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Towards more realistic QCD thermalization

Improving QCD kinetic theory with HTL screening and anisotropic gluon splitting

FWF Österreichischer Wissenschaftsfonds

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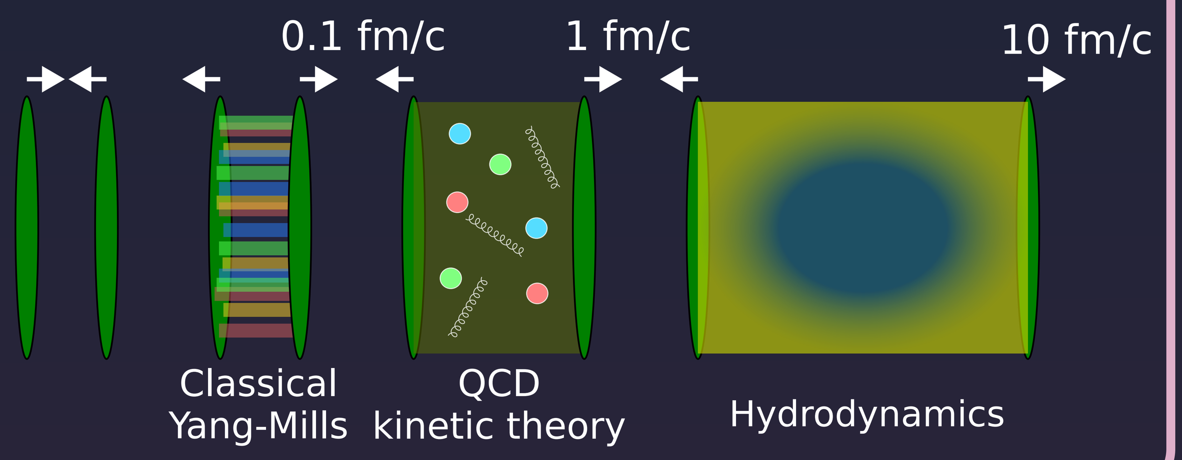
based on
Phys.Rev.D 110 (2024) (Boguslavski, Lindenbauer) [1]
arXiv:2509.09897 (Lindenbauer) [2]

Motivation

- **Quark-Gluon Plasma** (QGP) created in **heavy-ion collisions** at LHC or RHIC
- Several stages in time evolution of QGP *(see figure below)*
- **Kinetic theory**: Far-from-equilibrium $\to$ local equilibrium
- Implementations rely on simplifying assumptions:
  - Simplified Debye-like screening prescription
  - Isotropic approximation for splitting rate calculation
- Go **beyond these approximations**
- Use 't Hooft coupling $\lambda = g^2 N_C = 4\pi N_C \alpha_s$



Medium: Kinetic theory

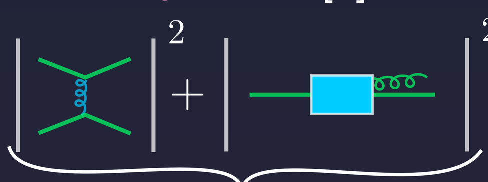
- **Quasi-particles** with **distribution function** $f(t,\vec{p})$
- Assume: Expanding system, mid-rapidity, homogeneous and isotropic in transverse $x-y$ plane
- Solve Boltzmann equation numerically for a purely gluonic system (dominant degrees of freedom for thermalization)
- Initial condition [3]

$$f(p_\perp, p_z) = \frac{2A}{\lambda}\frac{\langle p_T\rangle}{\sqrt{p_\perp^2+\xi^2 p_z^2}} \times e^{-\frac{2}{3\langle p_T\rangle^2}\left(p_\perp^2+\xi^2 p_z^2\right)}$$

$\xi \sim$ anisotropy, $\langle p_T\rangle = 1.8\,Q_s$, $Q_s \sim$ saturation scale

