

**Less or not at all investigated correlations
between different fragment and
prompt emission quantities**

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**WONDER-2026,
29 June – 3 July 2026, Aix-en-Provence, France**

This presentation includes an investigation of dependences and possible correlations between different physical quantities characterizing the pre-neutron (initial) fragments, the successive residual fragments, including the post-neutron fragments (which are the last residuals at the end of prompt emission) and the emission of prompt neutrons and prompt γ -rays.

Some of these correlations are already known and have been investigated, e.g. the correlation between ν and $E\gamma$, consisting in an almost linear increasing of $E\gamma(\nu)$. Other correlations have been investigated much less or not at all. And just these latter are investigated in this work.

Finding such dependences and correlations could be helpful for the prediction of nuclear data of prompt emission especially for fissioning nuclei without any experimental information.

The **DSE model code** provides almost all fragment and prompt emission quantities **as matrices** that depend on the **{A,Z,TKE} configuration** of initial (pre-neutron) fragments, i.e.

- for each emission sequence $q_k(A,Z,TKE)$ e.g. $\langle \varepsilon \rangle_k(A,Z,TKE)$, $\eta_k(A,Z,TKE)$, $\langle E^*_k \rangle(A,Z,TKE)$, $\langle E_{\text{post}} \rangle(A,Z,TKE)$, $T_k(A,Z,TKE)$, $\varphi_k(\varepsilon,A,Z,TKE)$ etc.
- as average over the emission sequences $q(A,Z,TKE)$ e.g. $\nu(A,Z,TKE)$, $\langle \varepsilon \rangle(A,Z,TKE)$, $\eta(A,Z,TKE)$, $E\gamma(A,Z,TKE)$, $\varphi(\varepsilon,A,Z,TKE)$, etc.

Due to the deterministic construction of the fragmentation and TKE ranges, **the number of {A,Z,TKE} configurations** usually employed in our calculations **is of about 18000 – 20000**.

The **probabilities of the {A,Z,TKE} configurations** (for which the DSE model code provides results as matrices) **expressed by the $Y(A,Z,TKE)$ distributions** of initial FF **allow to obtain all kinds of dependences (as histograms)**, many of them novel and interesting, leading to a deeper understanding of prompt emission in fission and beyond.

In this study the mentioned dependences are classified in the following categories referring to **prompt emission and fragment quantities as a function of:**

- I. the excitation energy of pre-neutron fragments **E^***
- II. the number of emission sequences (or of emitted prompt neutrons) **n**
- III. the kinetic energy of pre-neutron fragm. **KE** and post-neutron fragm. **KEp**
- IV. the well-known correlation between v and $E\gamma$ (this time referring to those of fragment instead of those of fragment pair involved up to now in the investigations of this correlation).

The following 4 fissioning nuclei with an important role in applications were chosen:

$^{235}\text{U}(n_{\text{th}},f)$ and **$^{239}\text{Pu}(n_{\text{th}},f)$** – as the fissile nuclei for the U-Pu fuel cycle

$^{233}\text{U}(n_{\text{th}},f)$ – as the fissile nucleus of the Th-U cycle

$^{252}\text{Cf}(SF)$ – as an important source of neutrons in many applications

for which **comprehensive prompt emission calculations** were performed, which have **validated**

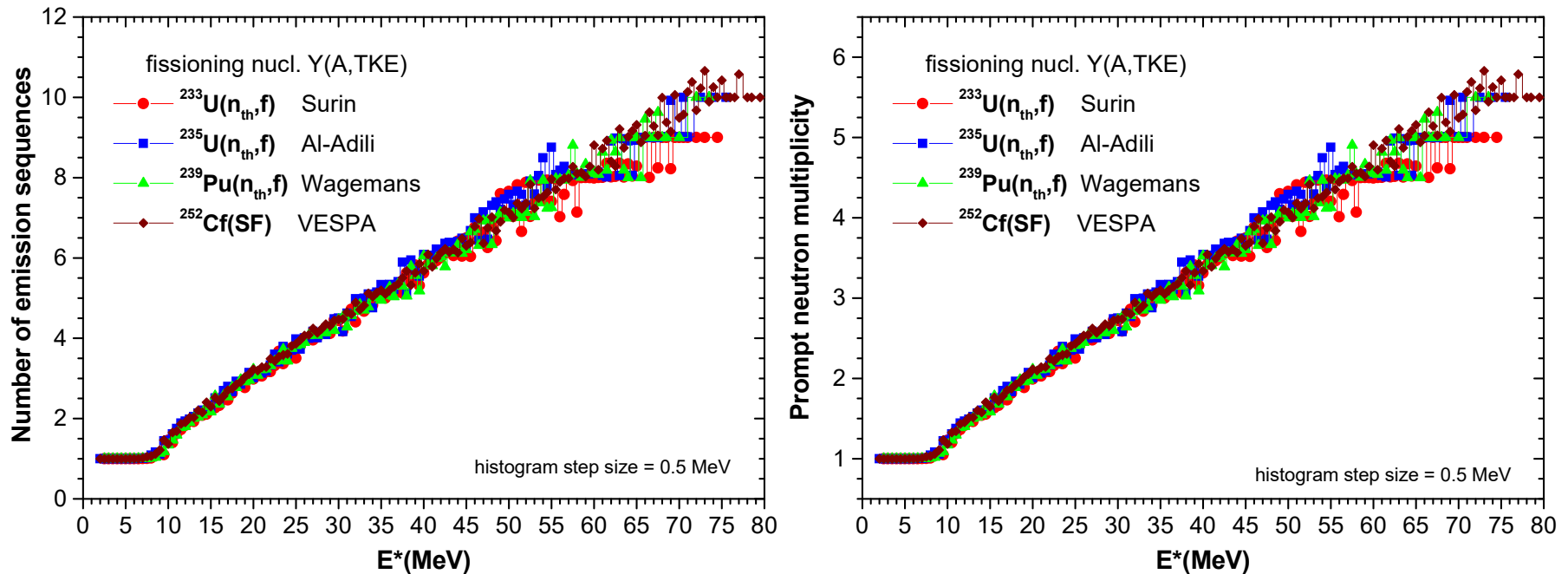
▪ **the DSE model itself** → the primary DSE results as matrices of e.g. $v(A, \text{TKE})$, $E\gamma(A, \text{TKE})$ describing well the existing experimental data (*e.g. Tudora Eur.Phys.J.A 55 (2019) 98, Eur.Phys.J.A 56 (2020) 128*) and

▪ **the DSE model together with the employed $Y(A, \text{TKE})$** → the good description of all exp. data of prompt emission and distributions of post-neutron fragments (independent FPY $Y(Z, A_p)$, $Y(A_p)$, $Y(N_p)$ etc.) by using different $Y(A, \text{TKE})$ (experimental and theoretical) (*e.g. Tudora et al. Eur.Phys.J.A 58 (2022) 126, 59 (2023) 283, 60 (2024) 25, 61 (2025) 210*)

The DSE output files (including the matrices $q_k(A, Z, \text{TKE})$ and $q(A, Z, \text{TKE})$) constitute the “row material” from which all investigated dependences and correlations are obtained

I. Prompt emission and fragment quantities as a function of excitation energy E^* of fully-accelerated pre-neutron fragments and as a function of TXE

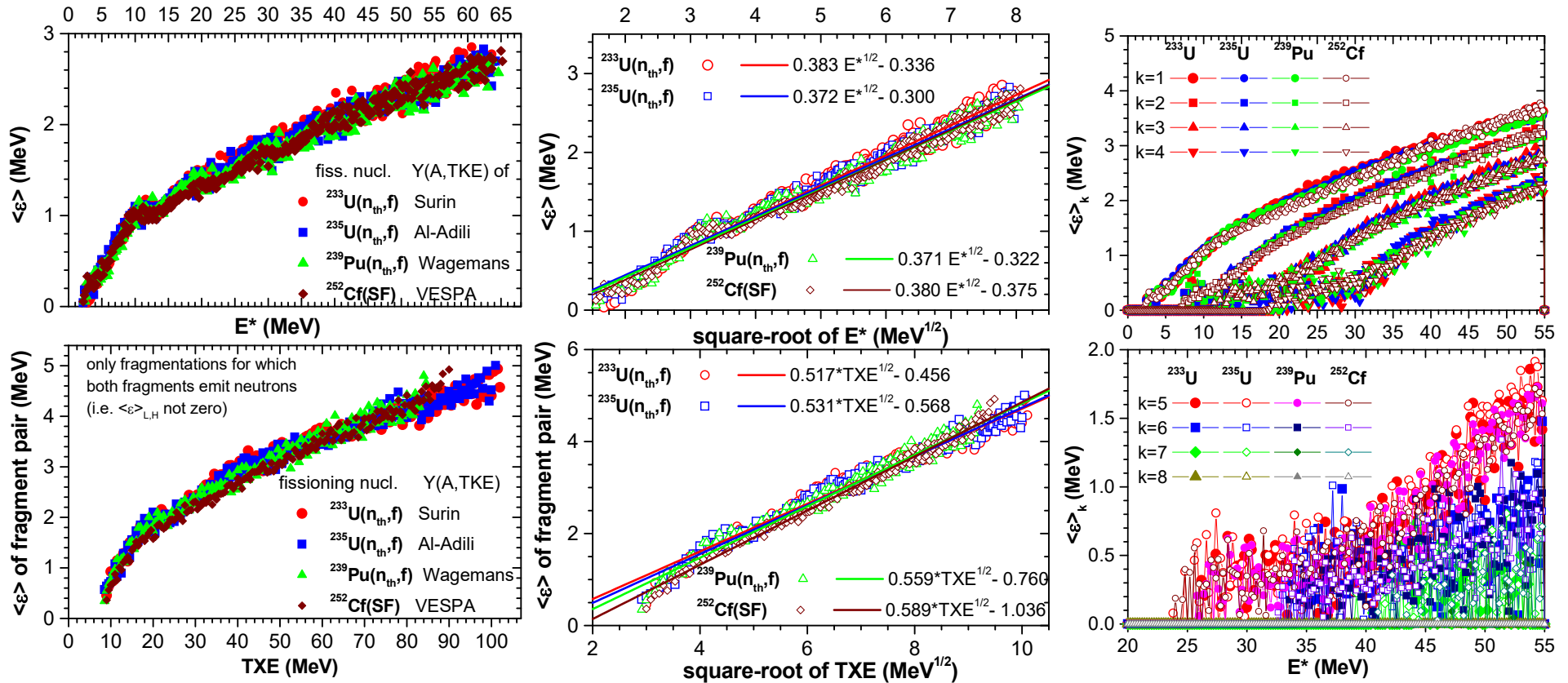
*I.1 Number of emission sequences and prompt neutron multiplicity as a function of E^**



The symbols corresponding to the four investigated fissioning nuclei cover each other in both cases of the number of emission sequences and the prompt neutron multiplicity as a function of E^* .

