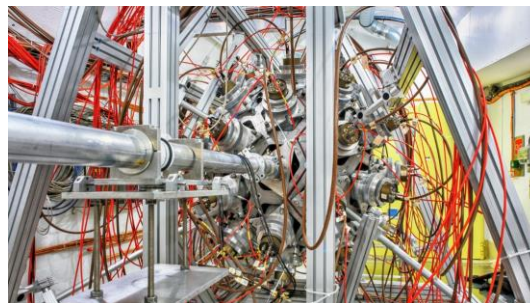


Nuclear theory

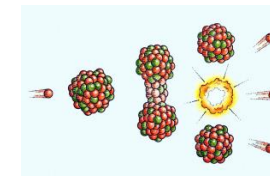


# Objective

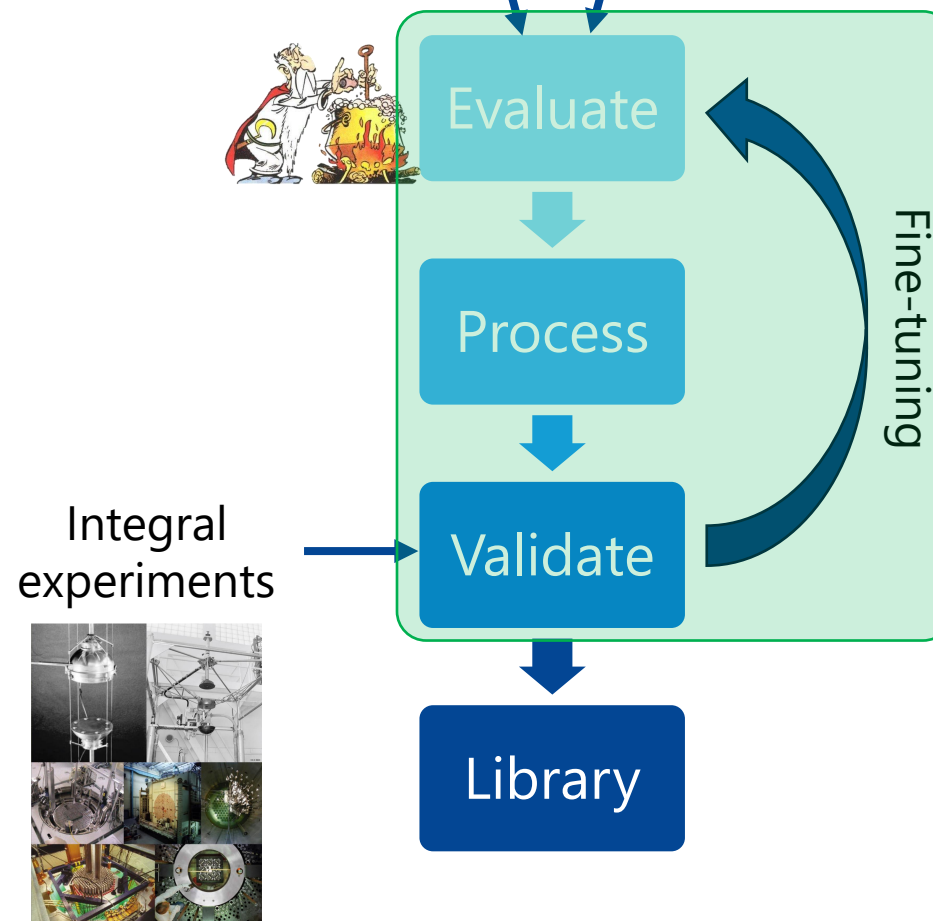
- Data Assimilation using both microscopic and integral measurement
- Identify limitations and challenges



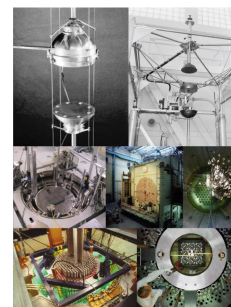
Microscopic experiments



Nuclear theory

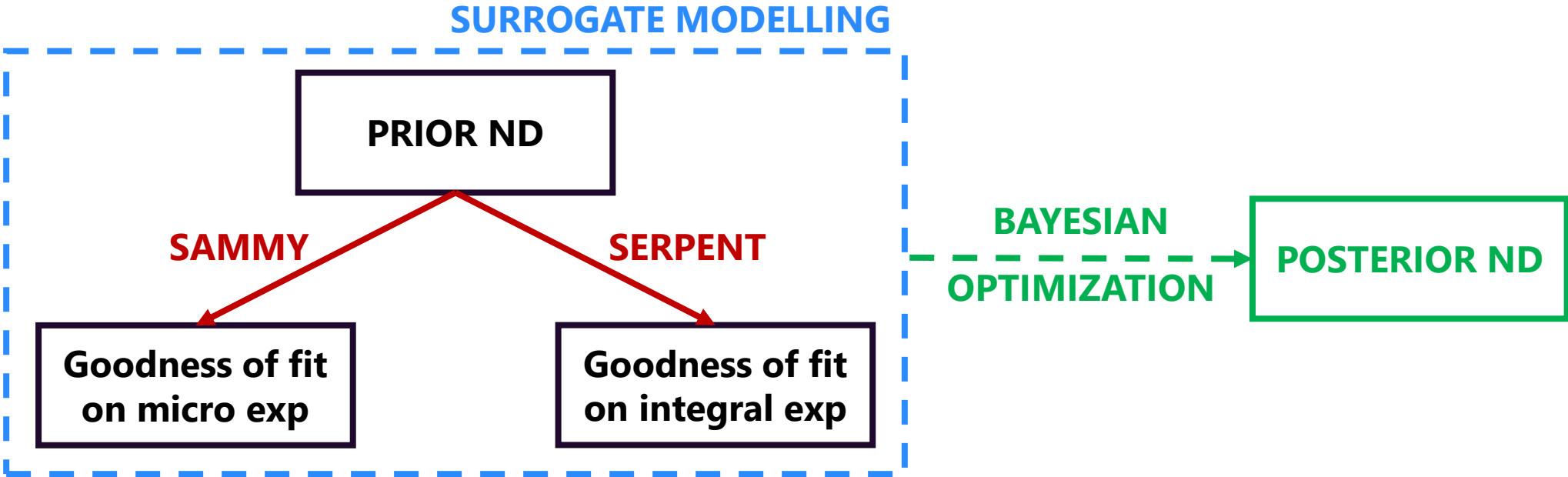


Integral experiments



# Methodology

## Overview



# Methodology

$$\chi^2 = \frac{(C - E)^2}{\Sigma}$$

Formulation unnormalized posterior

$\ln(p) =$

PRIOR

$$-\frac{1}{2} \chi_{\text{ND}}^2$$

MICROSCOPIC

$$-\frac{1}{2} \chi_{\text{m}}^2$$

INTEGRAL

$$-\frac{1}{2} \chi_{\text{i}}^2$$

# Case study: $^{53}\text{Cr}$

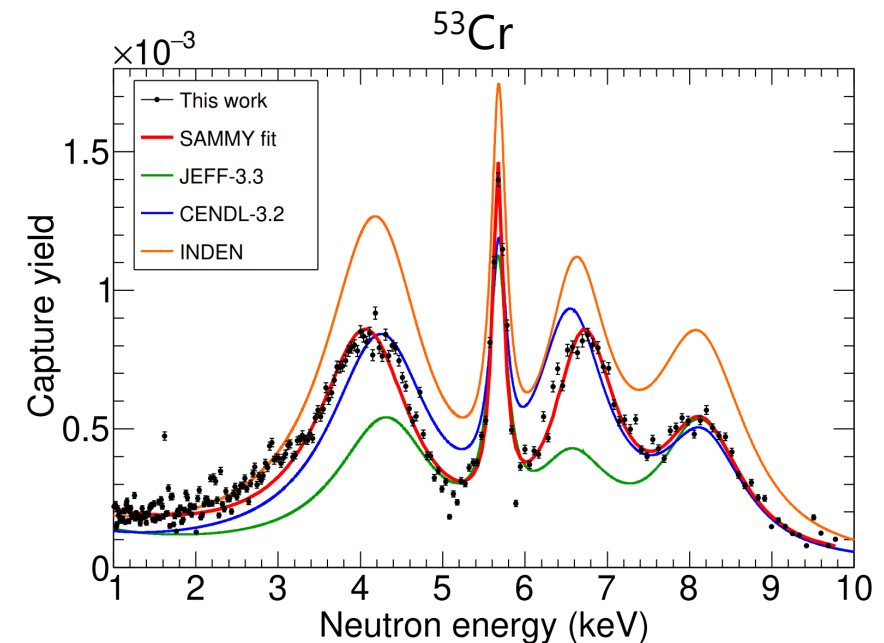


# Case study: $^{53}\text{Cr}$

## Why $^{53}\text{Cr}$ ?

- Stainless steel (11-26% chromium)
- Structural materials of reactors → corrosion resistance
- $^{50,52,53}\text{Cr}$  important for  $k_{\text{eff}}$  in some nuclear systems (mostly  $(n,\gamma)$ )
- $^{53}\text{Cr}$  was included in NEA High Priority Request List (1-100 keV)

Isotope	Abundance (%)
Cr-50	4.34
Cr-52	83.8
Cr-53	9.50
Cr-54	2.37

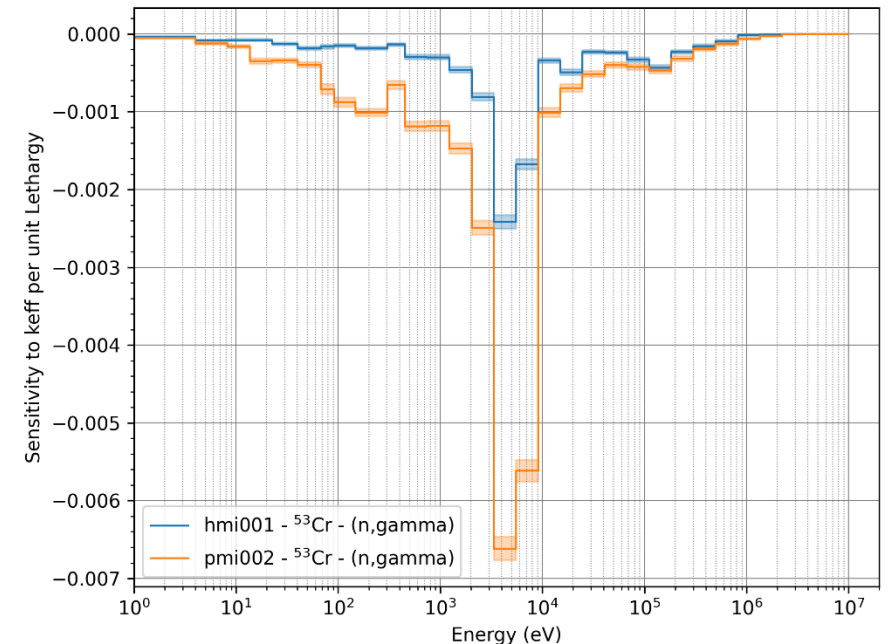
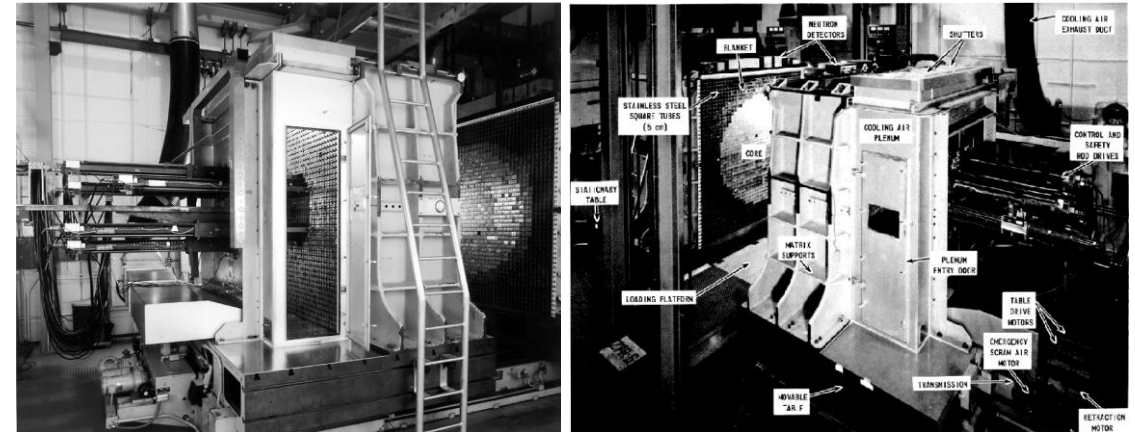


from P. Pérez-Maroto et al., " $^{50}\text{Cr}$  and  $^{53}\text{Cr}$  neutron capture cross sections measurement at the  $n_{\text{TOF}}$  facility at CERN," 2025

# Case study: $^{53}\text{Cr}$

## Selection of integral experiments

- Requirements:
  - Sensitive to  $^{53}\text{Cr}$  ( $n,\gamma$ ) in 1-10 keV range
- Two most sensitive criticality experiments selected from ICSBEP [4]:
  - HMI-001
  - PMI-002



# Case study: $^{53}\text{Cr}$

$$\chi^2 = \frac{(C - E)^2}{\Sigma}$$

Formulation unnormalized posterior

$\ln(p) =$

PRIOR

$$-\frac{1}{2} \chi_{\text{ND}}^2$$

MICROSCOPIC

$$-\frac{1}{2} \chi_{\text{m}}^2$$

INTEGRAL

$$-\frac{1}{2} \chi_{\text{i}}^2$$

# Case study: $^{53}\text{Cr}$

## Formulation unnormalized posterior

$$\chi^2 = \frac{(C - E)^2}{\Sigma}$$

$$\ln(p) =$$

### PRIOR

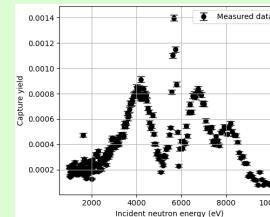
$$-\frac{1}{2} \chi_{\text{ND}}^2$$

- 4 input parameters:
  - $\Gamma_\gamma$  E=4.1 keV
  - $\Gamma_\gamma$  E=5.7 keV
  - $\Gamma_\gamma$  E=6.8 keV
  - $\Gamma_\gamma$  E=8.2 keV
- 20% rel. unc.  $\Gamma_\gamma$
- No correlation
- Multivariate normal