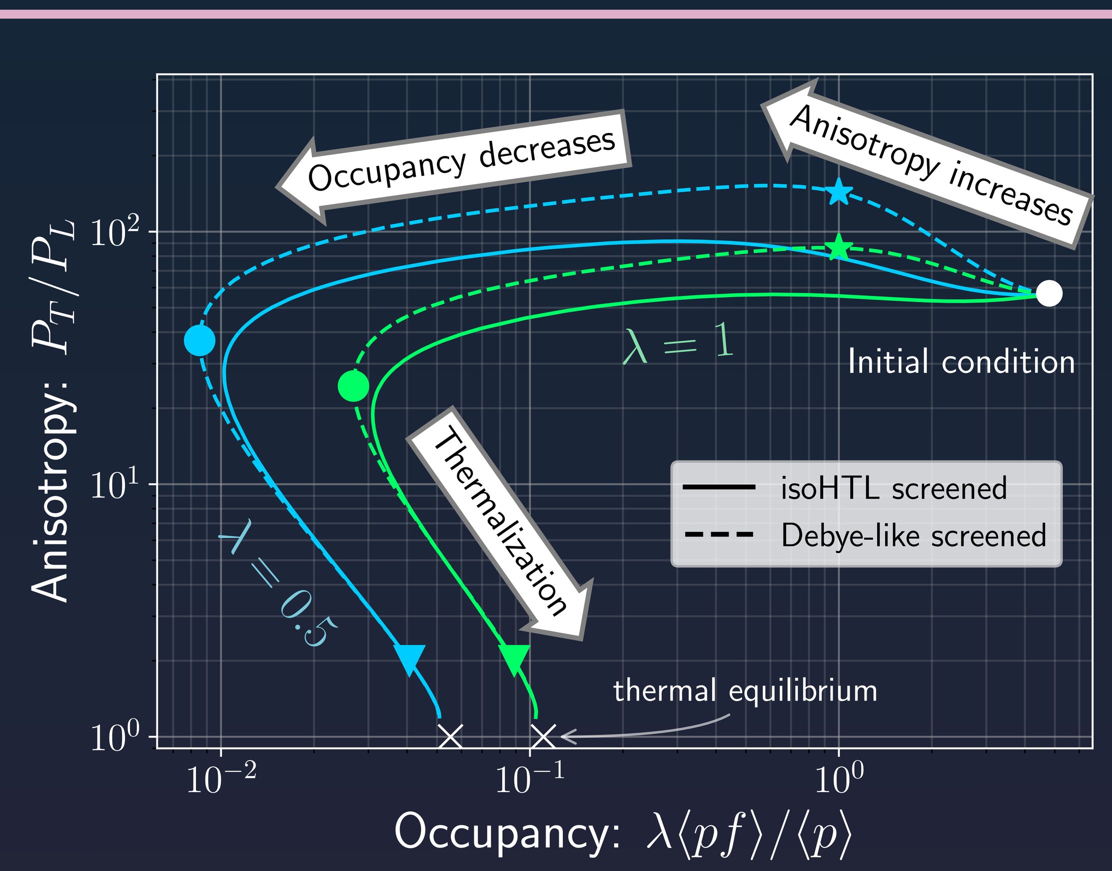
- Time evolution: **Boltzmann equation** [4]

$$(\partial_t + \vec{v}\cdot\vec{\nabla})f = \underbrace{\left|\;\cdot\;\right|^2 + \left|\;\cdot\;\right|^2}_{\text{Collision term}}$$



$$C^{22}(p) = \int |\mathcal{M}|^2 \left[f(p)f(k)(1+f(p'))(1+f(k')) - f(p')f(k')(1+f(p))(1+f(k))\right] \tag{1}$$

$$C^{12}(\tilde{p}) = \int \gamma^p_{p'k'} \left[f(p)(1+f(p'))(1+f(k')) - f(p')f(k')(1+f(p))\right]\left[\delta(\tilde{p}-p) - \delta(\tilde{p}-p') - \delta(\tilde{p}-k')\right] \tag{2}$$



- **Markers** ★ ● ▼ represent **different stages** of bottom-up thermalization [5] (arrows in figure)