This shows that the prompt neutron emission depends almost exclusively on the excitation energy of fully accelerated pre-neutron fragments E^* , regardless of the fissioning nucleus and, as consequence, of its fragmentation range.

I.2 Average center-of-mass energy of each emitted neutron $\langle \varepsilon \rangle_k$, of all emitted neutrons $\langle \varepsilon \rangle$ as a function of E^* and $\langle \varepsilon \rangle$ of fragment pair as a function of TXE

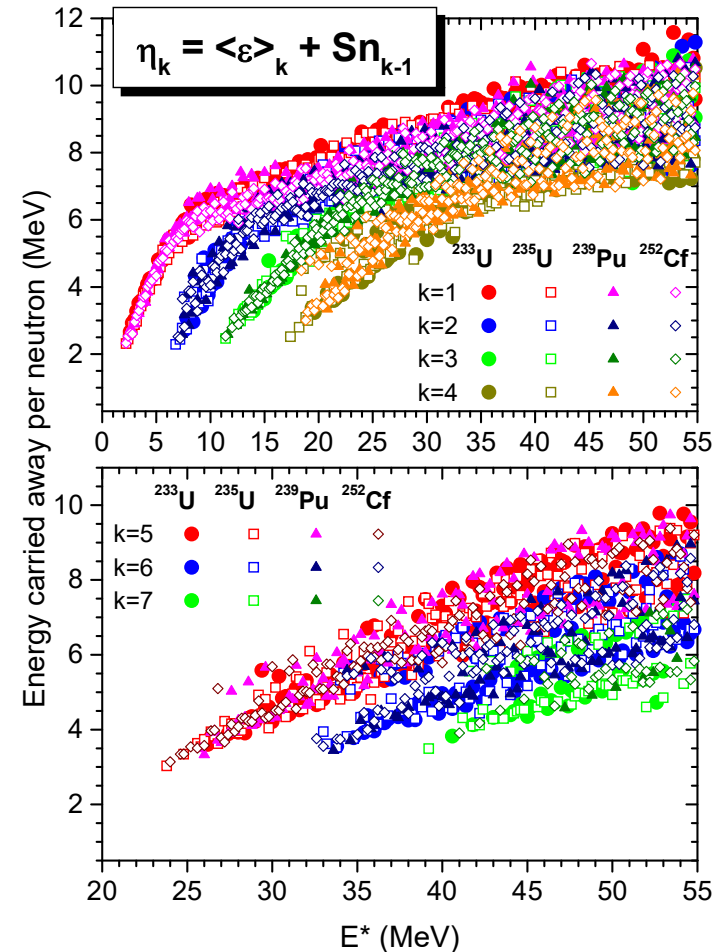
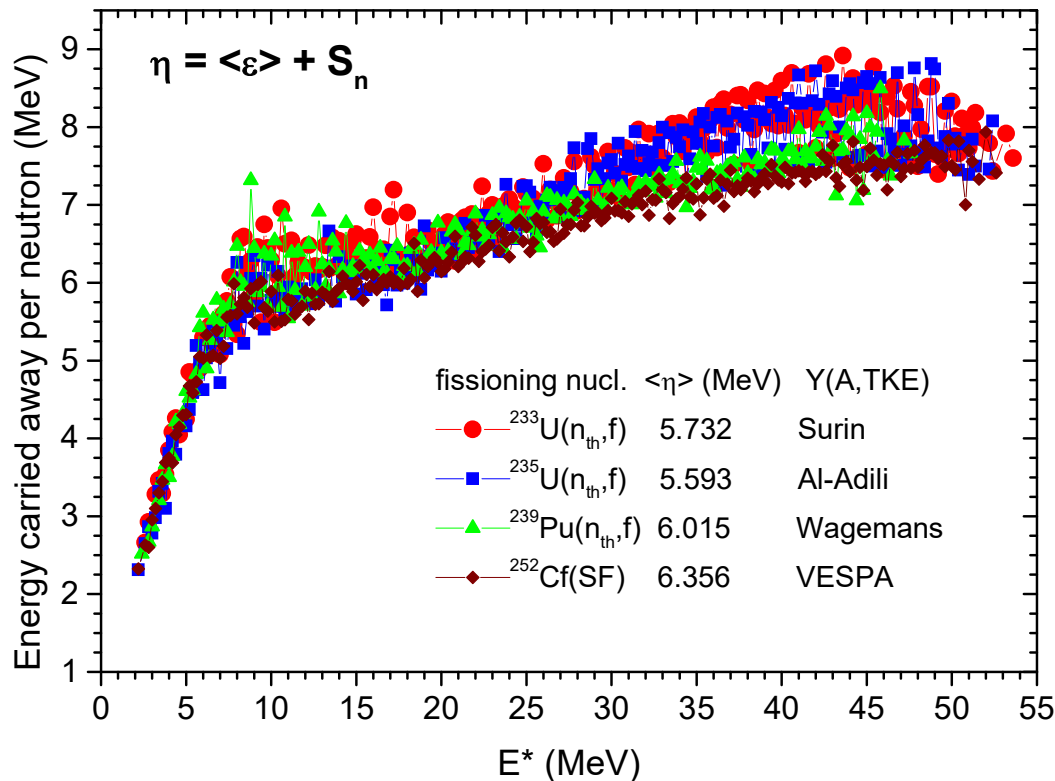


The linear increasing of both $\langle \varepsilon \rangle$ as a function of square-root of E^* and $\langle \varepsilon \rangle_{\text{pair}}$ as a function of square-root of TXE is not surprising because $\langle \varepsilon \rangle$ is almost proportional to the residual temp. T and the nuclear temperature is proportional to the square-root of E^* , i.e. $E^* = aT^2$, for nuclei in the Fermi-gas regime of their level densities as the fission fragments are.

$\langle \varepsilon \rangle_k(E^*)$ of the first 4 emitted neutrons are well delineated, this is not surprising because a great part of the $\{A, Z, \text{TKE}\}$ configurations emit 1, 2, 3 or 4 neutrons, while a higher number of neutrons is emitted by fewer and fewer $\{A, Z, \text{TKE}\}$ configurations so that the average c.m. energies $\langle \varepsilon \rangle_k$ for $k > 4$ are getting smaller and smaller and spread.

I.3 Energy carried away per neutron as a function of E^*

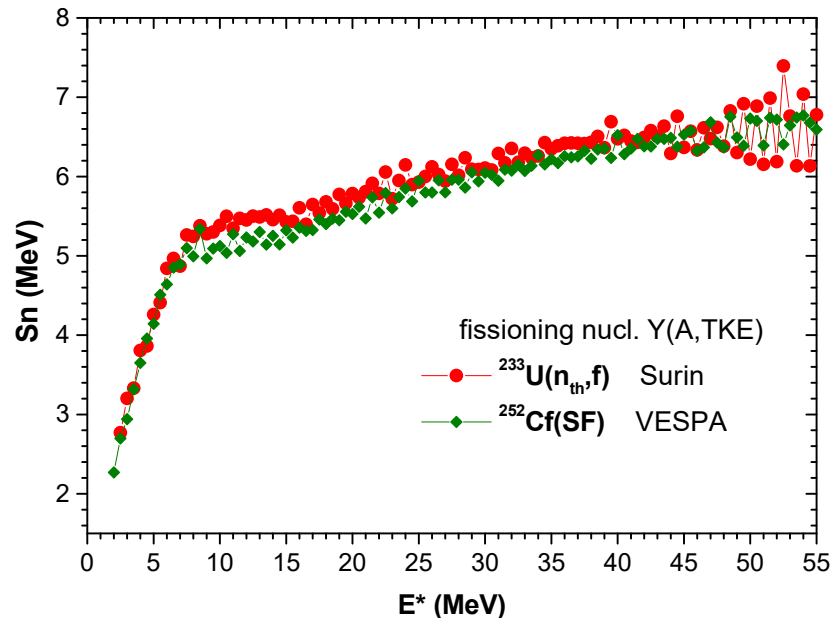
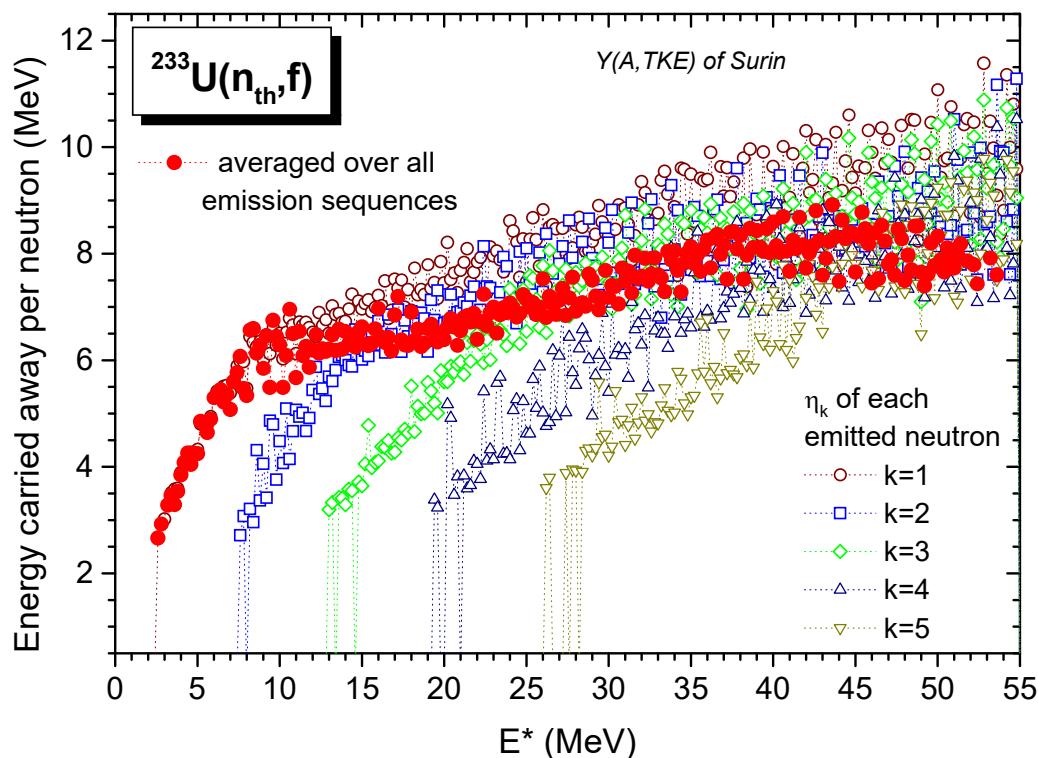
Defined as the sum of the average prompt neutron energy in the center-of-mass frame $\langle \varepsilon \rangle$ and the neutron separation energy from precursor fragment S_n