### MICROSCOPIC

$$-\frac{1}{2} \chi_m^2$$

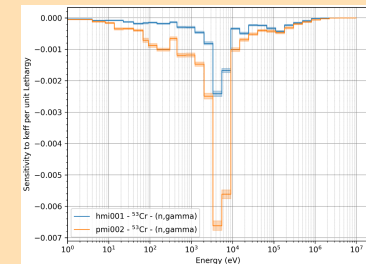
- 256 observations
- Treat normalization uncertainty as fitting parameter
- $\Sigma_m = \Sigma_{\text{exp}} + \Sigma_{\text{norm}} + \Sigma_\alpha$



### INTEGRAL

$$-\frac{1}{2} \chi_i^2$$

$$\Sigma_i = \Sigma_{\text{exp}} + \Sigma_{\text{GP}} + \Sigma_\alpha$$



# Case study: $^{53}\text{Cr}$

## Formulation unnormalized posterior

$$\chi^2 = \frac{(C - E)^2}{\Sigma}$$

### MICROSCOPIC

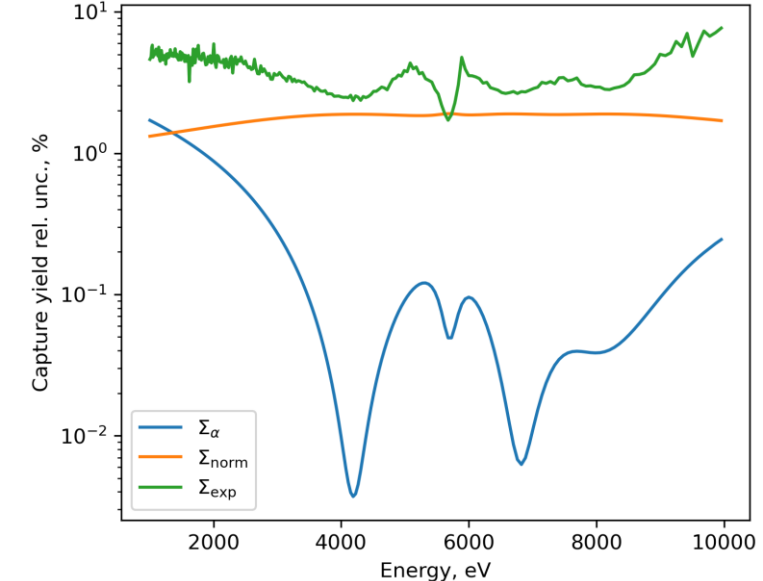
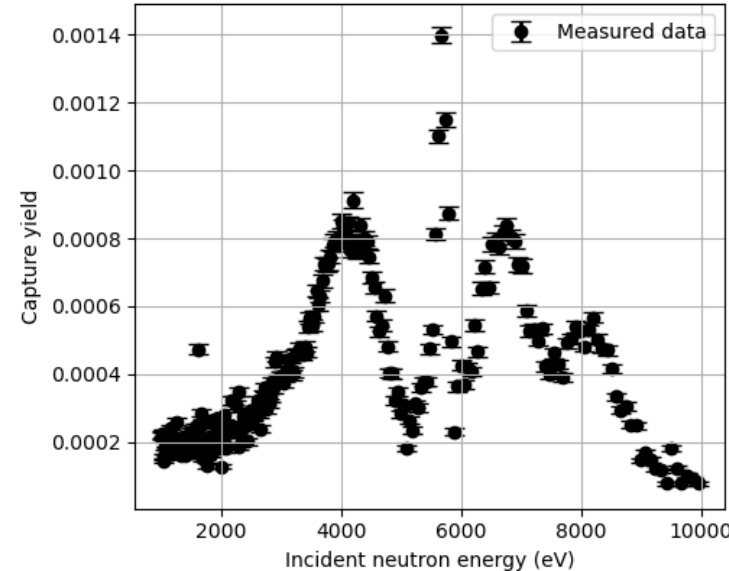
$$-\frac{1}{2} \chi_m^2$$

- 256 observations
- Treat normalization uncertainty as fitting parameter
- $\Sigma_m = \Sigma_{\text{exp}} + \Sigma_{\text{norm}} + \Sigma_{\alpha}$

$\Sigma_{\text{exp}}$ : Experimental setup

$\Sigma_{\text{norm}}$ : Normalization factor

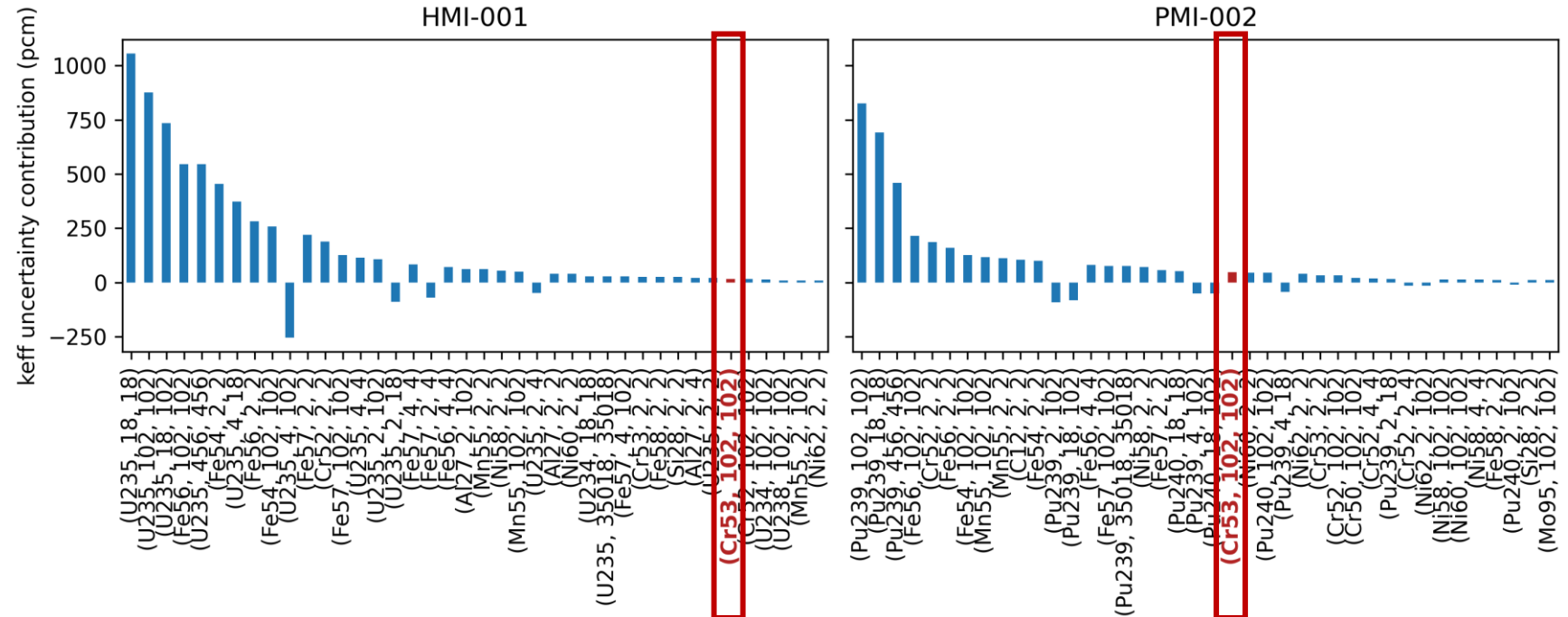
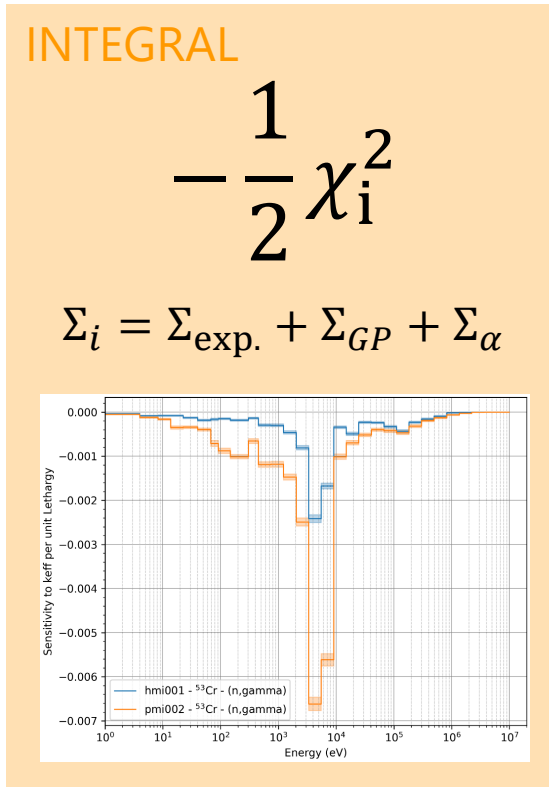
$\Sigma_{\alpha}$ : Other nuclear data



# Case study: $^{53}\text{Cr}$

$$\chi^2 = \frac{(C - E)^2}{\Sigma}$$

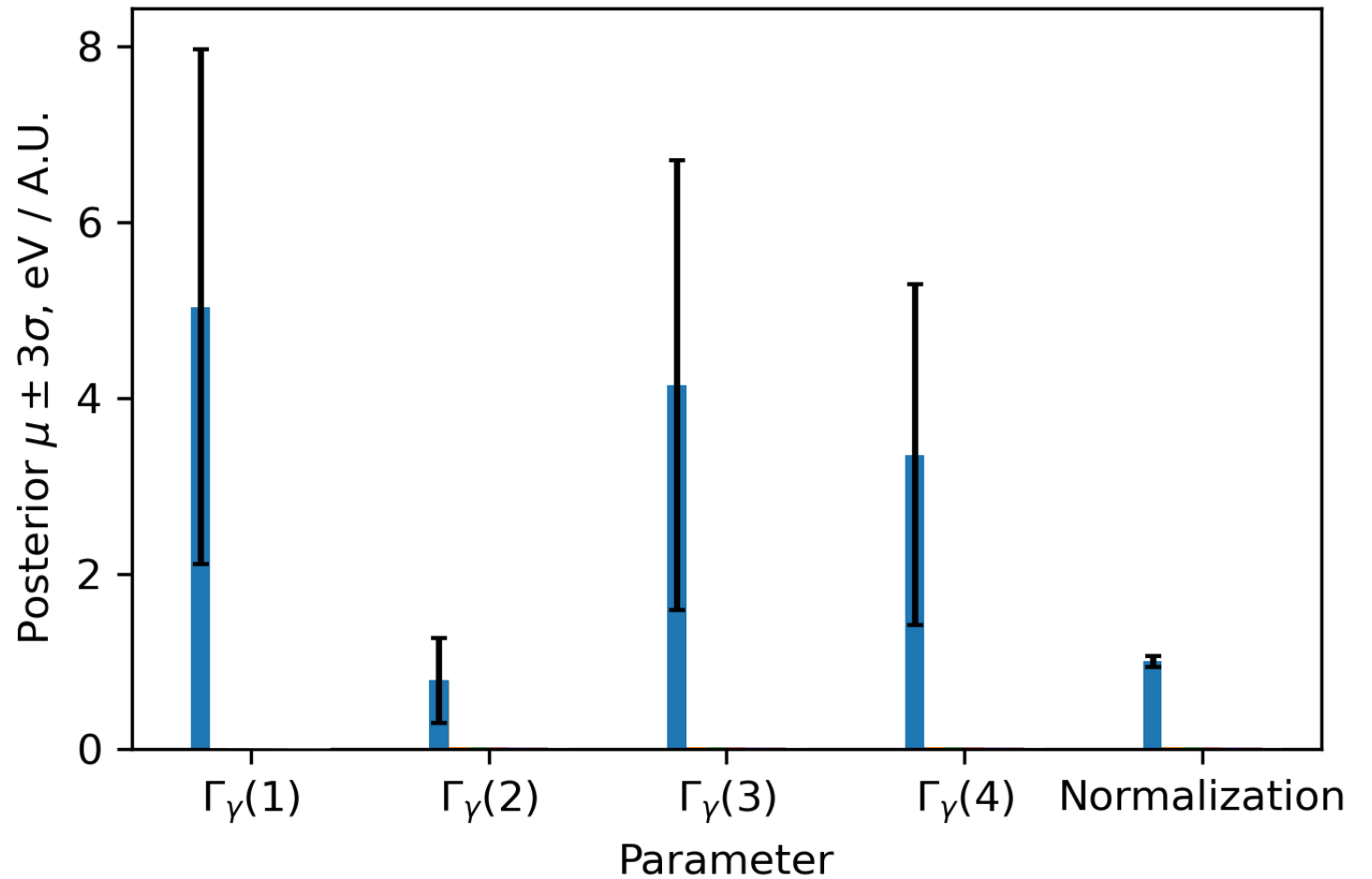
## Formulation unnormalized posterior



# Results

## Bayesian optimization

Description	
■	Prior Distribution
■	Integral Only
■	Micro Only
■	All experiments
■	All (Neglect ND)
■	All (Integral unc reduced)

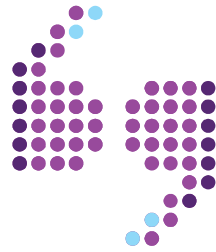


# Conclusions

- A methodology is presented to include both microscopic and integral measurements using surrogates
- Integral measurements can be included but uncertainties of other nuclear data should also be considered
- HMI-001 and PMI-002 are not sensitive enough to  $^{53}\text{Cr} (n,\gamma)$  for DA
- Improvement mostly comes from microscopic measurement