Elastic collision term

- $gg \to gg$ scattering matrix element

$$\frac{|\mathcal{M}|^2}{4\lambda^2 d_A} = 9 + \frac{(t-s)^2}{u^2} + \frac{(s-u)^2}{t^2} + \frac{(u-t)^2}{s^2} \tag{3}$$

- Replace t divergent free propagator with **retarded HTL propagator** (replacement gauge-invariant)

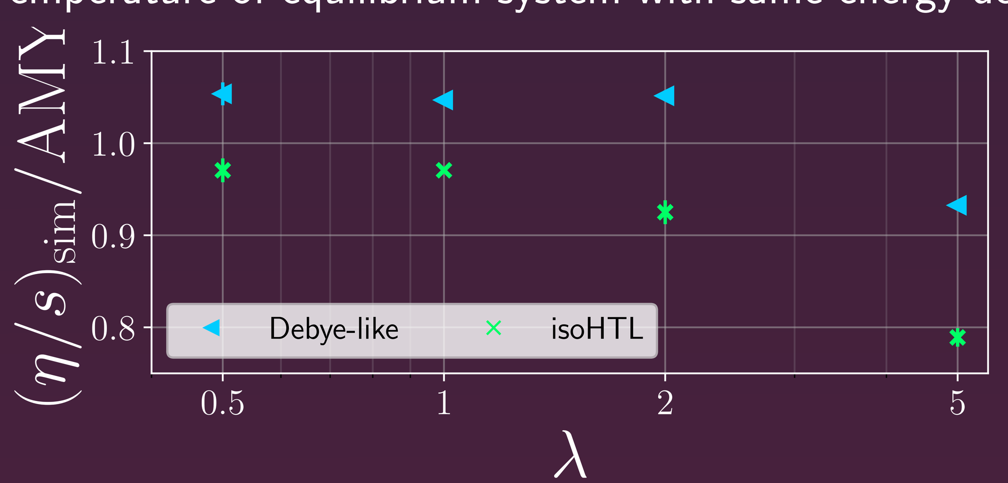
$$\frac{(s-u)^2}{t^2} \xrightarrow{\text{Debye-like}} \frac{(s-u)^2}{t^2}\frac{q^2}{q^2+\xi_g^2 m_D^2} \tag{4}$$

$$\frac{(s-u)^2}{t^2} \xrightarrow{\text{isoHTL}} \left|G^{\text{HTL}}_{\mu\nu}(P-P')(P+P')^\mu(K+K')^\nu\right|^2$$