Both $\eta(E^*)$ and $\eta_k(E^*)$ exhibit a **visible staggering**. This staggering is the most pronounced in the case of $^{235,233}\text{U}(n_{\text{th}},f)$ and it is the least pronounced in the case of $^{252}\text{Cf}(SF)$.

A slight **decrease of $\eta(E^*)$ with increasing mass of the fissioning nucleus** in also observed.

I.3 Energy carried away per neutron as a function of E^* (continuation)



The staggering of $\eta(E^*)$ is due to S_n (from precursor fragment) i.e. higher S_n values of even- N nuclei (fragments) compared to those of the neighboring odd- N ones. The even-odd effect in fragment charge of $Y(A, Z, TKE)$ plays a role, too. This is confirmed by the less pronounced staggering of $\eta(E^*)$ of $^{252}\text{Cf}(\text{SF})$ compared to those of $^{233,235}\text{U}(n_{\text{th}}, \text{f})$.

Consequently by taking into account that:

- the even-even and even-odd fragments appear with a higher probability than the neighboring odd-even and odd-odd fragments
- The higher S_n values of even- N fragments (even-even, even-odd) than those of neighboring odd- N ones (odd-odd, odd-even)

we can say that **the most important role in the staggering of $\eta(E^*)$ is played by the even-even fragments (reflected in higher η values) and even-odd fragments (reflected in lower η values).**

The slight decrease of $\eta(E^*)$ with increasing mass of the fissioning nucleus is due to the decrease of S_n values with increasing mass of fissioning nucleus.

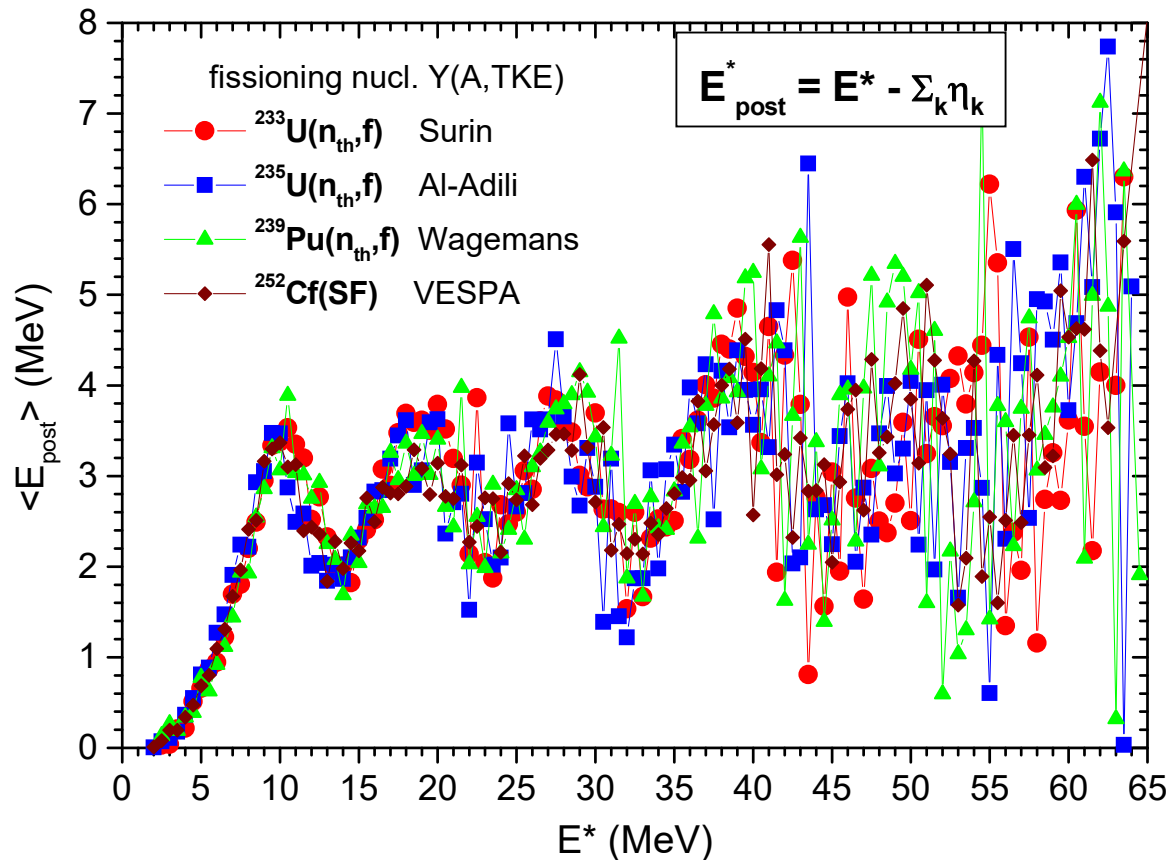
This is explained by the **well known decrease of S_n with increasing neutron excess (N/Z)** $\rightarrow S_n$ of the almost 3000 nuclei of the database AME2020 as a function of N/Z are well described by the descending part of the fitting concave parabola.

On the other hand **the heavier the fissioning nucleus is, the nuclei forming its fragmentation range have higher neutr. excesses N/Z and consequently they have lower S_n values.** This is the case of our example: $^{252}\text{Cf}(\text{SF})$ ($N/Z=1.571$) and ^{236}U ($N/Z=1.543$) with heir FF ranges constructed by taking Z at each A around $Z_p(A)$ which is based on UCD (i.e. the same N/Z as that of the fissioning nucleus) corrected with $\Delta Z(A)$ (which is around 0.5). So, the nuclei of the FF range of $^{252}\text{Cf}(\text{SF})$ have lower S_n values than those of $^{235}\text{U}(n_{\text{th}}, \text{f})$

I.4 Average excitation energy of post-neutron fragments as a function of E^*

$$E_{post}(A, Z, TKE) = E^*(A, Z, TKE) - \sum_{k=1}^{n(A, Z, TKE)} \eta_k(A, Z, TKE) =$$

