The chiral phase transition & ”beyond Landau”

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Chen, Cherman, & RDP = CCP, [2603.09977](2603.09977)

‘t Hooft anomalies in QCD at (imaginary) chemical potential, i μ_B.

In plane of T and i μ_B , chiral transition is *beyond Landau* (-Wilson-Fisher-Kadanoff)

Roberge-Weiss transition

Boundary conditions in imaginary time: in *pure* gauge theory,
Gauge transf.'s Ω can have Z(3) "twist":

$$\Omega(1/T) = \mathrm{e}^{2\pi i/3}\,\Omega(0)$$

Gluons are invariant:

$$A_\mu(1/T) \to \mathrm{e}^{-2\pi i/3} A_\mu(0)\,\mathrm{e}^{+2\pi i/3} = A_\mu(0)$$

~ Polyakov loop, order parameter for deconfinement. *"1-form" order parameter*.

Quarks (in fund. rep.) are *not*:

$$q(1/T) = \mathrm{e}^{2\pi i/3} q(0)$$

With chemical potential μ, b.c.'s

$$q(1/T) = \mathrm{e}^{\mu/T} q(0)$$

SO:

$$q(1/T) = \mathrm{e}^{2\pi i/3 - \mu/T} q(0) = q(0)\ ; \ \textit{if}\ \mu = \frac{2\pi}{3} i\,T$$

Roberge & Weiss '88.

$$\mathcal{Z}\left(\mu + \frac{2\pi}{3} i\right) = \mathcal{Z}(\mu)$$

Big deal…

‘t Hooft anomalies

‘t Hooft ’79….Cordova, Freed, Lam, Seiberg, 1905.09315 ; 1905.13361
Yonekura, 1901.08188 ; Nishimura & Tanizaki, 1903.04014;
Kobayashi, Yokokura & Yonekura, 2305.01217

Add background *gauge* fields for the *global* symmetries: $A_L{}^\mu$, $A_R{}^\mu$: *not* dynamical!

‘t Hooft anomaly: $U(1)_V$ x $SU(N_f)_{L,R}$ x $SU(N_f)_{L,R}$. Constant imag. μ like A_0 for $U(1)_V$

$$\mathcal{Z}(\mu + 2\pi i/3, A_L, A_R) = \mathcal{Z}(\mu, A_L, A_R)\, \mathcal{J}(A_L)\, \mathcal{J}^{-1}(A_R)$$

$$\mathcal{J}(A) = \exp\left(\frac{i}{4\pi}\int d^3x\, \epsilon^{ijk}\mathrm{tr}\left(A_i\partial_j A_k - \frac{2}{3} i A_i A_j A_k\right)\right)$$

↑ Chern-Simons term for the background gauge fields.

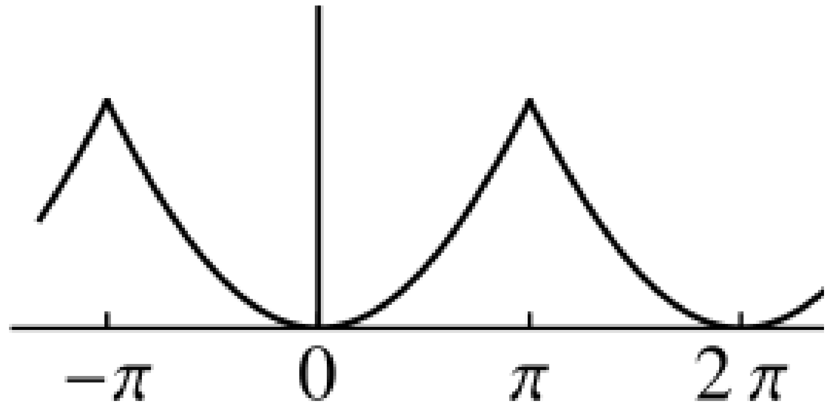
Computed perturbatively in the UV. Must hold for *any* effective theory in the IR;
‘t Hooft: introduce “spectator fermions”

Strong constraint on effective theories! “Anomaly in space of coupling constants”

High T: RW = 1^{st} order transition

At high T, compute potential for i μ_B in perturbation theory. *Easy peasy!*

$\mathcal{V}_{1-loop} \uparrow$



$-i\,\dfrac{3\,\mu_B}{T} \rightarrow$

Roberge & Weiss '88: 1^{st} order transition at $\mu_B = -i\dfrac{\pi}{3}T$

Quark density imaginary, flips sign.

