

Deriving a parton shower for jet thermalization in QCD plasmas

[2510.25837](#), [2607.26143](#)

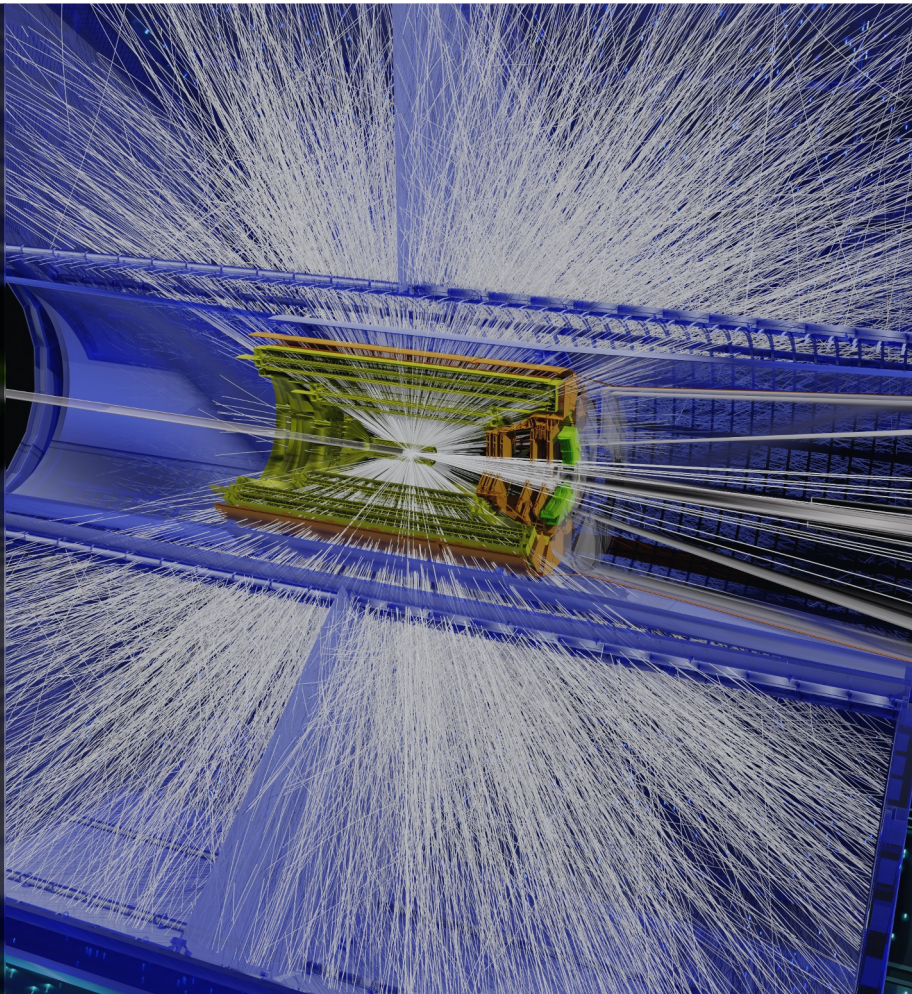
Adam Takacs

CERN TH,
University of Bergen



With Ismail Soudi

1. Heavy-ion collisions and the QGP

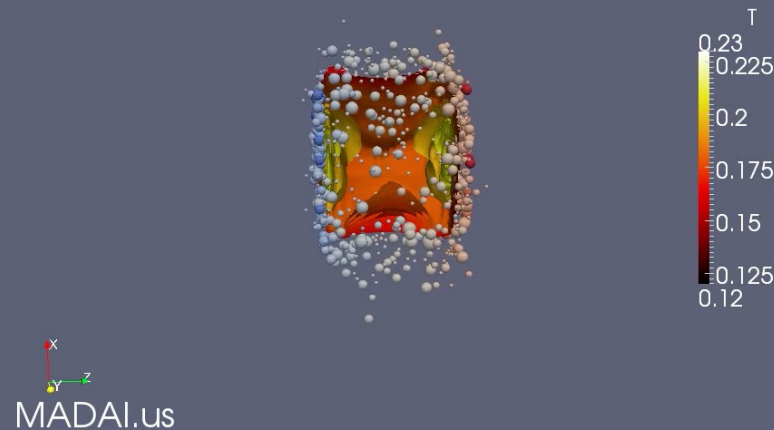


- Heavy-ion program at LHC and RHIC
- Nuclear matter in extreme conditions
- Signatures of the quark-gluon plasma:
 - Collective flow, etc.
 - **Jet quenching** (modification)

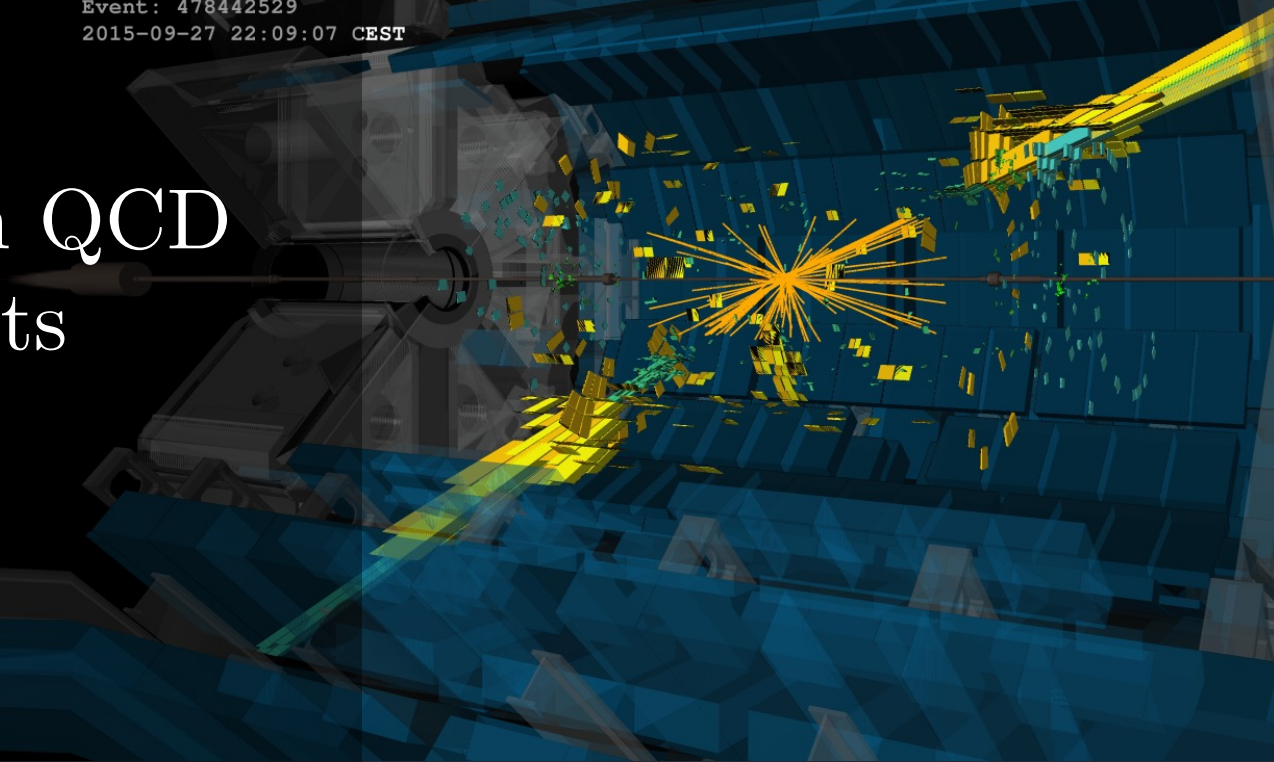
Hydrodynamic description!

