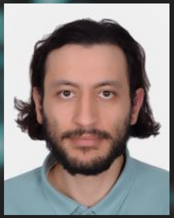
Deriving a parton shower for jet thermalization in QCD plasmas

2510.25837, 2607.26143

Adam Takacs

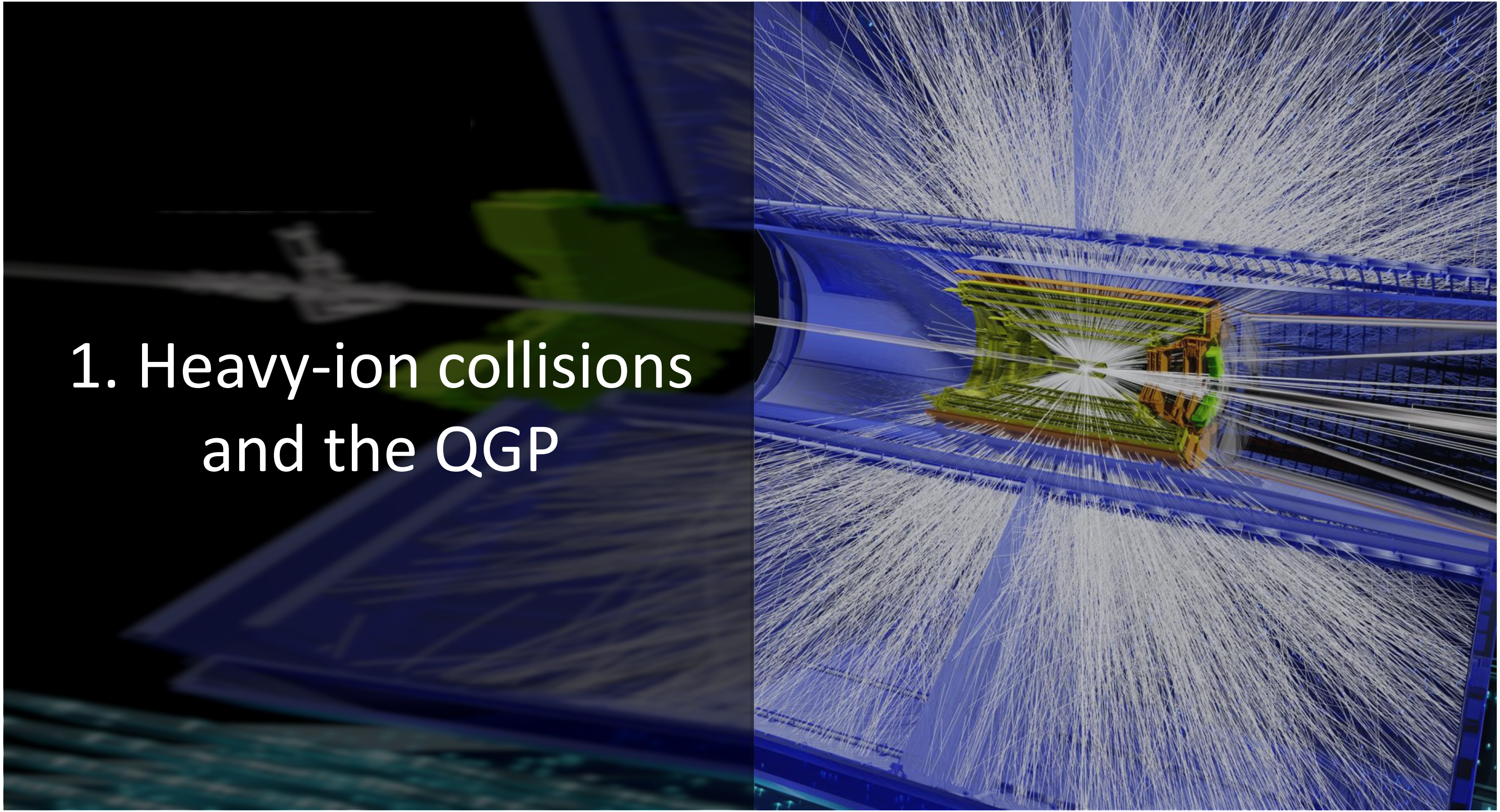
CERN TH,

University of Bergen



With Ismail Soudi

Strong and Electro-Weak Matter, 17-21 Aug. 2026, Helsinki

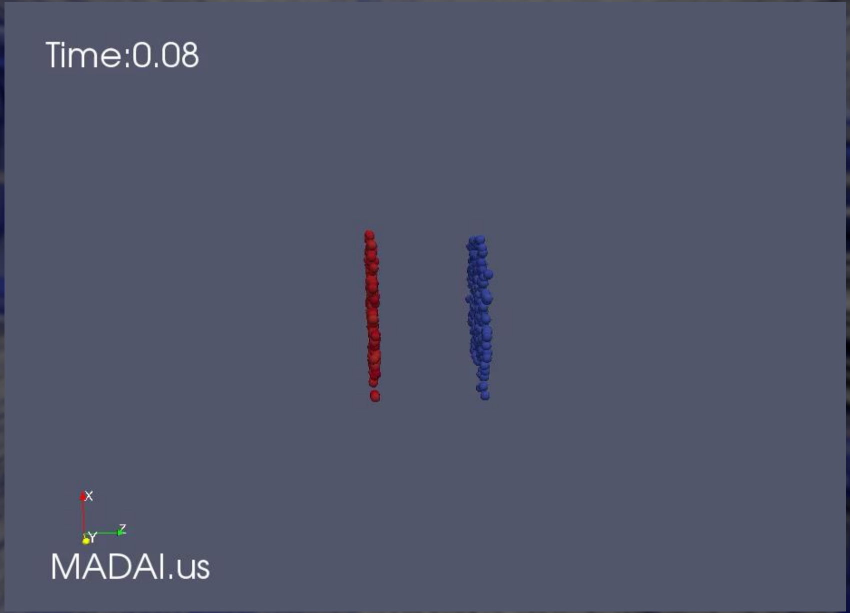


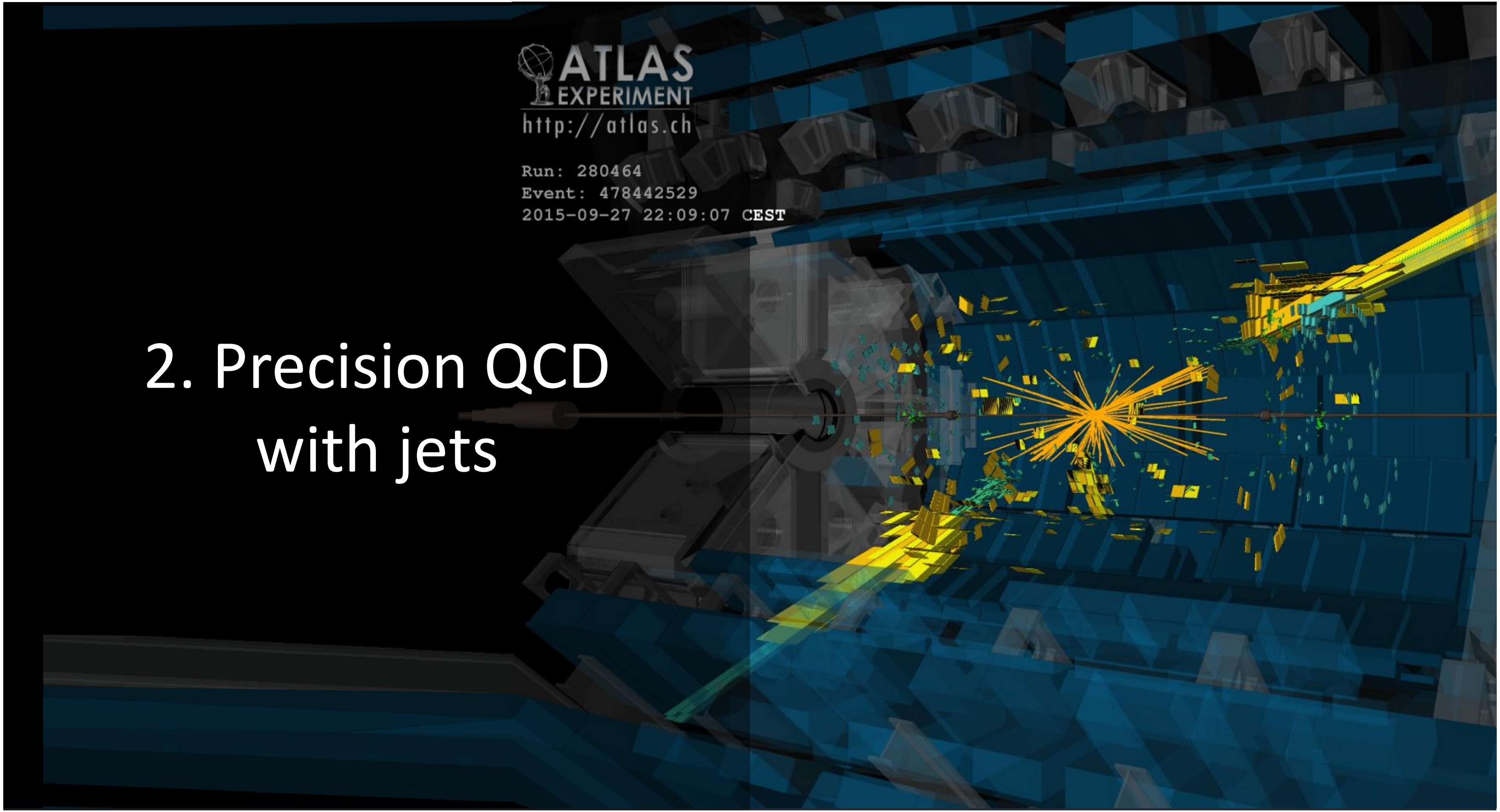
1. Heavy-ion collisions and the QGP

- Heavy-ion program at LHC and RHIC
- Nuclear matter in extreme conditions
- Signatures of the quark-gluon plasma:
  - Collective flow, etc.
  - **Jet quenching** (modification)