One way to satisfy 't Hooft anomaly is by 1^{st} order transition.

μ_B= 0 *not* analytically connected to $\mu_B = -i\dfrac{2\pi}{3}T$

RW transition exists at high T, presumably ends at some nonzero T. When?

t Hooft anomaly at *low* T: Wess-Zumino-Witten

At low T, chiral symmetry is broken, $\Phi = \sigma_0 \, U \; ; \; U^\dagger U = \mathbf{1}$

Can rewrite the effective Lagrangian with the Goldstone bosons, U = exp($i\pi/f_\pi$)

$$\mathcal{S}_{\rm GB} = \int d^3x \left(\frac{1}{g^2} {\rm tr} \, |\partial_i U|^2 + \mu \, \frac{\epsilon^{ijk}}{24\pi^2} \, {\rm tr}(B_i B_j B_k) \right) \; ; \; B_i = U^\dagger \partial_i U$$

The second term is baryon number, related to π_3(SU(N_f)) ≈ Z.

If 3 i μ_B/T shifts by 2 π, exp(-S_{GB}) shifts by exp(2πi)=1.

Gauging S_{GB}, this generates the correct anomaly (Witten '84)

Anomalies arise from fermion loops, *and* bosons with topological terms

In linear sigma model, with N_f xN_f matrix Φ

RDP, 2608.xxxxx

$$U \to \widetilde{U} = \Phi \frac{1}{\sqrt{\Phi^\dagger \Phi}} \; ; \; \det \Phi \neq 0$$

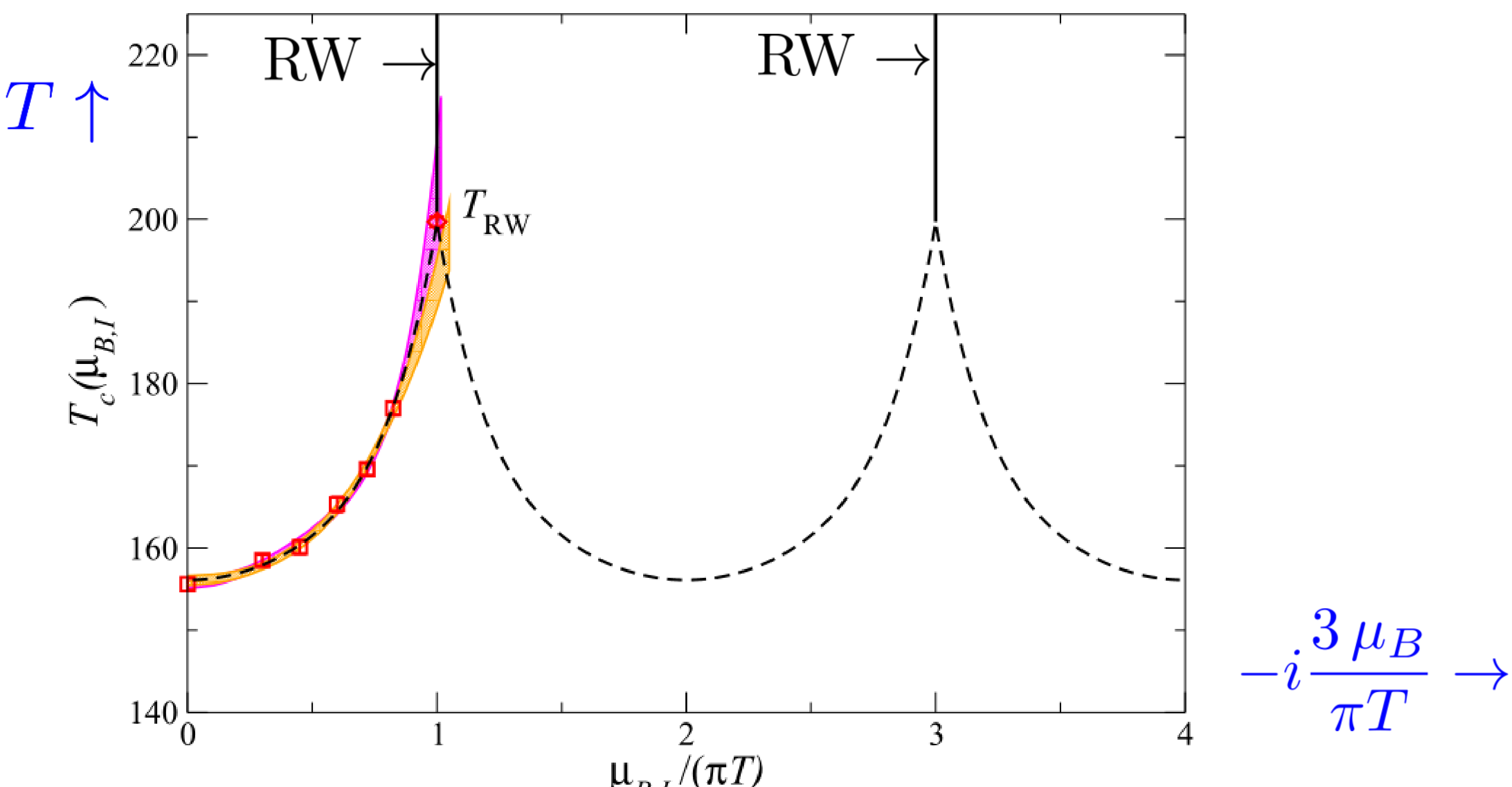
π_3(GL(N_f, C)) ≈ π_3(SU(N_f)) ≈ Z.