QGP $\approx$ perfect fluid

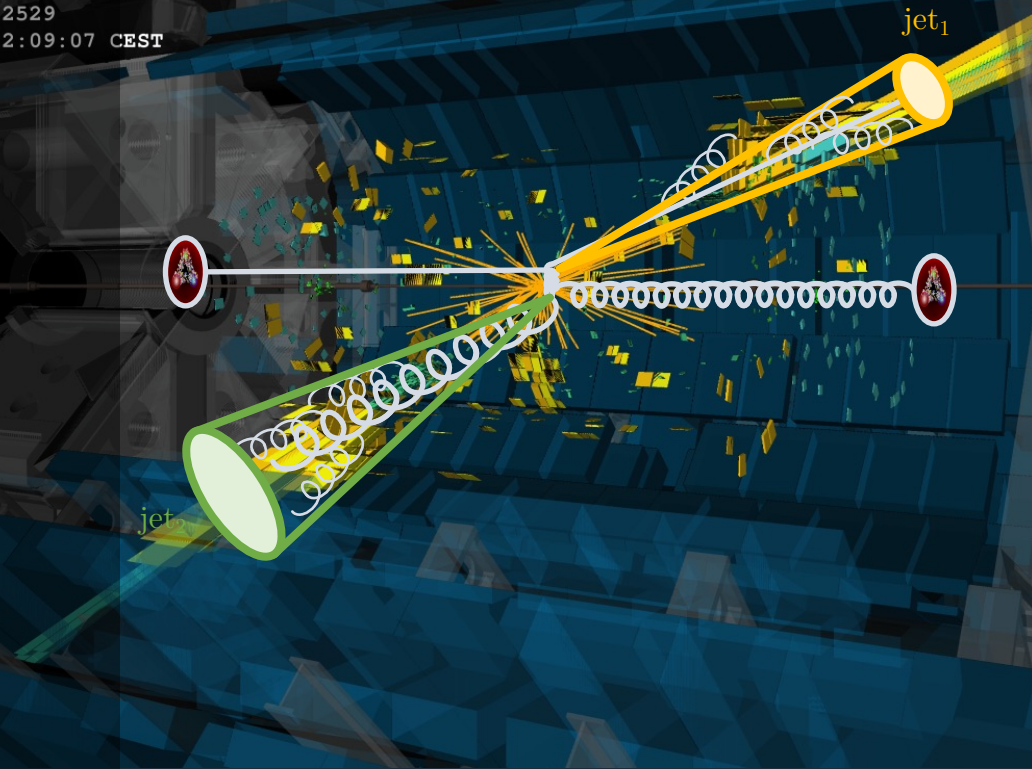
Time: 10.64



2. Precision QCD with jets



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Theory behind MC event generators

Collinear factorization:

$$\frac{d\sigma^{pp}}{dp_T^{jet}} = \sum_{a,b} \int dx_1 dx_2 \underbrace{f_a^p(x_1) \cdot f_b^p(x_2)}_{\text{Parton distribution functions}} \cdot \underbrace{\hat{\sigma}_{ab \rightarrow jet+X}}_{\text{Partonic cross-section}} \cdot \underbrace{f_{\text{corrections}}}_{\text{Higher order corrections}}$$

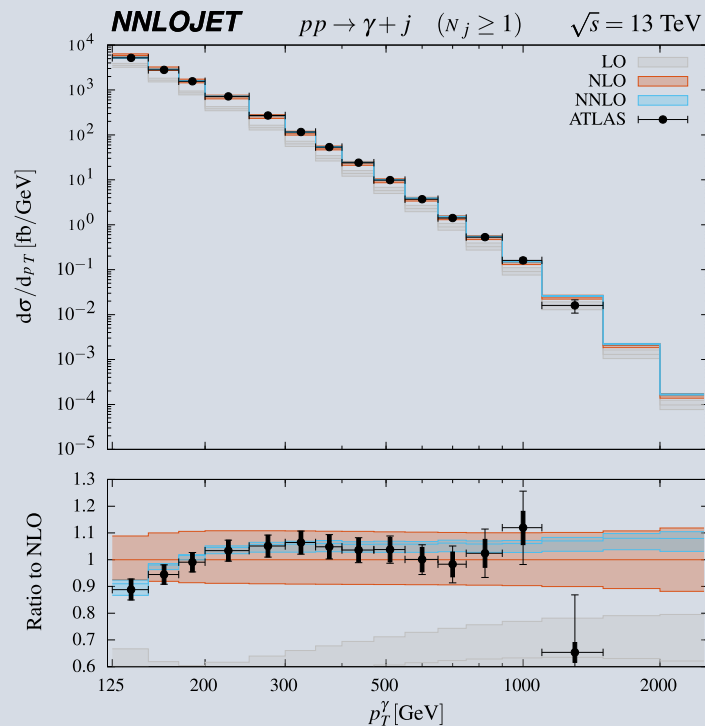
- Find parton a in proton.
- Non-perturbative (fitted to every collision).
- Hadronization, non-fact, ...
- Non-perturbative (fitted to every measurement).
- Want them to be small (jets, no hadrons, grooming).