# References

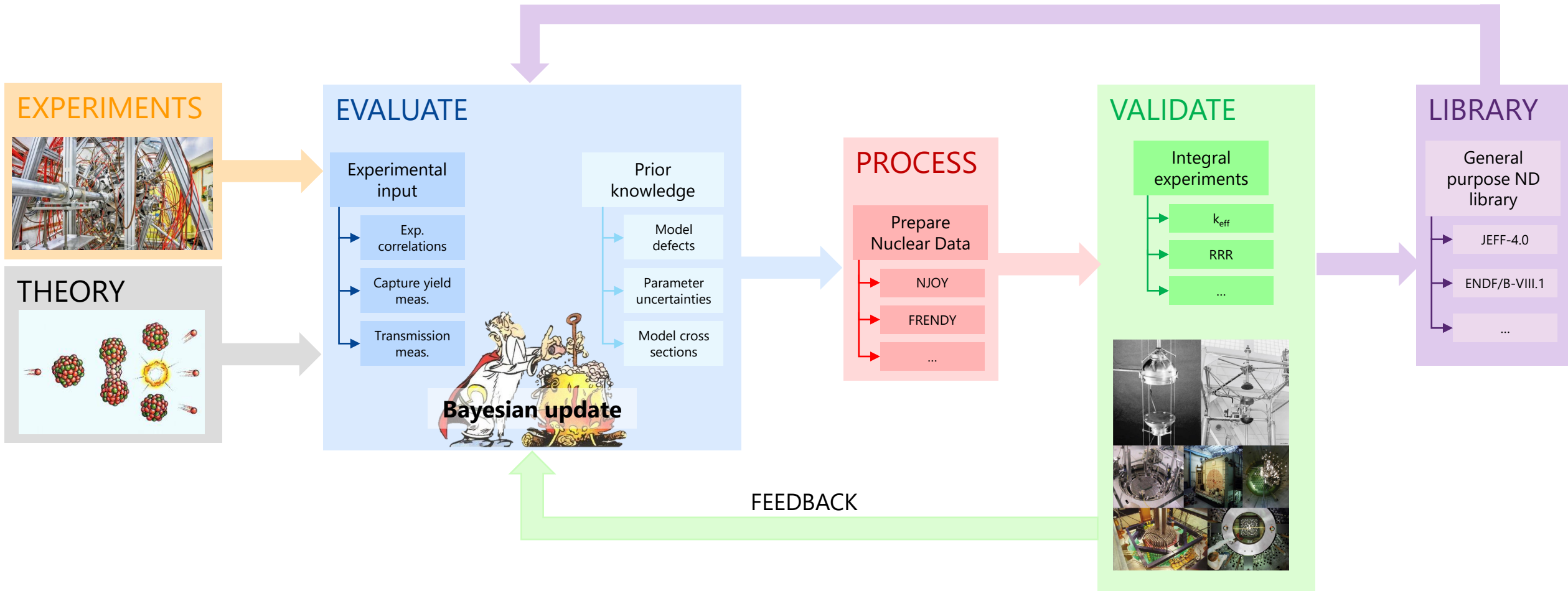
- N. M. Larson, "Updated User's Guide for Sammy: Multilevel R-Matrix Fits to Neutron Data Using Bayes' Equations," ORNL/TM-9179/R8, 941054, Oct. 2008. doi: [10.2172/941054](https://doi.org/10.2172/941054).
- J. Leppänen, V. Valtavirta, A. Rintala, and R. Tuominen, "Status of Serpent Monte Carlo code in 2024," *EPJ Nuclear Sci. Technol.*, vol. 11, p. 3, 2025, doi: [10.1051/epjn/2024031](https://doi.org/10.1051/epjn/2024031).
- P. Pérez-Maroto et al., "50Cr and 53Cr neutron capture cross sections measurement at the n\_TOF facility at CERN," 2025
- J. D. Bess, T. Ivanova, J. Martin, I. Hill, and L. Scott, "The 2020 edition of the ICSBEP handbook," *Nuclear Energy Agency, Organization for Economic Co-Operations and Development*, 2020.
- A. Trkov, O. Cabellos, and R. Capote, "Sensitivity of selected benchmarks to Cr-53 and Cr-50 capture".
- G. P. A. Nobre et al., "Newly Evaluated Neutron Reaction Data on Chromium Isotopes," *Nuclear Data Sheets*, vol. 173, pp. 1–41, Mar. 2021, doi: [10.1016/j.nds.2021.04.002](https://doi.org/10.1016/j.nds.2021.04.002).



# Additional slides

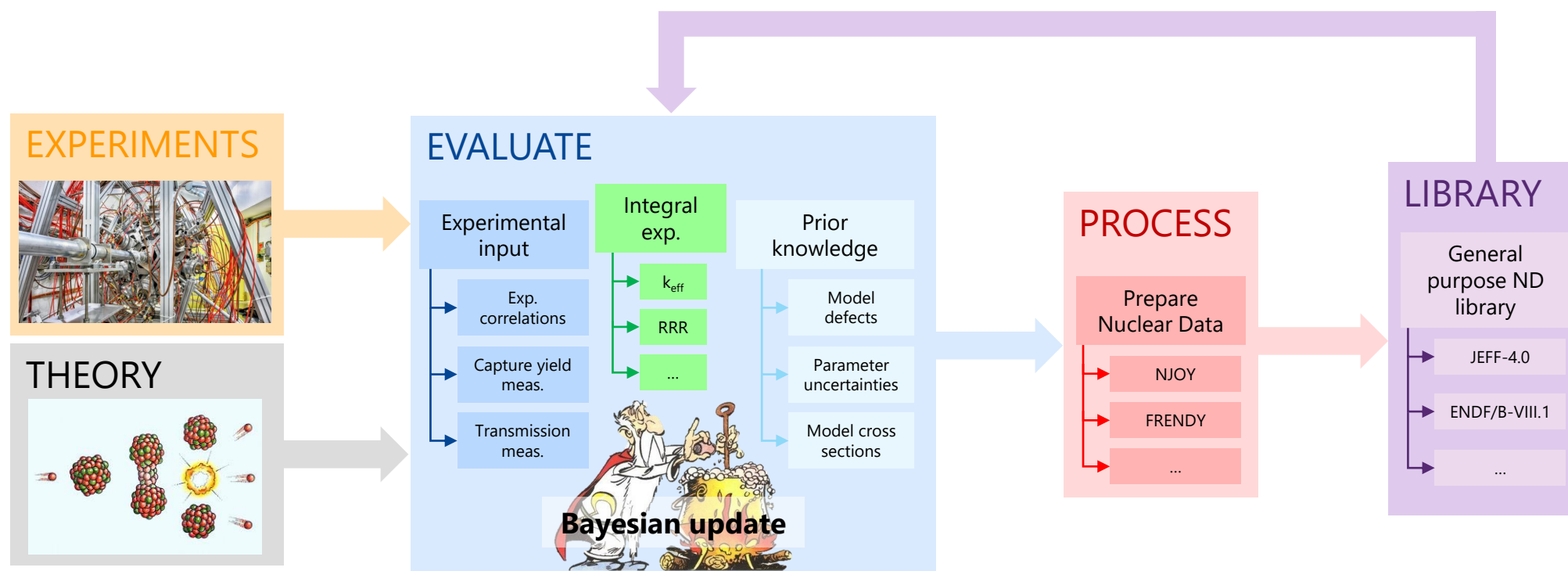
# Introduction

## Nuclear data evaluation process



# Objective

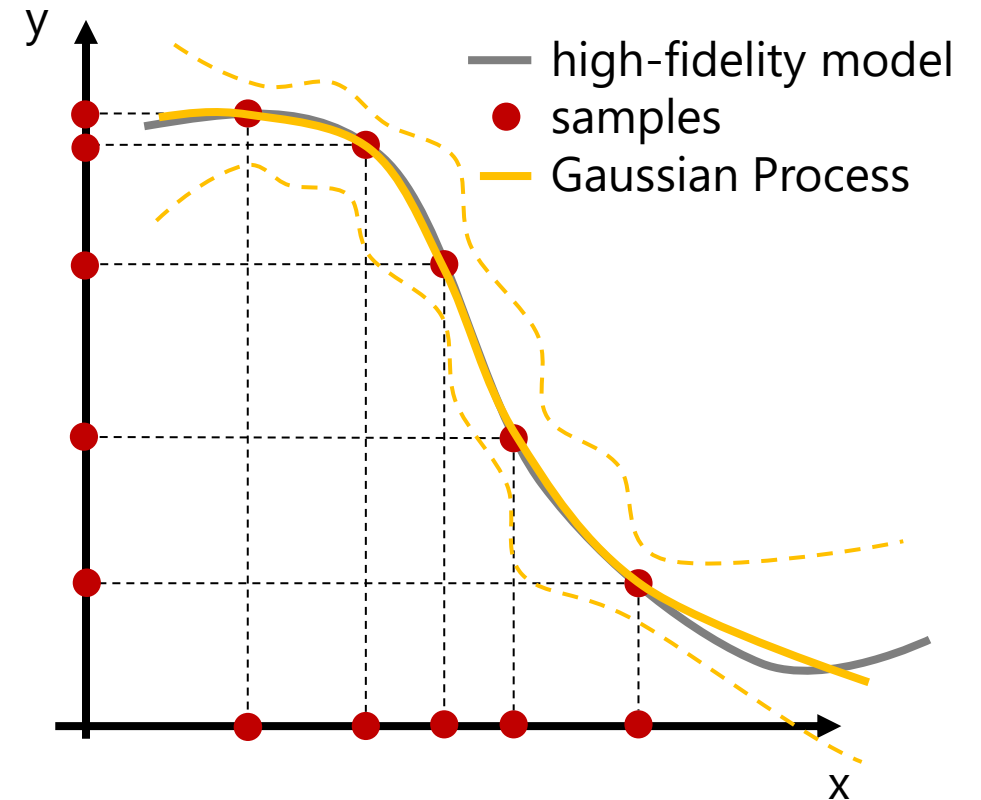
Combine integral and microscopic experiments in one step



# Methodology

## Surrogate modelling

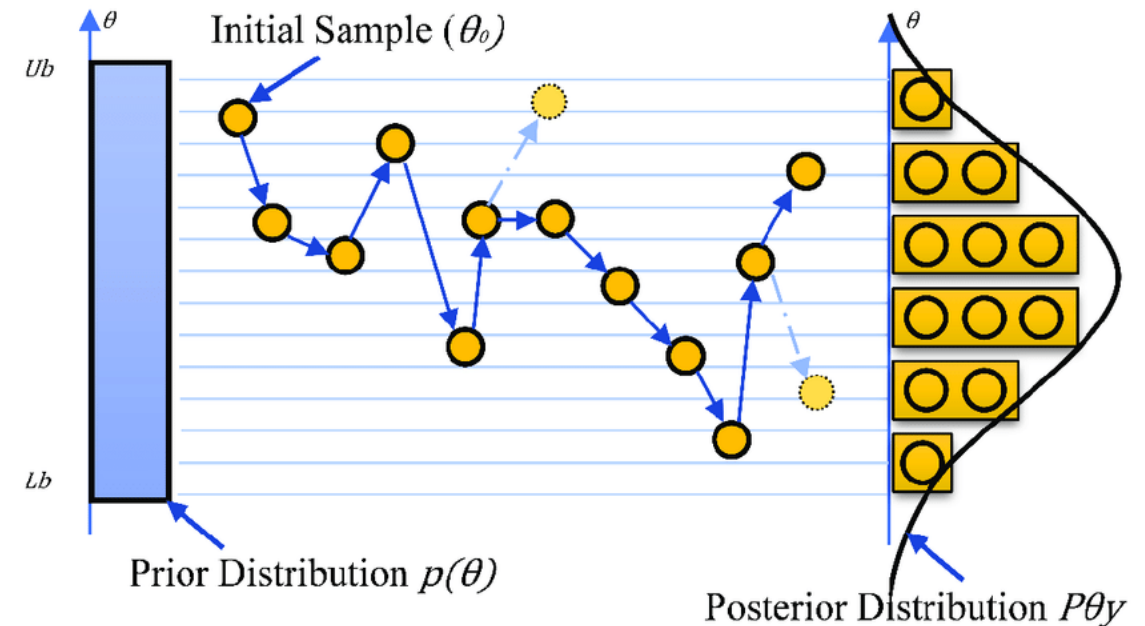
- SAMMY and mostly SERPENT evaluations are expensive
- Train easy to evaluate surrogate model
  - Sample input parameters
  - Evaluate sample in high-fidelity code
  - Split in training and testing set (80/20)
  - Train a Gaussian Process with training set
  - Test GP on testing set



# Methodology

## Markov Chain Monte Carlo

- Instead of randomly sampling the input space
- Carefully select next sample based on likelihood
- Propose new position:
  - If position is more likely  $\rightarrow$  Accept
  - If not  $\rightarrow$  sample to Accept / Reject
- If Accepted  $\rightarrow$  go there
- Rejected  $\rightarrow$  propose new position
- Eventually we sample from stationary distribution = target distribution



# Methodology

## Markov Chain Monte Carlo

- Accept proposed state based on ratio of unnormalized posterior:
  - Probability of observing proposed state in prior distribution
  - Probability of observing experiments given proposed state
- Probabilities can be small!
  - Take log for numerical stability

$$p(\theta \mid \text{data}) = \frac{p(\text{data} \mid \theta) \cdot p(\theta)}{p(\text{data})}$$

Posterior

Likelihood

Prior

Normalization

Proposed<sup>(n+1)</sup>  
Current<sup>(n)</sup>

$$p(\theta \mid \text{data}) = \frac{p(\text{data} \mid \theta) \cdot p(\theta)}{p(\text{data})}$$

Posterior

Likelihood

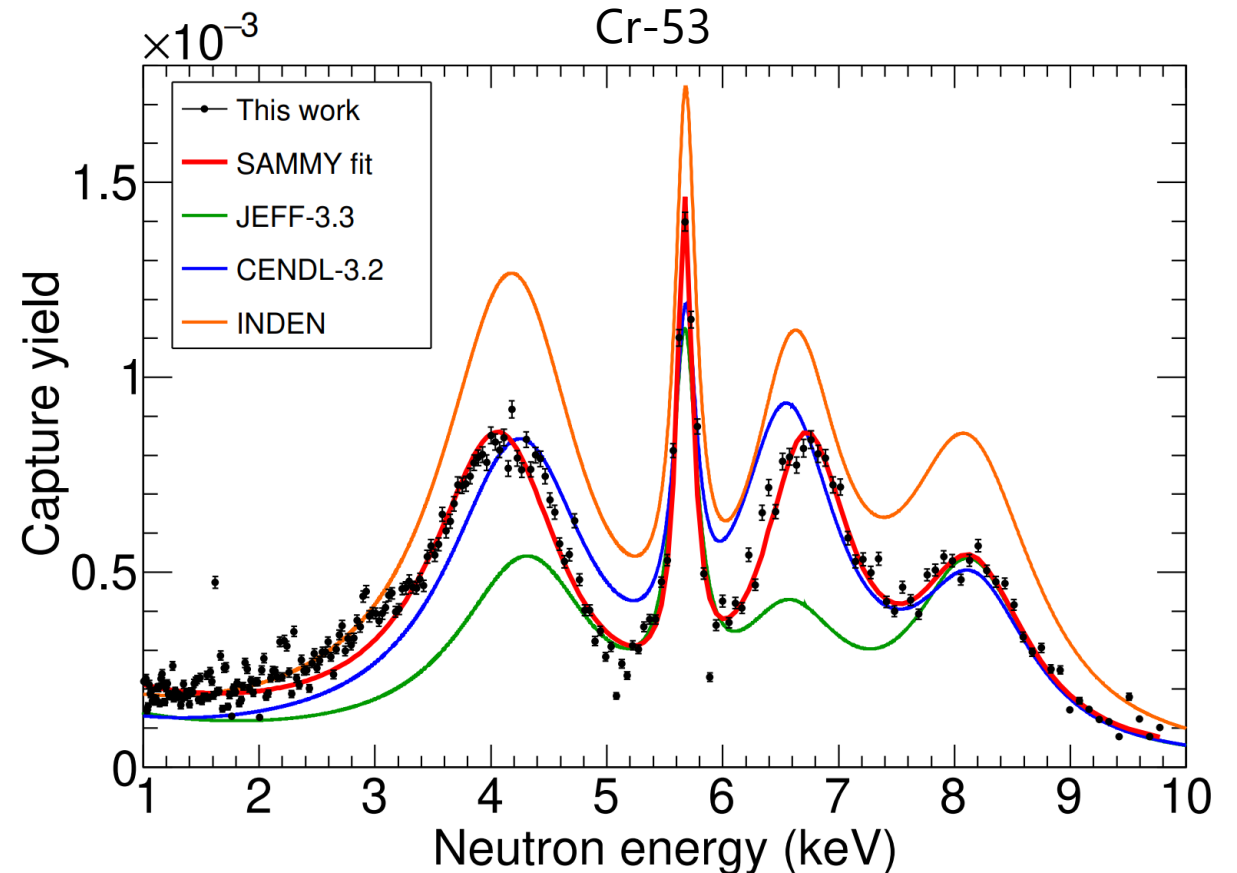
Prior

Normalization

# Case study: $^{53}\text{Cr}$

## New capture yield measurement

- Neutron time-of-flight
- Using thin target
- Not yet included in evaluation
- Focus on 1-10 keV range
  - **Infer  $\Gamma_\gamma$ -widths**