Hydrodynamic description!

QGP ≈ perfect fluid





2. Precision QCD with jets

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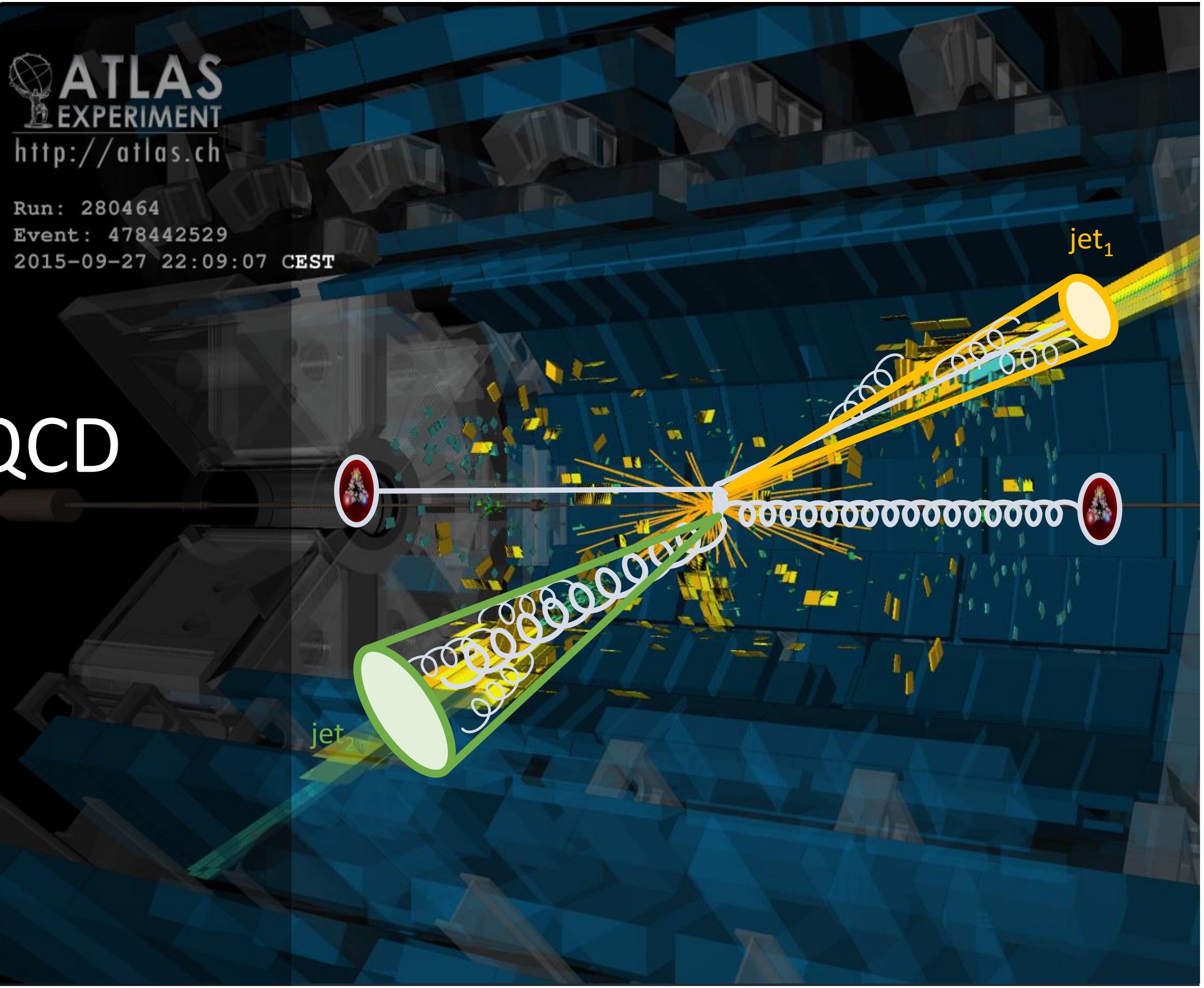


Theory behind MC event generators

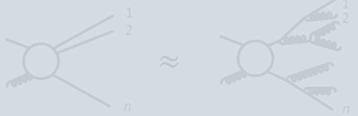
Collinear factorization:

$$\frac{d\sigma^{pp}}{dp_T^{jet}} = \sum_{a,b}\int dx_1 dx_2\, \underbrace{f_a^p(x_1)\cdot f_b^p(x_2)}\cdot \underbrace{\hat{\sigma}_{ab\to jet+X}}\cdot \underbrace{f_{\text{corrections}}}$$

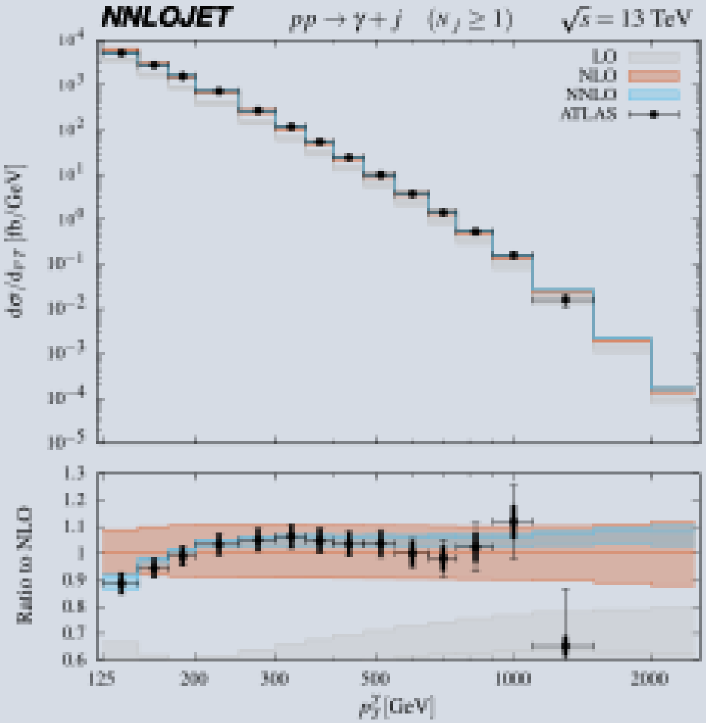
- Find parton a in proton.
- Non-perturbative
  (fitted to every collision).