Partonic cross-sec:

- improvable order-by-order
- loop expansion: LO → NLO → N²LO (only accessible for few legs 😞)
- logarithmic expansion: LL → NLL



← parton shower algorithm



[Chen, Ghermann, Glover, Hofer, Huss 1904.01044]

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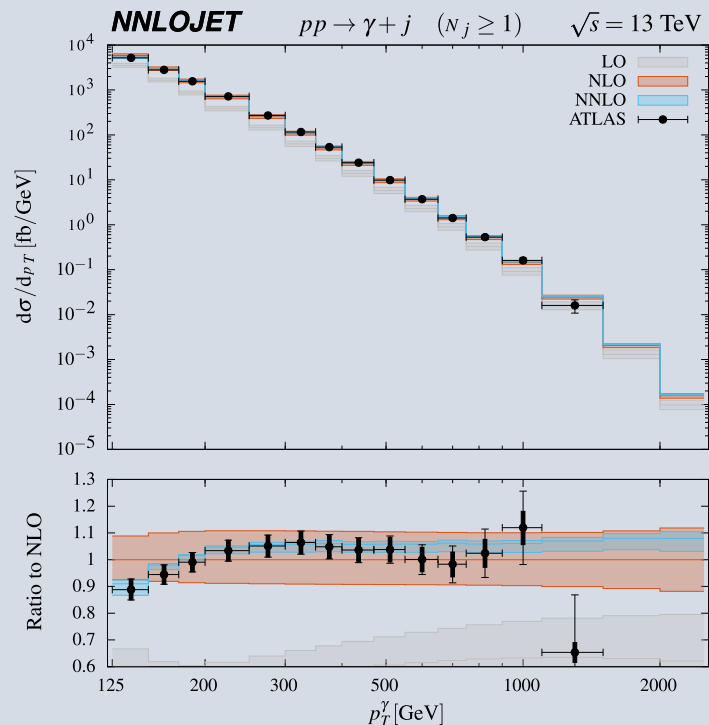
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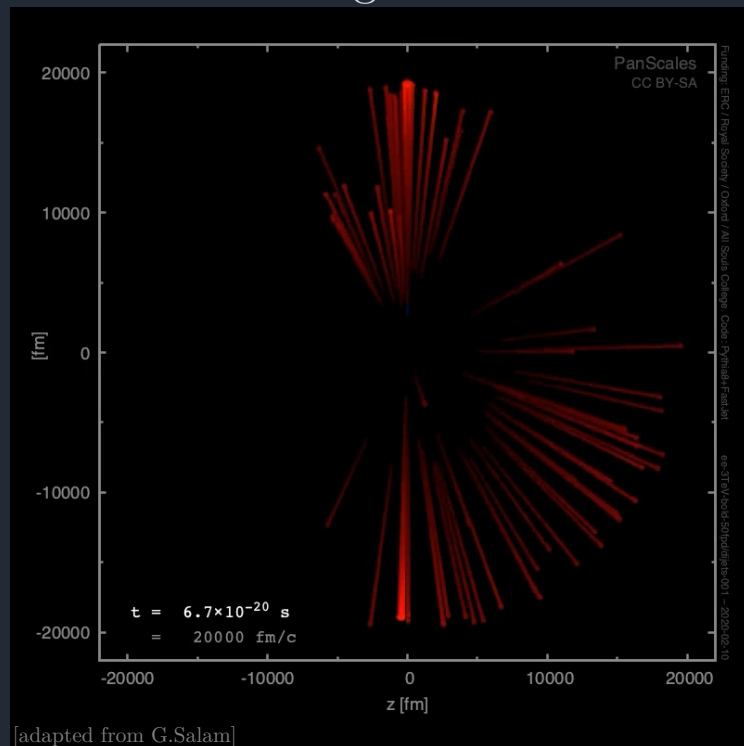


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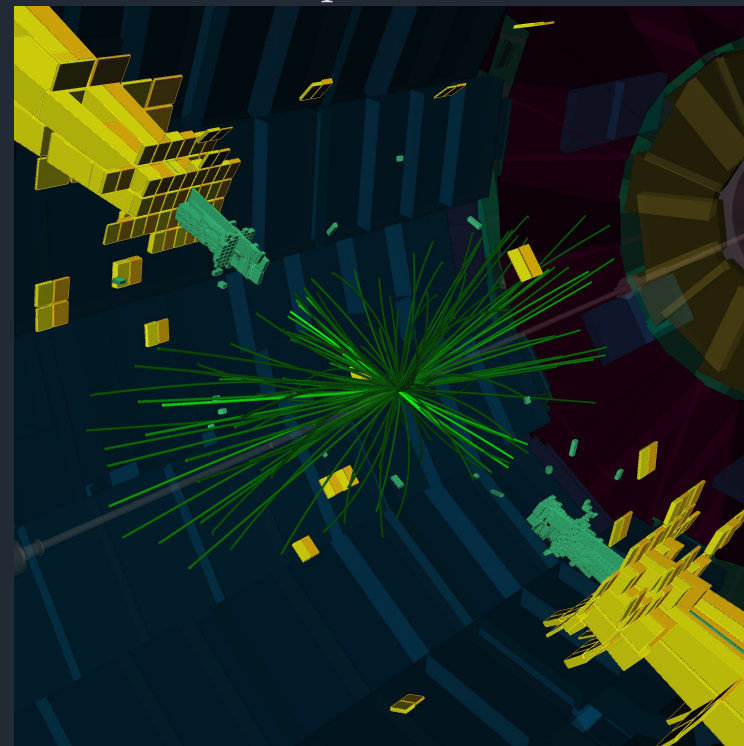
[Chen, Ghermann, Glover, Hofer, Huss 1904.01044]

