Diquarks from First Principles and Color-Superconducting Matter

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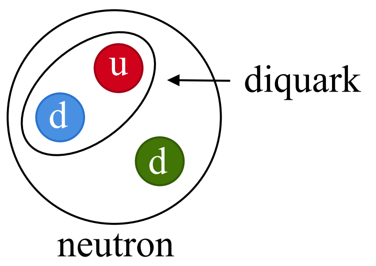
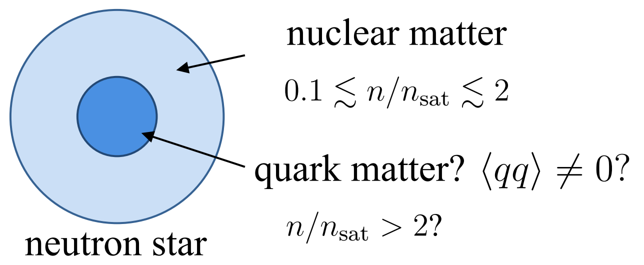
- **Motivation:**

  1. Hadron physics: nucleon ($3q$) in a quark-diquark picture ($\Delta + q$)



  2. Astrophysics: neutron star core with color-superconducting quark matter (diquark condensation $\langle qq \rangle \neq 0$)



- **Problem:** diquarks are not observables → no direct access to their masses and couplings, and few first-principles constraints for low-energy models of color-superconducting matter
- **Main goal:** properties of (scalar) diquarks $\Delta_{\mathfrak{a}} \sim q^{\mathsf{T}} C\gamma_5 i\epsilon_{\mathfrak{a}}\tau_2 q$ from first-principles QCD

$$S_{\rm QCD} = \;{}^{-1} + \;{}^{-1} + \; + \; + $$

  → constrain masses and couplings of low-energy models of color superconductivity
- **This work:** using the FRG with dynamical hadronization to compute diquark off-shell properties from QCD
- **Outcome:** consistent extraction of the inputs needed for effective models and construction of parameter-free models

General Framework

Functional renormalization group (FRG) including **dynamical hadronization** via $\dot{\Phi}_k[\Phi]$

$$\left(\partial_t + \int \dot{\Phi}_k[\Phi]\frac{\delta}{\delta\Phi}\right)\Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma^{(2)}_{\Phi\Phi,k}[\Phi] + R_k\right)\left(\partial_t + 2\frac{\delta\dot{\Phi}_k[\Phi]}{\delta\Phi}\right)R_k\right]$$

with the truncation

$$\Gamma_k[\Phi] = \Gamma_{{\rm glue},k}[A,\bar{c},c] + \Gamma_{{\rm quark-gluon},k}[A,\bar{q},q] + \Gamma_{{\rm qmd},k}[A,\bar{q},q,\phi,\Delta^\dagger,\Delta]$$

Gauge Sector

Expansion around the gauge-fixed classical tensor structure at zero momentum [1]

$$\Gamma_{{\rm glue},k}[A,\bar{c},c] = \frac{1}{2}\int_p A^a_\mu(p)\left[Z_{A,k}\left(p^2 + \boxed{m^2_{{\rm gap},k}}\right)\Pi^{\rm T}_{\mu\nu}(p) + \frac{1}{\xi}p^2\Pi^{\rm L}_{\mu\nu}(p)\right]A^a_\nu(-p)$$

$$+ Z_{c,k}\;{}^{-1} + \lambda_{A\bar{c}c,k} + \lambda_{A^3,k} + \lambda_{A^4,k}$$

- **Gauge invariance:** regulator R_k enforces a running gluon mass $m^2_{{\rm gap},k}$ [2]

  → closely related to gluon mass gap
- **Bootstrap approach:** fixing initial mass gap $m^2_{{\rm gap},k=\Lambda}$ to scaling solution:

$$Z_A \propto (k^2)^\kappa \qquad Z_c \propto (k^2)^{-2\kappa} \qquad \kappa = 0.63\text{–}0.66$$