- Hadronization, non-fact, …
- Non-perturbative
  (fitted to every measurement).
- Want them to be small
  (jets, no hadrons, grooming).

Partonic cross-sec:
- improvable order-by-order
- loop expansion: LO→NLO→N²LO
  (only accessible for few legs ☹)
  - ➢ logarithmic expansion: LL→NLL



← parton shower algorithm



[Chen,Ghermann,Glover,Hofer,Huss 1904.01044]

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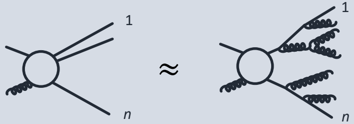

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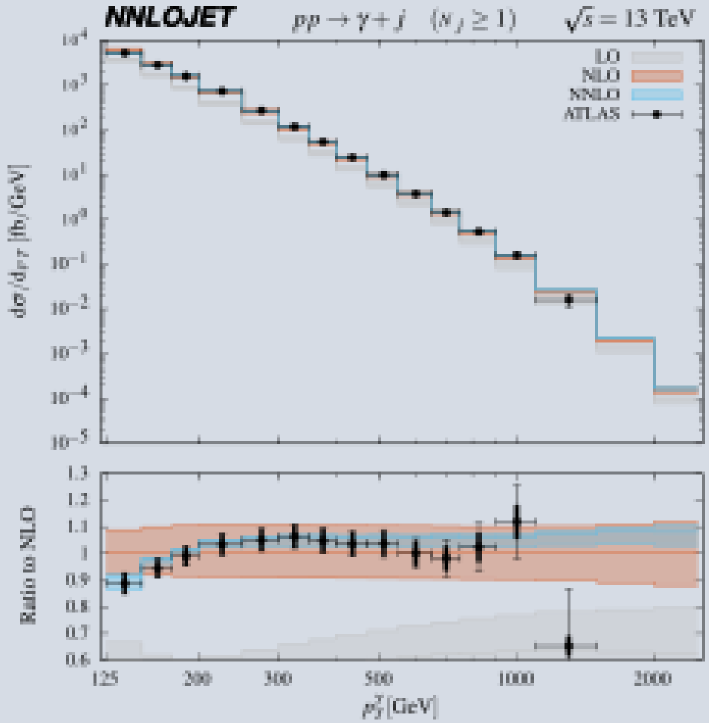

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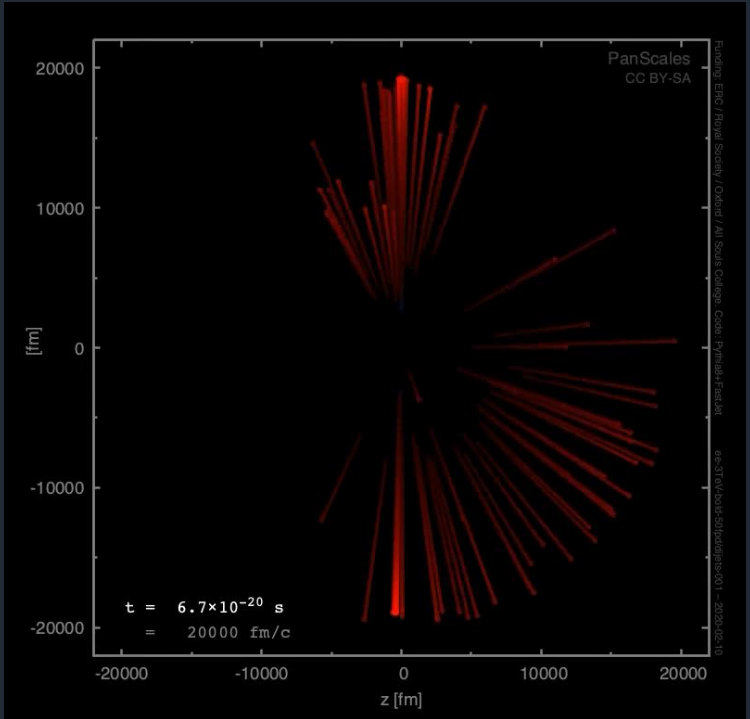
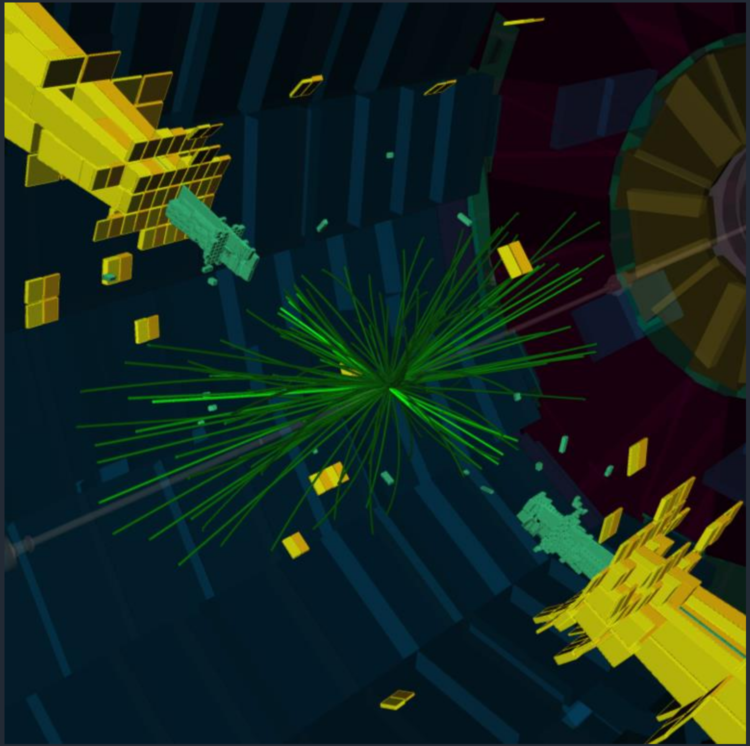
Event generator



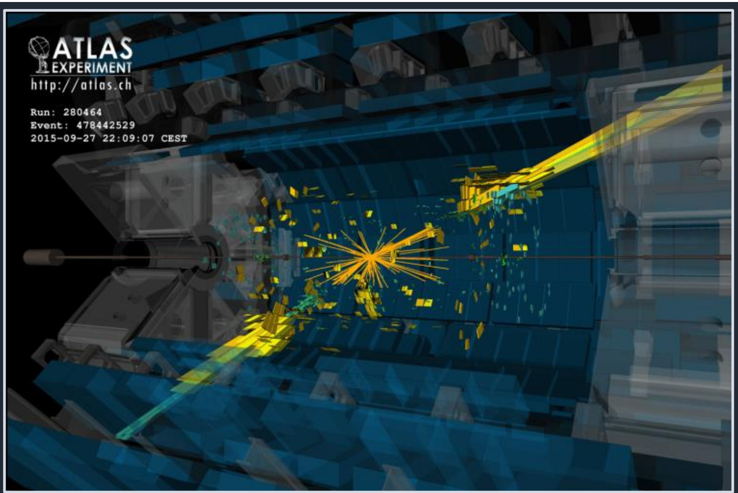
[adapted from G.Salam]

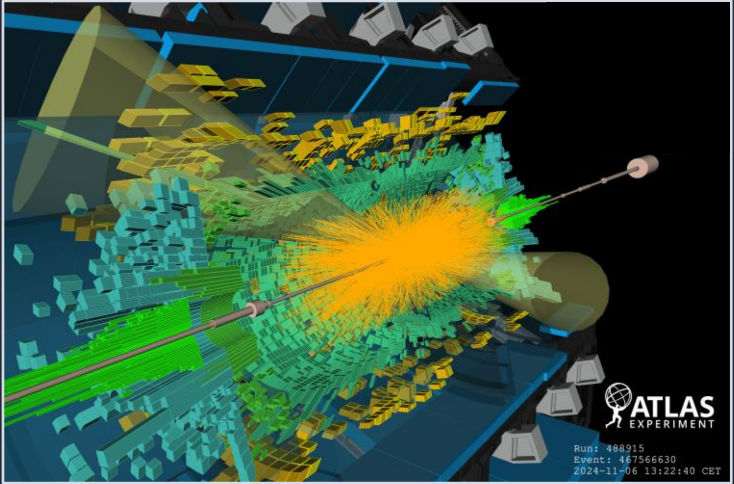
Experiment



- First-principle based, improvable order-by-order
- Describes (a certain class of) multi-differential observables.