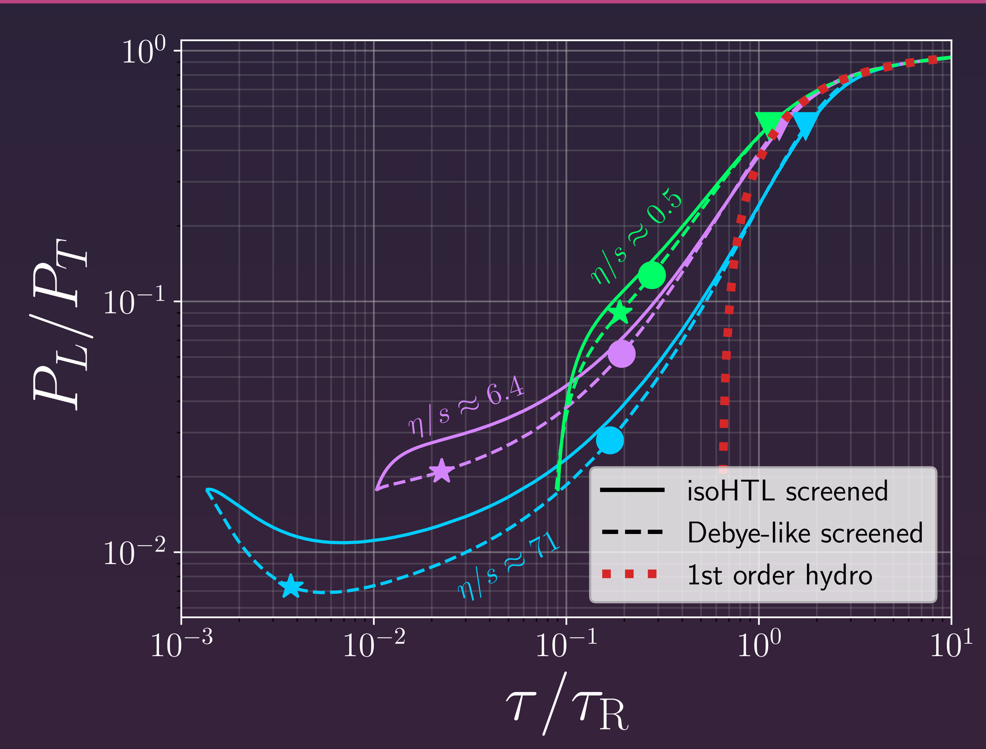
- For **hydro timescale** (relaxation time) $\tau_R = \frac{4\pi\eta/s}{T_\varepsilon(\tau)}$ need specific shear viscosity η/s
- Obtain from late-time pressure ratio $\frac{P_L}{P_T} \overset{\text{1st hydro}}{=} 1 - \frac{2\tau_R}{\pi\tau}$
- Temperature via **Landau matching**, $\varepsilon^{\text{eq}}(T_\varepsilon) = \varepsilon^{\text{simulation}}$

$$T_\varepsilon(\tau) = \left(\frac{30\,\varepsilon(\tau)}{\pi^2 \times \#\text{d.o.f.}}\right)^{1/4}$$

Temperature of equilibrium system with same energy density



Screening affects pressure ratio



Numerical method for the rate $\gamma^p_{p'k'}$

1. Position space with $D(z,b) = -\frac{C(b)+C((1-z)b))+C((1-z)b)}{2}$

$$(A - D(z,\mathbf{b}) - B\nabla^2)\mathbf{F}(\mathbf{b}) = -2i\nabla\delta^{(2)}(\mathbf{b}),$$

2. Solve for $\mathbf{g} = \mathbf{F}/b$ and expand in Fourier modes

$$\mathbf{g}(b,\phi_b) = \sum_n \mathbf{g}_n(b)e^{in\phi_b},$$

3. Coupled differential equations:

$$\left[\frac{A}{B} - \partial_b^2 - \frac{3}{b}\partial_b + \frac{n^2-1}{b^2}\right]\mathbf{g}_n(b) = \sum_m \frac{D_m(b)}{B}\mathbf{g}_{n-m}(b).$$

4. Linear and homogeneous: Linear combination of solutions to satisfy boundary conditions

$$\lim_{b\to 0}\mathbf{F}(\mathbf{b}) = \frac{i\mathbf{b}}{\pi B b^2}, \qquad \lim_{b\to\infty}\mathbf{F}(b) = \mathbf{0}$$

Inelastic collision term

$$\text{Rate: } \gamma^p_{p'k'} \sim \frac{p^4+p'^4+k'^4}{p^3p'^3k'^3}\int\frac{\mathrm{d}^2\mathbf{h}}{(2\pi)^2}2\mathbf{h}\cdot\text{Re}\mathbf{F}, \tag{5}$$

$$2\mathbf{h} = i\delta E(\mathbf{h})\mathbf{F}(\mathbf{h}) + \frac{1}{2}\int\frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2}C(\mathbf{q}_\perp) \times \left[(3\mathbf{F}(\mathbf{h}) - \mathbf{F}(\mathbf{h}-p\mathbf{q}_\perp) - \mathbf{F}(\mathbf{h}-k\mathbf{q}_\perp) - \mathbf{F}(\mathbf{h}+p'\mathbf{q}_\perp\right. \tag{6}$$