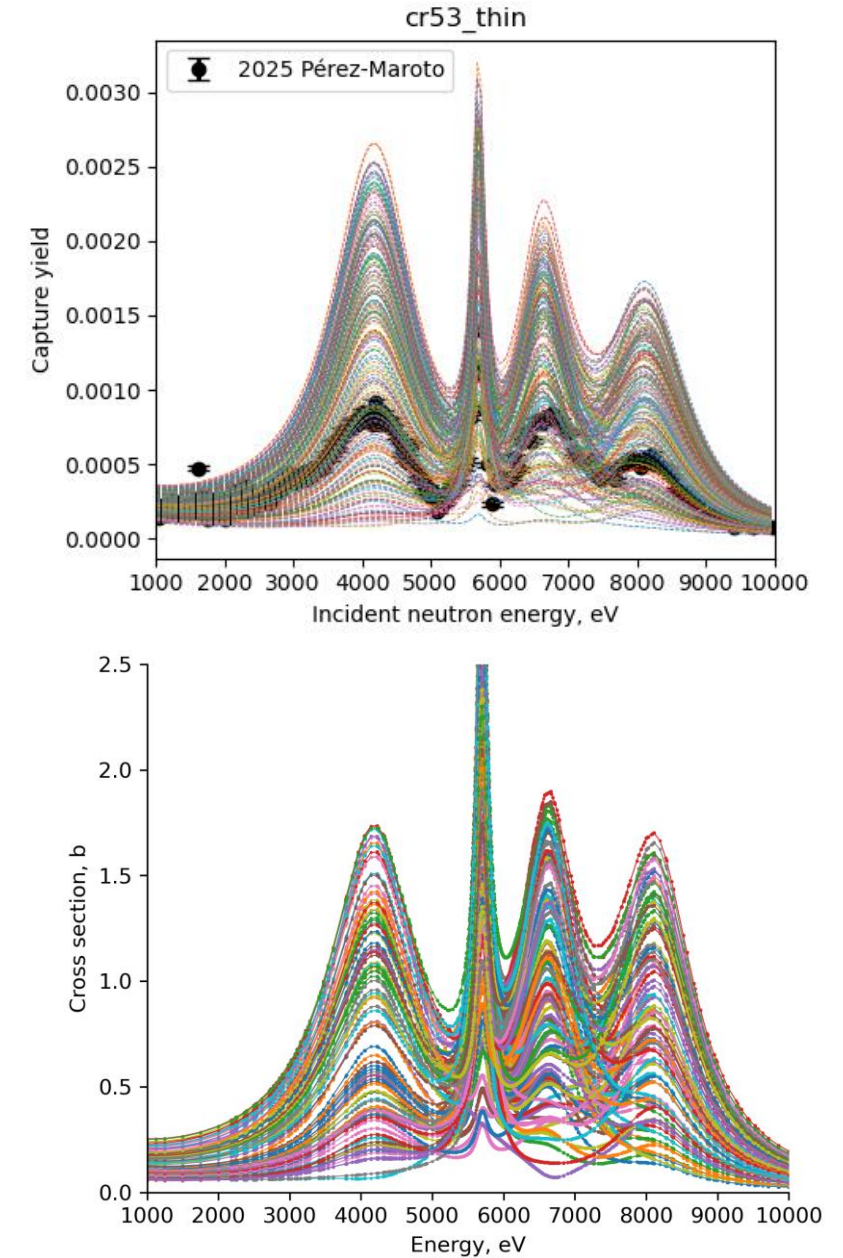


from P. Pérez-Maroto et al., “ $^{50}\text{Cr}$  and  $^{53}\text{Cr}$  neutron capture cross sections measurement at the  $n_{\text{TOF}}$  facility at CERN,” 2025

# Results

## Input data

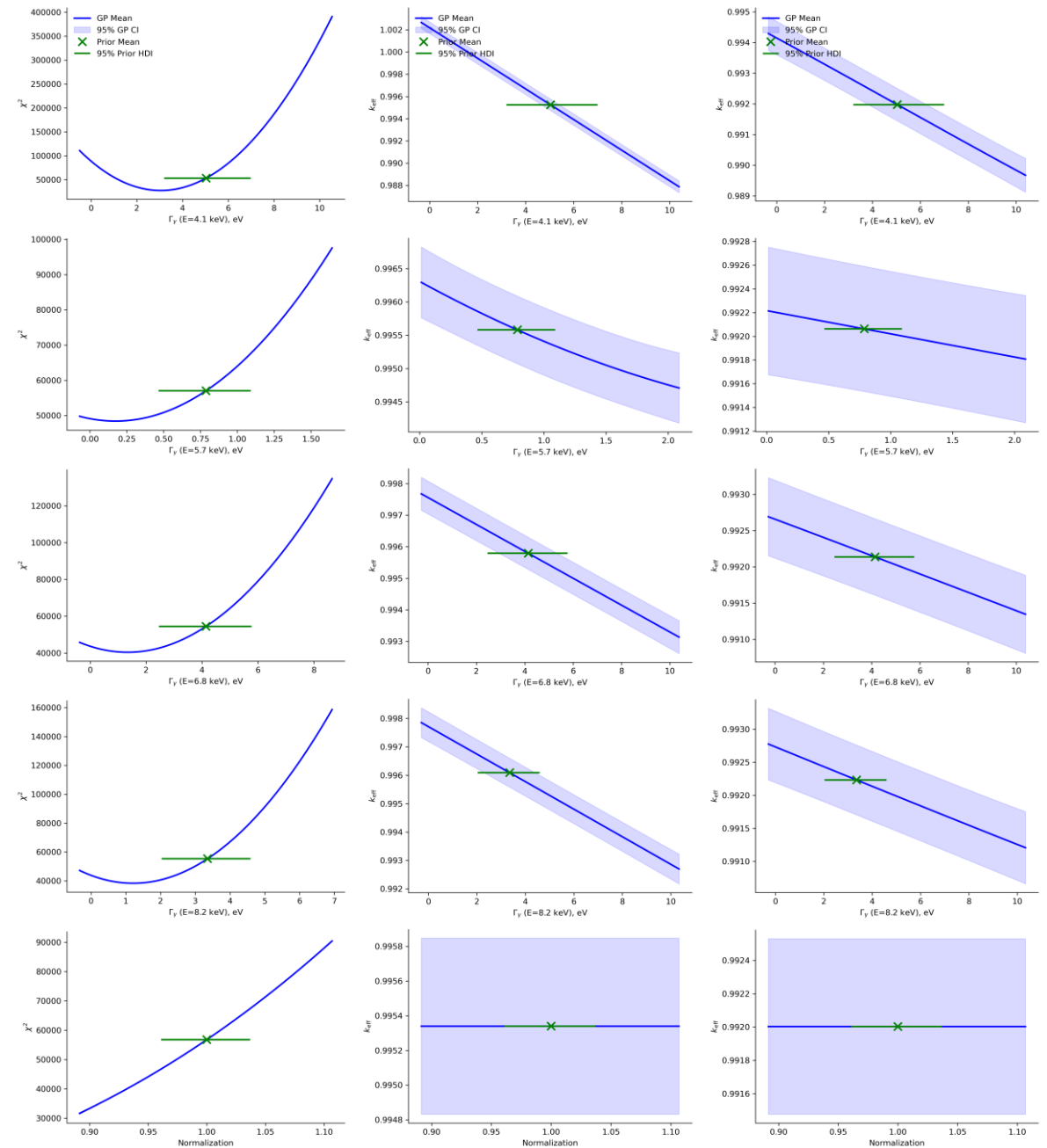
- Draw 100 uniform samples of:
  - $\Gamma_\gamma$ 's
  - Normalization for capture yield



# Results

## Surrogate behavior

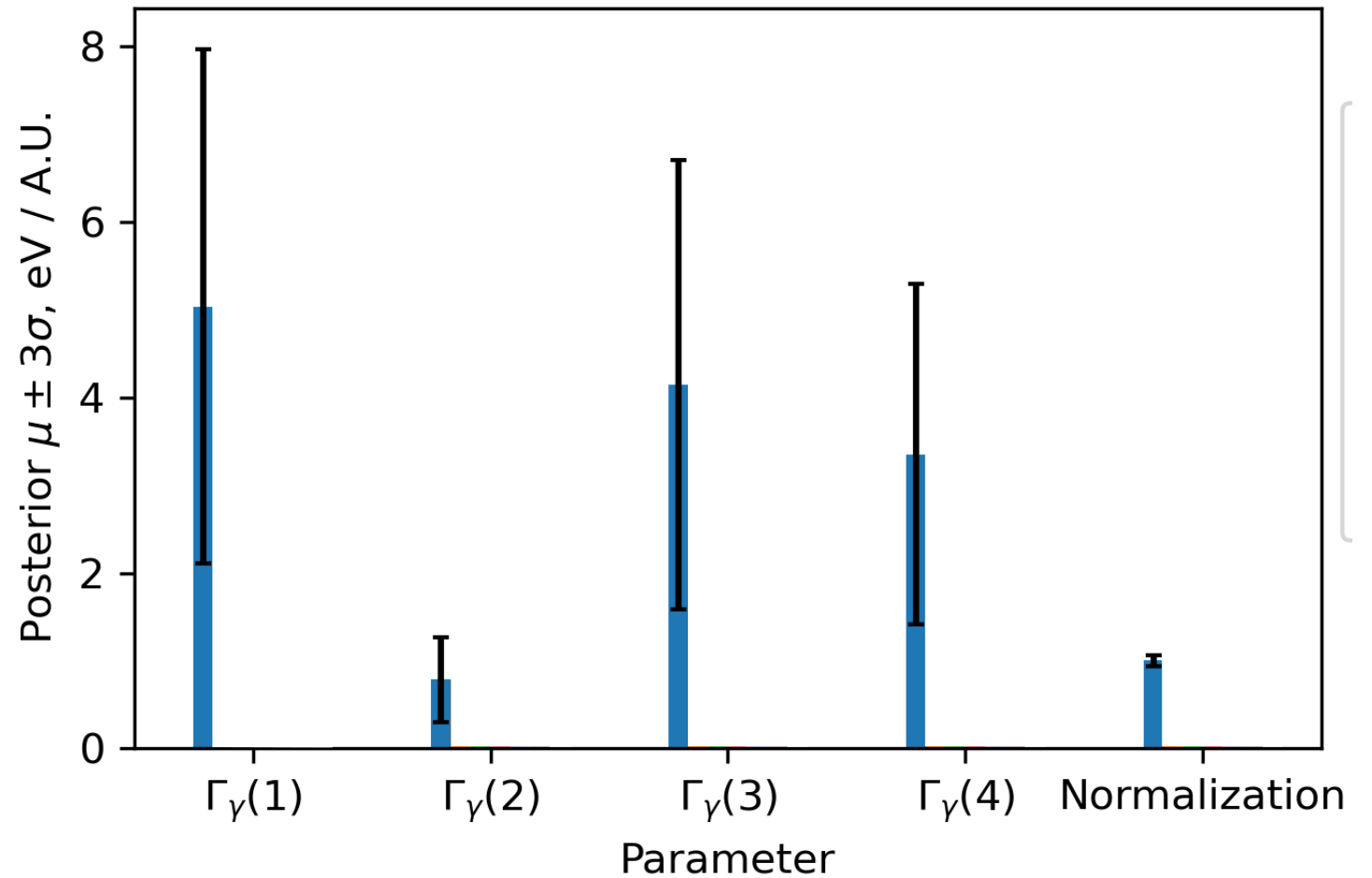
- Learn behavior of experiment
- GP performance on testing set
- SAMMY
- SERPENT