1. Jets in heavy-ion collisions





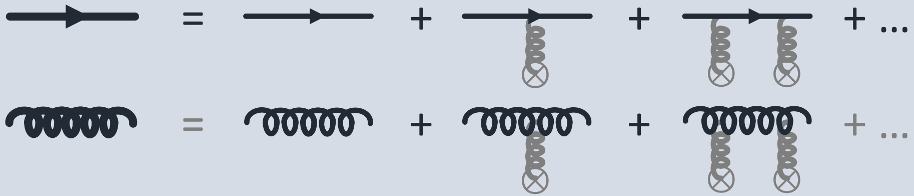
Scattering amplitudes in the QGP

see e.g. SCETg
[Mehtar-Tani, Ringer, Singh, Varun 2026]

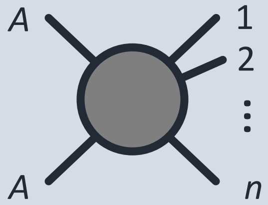
- Separate fields to hard and bkg. $(q = q_h + q_0, A = A_h + A_0)$

$$\mathcal{L}_{QCD}(q, A) = \mathcal{L}(q_h, A_h) + \mathcal{L}(q_0, A_0) + \mathcal{L}_{int}(q_h, A_h, q_0, A_0)$$
$$\approx \mathcal{L}(q_h, A_h) + g\bar{q}_h \hat{\mathcal{J}} q_h + g A_h \hat{\mathcal{J}} A_h$$

- Dressed propagators (and vertices):



Goal:



- Models for the background $\langle \hat{\mathcal{J}}(x^\mu)\hat{\mathcal{J}}^+(y^\mu)\rangle$:
    - High-temperature plasma $(T \gg \Lambda_{QCD})$
    - Color charge distribution
    - Non-perturbative "kernel"

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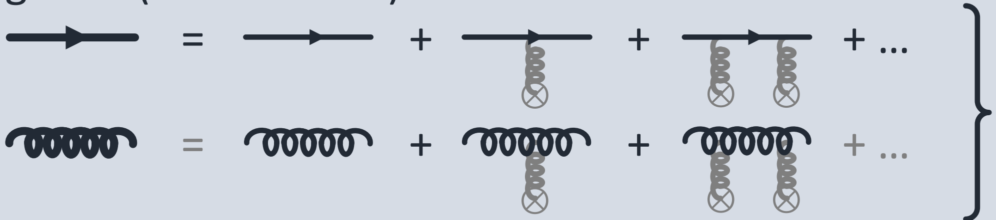

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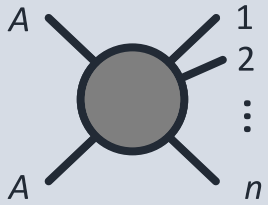

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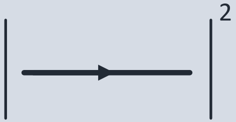
    - **High-temperature QCD plasma** ($T \gg \Lambda_{QCD}$)
    - Color charge distribution
    - Non-perturbative "kernel" to infer

[Baier, Dokshitzer, Mueller, Peigne, Schiff], [Zakharov], [Arnold, Moore, Yaffe]
[Blaizot, Dominguez, Iancu, Mehtar-Tani]
[Casalderrey-Solana, Mehtar-Tani, Tywoniuk, Salgado]
[Caucal, Iancu, Mueller, Soyez]

1. Broadening in large medium

Vacuum

- Straight propagation:

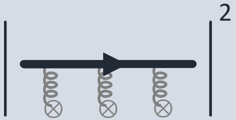


$\approx \ \delta_{ij}\delta(p' - p)$

- Color conservation:

Medium

- Broadening:



$\approx \ \mathcal{G}_{ij}(p', p)$

changing color
and momentum

- Color decoherence:

$\approx$

[Baier, Dokshitzer, Mueller, Peigne, Schiff], [Zakharov], [Arnold, Moore, Yaffe]
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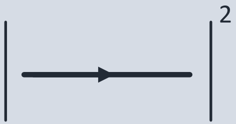

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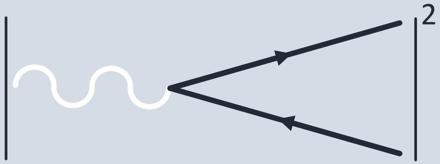

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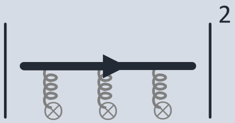

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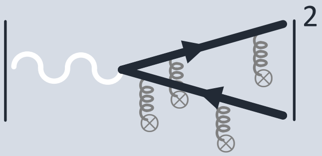

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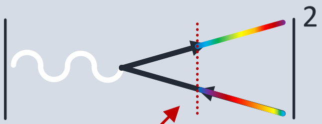

color decoherence:
legs become independent

2. Radiation in large medium

[Baier, Dokshitzer, Mueller, Peigne, Schiff], [Zakharov], [Arnold, Moore, Yaffe]
[Blaizot, Dominguez, Iancu, Mehtar-Tani]
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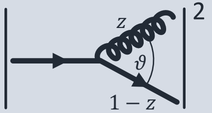
Vacuum

- Emission:



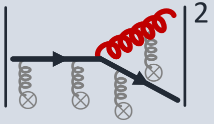
$$\Longrightarrow \quad \frac{dI_i^{vac}}{dz d\vartheta} \approx \frac{\alpha_s}{\pi}\frac{2C_i}{z}\frac{1}{\vartheta}$$

soft & collinear poles!

- Resumming emissions: collinear jet

Medium

- Vacuum + medium-induced emissions:



$$\Longrightarrow \quad \frac{dI_i^{med}}{dz d\vartheta} \approx \frac{\alpha_s}{\pi}\sqrt{\frac{Q_{med}}{E}}\frac{C_i}{\sqrt{z^3}}B(\vartheta)$$

$m < Q_{med}$
$z \ll Q_{med}/E$

soft & ~~collin~~ pole!

- Wide-angle medium-induced cascade:

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Vacuum

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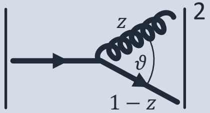

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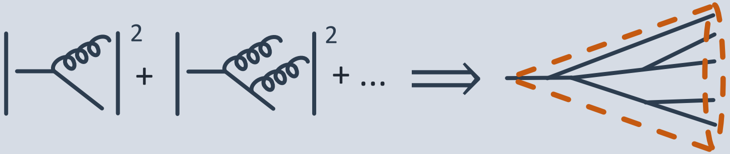
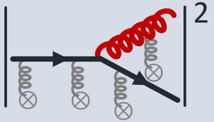


(parton shower)

Medium

- Vacuum + medium-induced emissions:



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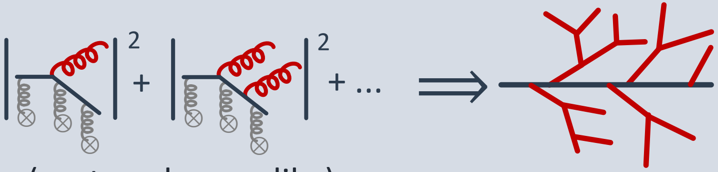
- Wide-angle medium-induced cascade:



(parton shower-like)

Also mixing of vacuum-like
and medium-induced terms!