Event generator



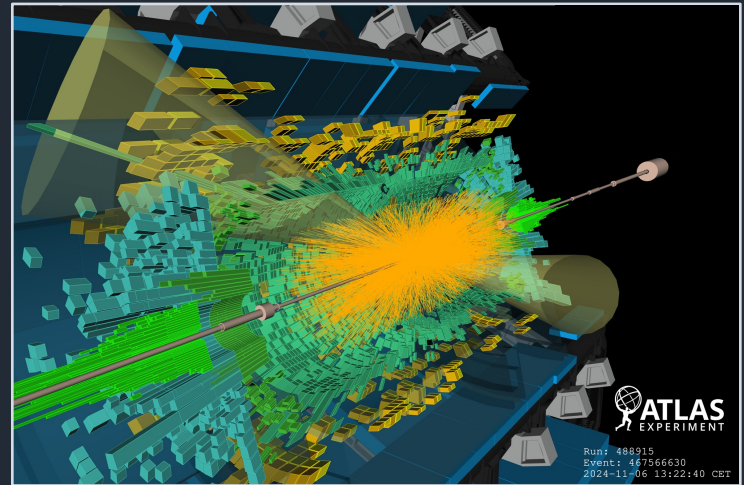
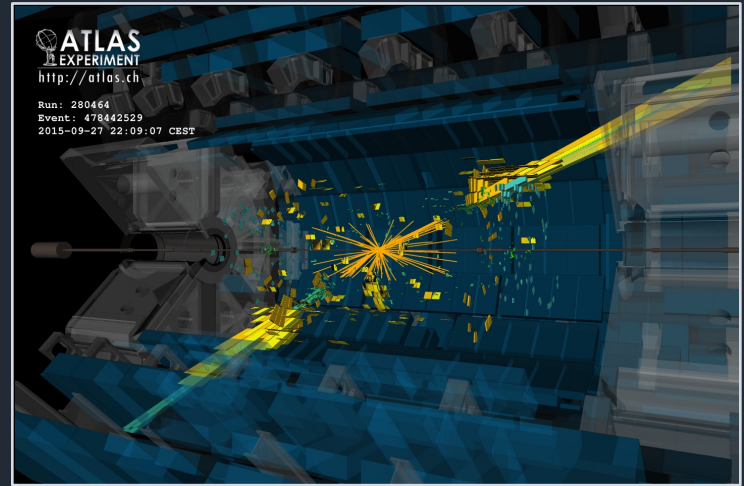
[adapted from G.Salam]

Experiment



- First-principle based, improvable order-by-order
- Describes (a certain class of) multi-differential observables.

1. Jets in heavy-ion collisions



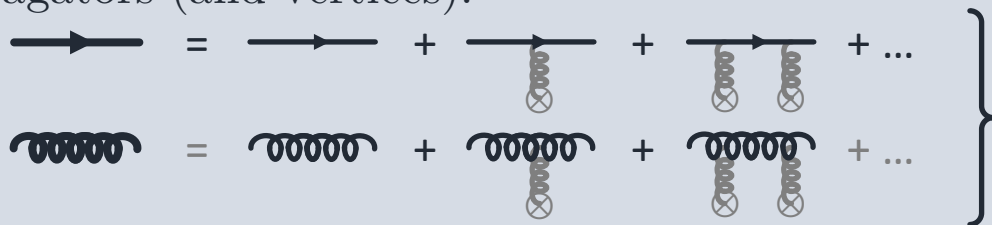
Scattering amplitudes in the QGP

see e.g. SCETg
[Mehtar-Tani, Ringer, Singh, Varun 2026]

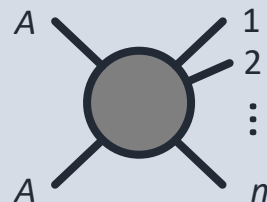
- Separate fields to hard and bkg. ($q = q_h + q_0$, $A = A_h + A_0$)

$$\begin{aligned}\mathcal{L}_{QCD}(q, A) &= \mathcal{L}(q_h, A_h) + \mathcal{L}(q_0, A_0) + \mathcal{L}_{int}(q_h, A_h, q_0, A_0) \\ &\approx \mathcal{L}(q_h, A_h) + g\bar{q}_h\hat{J}q_h + gA_h\hat{J}A_h\end{aligned}$$

- Dressed propagators (and vertices):



Goal:



- Models for the background $\langle \hat{J}(x^\mu)\hat{J}^+(y^\mu) \rangle$:

- High-temperature plasma ($T \gg \Lambda_{QCD}$)
- Color charge distribution
- Non-perturbative “kernel”

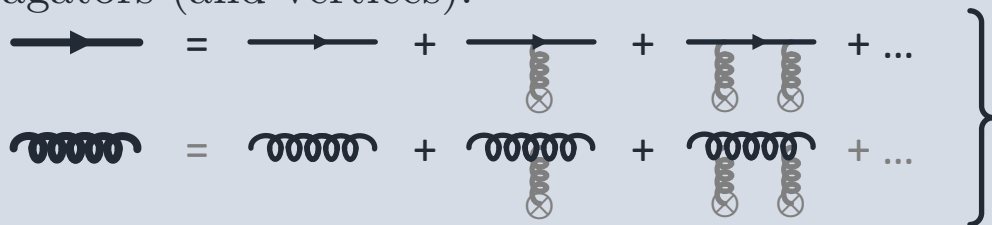
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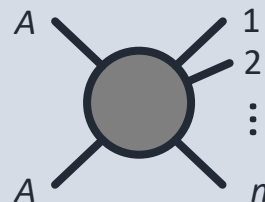
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- High-temperature QCD plasma ($T \gg \Lambda_{QCD}$)
- Color charge distribution
- Non-perturbative “kernel” to infer

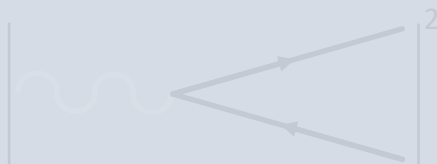
1. Broadening in large medium

Vacuum

- Straight propagation:

$$\left| \longrightarrow \right|^2 \approx \delta_{ij} \delta(p' - p)$$

- Color conservation:



Medium

- Broadening:

$$\left| \longrightarrow \right|^2 \approx g_{ij}(p', p)$$

changing color and momentum

- Color decoherence:



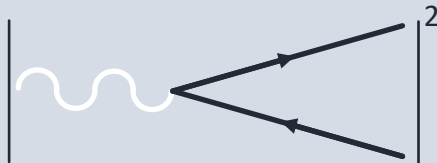
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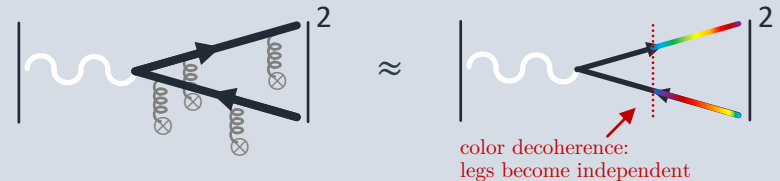
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← changing color and momentum

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2. Radiation in large medium

Vacuum

- Emission:

$$\left| \text{Diagram} \right|^2 \Rightarrow \frac{dI_i^{vac}}{dzd\vartheta} \approx \frac{\alpha_s}{\pi} \frac{2C_i}{z} \frac{1}{\vartheta}$$

soft & collinear poles!