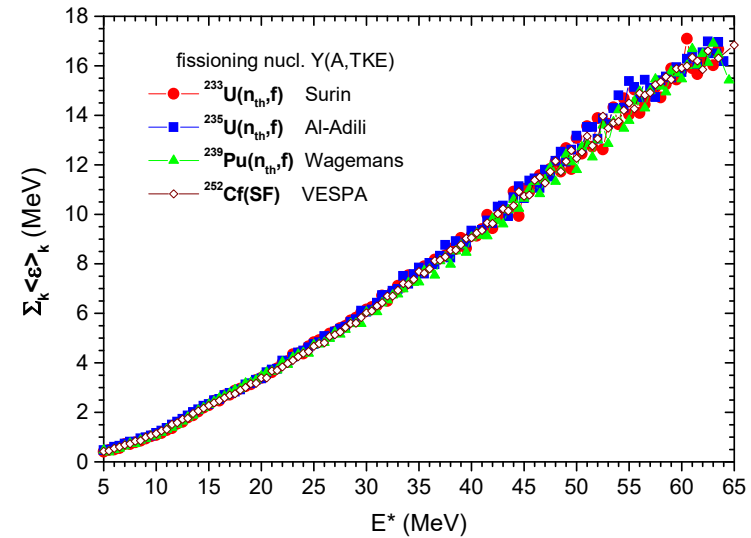
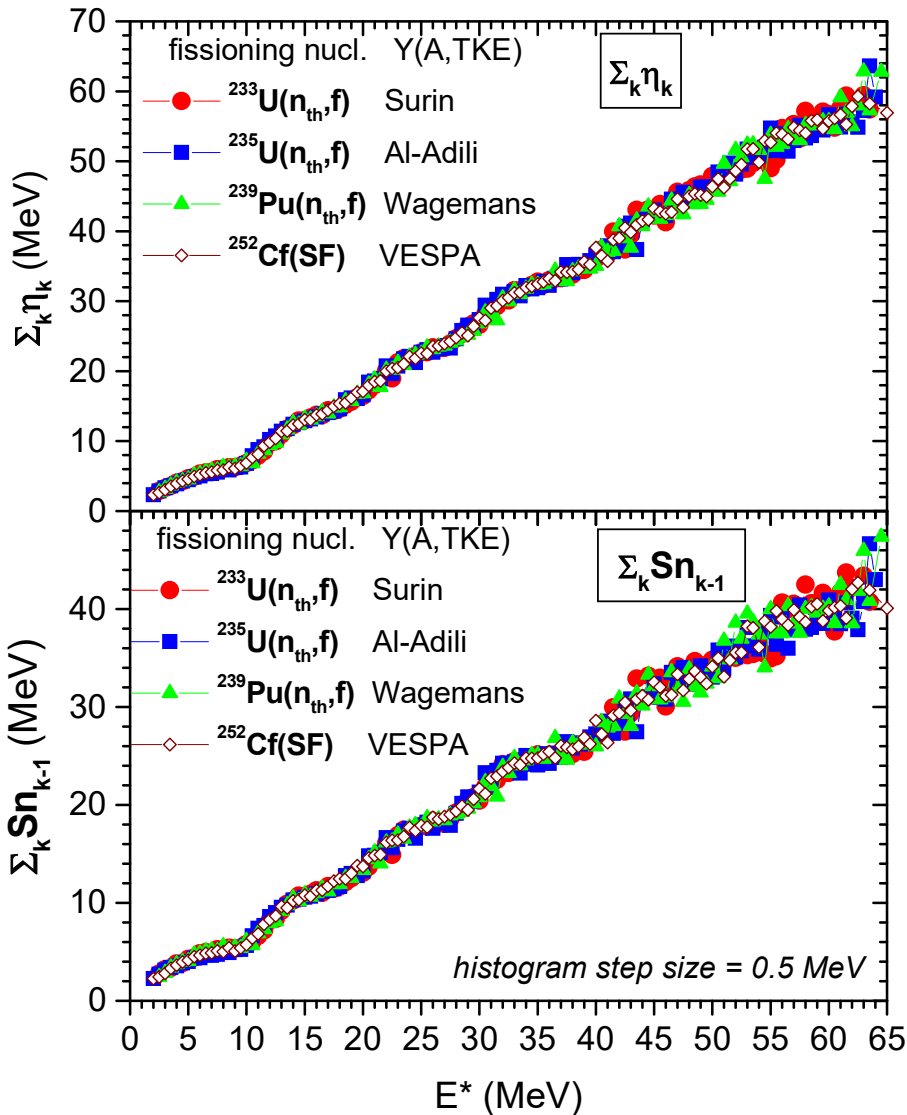
$$= E^*(A, Z, TKE) - \sum_{k=1}^{n(A, Z, TKE)} \langle \varepsilon \rangle_k(A, Z, TKE) - \sum_{k=1}^{n(A, Z, TKE)} S n_{k-1}(A, Z, TKE)$$



$\langle E_{post} \rangle(E^*)$ exhibits increases and decreases which look like oscillations with a periodicity of about 10 MeV. These oscillations are well delineated up to E^* of about 45 MeV.

I.4 Average excitation energy of post-neutron fragments as a function of E^* (continuation)

Explanation of the oscillating behaviour of $\langle E_{\text{post}} \rangle(E^*)$

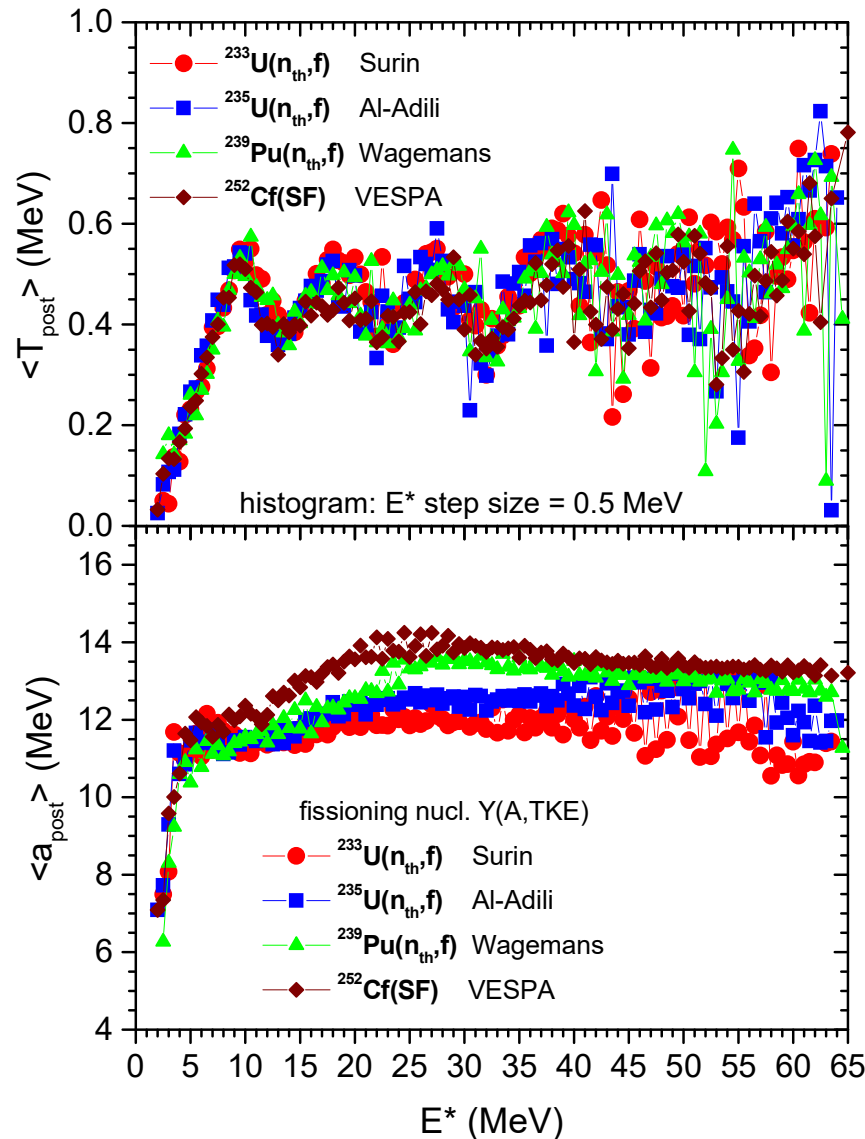


The sum $\sum_k \eta_k$ as a function of E^* exhibits a **visible wavy increasing**.

This behaviour is due exclusively to the sum of neutron separation energies from precursor fragments $\sum_k S n_{k-1}(E^*)$ **which exhibits a similar wavy increasing** (because the sum of center-of-mass energies of prompt neutrons $\sum_k \langle \epsilon \rangle_k(E^*)$ exhibits a smooth increase).

The wavy increasing of $\sum_k S n_{k-1}(E^*)$ is the result of the increasing number of emission sequences with increasing excitation energy E^* . In other words $\sum_k S n_{k-1}(E^*)$ shows waves as the number of terms in the sum is increasing.

I.4 Average excitation energy of post-neutron fragments as a function of E^* (continuation)



Another way to calculate E^*_{post} of each $\{A,Z,TKE\}$ configuration is as

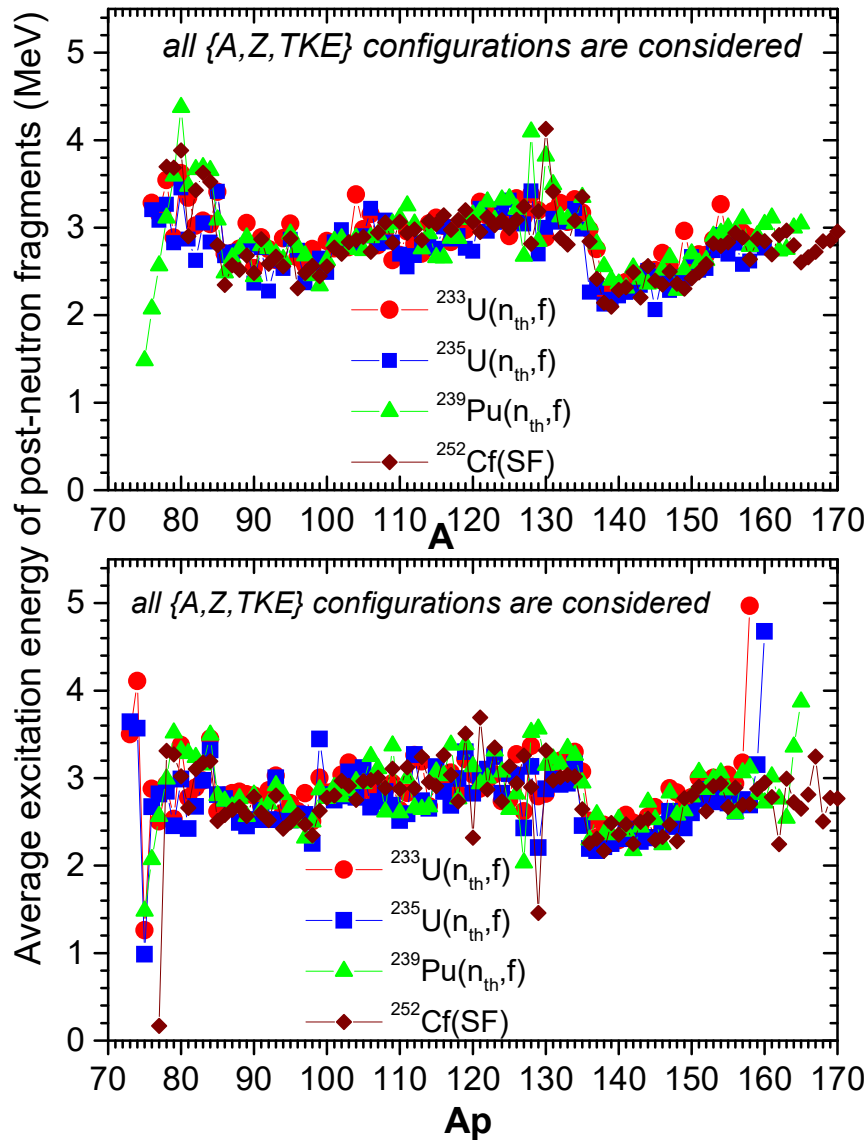
$$E^*_{post} = a_{post} T_{post}^2$$

in which a_{post} and T_{post} are those of the last residual fragment.