2. Radiation in large medium

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Vacuum

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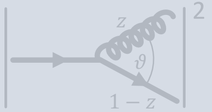

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soft & coll…

- Resumming emissions: co…

Medium

- Vacuum + m… …d emissions:

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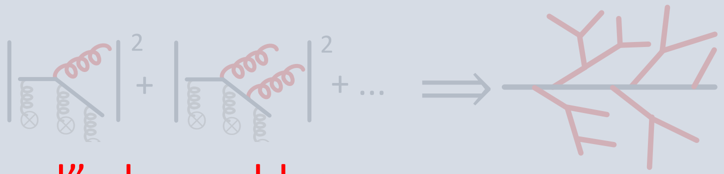


Jet evolution in the QGP

- Successful in pheno for “hard” observables.
- **Does not thermalize!**
- How to thermalize jets?

2. Jet thermalization and Medium response

[Wikipedia]

Medium response in phenomenology

Where does the lost energy go?

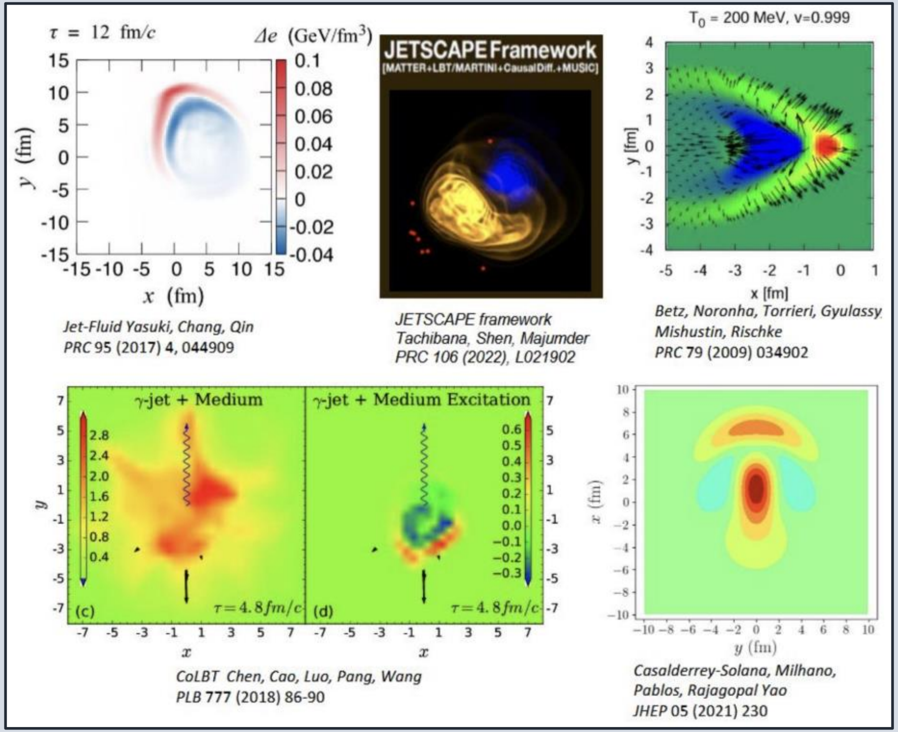
1. Jets evolve in pQCD,
2. Energy loss is modelled (e.g. see before).
3. Lost energy feeds hydro:

$$\partial_\nu T^{\mu\nu} = \Delta J^\mu(x,t)$$

hydro evolution

lost energy as source

- Does the lost energy really equilibrate?
- Weak vs strong coupling?
- Where are perturbative scatterings?
- Smaller collision systems: PbPb → OO → pPb?



Jet-Fluid Yasuki, Chang, Qin PRC 95 (2017) 4, 044909

JETSCAPE framework Tachibana, Shen, Majumder PRC 106 (2022), L021902

Betz, Noronha, Torrieri, Gyulassy Mishustin, Rischke PRC 79 (2009) 034902

CoLBT Chen, Cao, Luo, Pang, Wang PLB 777 (2018) 86-90

Casalderrey-Solana, Milhano, Pablos, Rajagopal Yao JHEP 05 (2021) 230

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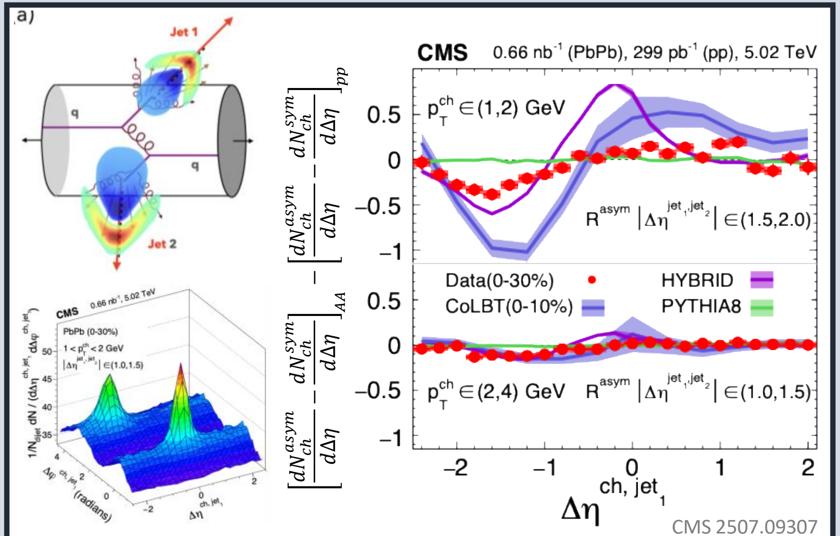


CMS 2507.09307
2602.19431

- New measurements on medium response.
- There is room for improvement.

Medium response in theory

In-medium jet evolution

Weakly coupled

QCD-effective kinetic theory: [Arnold,Moore,Yaffe]
thermalization from first principles. [Baier,Mueller,Schiff,Son]

$$(\partial_t + v \cdot \nabla_x) f(t,x,p) = C^{2\leftrightarrow 2} + C^{1\leftrightarrow 2}$$

How to thermalize jets?
$\updownarrow$
How to put jets in the EKT?

Strongly coupled

Instantaneous freeze-out/hydro
[Hybrid 2010.01140]

$$\partial_\nu T^{\mu\nu} = \Delta J^\mu$$

$\Downarrow$

Falling string + metric perturbation evolution in AdS/CFT
[Chesler, Yaffe]

Some other time

Medium response in theory

In-medium jet evolution

Weakly coupled

Currently: few scattering

QCD-kinetic theory: therm

$(\partial_t + v \cdot \nabla_x)\delta f(t, x,$

[Brewer,Mazeliauskas,Zhou]

How to put jets in the EKT?

Strongly coupled

hydro

[Hybrid 2010.01140]

$= \Delta J^\mu$

time

urbation evolution in

[Chesler, Yaffe]

In this talk:

(i) EKT ↔ jet quenching,

(ii) EKT ↔ parton showers,

(iii) EKT parton shower → med. resp

QCD kinetic theory

- Small perturbation: $f(t,\boldsymbol{x},\boldsymbol{p}) = n(p) + \delta f(t,\boldsymbol{x},\boldsymbol{p})$

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C^{1\leftrightarrow 2}[n,\delta f] + \delta C^{2\leftrightarrow 2}[n,\delta f].$$

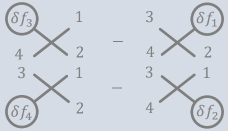
- $2 \leftrightarrow 2$ collisions:

$$\delta C^{2\leftrightarrow 2}[n,\delta f] = \int d\Omega\,|\mathcal{M}|^2\delta\mathcal{F}(p_1,p_2;p_3p_4)$$

scattering mx in QCD plasma

statistical factors (detailed balance)

$$\begin{aligned}\delta\mathcal{F} = {} & \delta f_3[n_4\bar{n}_1\bar{n}_2 \mp \bar{n}_4 n_1 n_2] \\ & + \delta f_4[n_3\bar{n}_1\bar{n}_2 \mp \bar{n}_3 n_1 n_2] \\ & - \delta f_2[n_1\bar{n}_3\bar{n}_4 \mp \bar{n}_1 n_3 n_4] \\ & - \delta f_1[n_2\bar{n}_3\bar{n}_4 \mp \bar{n}_2 n_3 n_4].\end{aligned}$$