- Resumming emissions: collinear jet



Medium

- Vacuum + medium-induced emissions:

$$\left| \text{Diagram} \right|^2 \Rightarrow \frac{dI_i^{med}}{dzd\vartheta} \approx \frac{\alpha_s}{\pi} \sqrt{\frac{Q_{med}}{E}} \frac{C_i}{\sqrt{z^3}} B(\vartheta)$$

$m < Q_{med}$
 $z \ll Q_{med}/E$ soft & collin pole!

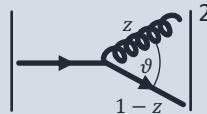
- Wide-angle medium-induced cascade:



2. Radiation in large medium

Vacuum

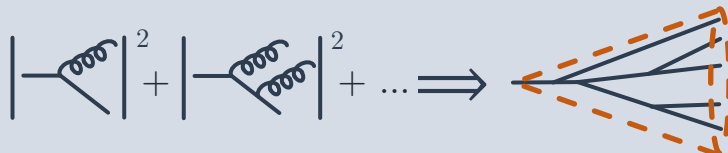
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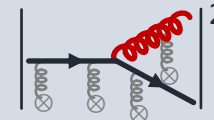


(parton shower)

Also mixing of vacuum-like
and medium-induced terms!

Medium

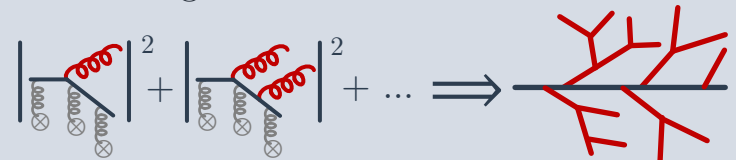
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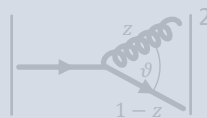


(parton shower-like)

2. Radiation in large medium

Vacuum

- Emission:



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soft & collinear

- Resumming emissions:



Jet evolution in the QGP



- Successful in pheno for “hard” observables.
- Does not thermalize!
- How to thermalize jets?

Medium

- Vacuum + medium-induced emissions:



$$\frac{dI_i^{med}}{dzd\vartheta} \approx \frac{\alpha_s}{\pi} \sqrt{\frac{Q_{med}}{E}} \frac{C_i}{\sqrt{z^3}} B(\vartheta)$$

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- Wide-angle medium-induced cascade:




2. Jet thermalization and Medium response



[Wikipedia]

Medium response in phenomenology

Where does the lost energy go?

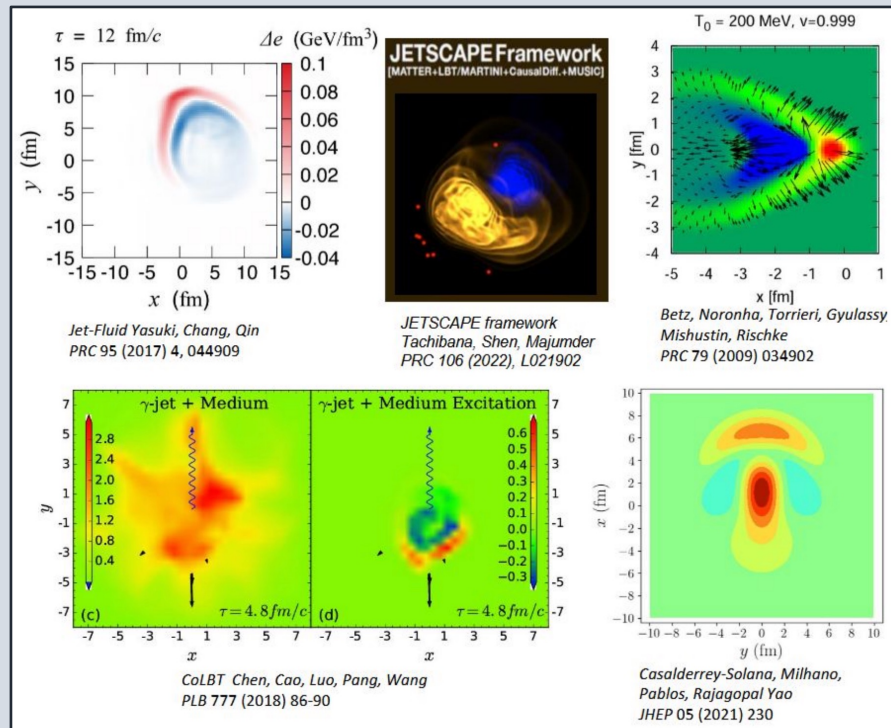
1. Jets evolve in pQCD,
2. Energy loss is modelled (e.g. see before).
3. Lost energy feeds hydro:

$$\partial_\nu T^{\mu\nu} = \Delta J^\mu(x, t)$$

hydro evolution

lost energy as source

- Does the lost energy really equilibrate?
- Weak vs strong coupling?
- Where are perturbative scatterings?
- Smaller collision systems: PbPb $\rightarrow$ OO $\rightarrow$ pPb?



Medium response in phenomenology

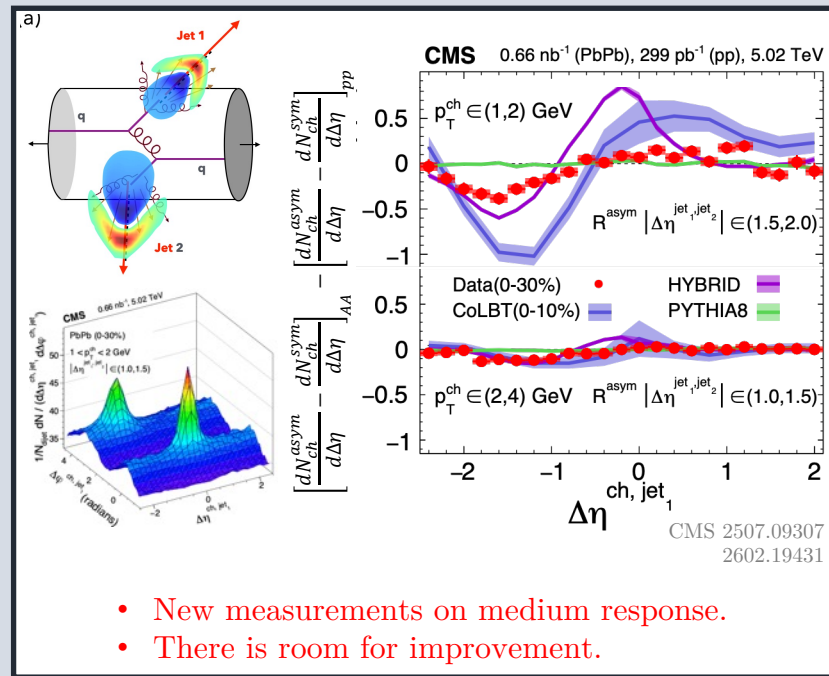
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hydro evolution
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- New measurements on medium response.
- There is room for improvement.