Lattice, 2+1 flavors: $T_{RW} > T_{ch}$.

Lattice can compute at imaginary chemical potential, A_0 is real.

Bonati, D'Elia, Mariti, Mesiti, Negro 1602.01426, Cuteri + (Bielefeld) 2205.12707
D'Elia, Siliberto, Zambello 2608.12883

QCD, 2+1 flavors: *crossover* at μ=0, remains crossover until $T_{RW} > T_{ch}$.



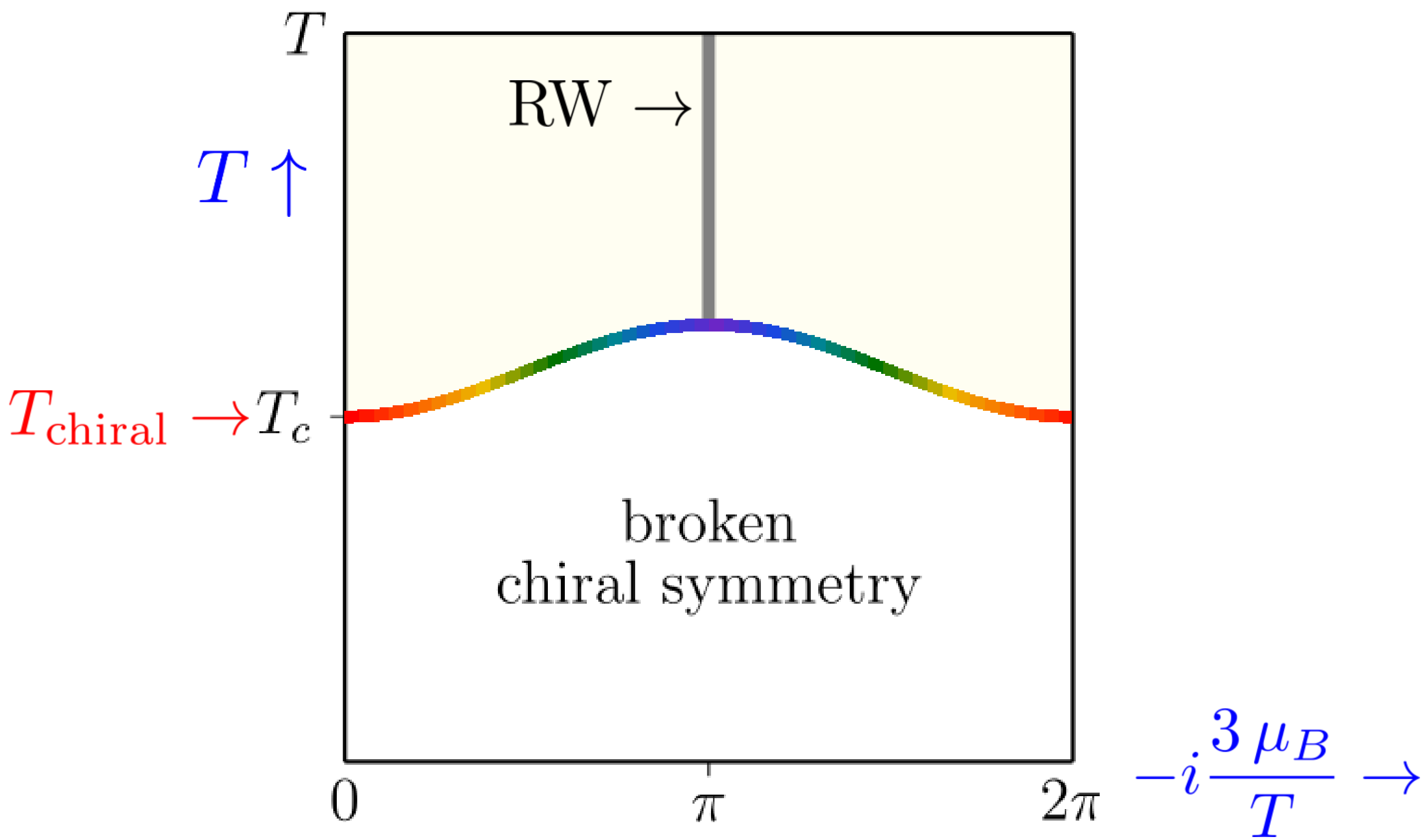
CCP: probably continuous manifold, $T_{RW} = T_{ch}$.

Roberge-Weiss ends on critical line, but universality class changes *continuously* with μ_B

Requires that the baryon density is *exactly* marginal for *all* μ_B.

Manifestly "beyond Landau" (Wilson-Fischer-Kadanoff)