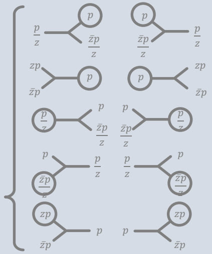
- $1 \leftrightarrow 2$ collisions:

$$\delta C^{1\leftrightarrow 2}[n,\delta f] = \int dz\left[\frac{1}{z^3}\Gamma\left(z,\frac{p}{z}\right)\delta\mathcal{F}\left(\frac{p}{z};p,\frac{\bar{z}p}{z}\right) - \frac{1}{2}\Gamma(z,p)\delta\mathcal{F}(p;zp,\bar{z}p)\right]$$

splitting rate

statistical factors (detailed balance)

- Two type of splitting
- Merging (holes)



QCD kinetic theory at high-energy $p \gg T$

- Small perturbation: $f(t,\boldsymbol{x},\boldsymbol{p}) = n(p) + \delta f(t,\boldsymbol{x},\boldsymbol{p})$

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C^{1\leftrightarrow 2}[n,\delta f] + \delta C^{2\leftrightarrow 2}[n,\delta f].$$

- $2 \leftrightarrow 2$ collisions:

$$\delta C^{2\leftrightarrow 2}[n,\delta f] \approx \int_{q_\perp} dq_\perp\, C(q_\perp)[\delta f(p+q_\perp) - \delta f(p)] \longrightarrow$$

medium-induced broadening

from before!

(small angle)
scattering mx
in QCD plasma

statistical factors (detailed balance)

$$\begin{aligned}\delta\mathcal{F} = {} & \delta f_3[n_4\bar{n}_1\bar{n}_2 \mp \cancel{\bar{n}_4 n_1 n_2}] \\ & + \delta f_4[\cancel{n_3\bar{n}_1\bar{n}_2} \mp \cancel{\bar{n}_3 n_1 n_2}] \\ & - \delta f_2[\cancel{n_1\bar{n}_3\bar{n}_4} \mp \cancel{\bar{n}_1 n_3 n_4}] \\ & - \delta f_1[n_2\bar{n}_3\bar{n}_4 \mp \cancel{\bar{n}_2 n_3 n_4}].\end{aligned}$$

- $1 \leftrightarrow 2$ collisions:

$$\delta C^{1\leftrightarrow 2}[n,\delta f] \approx \int dz\left[\frac{1}{z^3}\Gamma\left(z,\frac{p}{z}\right)\delta f\left(t,\frac{p}{z}\right) - \frac{1}{2}\Gamma(z,p)\delta f(t,p)\right] \longrightarrow$$

medium-induced cascade

from before!

- Using this connection:

(i) High energy AMY-EKT $\leftrightarrow$ jet quenching!
(recipe to put jets into EKT)

(ii) EKT $\leftrightarrow$ parton shower with detailed balance
(recipe to enforce thermalization in jets)

QCD kinetic theory at high-energy $p \gg T$

- Small perturbation: $f(t, \boldsymbol{x}, \boldsymbol{p}) = n(p) + \delta f(t, \boldsymbol{x}, \boldsymbol{p})$

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t, \boldsymbol{x}, \boldsymbol{p}) = \delta C^{1\leftrightarrow 2}[n, \delta f] + \delta C^{2\leftrightarrow 2}[n, \delta f].$$

- $2 \leftrightarrow 2$ collisions:

$$\delta C^{2\leftrightarrow 2}[n, \delta f] \approx \int_{q_\perp} dq_\perp\, C(q_\perp)[\delta f(p + q_\perp) - \delta f(p)] \longrightarrow$$

medium-induced broadening from before!

(small angle) scattering mx in QCD plasma

statistical factors (detailed balance)

$$\begin{aligned} \delta\mathcal{F} = \delta f_3[n_4\bar{n}_1\bar{n}_2 &\mp \cancel{\bar{n}_4 n_1 n_2}] \\ + \delta f_4[\cancel{n_3\bar{n}_1\bar{n}_2} &\mp \cancel{\bar{n}_3 n_1 n_2}] \\ - \delta f_2[\cancel{n_1\bar{n}_3\bar{n}_4} &\mp \cancel{\bar{n}_1 n_3 n_4}] \\ - \delta f_1[n_2\bar{n}_3\bar{n}_4 &\mp \cancel{\bar{n}_2 n_3 n_4}]. \end{aligned}$$

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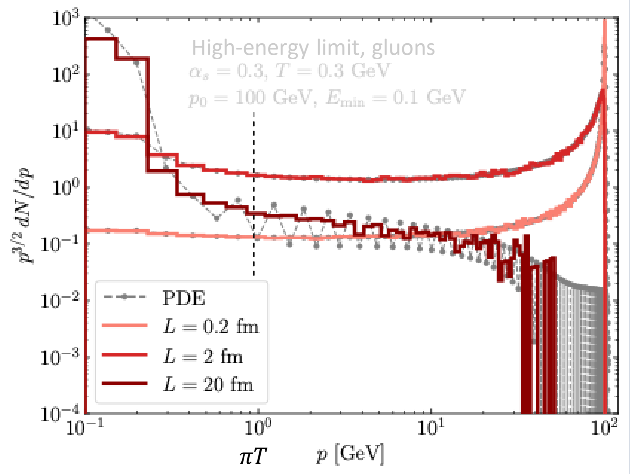
Numerical solution to EKT

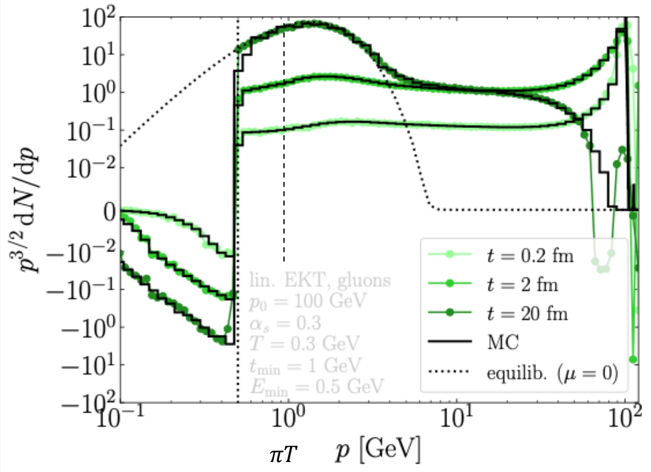
see also PDEs [Mehtar-Tani,Schlichting,Soudi] [Brewer,Mazeliauskas,Zhou]

High-energy limit



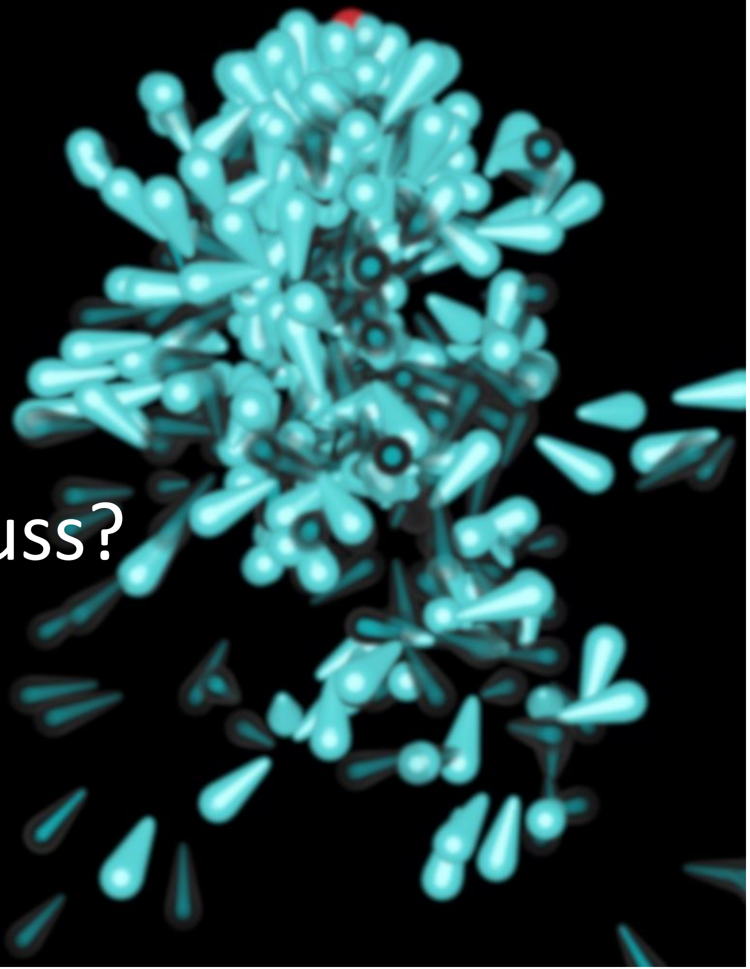
- High-energy limit is analogous to traditional parton showers.
- Doesn't thermalize!
- Parton shower = differential eq. solver

Full EKT



- Including all statistical factors to the parton shower (ensure detailed balance)
- Thermalizes!
- Parton shower = differential eq. solver

Outlook:
Why all the fuss?