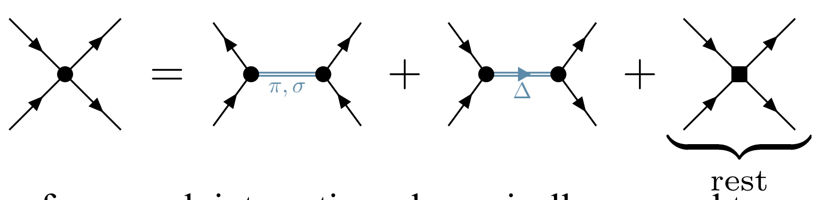
Quark-Gluon Sector

$$\Gamma_{{\rm quark-gluon}}[A,\bar{q},q] = Z_{q,k}\;{}^{-1} + \lambda_{A\bar{q}q,k} + \lambda_{{\rm sps},k} + \lambda_{{\rm csc},k}$$

Non-perturbative low-energy behavior of QCD encoded in four-quark correlations:

scalar-pseudoscalar (σ-meson and pions) and **color-superconducting** (scalar diquarks)

Dynamical hadronization: generation of mesons, diquark correlations



Low-energy physics: four-quark interactions dynamically mapped to composite degrees of freedom:

$$\sigma \sim \bar{q}q \qquad \text{and} \qquad \vec{\pi} \sim \bar{q}\gamma_5\vec{\tau}q \qquad \text{and} \qquad \Delta_{\mathfrak{a}} \sim q^{\mathsf{T}} C\gamma_5 i\epsilon_{\mathfrak{a}}\tau_2 q$$

Composite Sector

$$\Gamma_{{\rm qmd},k}[A,\bar{q},q,\phi,\Delta^\dagger,\Delta] = \int_x \Big\{ \frac{Z_{\phi,k}}{2}(\partial_\mu\phi)(\partial_\mu\phi) + Z_{\Delta,k}(\partial_\mu\Delta)^\dagger(\partial_\mu\Delta) + U_k(\phi^2,|\Delta|^2) - h\sigma$$

$$+ g_{\phi,k}\bar{q}\left(\sigma + i\gamma_5\vec{\pi}\cdot\vec{\tau}\right)q + \frac{1}{2}g_{\Delta,k}\left(\Delta_{\mathfrak{a}}\bar{q}\gamma_5\tau_2 i\epsilon_{\mathfrak{a}}C\bar{q}^{\mathsf{T}} - \Delta^*_{\mathfrak{a}}q^{\mathsf{T}}C\gamma_5\tau_2 i\epsilon_{\mathfrak{a}}q\right)$$

$$- i\lambda_{A\Delta^\dagger\Delta,k}A^a_\mu\left[(\partial_\mu\Delta^{\mathsf{T}})T^a\Delta^* - \Delta^{\mathsf{T}}T^a(\partial_\mu\Delta^*)\right] + \lambda_{A^2\Delta^\dagger\Delta,k}A^a_\mu A^b_\mu\Delta^{\mathsf{T}}T^aT^b\Delta^* \Big\}$$

with $\phi = (\sigma, \vec{\pi})$

meson and diquark exchange with self-interactions (+ diquark-gluon interactions)

Diquark Pole and Curvature Mass

Curvature mass:

$$m^2_{\Delta,{\rm curv}} = G^{-1}_\Delta(p_0 = 0, \vec{p} = 0)/Z_\Delta$$

- often used in low-energy models
- identical to pole mass in free theory

Pole mass:

$$G^{-1}_\Delta(p_0 = im_{\Delta,{\rm pole}}, \vec{p} = 0) = 0$$

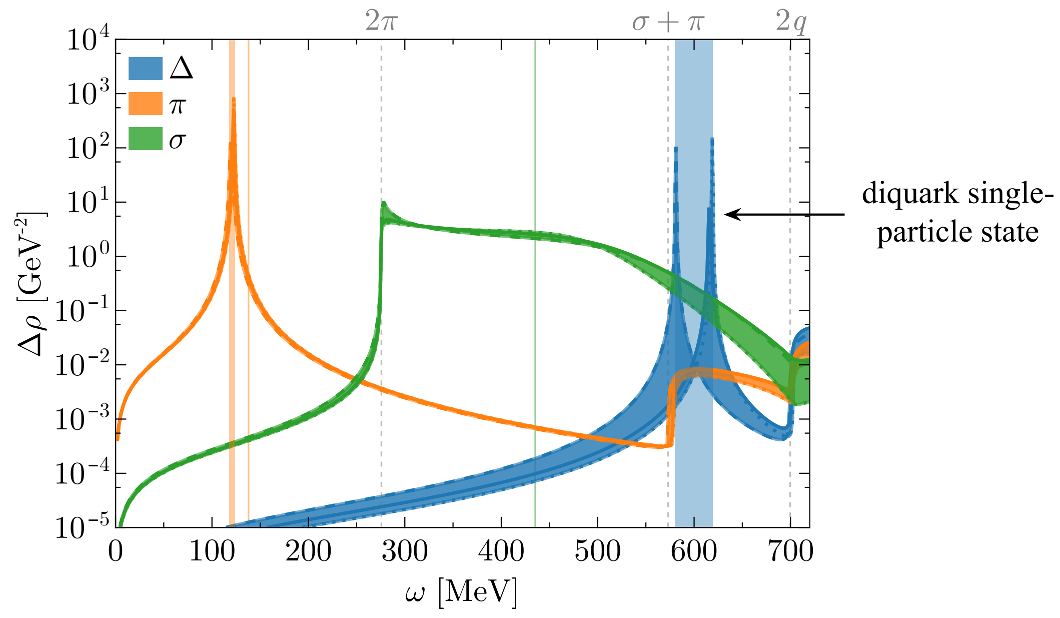
- physical, observable mass
- requires real-frequency information

Pole mass via analytically continued FRG [3]:

$$\partial_t\Gamma^{(2),R}(\omega) \equiv \lim_{\epsilon\to 0}\partial_t\Gamma^{(2),E}(p_0 = -i(\omega + i\epsilon),\, \vec{p} = 0)$$

spectral function

$\Delta\rho \sim {\rm Im}\left(\Gamma^{(2),R}\right)^{-1}$



Conclusion: In a first-principles and self-consistent approach, we find

- $m_{\Delta,{\rm pole}} = 580\text{–}619$ MeV → consistent with lattice / DSE estimates and quark-diquark picture of nucleon
- $m_{\Delta,{\rm curv}} \sim 1$ GeV $\gg m_{\Delta,{\rm pole}}$ → relevant for low-energy model parameter fixing

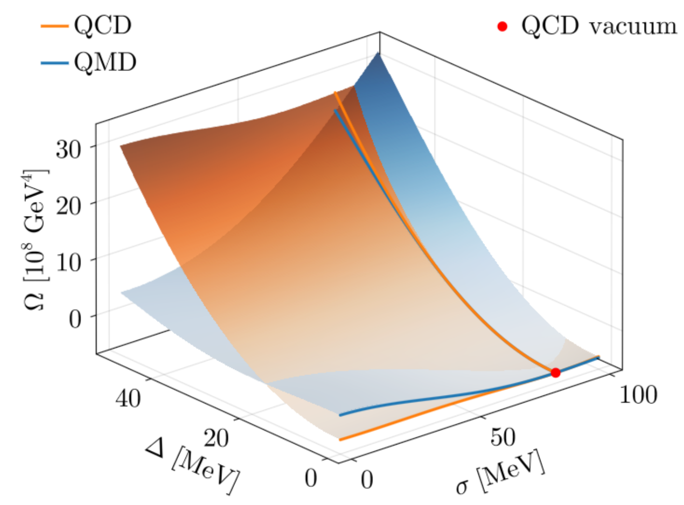
Application: Finite Density Quark Matter