~ Deconfined Quantum Critical Point in 2+1 antiferromagnets: Chen, 2511.19793



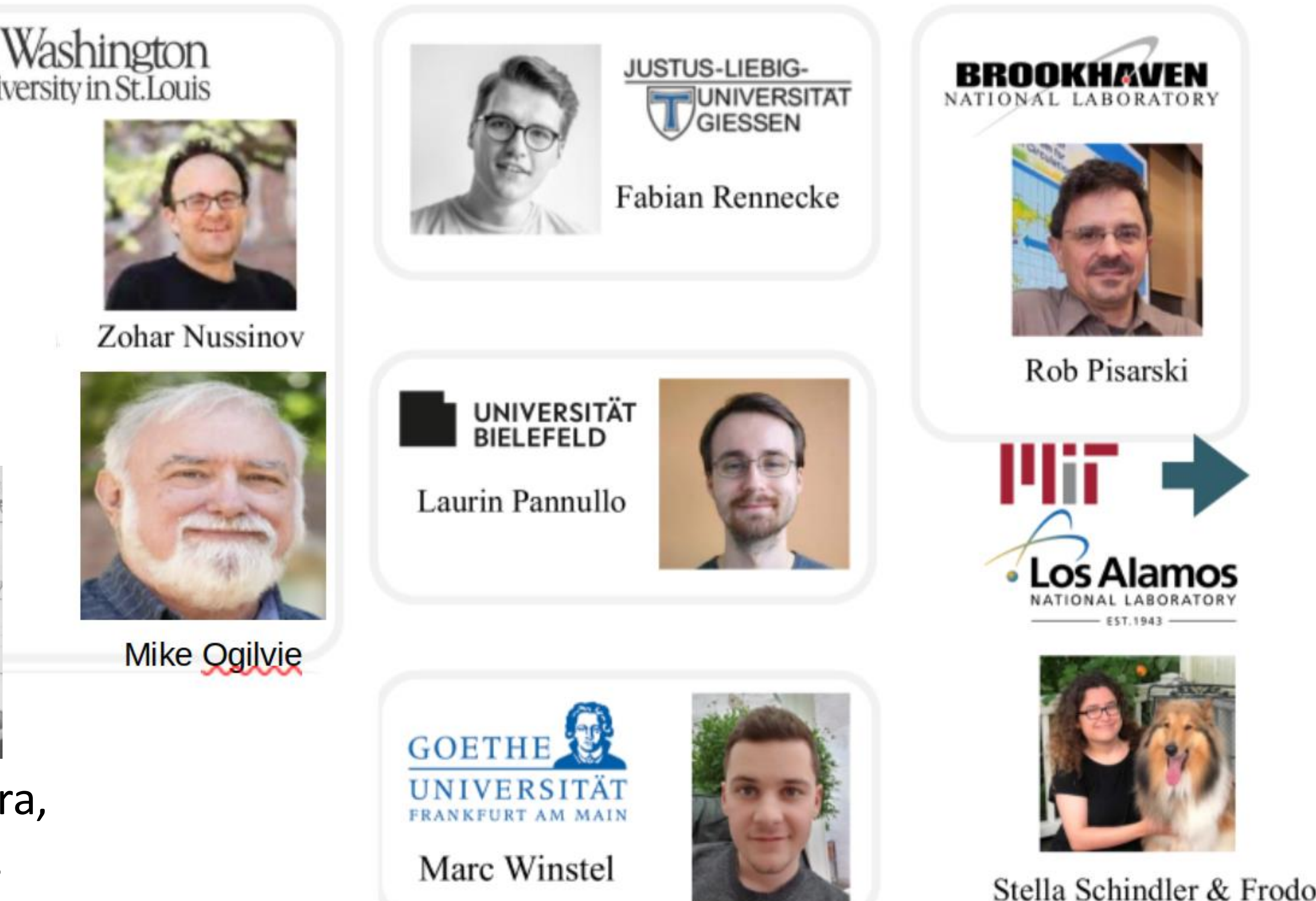
A moat

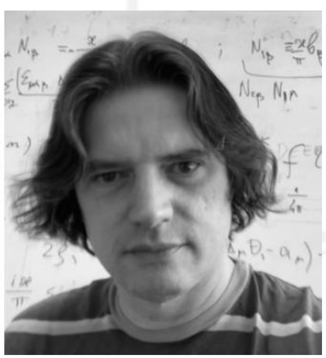
Bodiam Castle in East Sussex

A new bump in dileptons from moaty pions

Zohar Nussinov, Mike Ogilvie, Lauren Pannullo, RDP, Fabian Rennecke, Stella Schindler, and Marc Winstel, [2410.22418](https://arxiv.org/abs/2410.22418) , PRL 135, 101904 (2025)

+ work in progress (& Flavio Nogueira)





Flavio Nogueira,
Dresden Univ.