Applications

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C[n,\delta f].$$

1. **New phenomenological applications of EKT!**

   Interface with jets, hadronization, KøMPøST, etc.

2. **Multi-particle correlations.**

   Parton-shower is an n-particle cross-section generator:

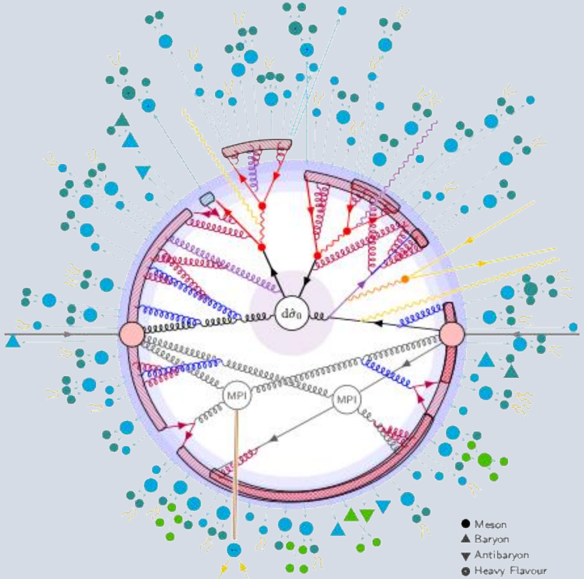
   Unexplored topic in EKT, great relevance e.g. EEC.

3. **Incorporate inhomogeneities.**

   Previously neglected! (parton shower speedup)

   Hydrodynamization: $\delta f(t,x,p) \to \delta f_{eq}(p, \delta T(t,x), \delta u(t,x))$

4. **Applicable for any kinetic theory!**



[Pythia 8.3 2203.11601]

Applications

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C[n,\delta f].$$

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   Interface with jets, hadronization, KøMPøST, etc.

2. **Multi-particle correlations:**

   Parton-shower is an n-particle cross-section generator:

   Unexplored topic in EKT, great relevance e.g. EEC.

   $$\delta f(p_1, p_2, \dots) = \frac{1}{N}\frac{dN}{dp_1 dp_2 \dots}$$

3. **Incorporate inhomogeneities (iii):**

   Previously neglected! (parton shower speedup)

   Hydrodynamization: $\delta f(t,x,p) \to \delta f_{eq}(p, \delta T(t,x), \delta u(t,x))$

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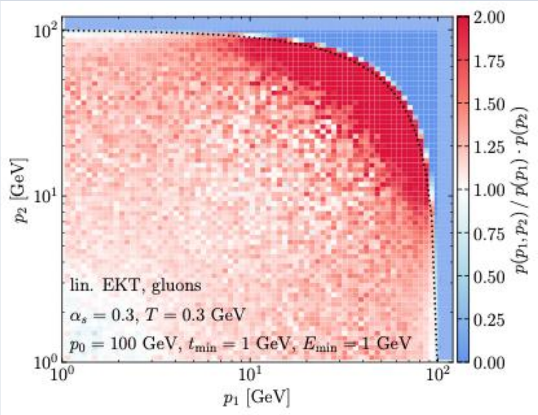

Correlations beyond molecular chaos!

Applications

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C[n,\delta f].$$

1. **New phenomenological applications of EKT:**

   Interface with jets, hadronization, KøMPøST, etc.

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$$\delta f(x) = \frac{1}{N}\frac{dN}{dx}$$

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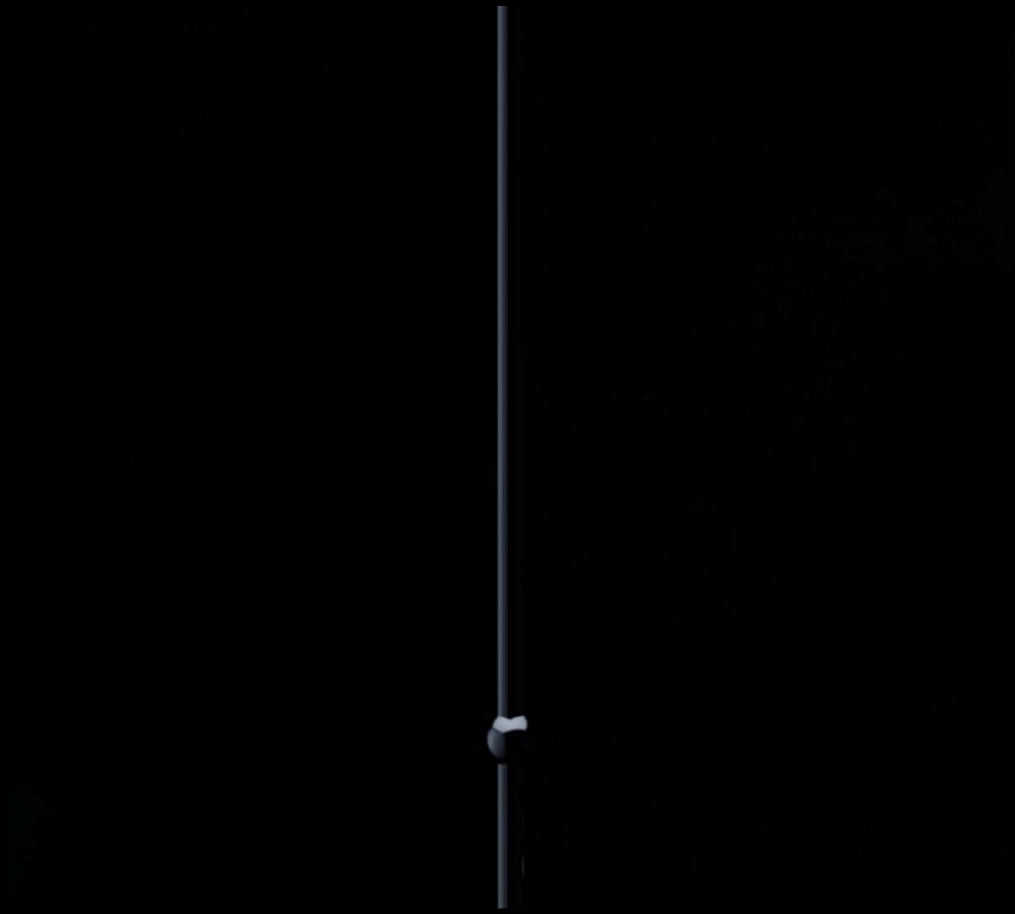
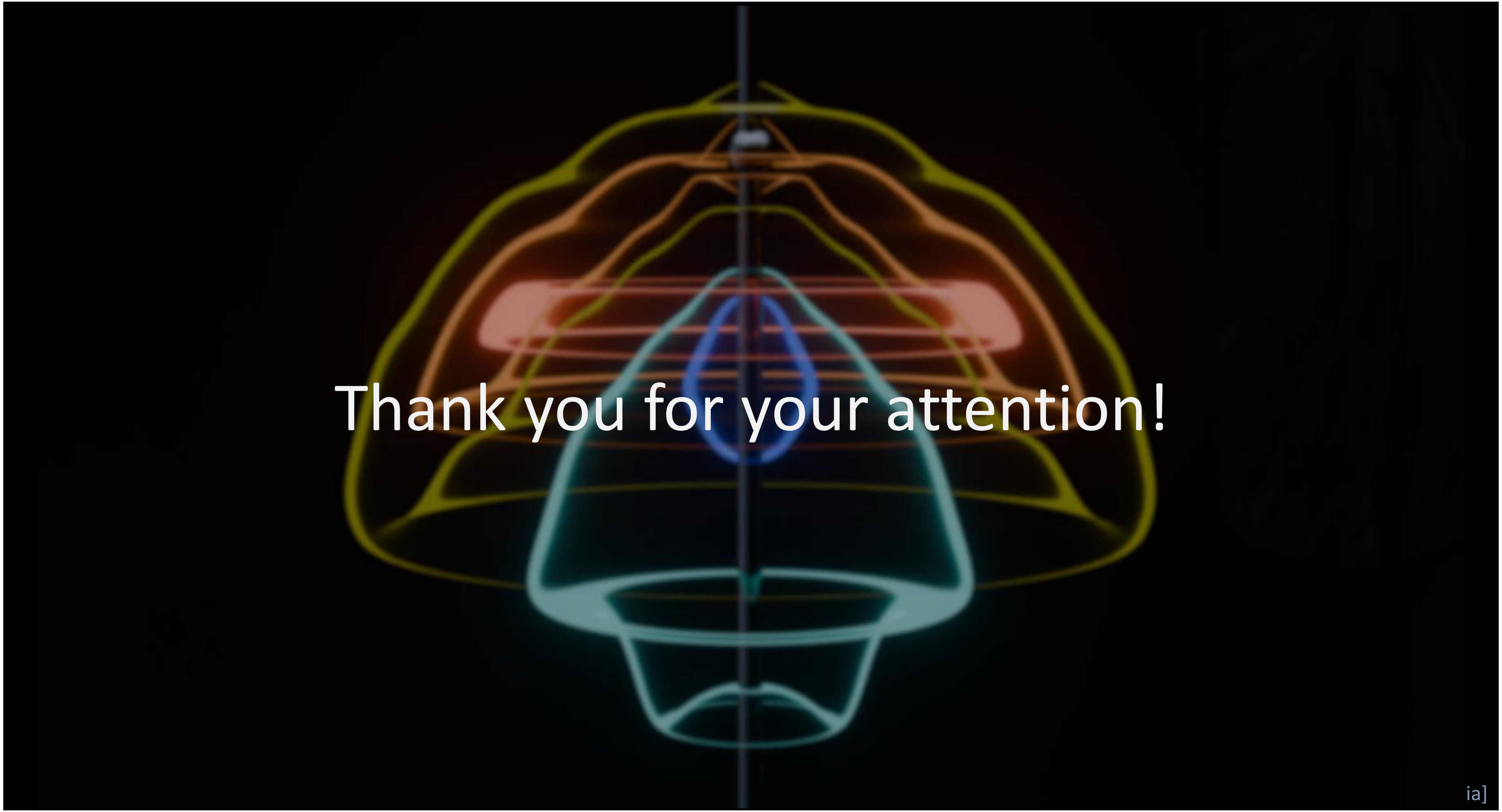