The oscillating behaviour of $\langle T_{post} \rangle(E^*)$ is not surprising taking into account that T_{post} is the solution of the last equation of residual temperature, the iterative solving of these equations of the DSE modeling including implicitly the sum of neutron separation energies from the precursor fragments $\sum_k S_{n_{k-1}}$.

Note, in the calculation of $\langle E_{post} \rangle(E^*)$ only the $\{A,Z,TKE\}$ configurations which emit prompt neutrons were taken into account.

Average excitation energy of post-neutron fragments as a function of pre-neutron fragment mass A and post-neutron fragment mass A_p



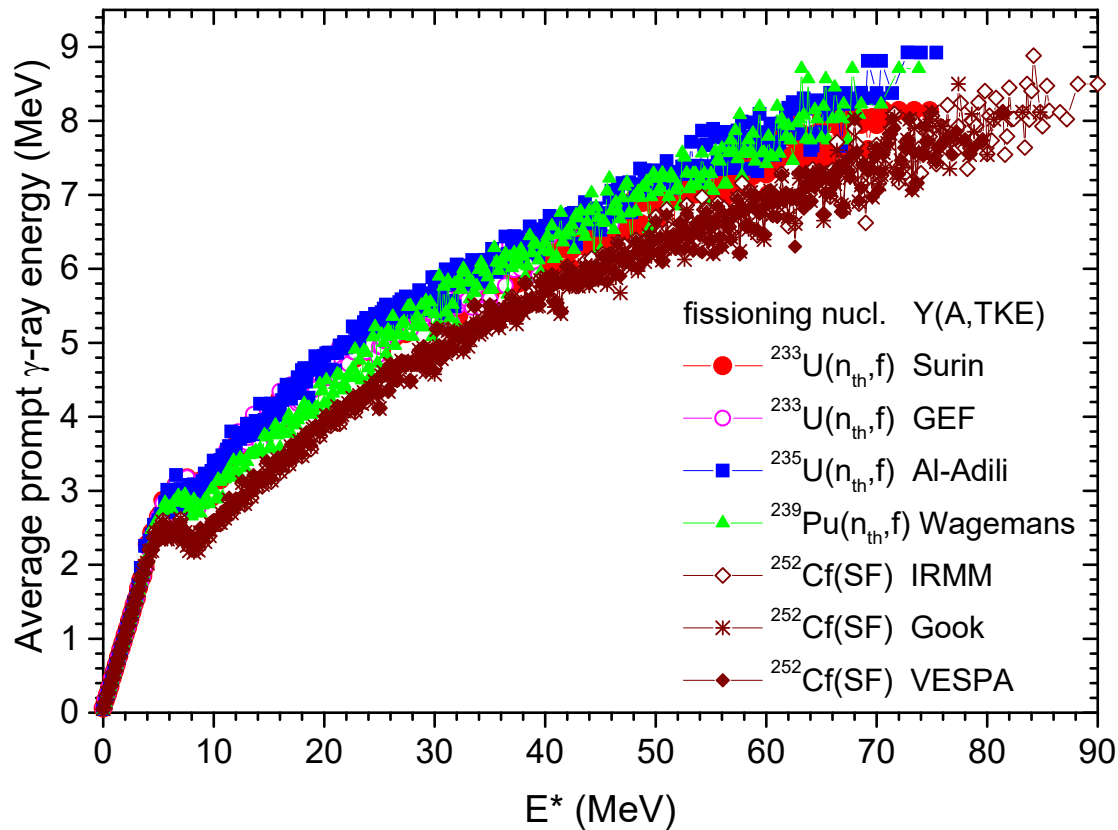
← This time **all {A,Z,TKE} configurations** (those which emit prompt neutrons and those which do not) **are taken into account**

It can be observed that both $\langle E_{\text{post}} \rangle(A)$ and $\langle E_{\text{post}} \rangle(A_p)$ do not exhibit a sawtooth shape as other physical quantities as a function of A (or A_p) do. (e.g. $v(A)$, $v(A_p)$, $\langle \epsilon \rangle(A)$, $E^*(A)$ etc.)

$\langle E_{\text{post}} \rangle(A)$ and $\langle E_{\text{post}} \rangle(A_p)$ of the 4 fissioning nuclei do not differ significantly from each others.

Y(A,TKE)		$\langle E_{\text{post}}^* \rangle_{\text{LF}}$ (MeV)	$\langle E_{\text{post}}^* \rangle_{\text{HF}}$ (MeV)
²³³ U(<i>n</i> _{th} ,f)	Surin	2.8059	2.6909
	GEF	2.8162	2.7085
²³⁵ U(<i>n</i> _{th} ,f)	Al-Adili	2.6759	2.4849
²³⁹ Pu(<i>n</i> _{th} ,f)	Wagemans	2.7873	2.7590
²⁵² Cf(SF)	IRMM	2.9035	2.5226
	Gook	2.9143	2.5310
	VESPA	2.8872	2.5316

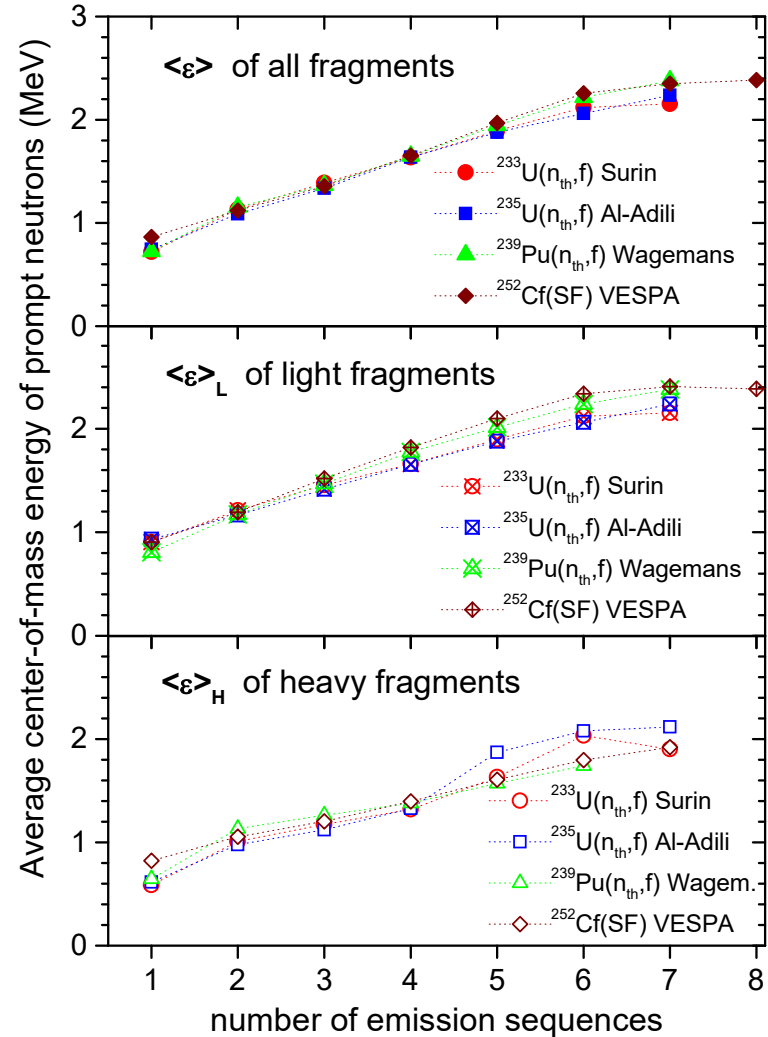
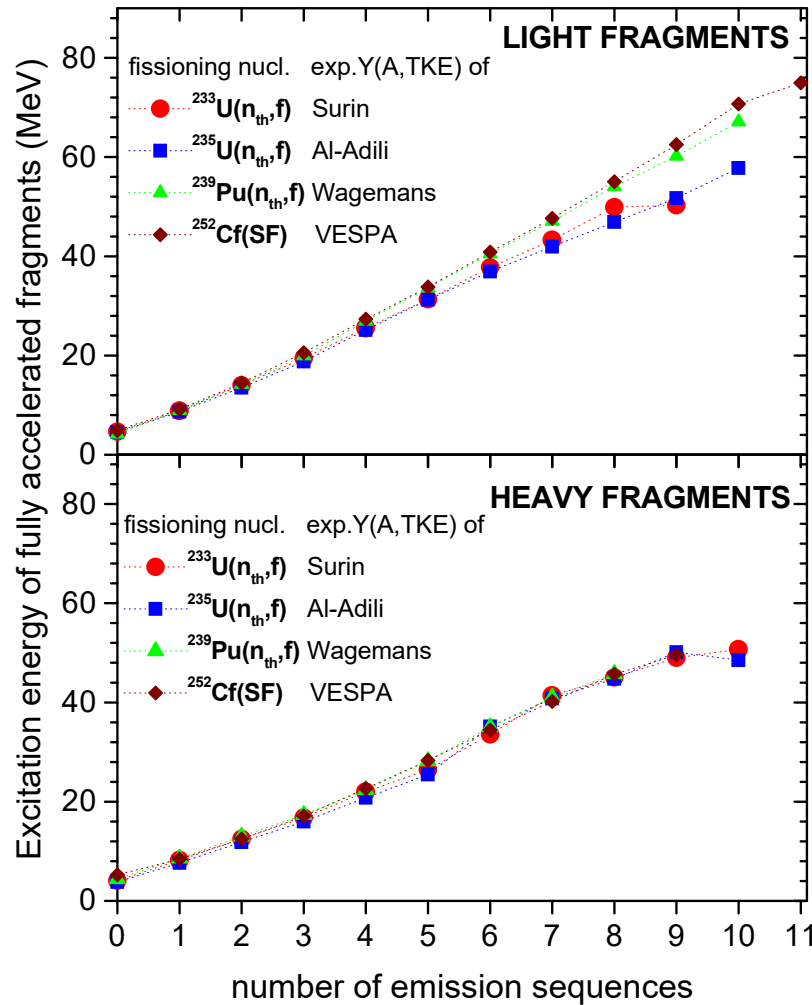
1.5 Average prompt gamma-ray energy as a function of E^*



$\langle E_{\gamma} \rangle(E^*)$ of $^{252}\text{Cf}(\text{SF})$ is the lowest. The highest ones are those of $^{235}\text{U}(n_{\text{th}},f)$ and $^{239}\text{Pu}(n_{\text{th}},f)$.