Medium response in theory

In-medium jet evolution

Weakly coupled

QCD- effective kinetic theory:
thermalization from first principles.

[Arnold,Moore,Yaffe]

[Baier,Mueller,Schiff,Son]

$$(\partial_t + v \cdot \nabla_x) f(t, x, p) = C^{2 \leftrightarrow 2} + C^{1 \leftrightarrow 2}$$

How to thermalize jets?



How to put jets in the EKT?

Strongly coupled

Instantaneous freeze-out/hydro

[Hybrid 2010.01140]

$$\partial_\nu T^{\mu\nu} = 0$$

Falling string + metric perturbation evolution
in AdS/CFT

[Chesler, Yaffe]

Some other
time

Medium response in theory

In-medium jet evolution

Weakly coupled

Currently: few scattering

In this talk:

- (i) EKT $\leftrightarrow$ jet quenching,
- (ii) EKT $\leftrightarrow$ parton showers,
- (iii) EKT parton shower $\rightarrow$ med. resp

QCD-kinetic theory: th

$$(\partial_t + v \cdot \nabla_x) \delta f(t, x,$$

How to put jets in the EKT?

Strongly coupled

/hydro
[Hybrid 2010.01140]

Δ/Λ

perturbation evolution

[Chesler, Raffe]

QCD kinetic theory

- Small perturbation: $f(t, \mathbf{x}, \mathbf{p}) = n(p) + \delta f(t, \mathbf{x}, \mathbf{p})$

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_{\mathbf{x}}\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta \mathcal{C}^{1 \leftrightarrow 2}[n, \delta f] + \delta \mathcal{C}^{2 \leftrightarrow 2}[n, \delta f].$$

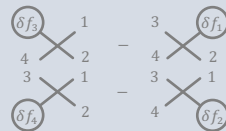
- $2 \leftrightarrow 2$ collisions:

$$\delta \mathcal{C}^{2 \leftrightarrow 2}[n, \delta f] = \int d\Omega |\mathcal{M}|^2 \delta \mathcal{F}(p_1, p_2; p_3 p_4)$$

scattering mx
in QCD plasma

statistical factors (detailed balance)

$$\begin{aligned} \delta \mathcal{F} = & \delta f_3 [n_4 n_1 n_2 \mp n_4 n_1 n_2] \\ & + \delta f_4 [n_3 n_1 n_2 \mp n_3 n_1 n_2] \\ & - \delta f_2 [n_1 n_3 n_4 \mp n_1 n_3 n_4] \\ & - \delta f_1 [n_2 n_3 n_4 \mp n_2 n_3 n_4]. \end{aligned}$$

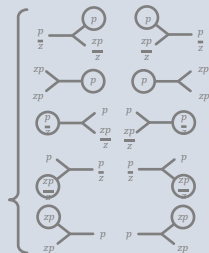


- $1 \leftrightarrow 2$ collisions:

$$\delta \mathcal{C}^{1 \leftrightarrow 2}[n, \delta f] = \int dz \left[\frac{1}{z^3} \Gamma\left(z, \frac{p}{z}\right) \delta \mathcal{F}\left(\frac{p}{z}; p, \frac{\bar{z}p}{z}\right) - \frac{1}{2} \Gamma(z, p) \delta \mathcal{F}(p; zp, \bar{z}p) \right]$$

splitting rate

statistical factors (detailed balance)



- Two type of splitting
- Merging (holes)

QCD kinetic theory at high-energy $p \gg T$

- Small perturbation: $f(t, \mathbf{x}, \mathbf{p}) = n(p) + \delta f(t, \mathbf{x}, \mathbf{p})$

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$$\delta \mathcal{C}^{2 \leftrightarrow 2}[n, \delta f] \approx \int_{q_{\perp}} dq_{\perp} C(q_{\perp}) [\delta f(p + q_{\perp}) - \delta f(p)] \longrightarrow \text{medium-induced broadening from before!}$$

(small angle)
scattering mx
in QCD plasma

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- Using this connection:

- High energy AMY-EKT $\leftrightarrow$ jet quenching!
(recipe to put jets into EKT)
- EKT $\leftrightarrow$ parton shower with detailed balance
(recipe to enforce thermalization in jets)

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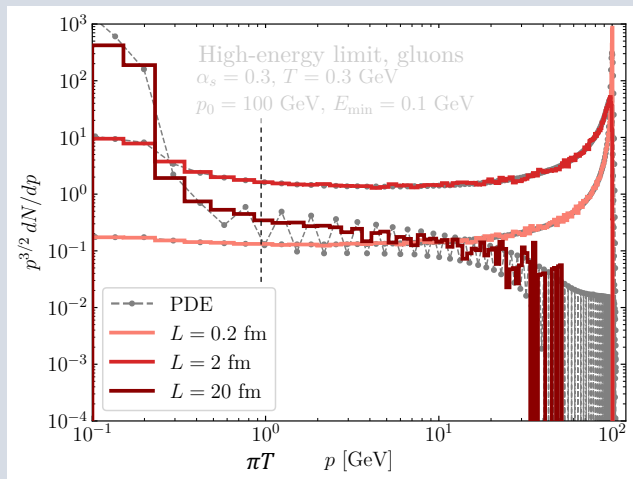
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Numerical solution to EKT