- **Input:** FRG infrared correlators from QCD fix g_Δ, g_ϕ, and $\Omega(\sigma,\Delta)$

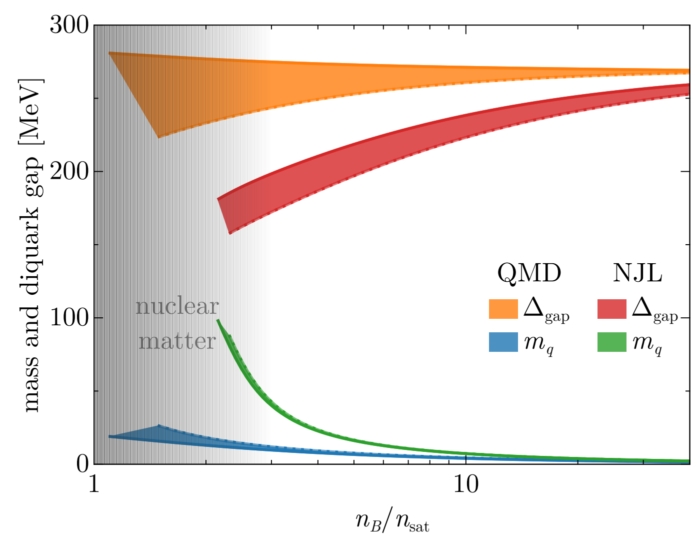
  → fit the shape of the effective potential around the vacuum

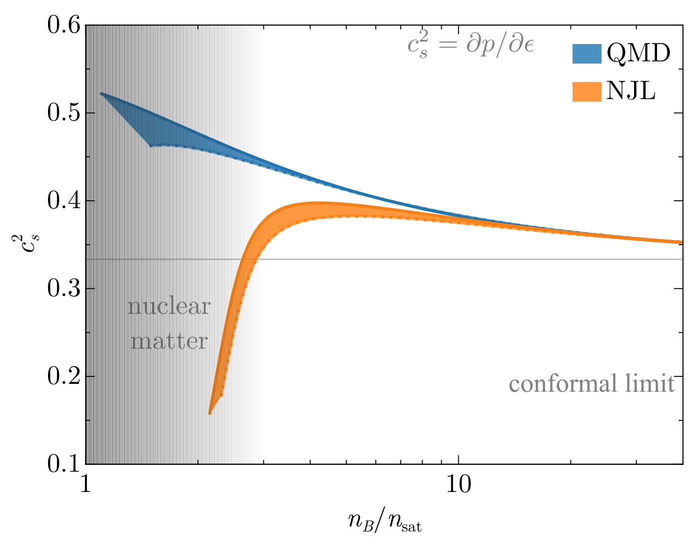
  e.g. renormalized quark-meson-diquark (QMD) model [4]

  or RG-consistent NJL model [5]
- **Output:** parameter-free mean-field predictions at finite density



Large diquark gap $\Delta \sim 250$ MeV at astrophysical densities, with $c_s^2 > 1/3$ at intermediate densities → consistent with neutron-star observations [6]





- **Outlook:** incorporate other degrees of freedom into this procedure, such as vector mesons relevant for neutron-star matter

Take-home message

- Extracting vacuum inputs from QCD can be used to fix parameters in low-energy models with degrees of freedom not directly accessible via observables such as diquarks.
- First-principles FRG with dynamical hadronization yields a scalar diquark pole mass $m_{\Delta,{\rm pole}} = 580\text{–}619$ MeV, consistent with lattice / DSE estimates and the quark-diquark picture of the nucleon.
- Our method yields parameter-free models, giving predictions consistent with neutron-star observations.

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