Fissioning nucleus	$\langle E_{\gamma} \rangle$ using Y(A,TKE) of	Experimental data
$^{233}\text{U}(n_{\text{th}},f)$	6.718 MeV Surin	(6.69 \pm 0.30) MeV [F.Pleasanton, NPA 213, 1973]
	6.756 MeV GEF	
$^{235}\text{U}(n_{\text{th}},f)$	6.915 MeV Al-Adili	(6.92 \pm 0.09) MeV [Oberstedt et al. PRC 87, 2013, EXFOR]
$^{239}\text{Pu}(n_{\text{th}},f)$	6.850 MeV Wagemans	6.81 MeV [Verbinski et al., PRC 7, 1972]
$^{252}\text{Cf}(\text{SF})$	6.909 MeV Gook	(6.81 \pm 0.14) MeV [Oberstedt et al. PRC 92, 2015, EXFOR] (6.84 \pm 0.30) MeV [Verbinski et al., PRC 7, 1972, EFOR]
	6.907 MeV IRMM	
	6.837 MeV VESPA	

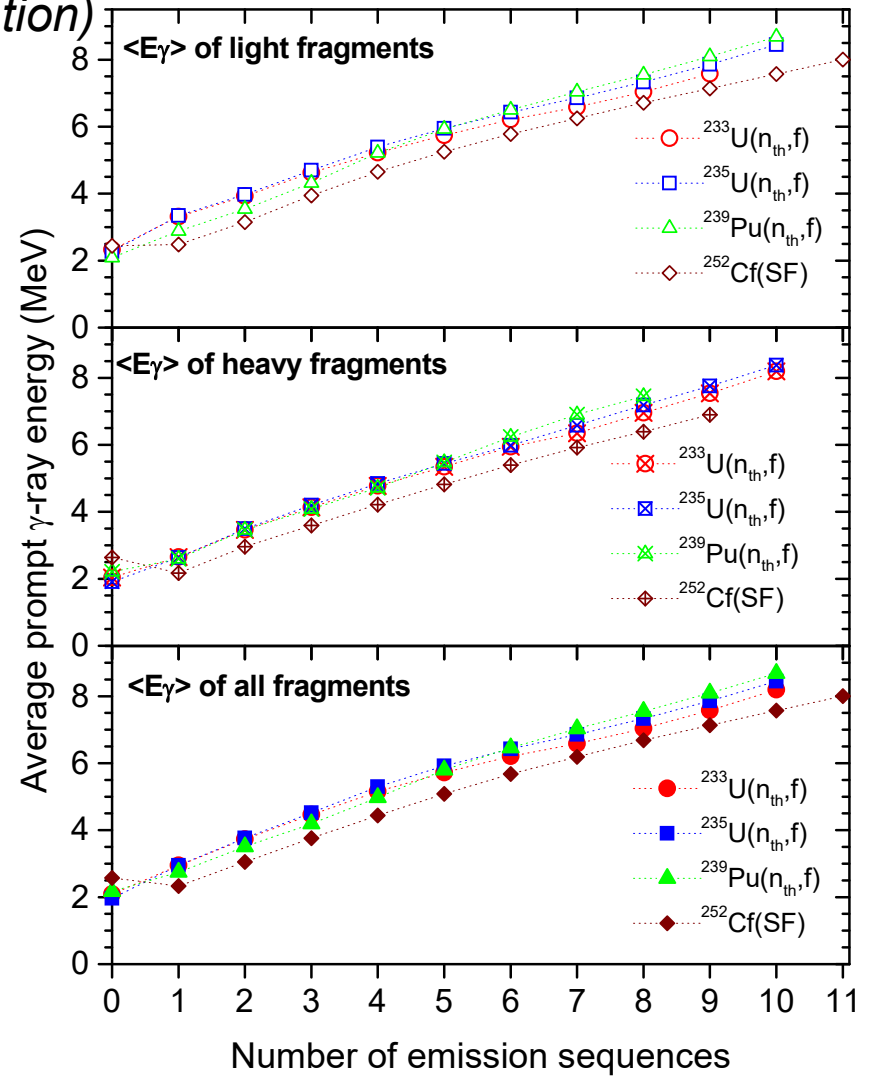
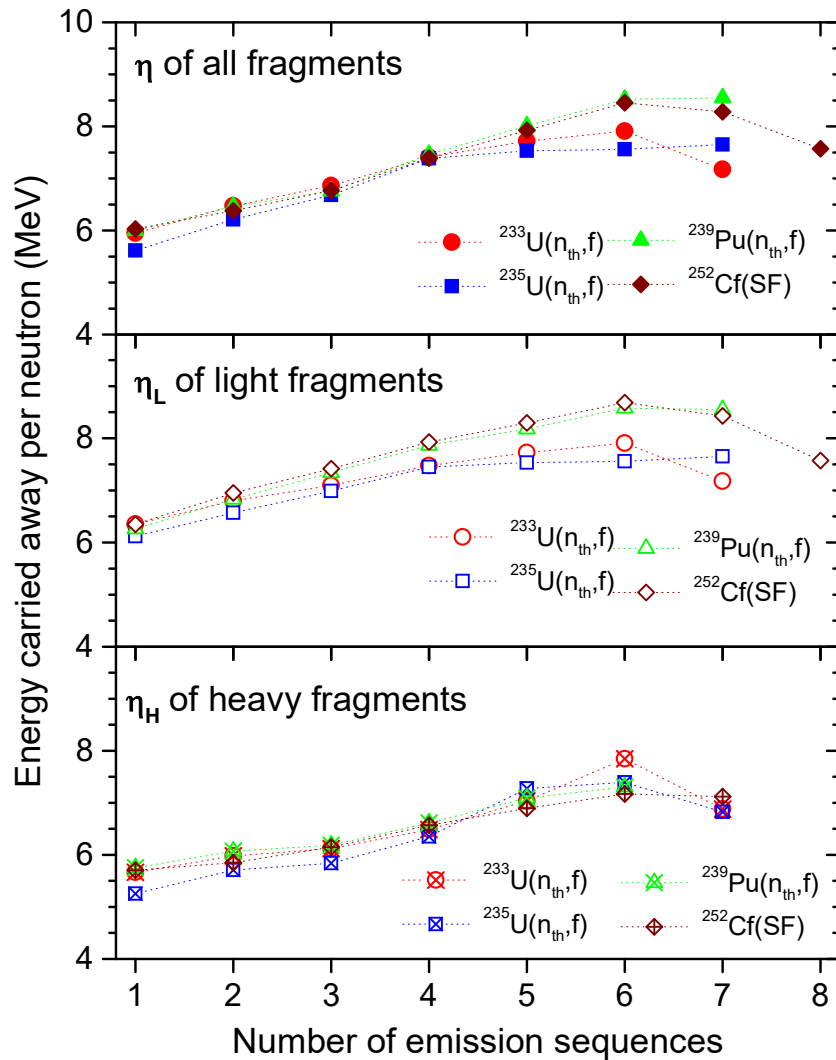
II. Prompt emission and fragment quantities as a function of the number of emission sequences (number of emitted prompt neutrons)



All investigated cases have shown an increase of all quantities with increasing number of emission sequences, a part of these increases being almost linear up to a number of emission sequences of 4, 5 or even 6.

II. Prompt emission and fragment quantities as a function of the number of emission sequences (number of emitted prompt neutrons)

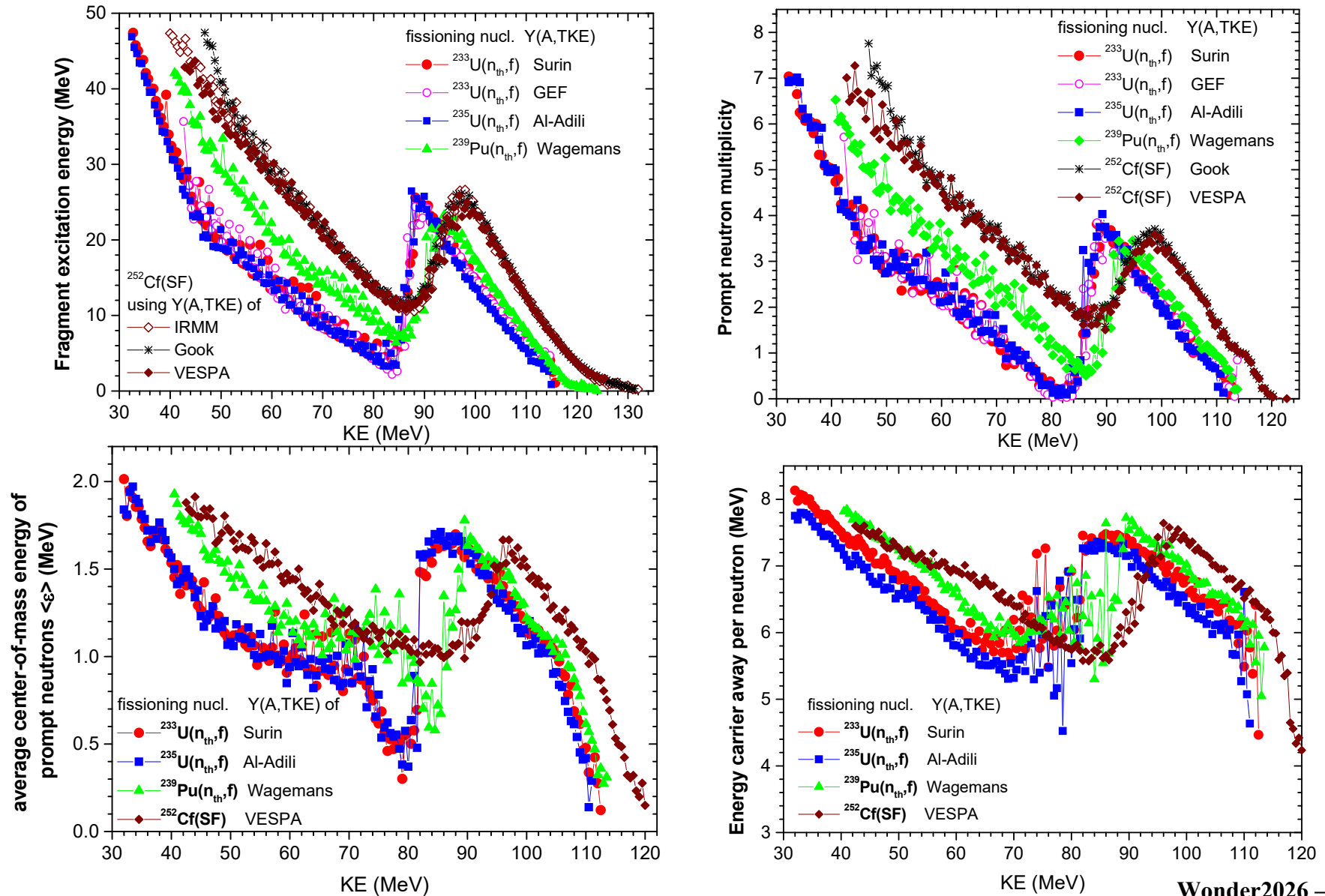
(continuation)