# Results

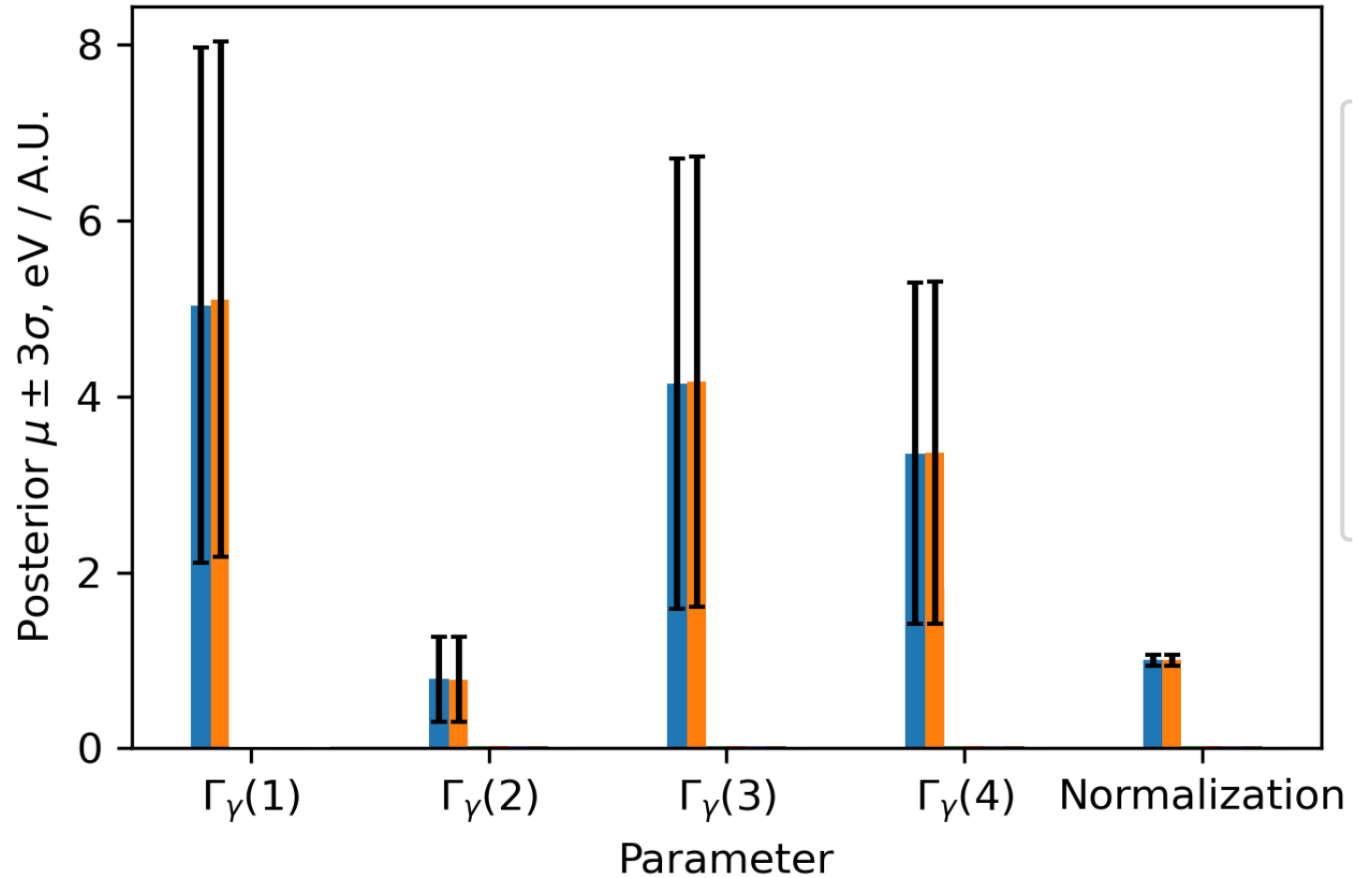
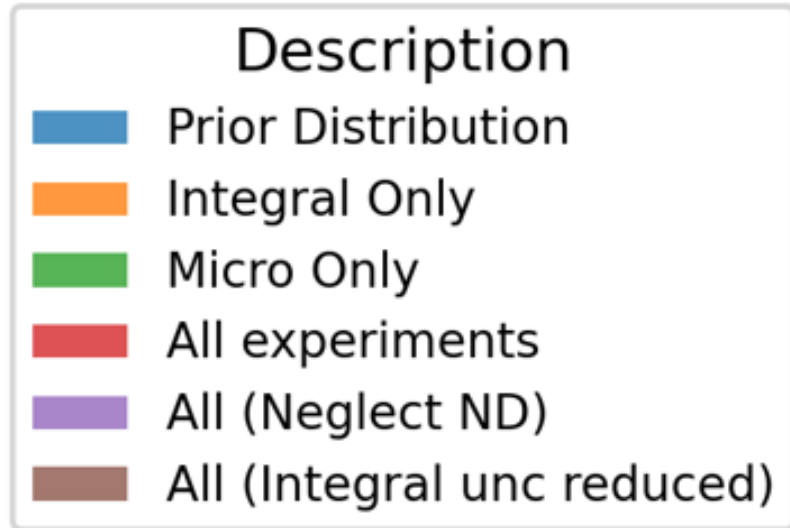
## Bayesian optimization

Description	
■	Prior Distribution
■	Integral Only
■	Micro Only
■	All experiments
■	All (Neglect ND)
■	All (Integral unc reduced)



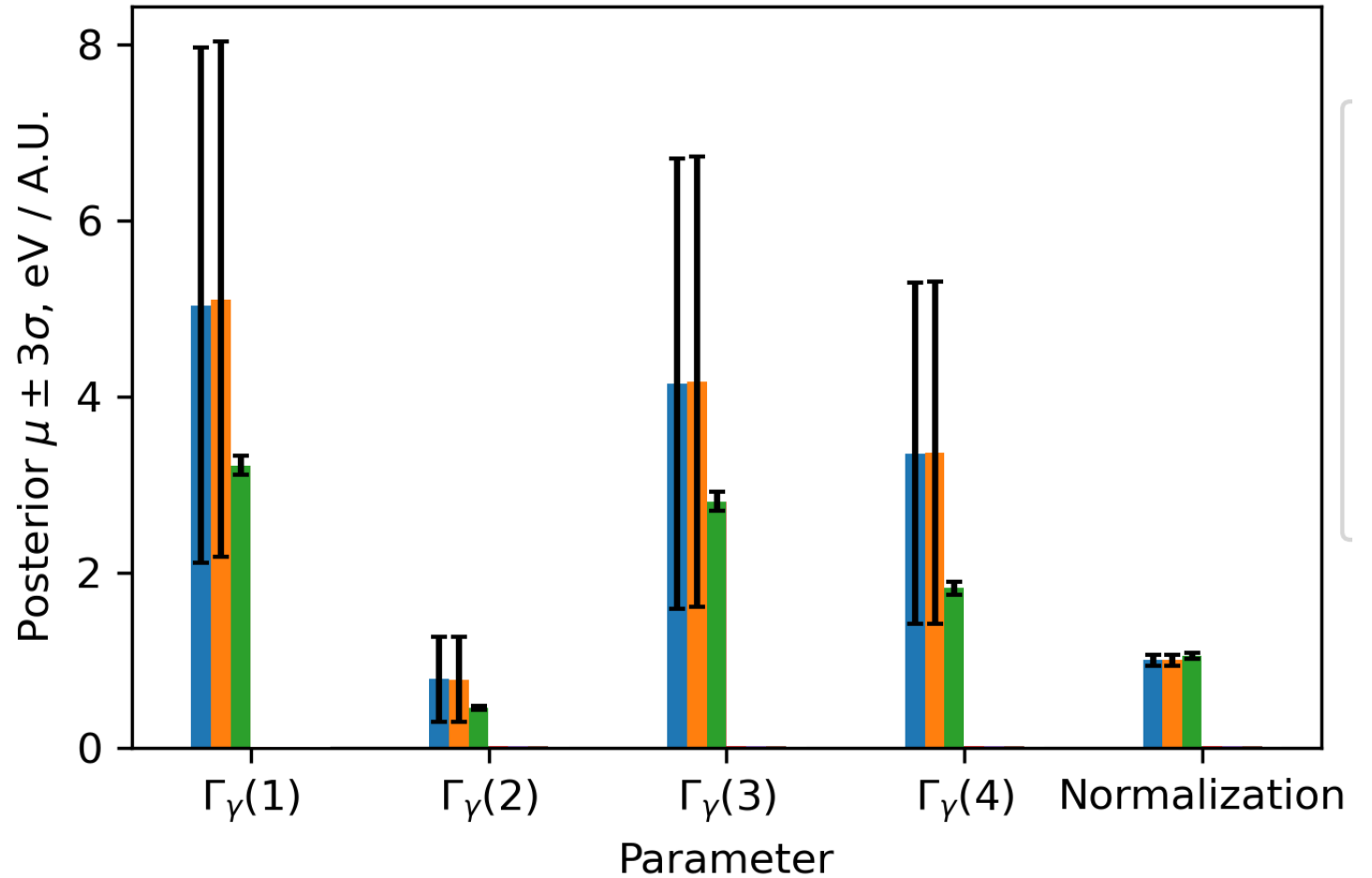
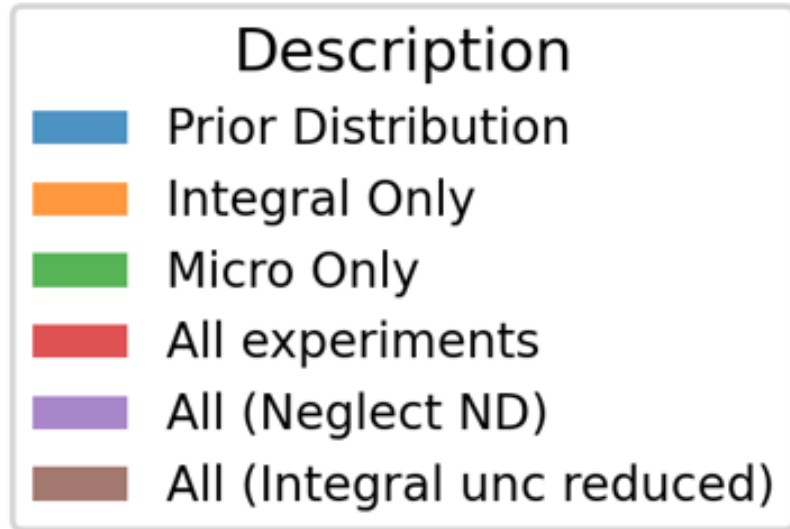
# Results

## Bayesian optimization



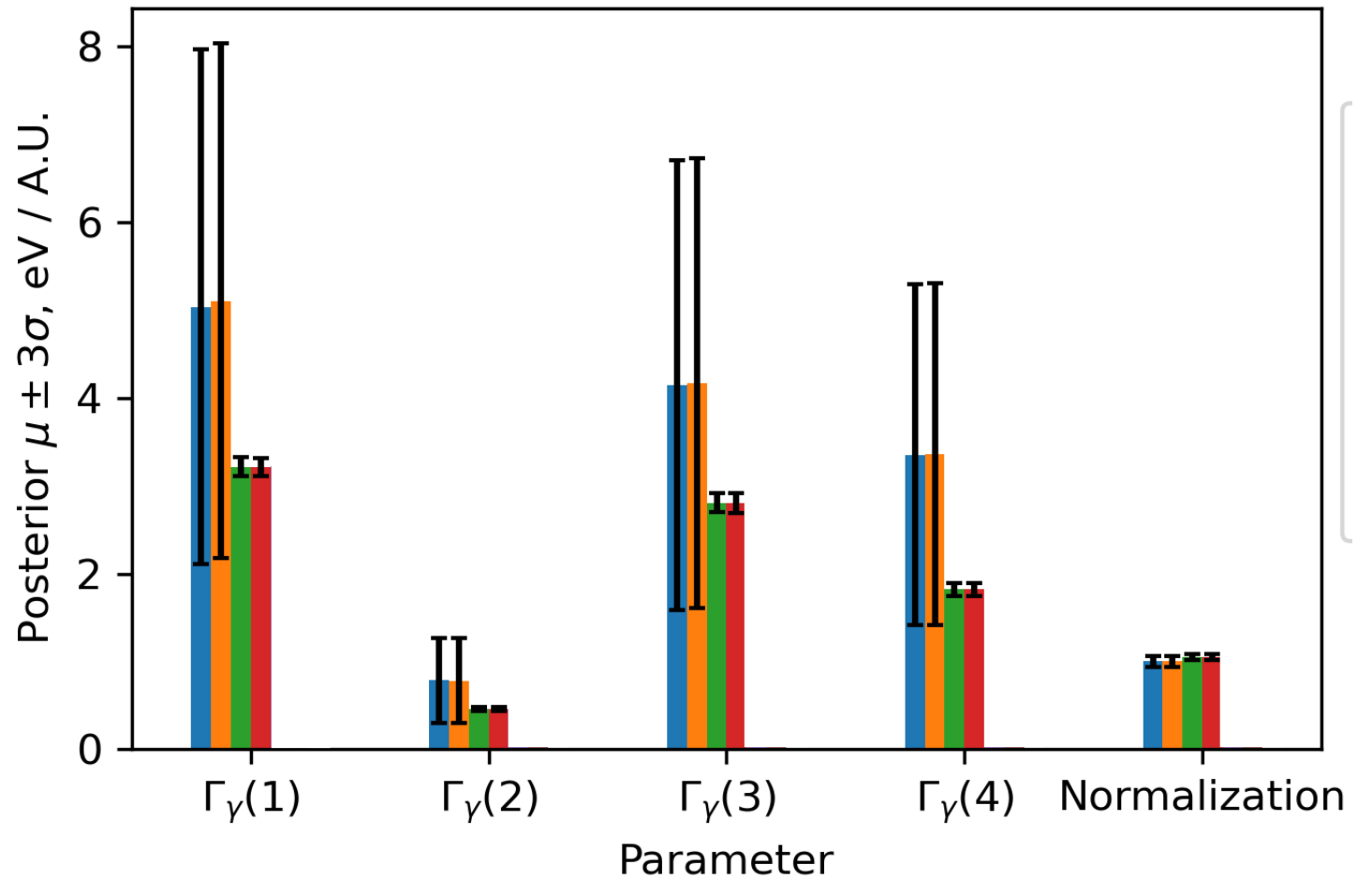
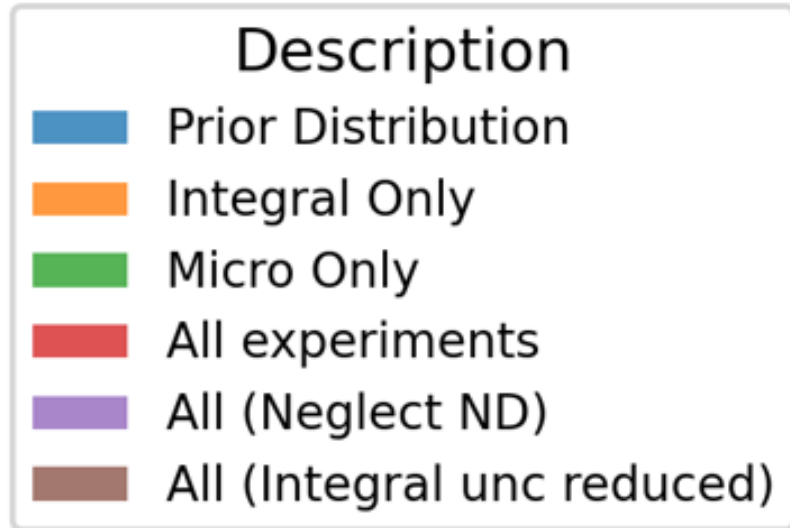
# Results