- **Medium input**: "Collision kernel" $C(\mathbf{q}_\perp)$

$$C^{\text{iso}}(q_\perp) \overset{q_\perp \ll T_*}{=} \frac{C_R g^2 m_D^2 T_*}{q_\perp^2(q_\perp^2+m_D^2)} \tag{7}$$

- **Novel method** for solving Eq. (6) for **anisotropic $C(\mathbf{q}_\perp$** (generalizes [6,7])
- To position space:

$$C(\mathbf{b}) = \int\frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2}(1-e^{i\mathbf{b}\cdot\mathbf{q}_\perp})C(\mathbf{q}_\perp), \tag{8}$$

$$C_{\text{ref}}(b) = \frac{C_R g_s^2 T_*}{2\pi}\left(\gamma_E + K_0(bm_D) + \log\frac{bm_D}{2}\right), \tag{9}$$

- Test with **squeezed thermal distribution**

$$f(\mathbf{p};\xi) = A(\xi)\,n_B(p_x^2+p_y^2+p_z^2(1+\xi)).$$

and **angular-dependent Debye mass** in (7)

$$m_D^2(\phi) = \left(1 - \frac{2\xi}{3} + \xi\cos^2\phi\right)\bar{m}_D^2.$$

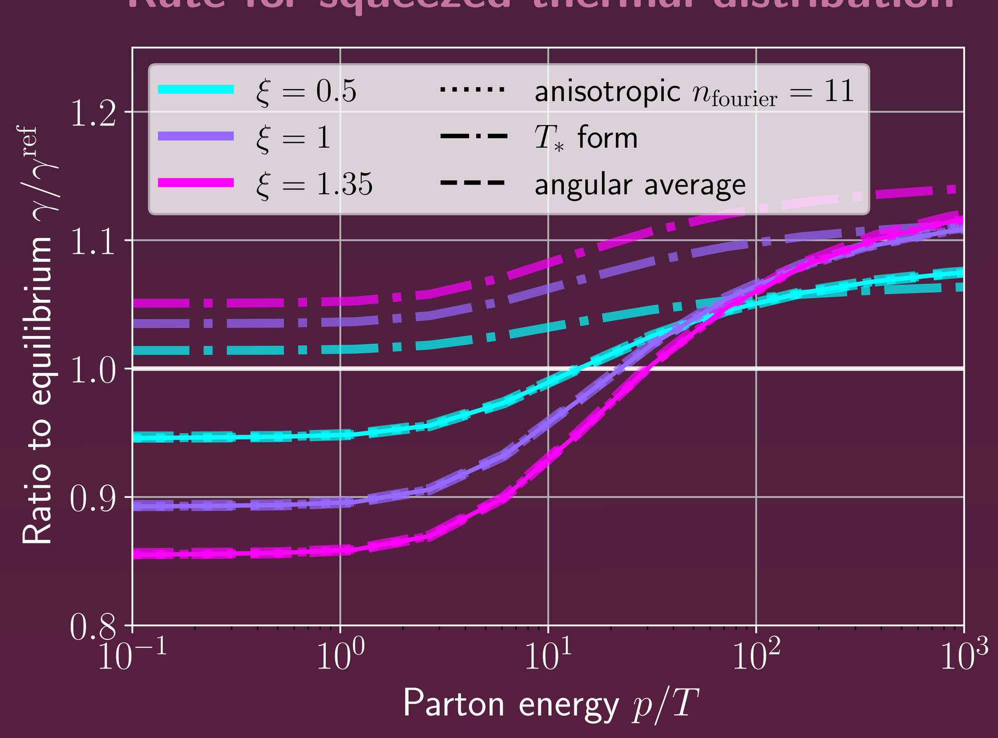
Conclusions

- **Elastic collision term**:
  - HTL screening decreases maximum pressure anisotropy
  - IsoHTL reproduces both transverse and longitudinal momentum broadening [9]
  - Slight decrease in η/s
- **Inelastic collision term**: Rates:
  - New method for obtaining rate for anisotropic $C(q_\perp)$
  - Angular averaged $\langle C(q_\perp)\rangle_\phi$ leads to almost identical rate
  - Isotropic approximated form using T_* and m_D $\to$ qualitatively different rates