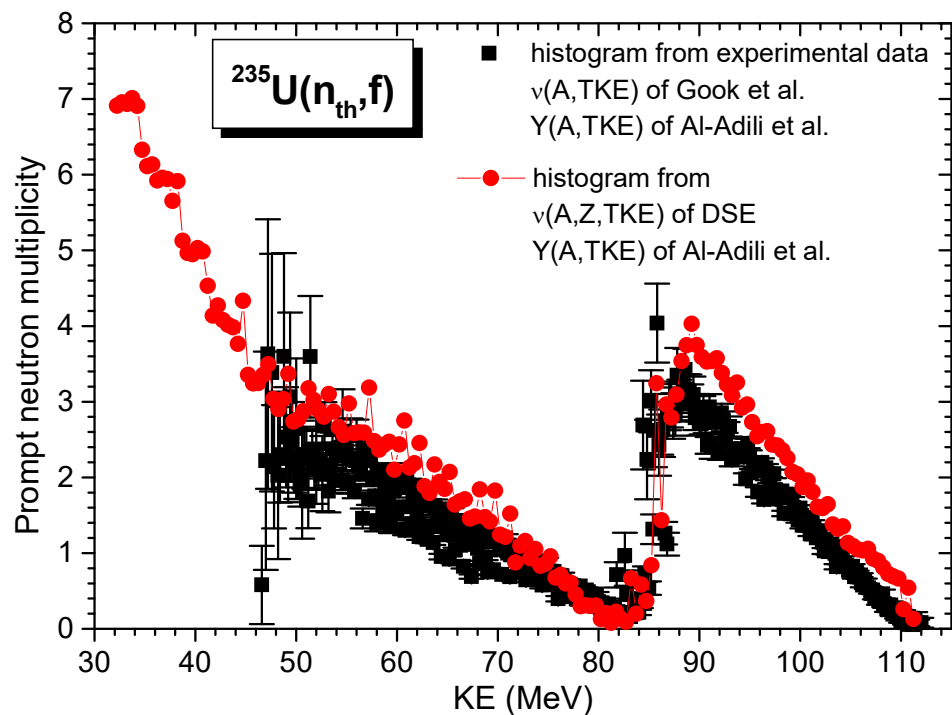
$\langle E_\gamma \rangle(n)$ of $^{252}\text{Cf}(SF)$ is the lowest. The highest ones are those of $^{235}\text{U}(n_{\text{th}},f)$ and $^{239}\text{Pu}(n_{\text{th}},f)$.

III. Prompt emission and fragment quantities as a function of the kinetic energy of pre-neutron fragments KE

They exhibit a sawtooth shape which looks as a reflection in mirror of the well-hown sawtooth shape of fragment and prompt emission quantities as a function of A or A_p (e.g. $E^*(A)$, $v(A)$, $v(A_p)$, $\langle \varepsilon \rangle(A)$ etc.)

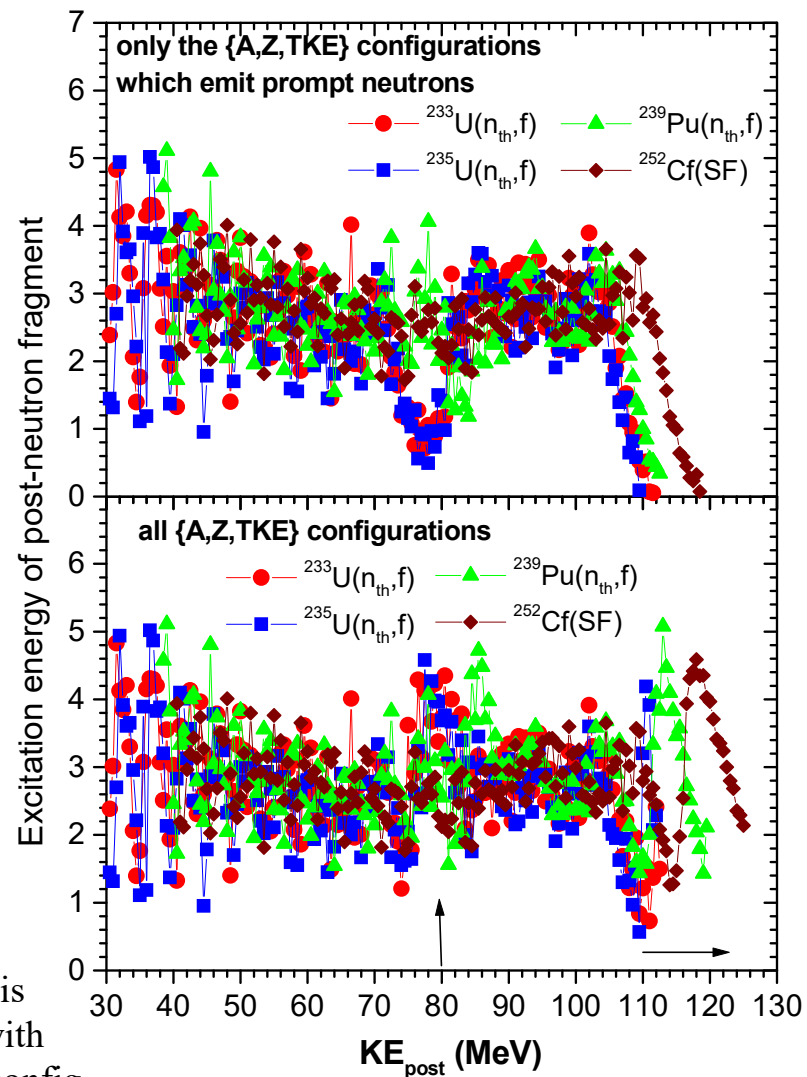


Experimental correlation $\nu(\text{KE})$ based on experimental $\nu(\text{A},\text{TKE})$ data of Gök et al. and experimental $Y(\text{A},\text{TKE})$ data of Al-Adili et al.



The experimental $\nu(\text{KE})$ exhibits the same sawtooth shape. The underestimation in the descending parts of the sawtooth shape is explained by the fact that the experim. $\nu(\text{A},\text{TKE})$ data (appearing with the probability $Y(\text{A},\text{TKE})$) refer to a smaller number of $\{\text{A},\text{TKE}\}$ config., of about 2200, while in DSE the number of $\{\text{A},\text{Z},\text{TKE}\}$ config. is much higher, of about 18000. **I.e. in the case of DSE there are much more prompt neutron multiplicities (coming with different probabilities) which contribute in each KE bin compared to the case when experimental $\nu(\text{A},\text{TKE})$ data are used.**

$\langle E_{\text{post}} \rangle$ as a function of KE_p



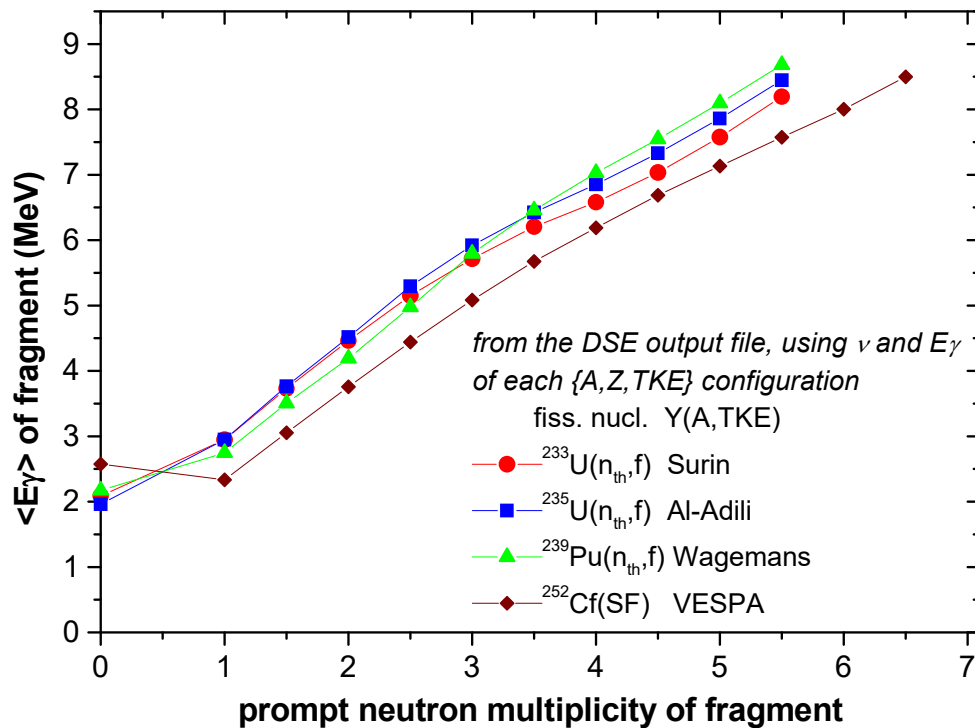
The majority of FF which do not emit neutrons are those from near symmetric fragmentations (KE_p of 75–85 MeV), and the light fragments coming from very asymmetric fragmentations ($\text{KE}_p > 110$ MeV).