## Bayesian optimization



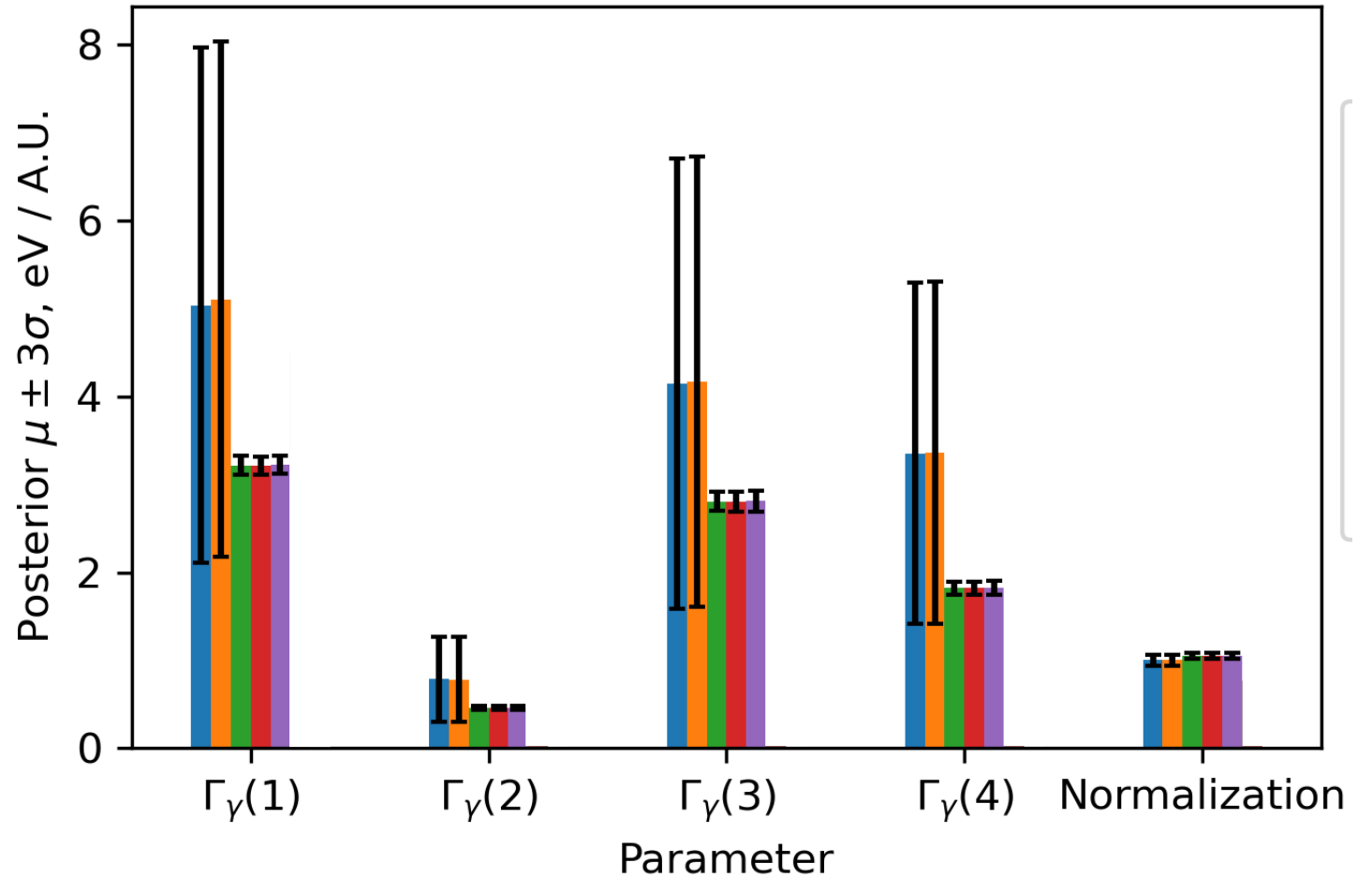
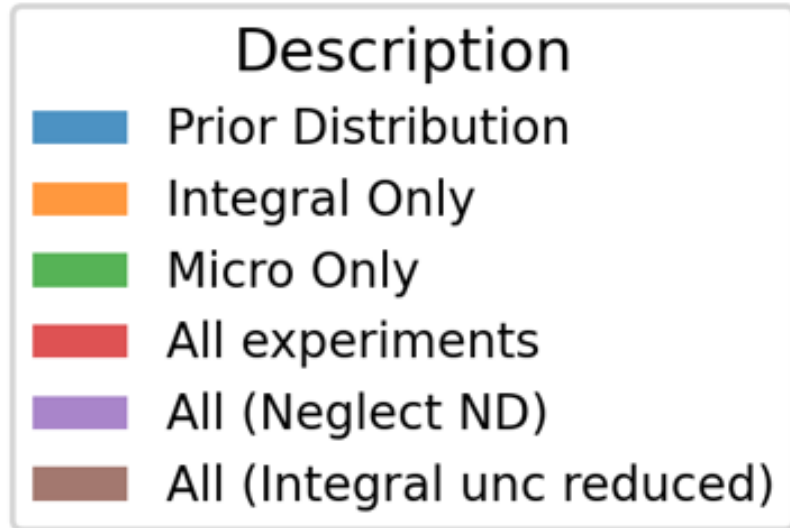
# Results

## Bayesian optimization



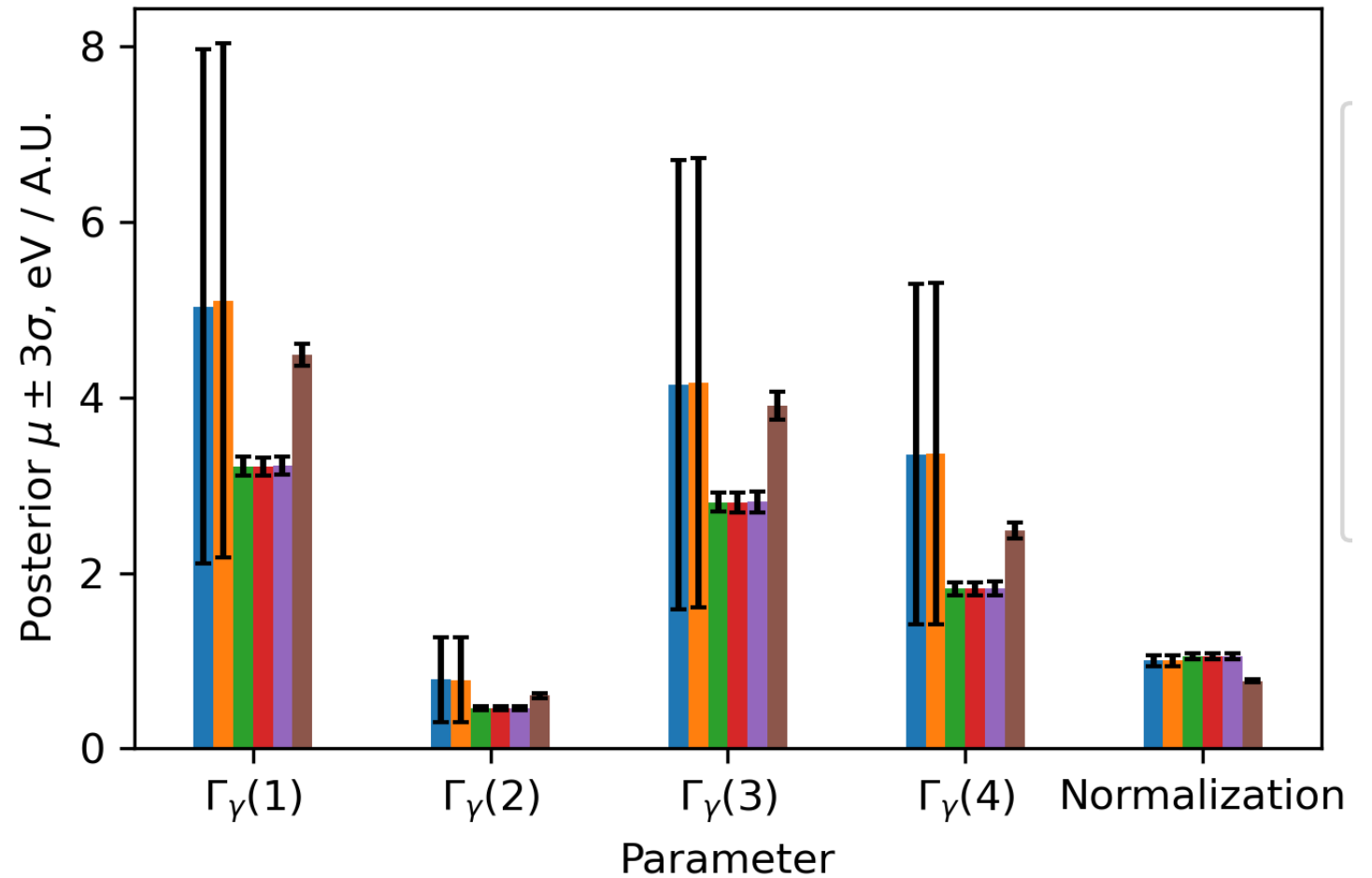
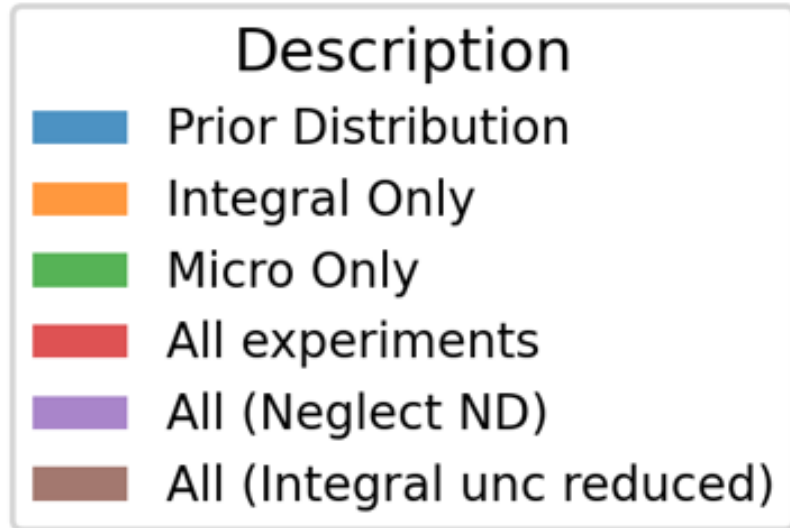
# Results

## Bayesian optimization



# Results

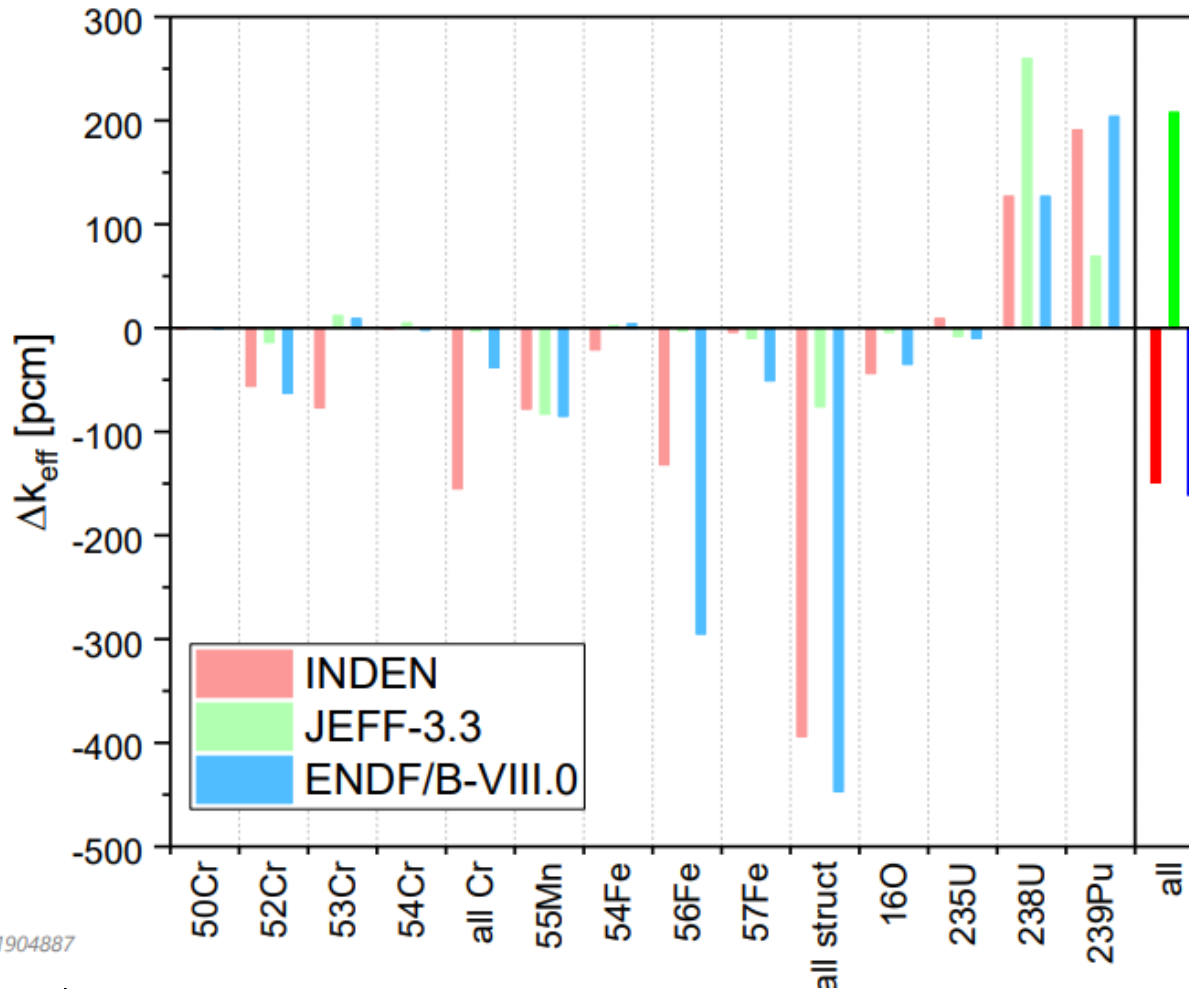
## Bayesian optimization



# Keff sensitivity to ND library

Nuclear data files were processed at relevant temperatures (1300 K for fuel, 650 K structural materials)  
Reference library JEFF-3.1.2. Replacement per isotope or whole data set