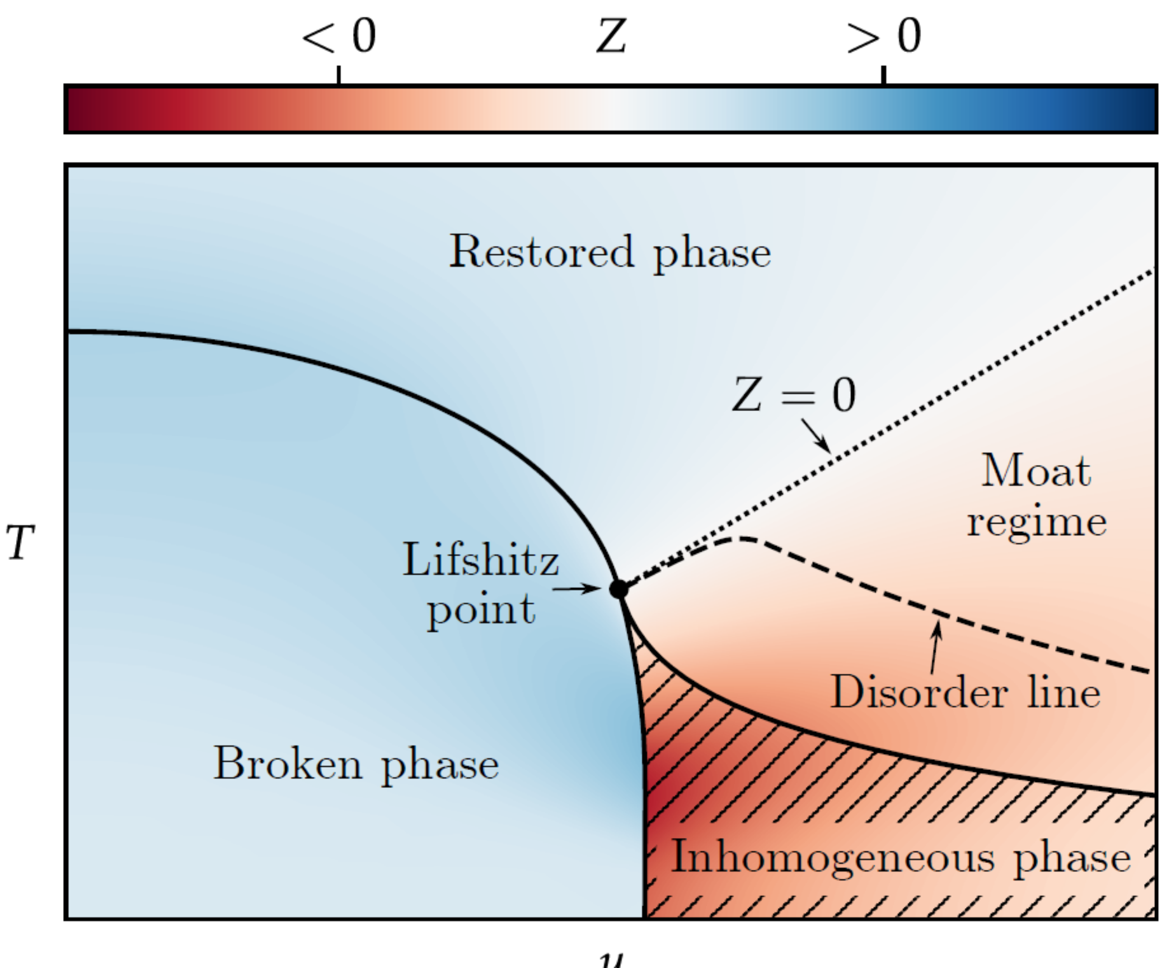
Moaty sigmas in the Gross-Neveu model

Asymp.'y free, soluble $N_f \to \infty$.

$$\Delta_\pi^{-1} = -\omega^2 + \frac{(\vec{k}^2)^2}{M^2} + Z\,\vec{k}^2 + m_{\rm eff}^2$$

Blue: Z > 0   Red: Z < 0 Moat: Z < 0!

Large moat regime in T & μ



1+1dim.'s: no CEP,

Lifshitz point, Z = 0

Disorder line, m_{im}=0:

$$\langle \overline{\psi}\psi(x)\, \overline{\psi}\psi(0)\rangle \sim \cos(m_{\rm im})\, e^{-m_{\rm re}}$$