IV. Correlation between the prompt neutron multiplicity and average prompt gamma-ray energy (this time related to fragment instead of fragment pair)

- **Correlation referring to the total average $\langle \nu \rangle$ and $\langle E\gamma \rangle$ in (n,f)** (at E_n below the threshold of second chance fission) Experimentally observed by **J. Fréhaut** ((n,f) of ^{232}Th , ^{235}U , ^{237}Np) i.e. the almost linear increasing of $\langle \nu \rangle \approx a_1 E_n + b_1$ and $\langle E\gamma \rangle \approx a_2 E_n + b_2$, by eliminating $E_n \rightarrow \langle E\gamma \rangle \approx (a_2/a_1)\langle \nu \rangle + b_2 - a_2 b_1/a_1$ The slope and intercept of this linear dependence were parameterized as a function of the fissility parameter (Z^2/A) of fissioning nucleus (**Tudora and Vladuca** 20 y ago)

- **Correlation obtained from the linear decreasing of $\nu(\text{TKE})$ and $E\gamma(\text{TKE})$** which obviously refer to the fragment pair, e.g. experimentally **H.Nifenecker** $^{252}\text{Cf}(\text{SF})$, and in Ref.[**Tudora Eur.Phys.J.A 56 (2020) 125**] and references therein, i.e. $\langle \nu \rangle \approx \alpha_1 \text{TKE} + \beta_1$ and $\langle E\gamma \rangle \approx \alpha_2 \text{TKE} + \beta_2$ by eliminating TKE $\rightarrow \langle E\gamma \rangle \approx (\alpha_2/\alpha_1)\langle \nu \rangle + \beta_2 - \alpha_2 \beta_1/\alpha_1$

Now the correlation refers to the fragment, because it is obtained from the matrices $\nu(A,Z,\text{TKE})$ and $E\gamma(A,Z,\text{TKE})$ provided by the DSE code, these quantities appearing with the probability expressed by the $Y(A,Z,\text{TKE})$ distribution.



← the almost linear increasing of $\langle E\gamma \rangle(\nu)$ is maintained.

The magnitudes of $\langle E\gamma \rangle(\nu)$ follow the same order as in previous cases of $\langle E\gamma \rangle(E^*)$ and $\langle E\gamma \rangle(n)$. I.e. $\langle E\gamma \rangle(\nu)$ of $^{235}\text{U}(n_{\text{th}},f)$ and $^{239}\text{Pu}(n_{\text{th}},f)$ are the highest, followed by that of $^{233}\text{U}(n_{\text{th}},f)$, while that of $^{252}\text{Cf}(\text{SF})$ is the lowest.

This is not surprising and it is quite intuitive because of the clear correlation between E^* and the number of emission sequences and prompt neutron multiplicity (i.e. **an almost linear increasing of $E^*(\nu)$**) and the **almost linear increasing of $\langle E\gamma \rangle(E^*)$**).

Synthesis of the features of investigated dependences and correlations

The prompt neutron emission depends decisively on the magnitude of fragment excitation energy at full acceleration E^* (regardless of the fissioning nucleus and, as consequence, its fragmentation range). This is proved by the overlap of the prompt neutr. multiplicity $\nu(E^*)$ and number of emiss. sequences $n(E^*)$ respectively, of the 4 investigated fissioning nuclei.

The average center-of-mass energy of prompt neutrons $\langle \epsilon \rangle$ as a function of the square-root of E^* and $\langle \epsilon \rangle_{\text{pair}}$ as a function of the square-root of TXE exhibit a linear increasing behaviour explained by the level densities of fragments in the Fermi-gas regime.

The energy carried away per neutron as a function of E^* exhibits a visible staggering which is due to the well-known behaviour of Sn (higher Sn values of even-N fragments compared to those of neighboring odd-N ones) and to a lesser extent to the even-odd effect in fragment charge in the $Y(A,Z,TKE)$ distribution.

The slight decrease of $\eta(E^*)$ with increasing mass of the fissioning nucleus is explained by the slight decrease of Sn(E^*) due to the known decreasing behaviour of Sn as a function of neutron excess N/Z , so that the heavier the fissioning nucleus is, the nuclei forming its fragmentation range have higher neutron excesses N/Z and consequently they have lower Sn.

The average energy of post-neutron fragments $\langle E_{\text{post}} \rangle$ as a function of E^* exhibits successive increases and decreases which look like oscillations with a periodicity of about 10 MeV.

This oscillating behaviour of $\langle E_{\text{post}} \rangle(E^*)$ is due to the sum of energies carried away by all prompt neutrons successively emitted $\sum_k \eta_k(E^*)$ which exhibits a wavy increasing. This behaviour of $\sum_k \eta_k(E^*)$ is explained by the similar wavy increasing of the sum of neutron separation energies from precursor fragments $\sum_k S_{n_{k-1}}(E^*)$, each wavelet being due to the increase of the number of terms in this sum due to the increasing number of $\{A,Z,TKE\}$ configurations which emit more neutrons as E^* is increasing.

$\langle E_{\text{post}} \rangle(E^*)$ and $\langle E_{\text{post}} \rangle(A)$ or $\langle E_{\text{post}} \rangle(A_p)$ do not differ significantly for one fissioning nucleus to another.

An increase of all quantities with increasing number of emission sequences is observed, a part of these increases being almost linear up to a number of emission sequences of about 4, 5 or even 6.

In both cases $\langle E_\gamma \rangle(E^*)$ and $\langle E_\gamma \rangle(n)$ it was found that the average E_γ of $^{252}\text{Cf}(\text{SF})$ is the lowest, while those of $^{235}\text{U}(n_{\text{th}},f)$ and $^{239}\text{Pu}(n_{\text{th}},f)$ are the highest, followed by that of $^{233}\text{U}(n_{\text{th}},f)$.

All investigated quantities as a function of KE (i.e. $E^*(KE)$, $\nu(KE)$, $\langle \epsilon \rangle(KE)$, $\eta(KE)$) exhibit similar sawtooth shapes which look as a reflection in mirror of the better known sawtooth shapes of such quantities as a function of A (or Ap) (such as $\nu(A)$, $\nu(Ap)$, $E^*(A)$, $\langle \epsilon \rangle(A)$ etc.). The descending parts of the sawtooth shapes of quantities as a function of KE are mostly due to FF coming from asymmetric fragmentations (the HF contributing to the left descending part at lower KE values and the LF to the right descending part at higher KE). The steep growth in the middle part (which connect the descending parts) is mostly due to both LF and HF coming from near symmetric fragmentations.

The existence of experimental $\nu(A, TKE)$ data allows to correlate the prompt neutron multiplicity with KE without resorting to prompt emission model calculations. It was shown in this work that such $\nu(KE)$ correlations based on experimental $\nu(A, TKE)$ data maintain the sawtooth shape and agree with $\nu(KE)$ obtained from DSE model calculations.

$\langle E_{\text{post}} \rangle(KE_p)$ (which does not exhibit a sawtooth shape) calculated in 2 cases, i.e. by taking into account only the $\{A, Z, TKE\}$ configurations which emit prompt neutrons and by considering all $\{A, Z, TKE\}$ configurations, has shown that most of $\{A, Z, TKE\}$ configurations which do not emit neutrons are those corresponding to LF and HF coming from near symmetric fragmentations and also to the $\{A, Z, TKE\}$ configurations corresponding to LF coming from very asymmetric fragmentations.

Again there are no significant differences between $\langle E_{\text{post}} \rangle(KE_p)$ of the 4 investigated fissioning nuclei.

Up to now, the correlation between E_γ and ν - consisting of an almost linear increasing of $\langle E_\gamma \rangle$ as a function of $\langle \nu \rangle$ - was investigated by taking into account $\langle E_\gamma \rangle$ and $\langle \nu \rangle$ of fragment pair: i.e. *a)* (n,f) at E_n below the threshold of the second chance fission by considering the almost linear increasing of both $\langle \nu \rangle(E_n)$ and $\langle E_\gamma \rangle(E_n)$ and *b)* in SF or the fission at a single E_n value by considering the almost linear decreasing behaviour of both $\nu(TKE)$ and $E_\gamma(TKE)$.

This time the dependence of the average E_γ emitted by fragment on the prompt neutron multiplicity ν emitted by fragment was investigated. And it was found that the almost linear increasing of the average E_γ as a function of ν is maintained.

It was found that $\langle E_\gamma \rangle(\nu)$ of $^{235}\text{U}(n_{\text{th}}, f)$ and $^{239}\text{Pu}(n_{\text{th}}, f)$ are the highest followed by that of $^{233}\text{U}(n_{\text{th}}, f)$, while $\langle E_\gamma \rangle(\nu)$ of $^{252}\text{Cf}(SF)$ is the lowest. Consequently the magnitudes of $\langle E_\gamma \rangle(\nu)$ follow the same order as those of $\langle E_\gamma \rangle(E^*)$ and obviously of $\langle E_\gamma \rangle$ as a function of the number of emission sequences. This is not surprising and quite intuitive because of the clear correlations between E^* and ν (consisting in an almost linear increasing of $\nu(E^*)$).