Update of INDEN (higher statistics) + JEFF-3.3 and ENDF/B-VIII.0



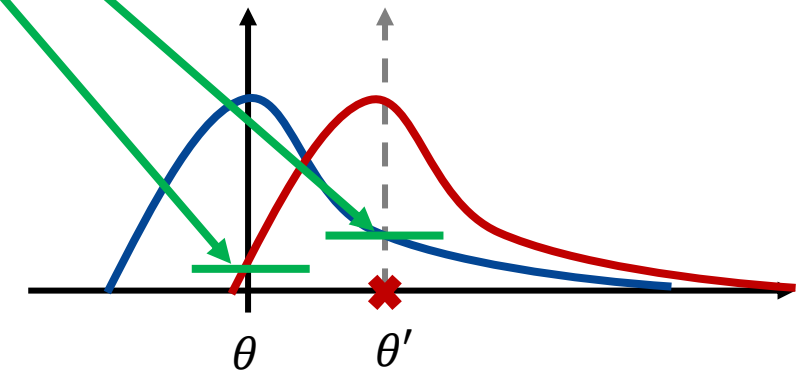
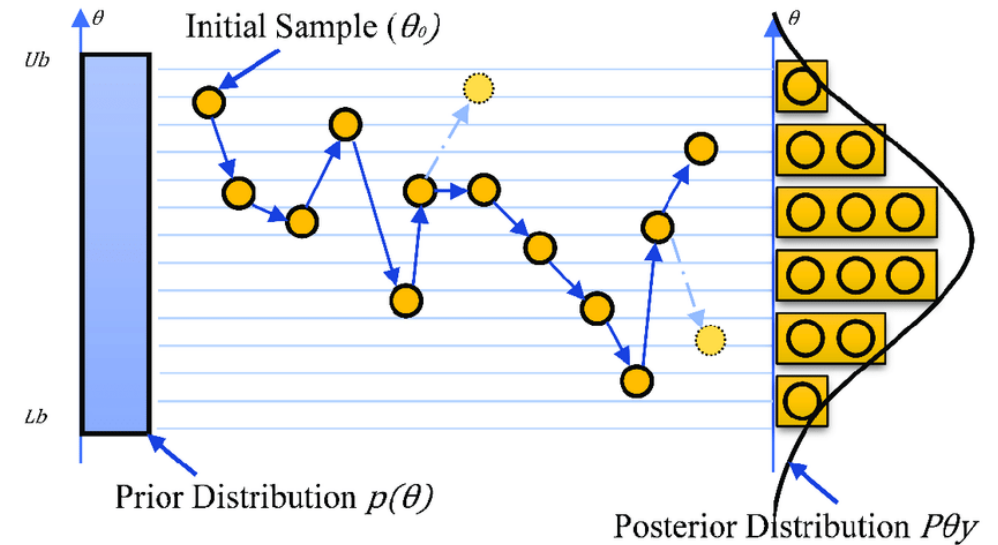
- Results of Cr (all isotopes) are confirmed
- Individual Cr isotopes became consistent, so that sensitivity to all is linear combination of individual
- Negative effect of all structural materials is even more pronounced in case of ENDF/B-VIII.0
- Overall, INDEN and ENDF/B-VIII.0 provide lower  $k_{\text{eff}}$  that reference JEFF-3.1.2 while JEFF-3.3 results in highest  $k_{\text{eff}}$

# Methodology

## Metropolis-Hastings algorithm

1. Start from a random starting position  $\theta$
2. Propose new position  $\theta'$  by sampling from lognormal distribution centered in  $\theta$
3. Accept or reject new position:  $\alpha(\theta'|\theta) = \min\left(1, \frac{\pi(\theta')}{\pi(\theta)} \cdot \frac{q(\theta|\theta')}{q(\theta'|\theta)}\right)$ 
  - $\frac{\pi(\theta')}{\pi(\theta)}$  is ratio of unnormalized posterior distributions
    - Likelihood x prior (normalizing constant cancels in ratio)
  - If reject  $\rightarrow$  propose another position
4. Sample many times until stationary distribution is reached

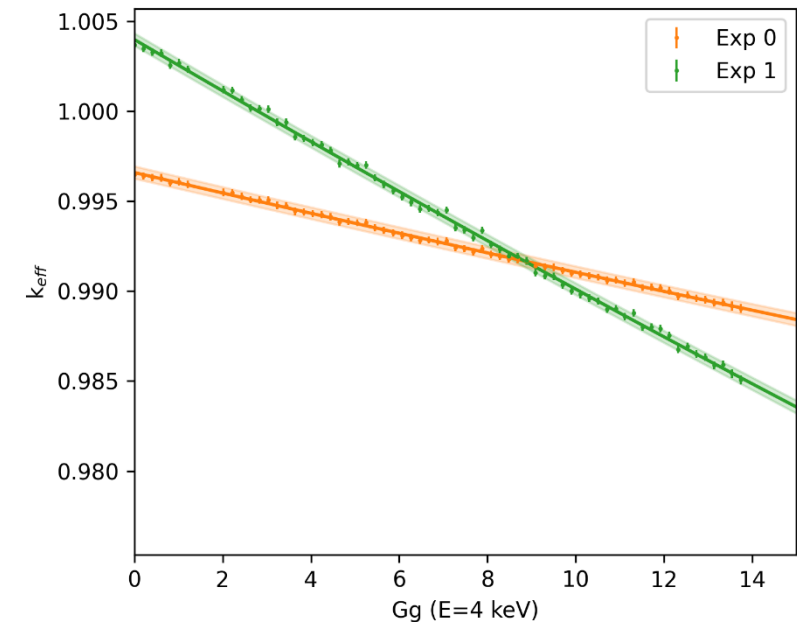
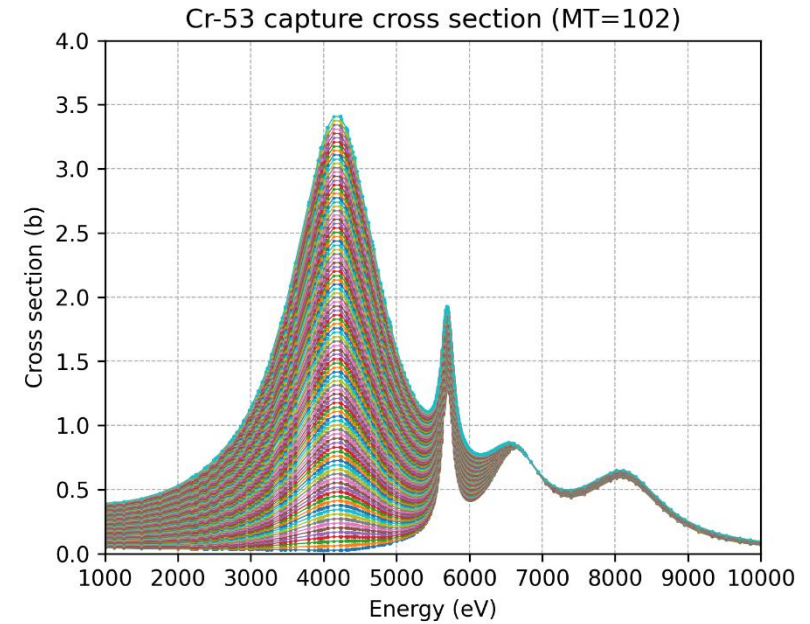
Stationary distribution = target distribution



# PRELIMINARY

## Results

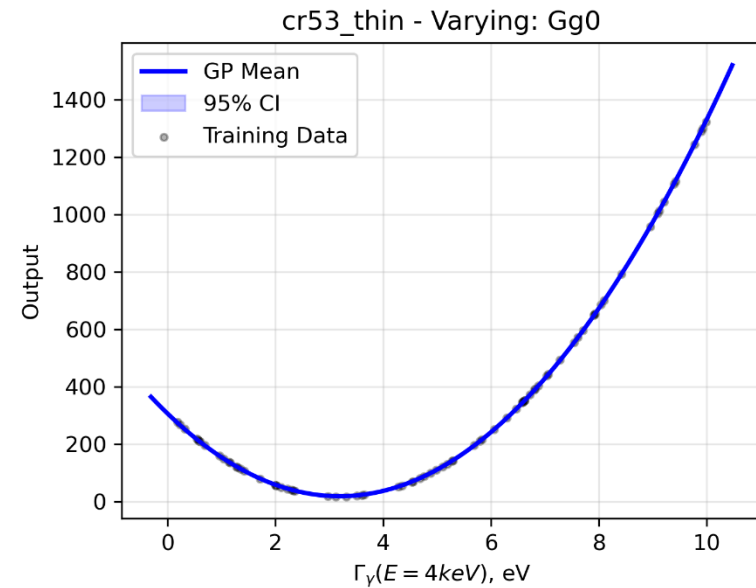
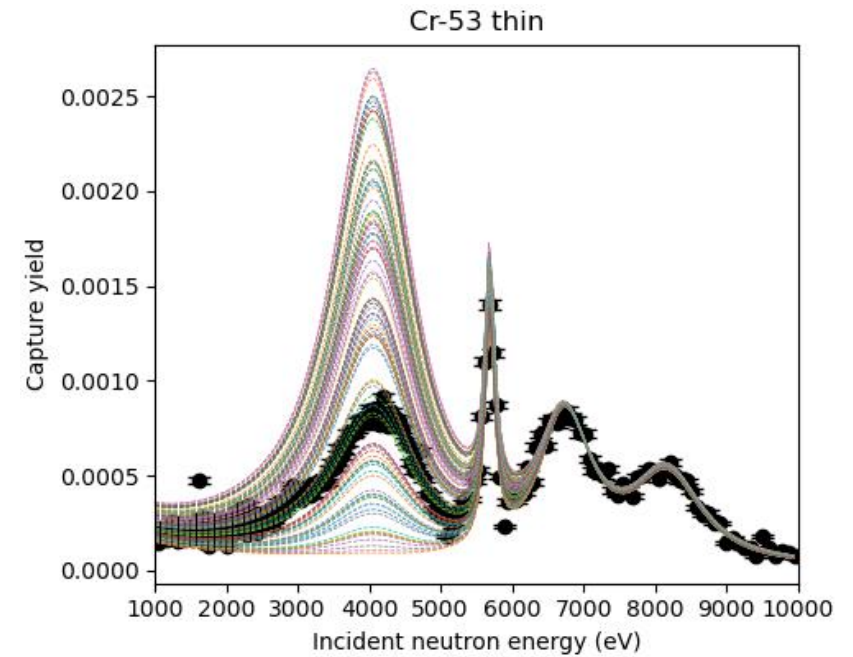
- Surrogate integral measurement
- Vary the first  $\Gamma_\gamma$  width (4 keV)
- For 2 criticality experiments:
  - Sensitive to Cr-53



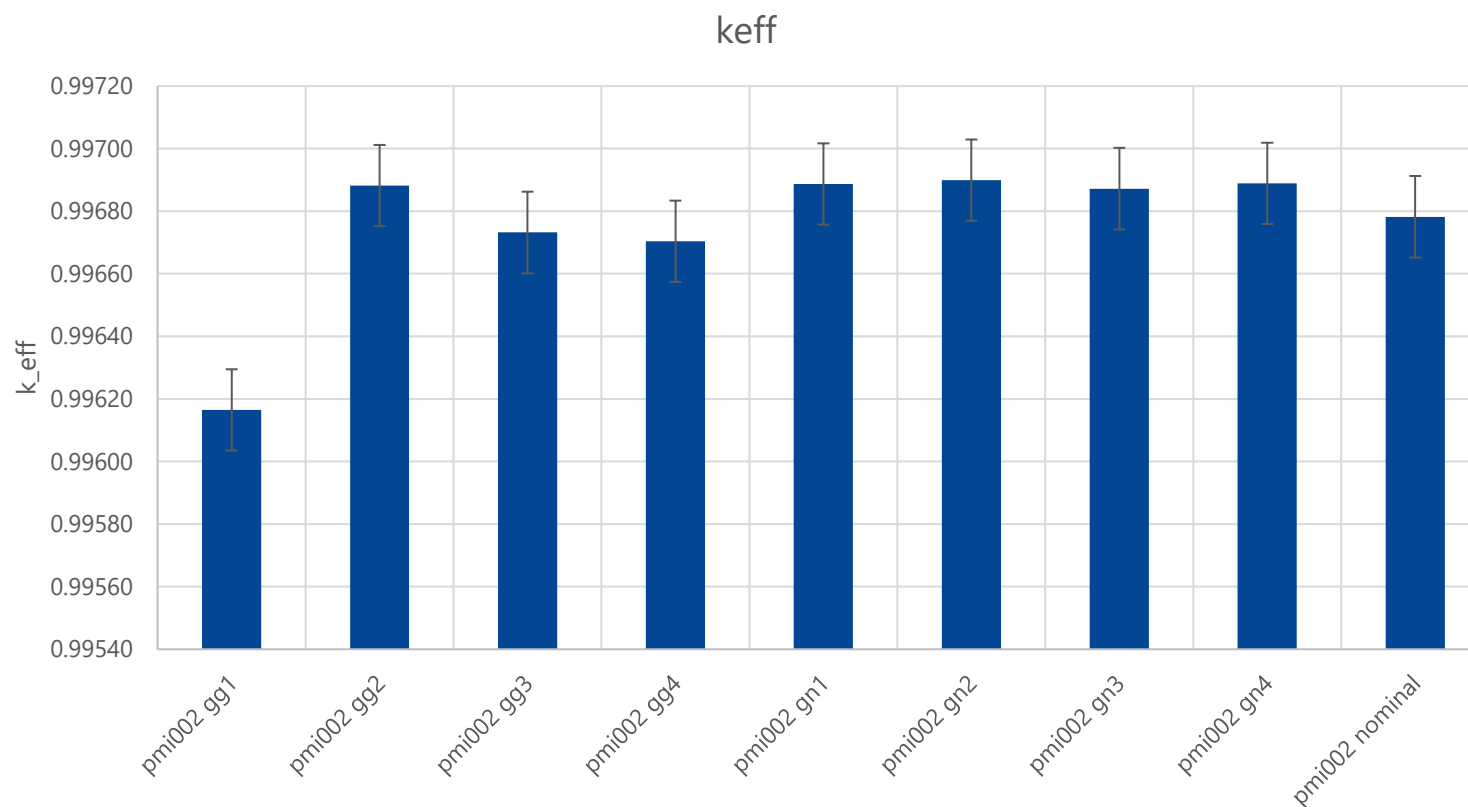
# PRELIMINARY

## Results

- Microscopic measurement: nTOF
- Vary the first  $\Gamma_\gamma$  width ( $E=4$  keV)
- Calculate chi-squared on measurement