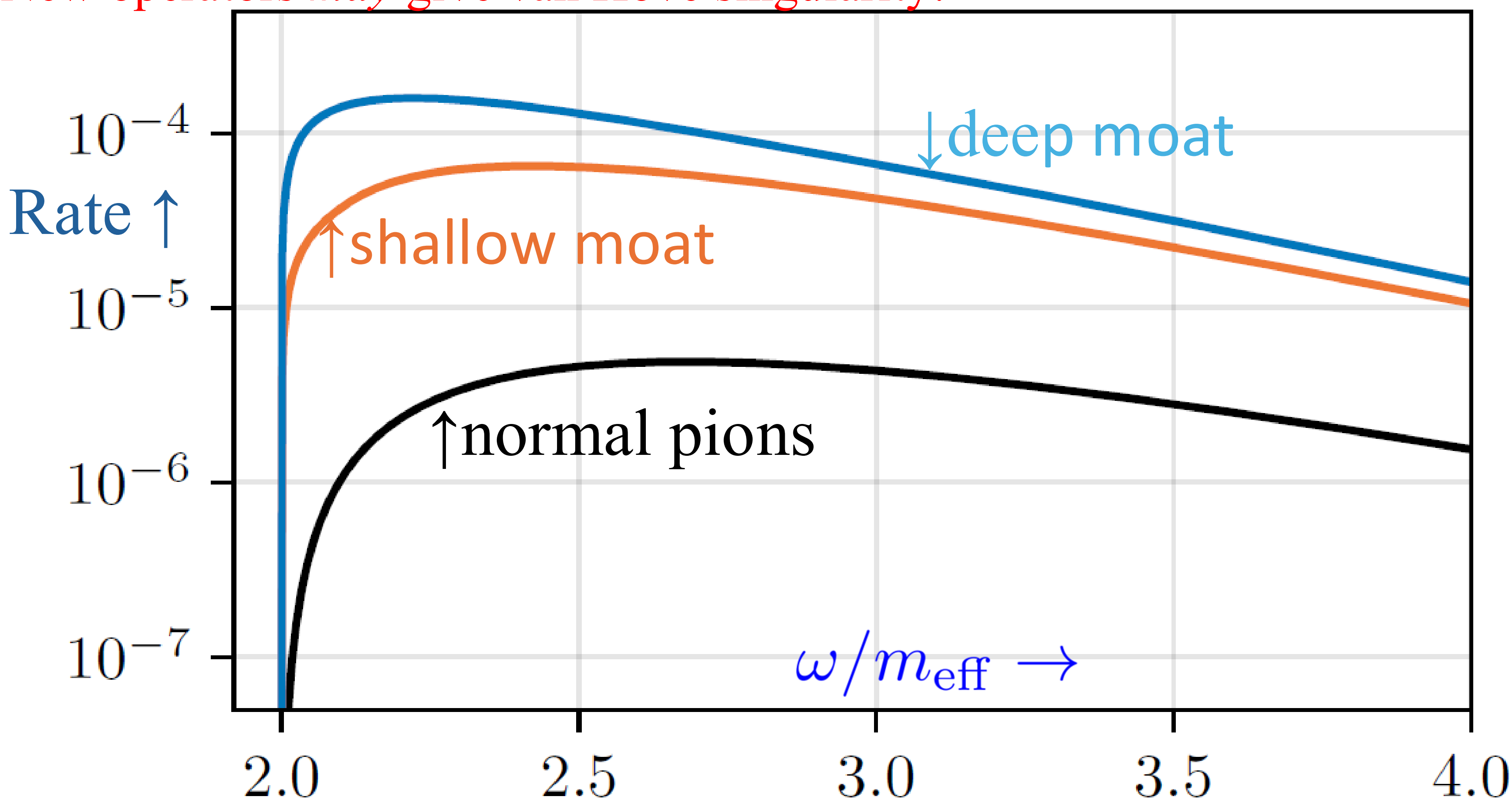
Inhomogenous phase =
kink crystal

Moaty pions: *big* threshold in dileptons

Near threshold, $\omega \geq 2m_{eff}$, each dilepton has small $p \neq 0$. Computed for pair mom.=0.
Moaty rates enhanced by ~ 100 *near* threshold for *either* shallow *or* deep moat
To do: compute with *all* op.'s, pair mom. $p \neq 0$
New operators *may* give van Hove singularity!

$$R_{\text{normal}} \sim \int d^3p \sim p^2$$

$$R_{\text{moaty}} \sim \int d^3p \sim p^2_{\text{moat}}$$



How to cool a neutron star *very* fast

L. Brodie & RDP, *BP*, 2501.02055 , PRL 135 .

L. Brodie, R. Negreiros, J. Steinheimer, V. Dexheimer, & RDP, *BNSDP*, 2603.06789

Punchline: in parity doubled models, for *limited* region of density, in *heavy* stars,
Urca with parity *doubled* nucleons opens up, *neutrino emission increases by 10^3 !*

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