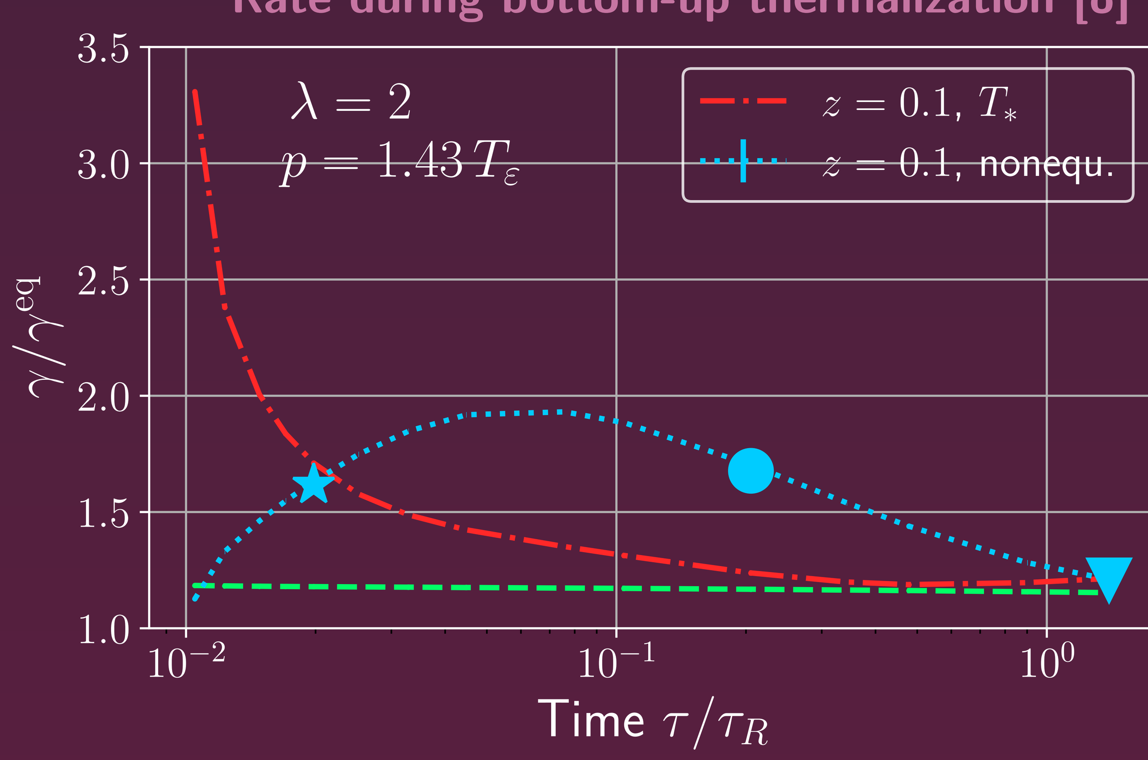
$\to$ Possible consequences for QCD kinetic theory simulations and energy loss/jet quenching!

Inelastic collision term: rates

Rate for squeezed thermal distribution



Rate during bottom-up thermalization [8]



[1] Phys.Rev.D 110 (2024) [Boguslavski, Lindenbauer]
[2] arXiv:2509.09897 [Lindenbauer]
[3] Phys.Rev.Lett. 115 (2015) [Kurkela, Zhu]
[4] JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]
[5] Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]
[6] JHEP 12 (2002) [Aurenche, Gelis, Moore, Zaraket]
[7] JHEP 10 (2021) [Moore, Schlichting, Schlusser, Soudi]
[8] arXiv:2509.03868 [Altenburger, Boguslavski, Lindenbauer]
[9] JHEP 05 (2026) [Boguslavski, Lindenbauer, Mazeliauskas, Takacs, Zhou]

[1] [2]

Strong and Electro-Weak Matter 2026, Helsinki

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