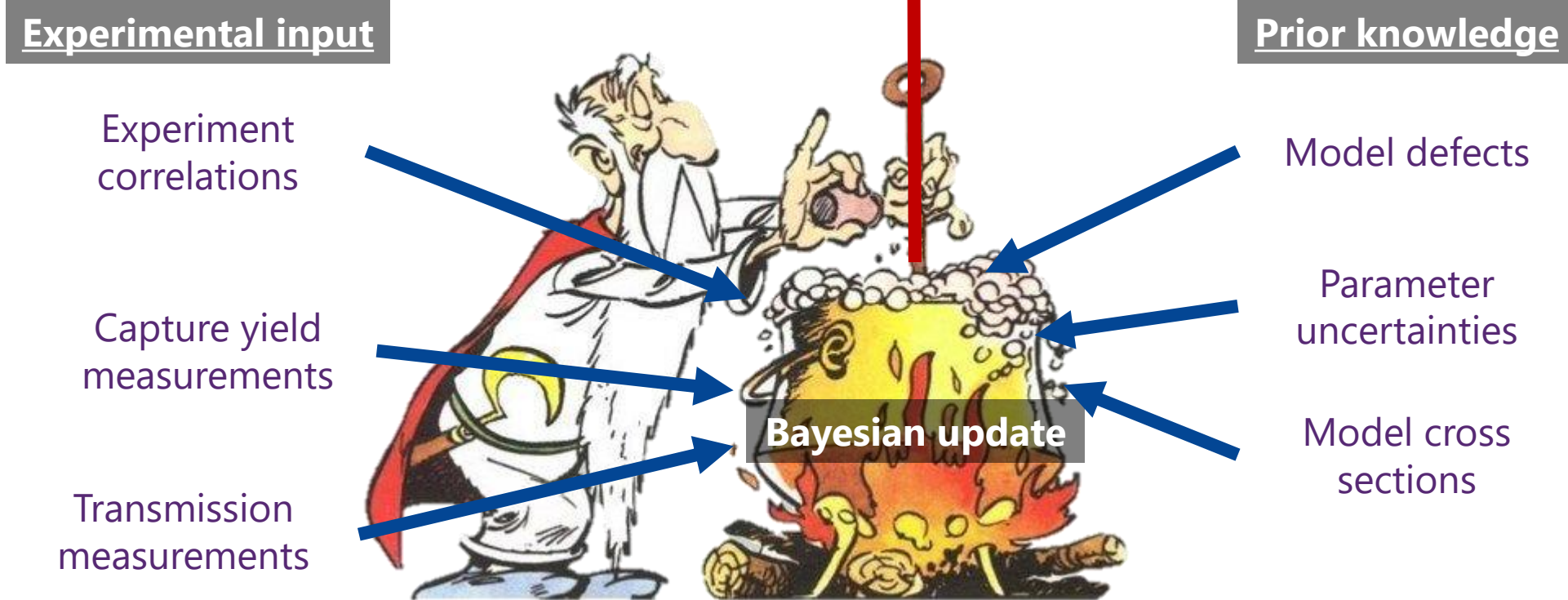


# Sensitivity to Gg (PMI002)



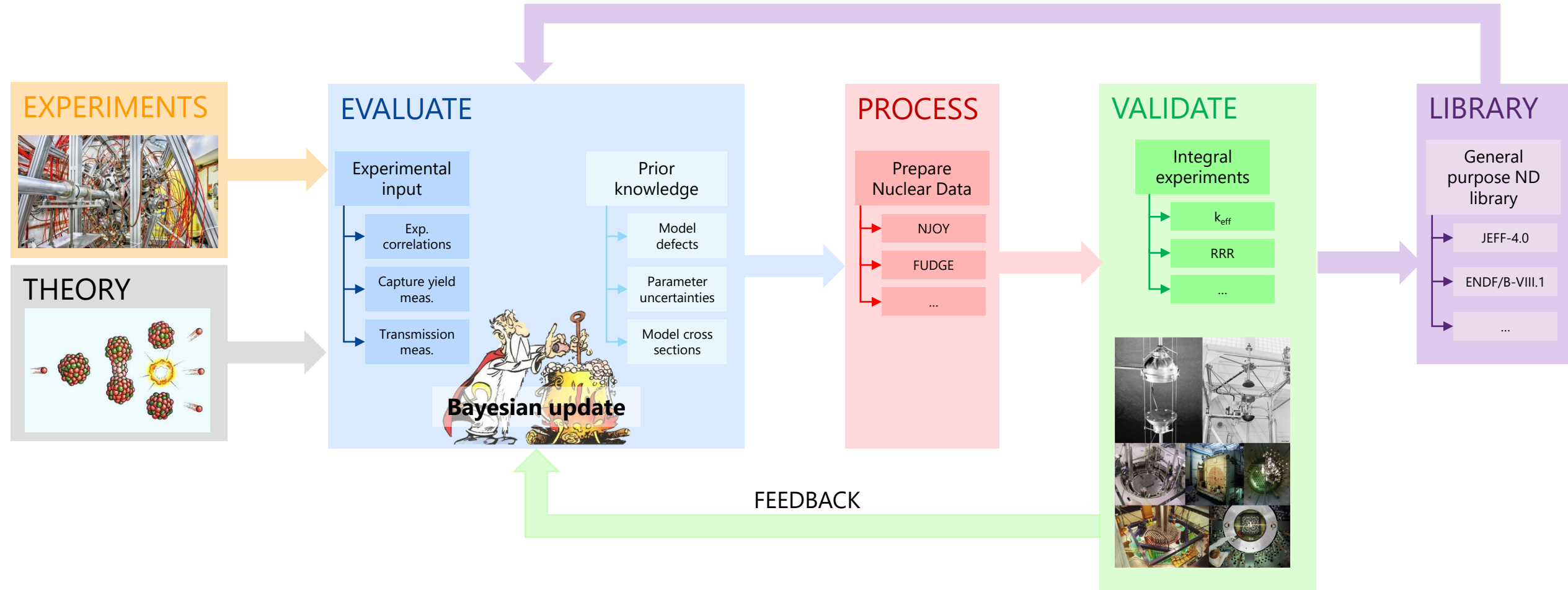
# Introduction

## ND Evaluation



from D. Neudecker et al.

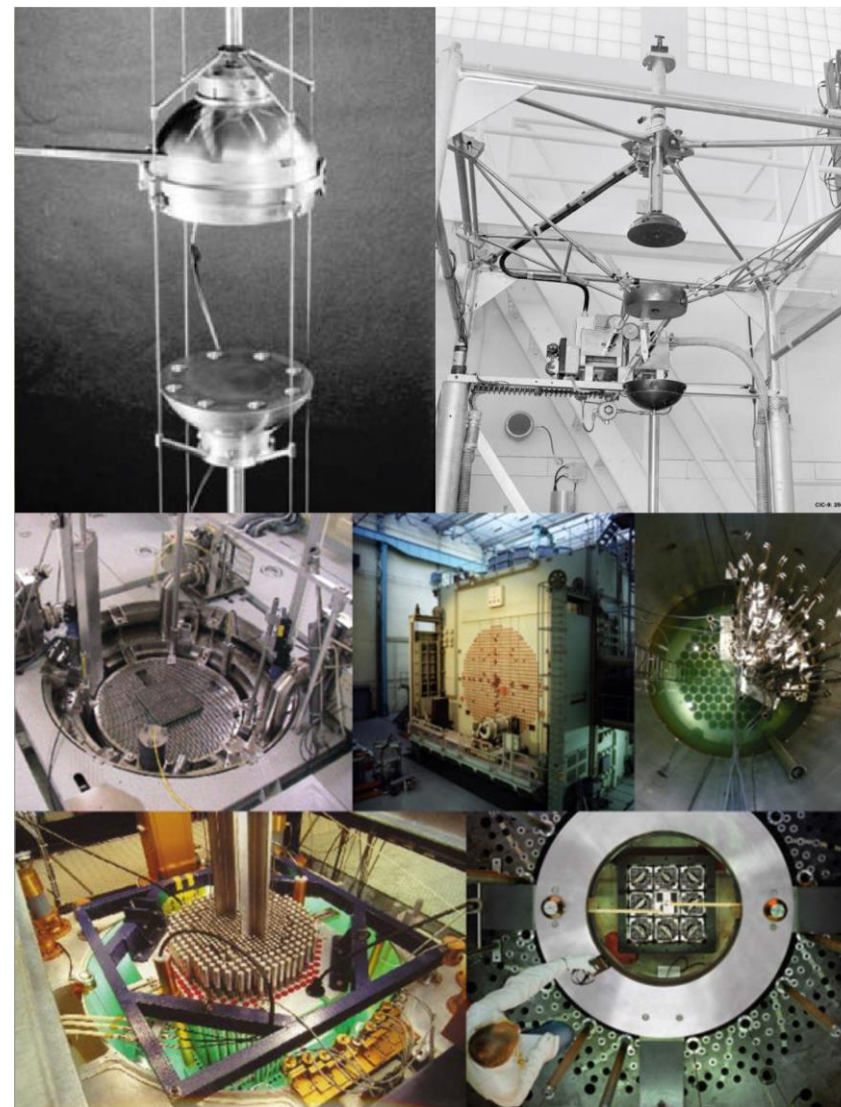
# Introduction



# Introduction

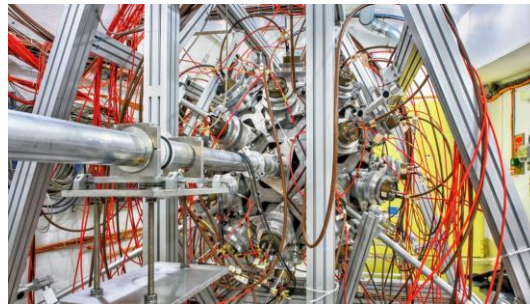
## Validation on integral experiments

- Well-known benchmarks
- Calculate C/E:
  - Critical assemblies
  - Pulsed spheres
  - Shielding benchmarks
- Inverse modelling to provide feedback

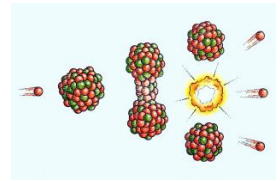


# Introduction

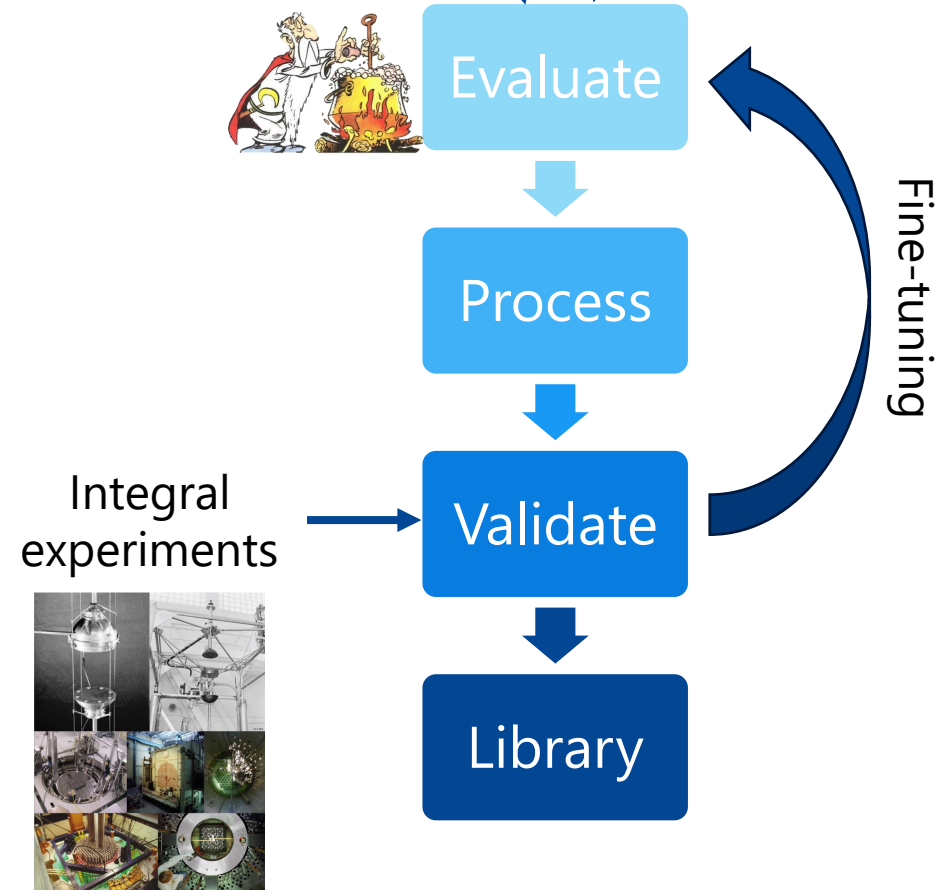
- Nuclear data evaluation process
  - Multi-step process
  - Evaluation → Bayesian approach
  - Integral experiments used for validation



Microscopic experiments



Nuclear theory



## Experimental observables

		$\Gamma_r \gg \Gamma_n$	$\Gamma_r \ll \Gamma_n$
• Transmission	$A_{t,\text{thin}} \propto g\Gamma_n$	$g\Gamma_n$	$g\Gamma_n$
$\ell = 0$ assignment scattering radius	$A_{t,\text{thick}} \propto \sqrt{g\Gamma_n \Gamma}$	$\sqrt{g\Gamma_n \Gamma_r}$	$\sqrt{g\Gamma_n}$
	$\Rightarrow g\Gamma_n, \Gamma$	<u><math>g\Gamma_n, \Gamma_r</math></u>	<u><math>g, \Gamma_n</math></u>
• Reaction	$A_{r,\text{thin}} \propto g \frac{\Gamma_n \Gamma_r}{\Gamma}$	$g\Gamma_n$	<u><math>g\Gamma_r</math></u>
Higher sensitivity for weak resonances	$\Rightarrow$	<u><math>g\Gamma_n, \Gamma_r</math></u>	<u><math>g, \Gamma_n, \Gamma_r</math></u>

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### **SCK CEN**

Belgian Nuclear Research Centre

Foundation of Public Utility

Registered Office: Avenue Herrmann-Debrouxlaan 40 – BE-1160 BRUSSELS

Operational Office: Industriezone Boeretang Zuid - Boeretang 190 – BE-2400 MOL