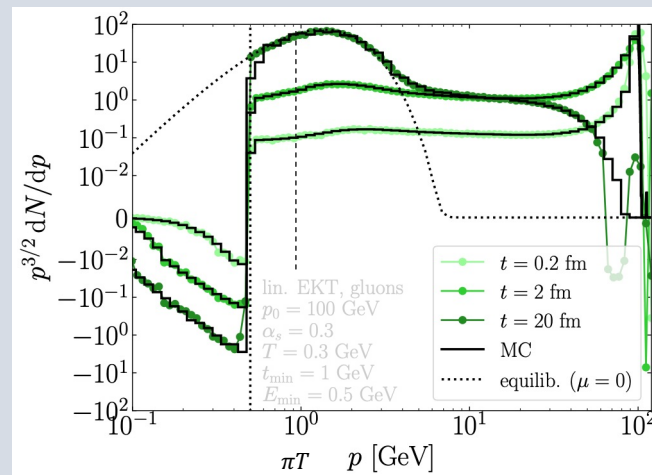
see also PDEs [Mehtar-Tani,Schlichting,Soudi]
[Brewer,Mazeliauskas,Zhou]

High-energy limit

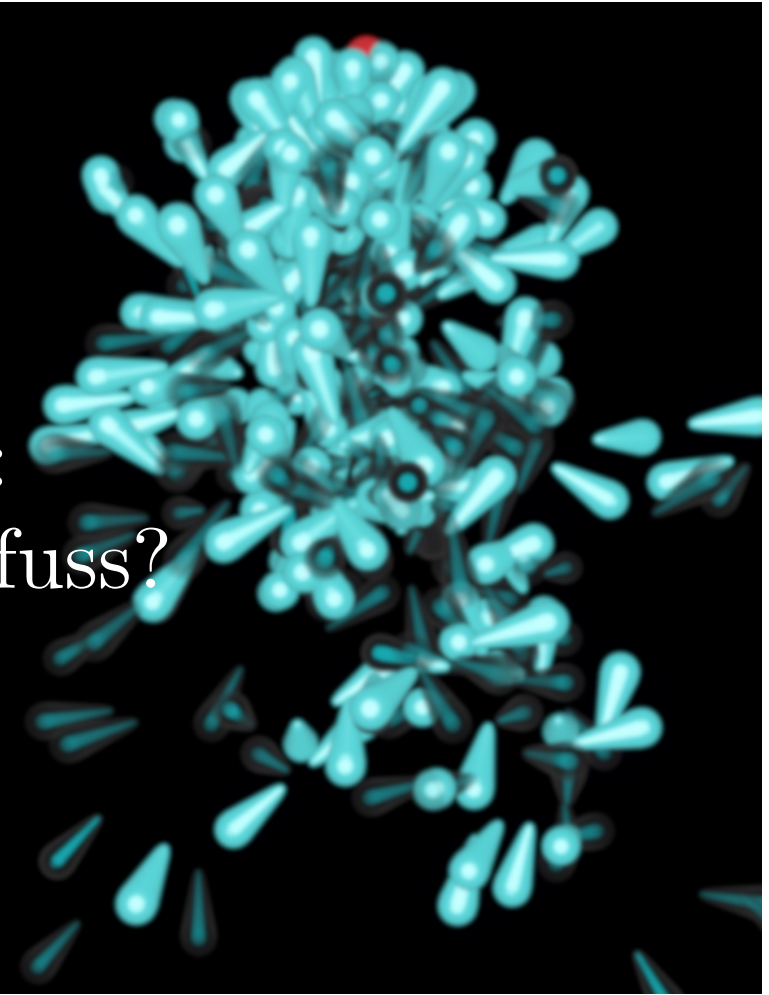


- High-energy limit is analogous to traditional parton showers.
- **Doesn't thermalize!**
- Parton shower = differential eq. solver

Full EKT



- Including all statistical factors to the parton shower (ensure detailed balance)
- **Thermalizes!**
- Parton shower = differential eq. solver



Outlook: Why all the fuss?

Applications

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_{\mathbf{x}}\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

1. New phenomenological applications of EKT!

Interface with jets, hadronization, KØMPØST, etc.

2. Multi-particle correlations.

Parton-shower is an n -particle cross-section generator:

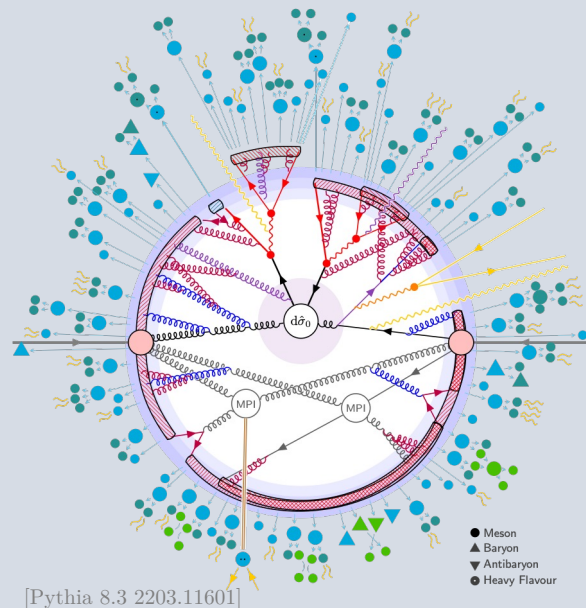
Unexplored topic in EKT, great relevance e.g. EEC.

3. Incorporate inhomogeneities.

Previously neglected! (parton shower speedup)

Hydrodynamization: $\delta f(t, \mathbf{x}, \mathbf{p}) \rightarrow \delta f_{eq}(p, \delta T(t, \mathbf{x}), \delta u(t, \mathbf{x}))$

4. Applicable for any kinetic theory!



Applications

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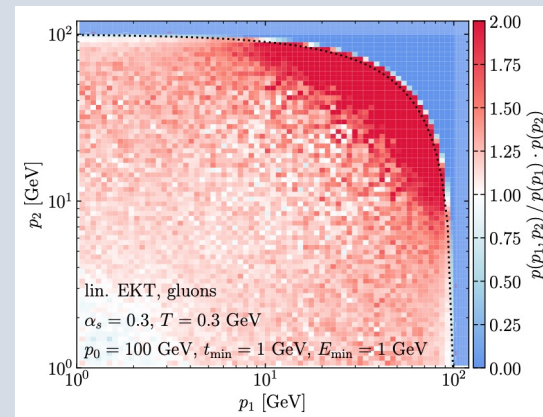
Unexplored topic in EKT, great relevance e.g. EEC. $\delta f(p_1, p_2, \dots) = \frac{1}{N} \frac{dN}{dp_1 dp_2 \dots}$

3. Incorporate inhomogeneities (iii):

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Correlations beyond molecular chaos!