https://youtu.be/9sLDOXvyQVs?si=hFhoUW9E16dgFy4H

Summary:

1. Perturbative grounds of jet quenching
2. QCD-EKT $\leftrightarrow$ parton energy loss and broadening
3. Lin. Boltzmann eq. $\leftrightarrow$ parton shower
4. New opportunities for EKT:
    - Jet thermalization,
    - Multi-particle correlations, hydrodynamization, fluctuations
    - Including in phenomenological workflows

2026 A Decade of Jet Substructure in Heavy-Ion Collisions
Theory Institute at CERN
Dec 7 – 18, 2026



Thank you for your attention!

QCD kinetic theory as a parton shower

$$\left(\partial_t + \frac{\boldsymbol{p}}{p}\nabla_{\boldsymbol{x}}\right)\delta f(t,\boldsymbol{x},\boldsymbol{p}) = \delta C[n,\delta f].$$

- Spatially homogeneous case:

    1. Separate "real" and "virtual" collisions: $\delta C = \delta C^{\mathrm{r}}[n,\delta f] - \delta C^{\mathrm{v}}[n]\,\delta f$.

    2. Integrate the Boltzmann eq: where $\Delta(t) = \exp\left[\int^t dt'\,\delta C^{\mathrm{v}}[n]\right]$

$$\delta f(t) = \Delta(t)\delta f_0 + \int^t dt' \frac{\Delta(t)}{\Delta(t')}\delta C^{\mathrm{r}}[n,\delta f].$$

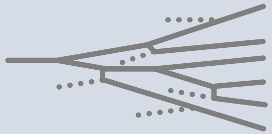
4. Iterate: $\delta f_0 = \delta(p - p_0)$.

5. Histogram the particles: $\delta f(t,x,p) = \frac{1}{N_{\mathrm{ev}}}\frac{\mathrm{d}N}{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{x}}$

(*ii*) kinetic theory ↔ parton shower!

- Inhomogeneous case is trivial: free streaming between collisions.

(*iii*) inhomogeneous perturbations

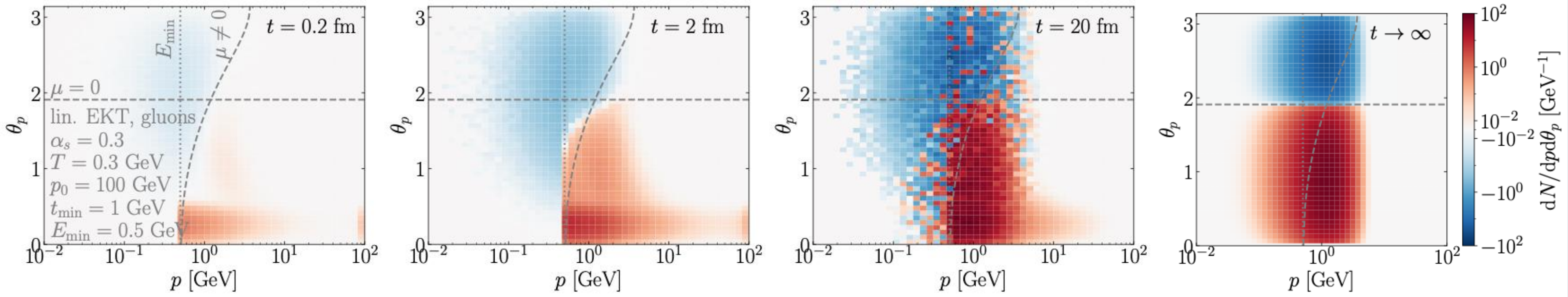


```
Initial condition:#[ID, p, x, weight]
    [21 (100,0,0,100) (0,0,0,0) 1]


Output:
    [[21 (1.5,0.0,00,1.5) (0.1,0.0,00,0.1) 1],
    [21 (2.8,0.0,0.0,2.8) (2.0,0.0,00,2.0) 1],
    [21 (5.0,0.0,0.0,5.0) (9.4,0.0,0.0,9.4) 1],
    [21 (1.7,1.1,0.6,1.1) (9.7,0.0,0.0,9.7) -1],
    [21 (82.,-0.9,-0.2,82) (9.7,0.0,0.0,9.7) 1],
    [21 (1.0,0.4,0.2,0.9) (12.0,1.4,0.5,12.0) 1],
    [21 (2.2,0.0,0.0,2.2) (15.0,0.0,0.0,15.0) 1],
    [21 (3.3,0.0,0.0,3.3) (15.0,0.0,0.0,15.0) 1],
    [21 (1.5,0.7,0.2,1.3) (15.5,2.6,1.0,14.8) 1],
    [21 (2.1,0.9,0.3,1.8) (15.5,2.6,1.0,14.8) 1]]
```

Numerical solution to EKT

see also PDEs [Mehtar-Tani,Schlichting,Soudi]
[Brewer,Mazeliauskas,Zhou]



1. Collinear radiation of soft gluons.
2. Broadening of soft fragments
3. $\mu \neq 0$ equilibrium $\rightarrow \mu = 0$ equilibrium.

- (+) and (−) medium response.
- Soft and hard medium response.
- Equilibrium only at very late times.