Applications

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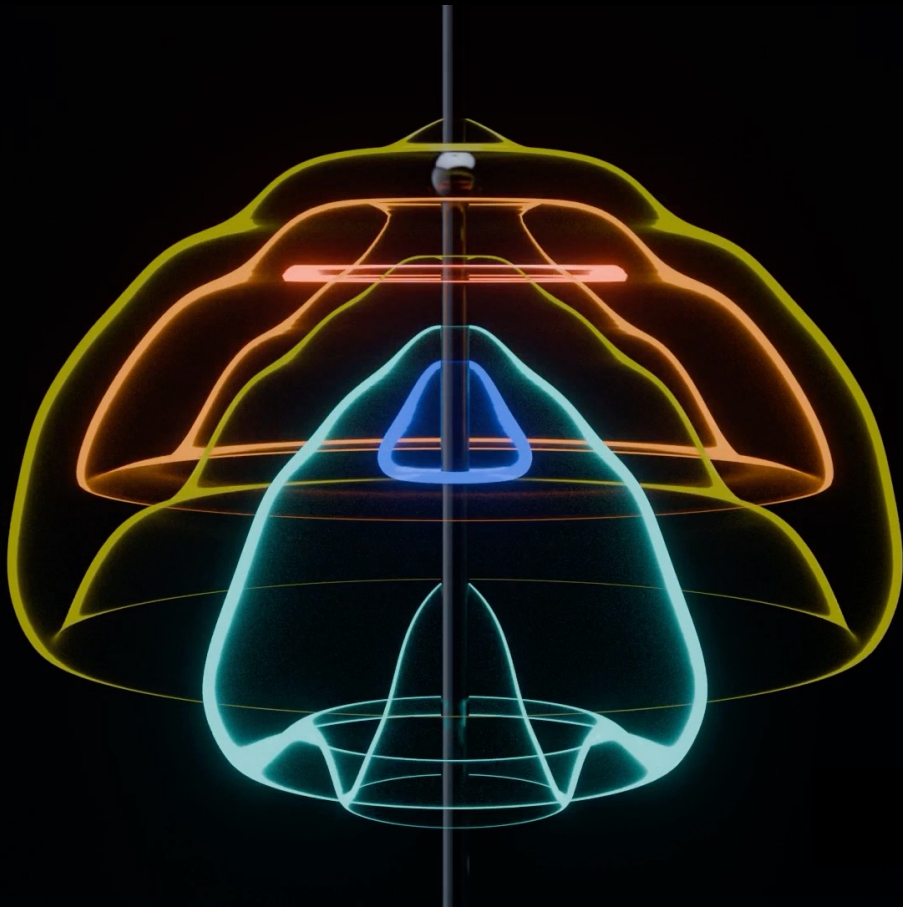
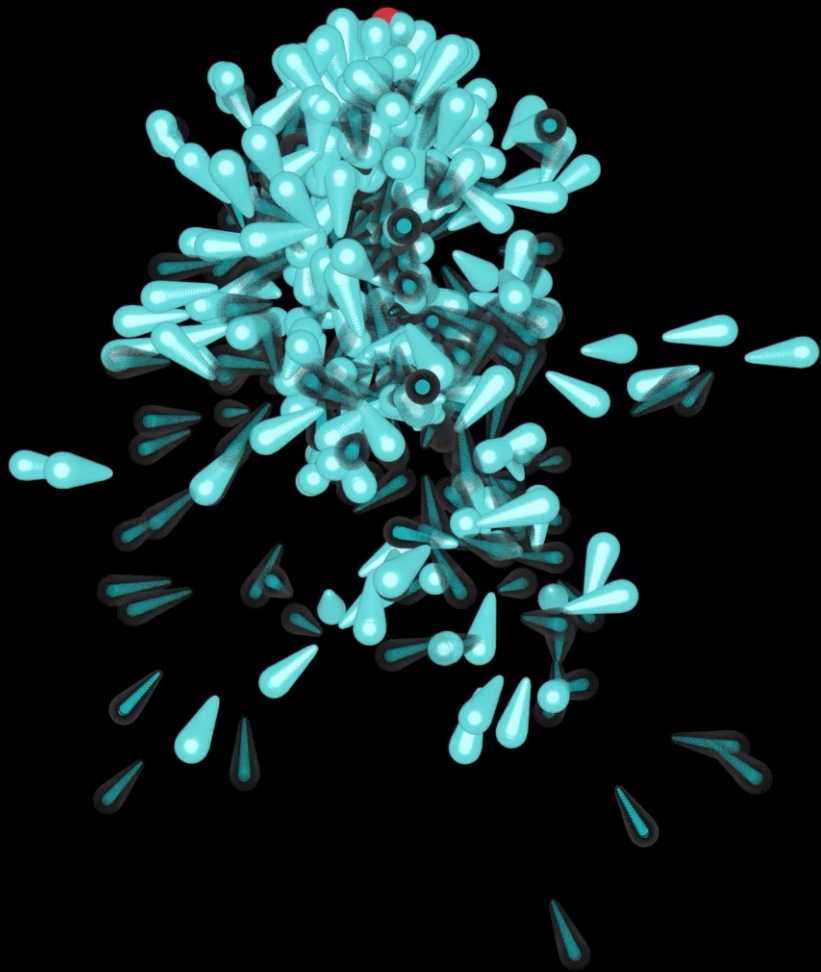
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Previously neglected! (parton shower speedup)

Hydrodynamization: $\delta f(t, \mathbf{x}, \mathbf{p}) \rightarrow \delta f_{eq}(p, \delta T(t, \mathbf{x}), \delta u(t, \mathbf{x}))$

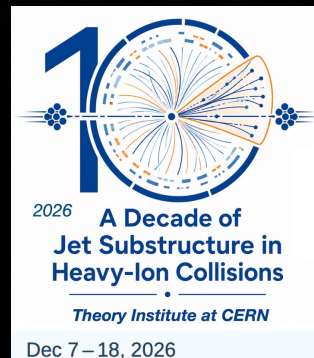
$$\delta f(x) = \frac{1}{N} \frac{dN}{dx}$$

4. Applicable for any kinetic theory!



Summary:

1. Perturbative grounds of jet quenching
2. QCD-EKT $\leftrightarrow$ parton energy loss and broadening
3. Lin. Boltzmann eq. $\leftrightarrow$ parton shower
4. New opportunities for EKT:
 - Jet thermalization,
 - Multi-particle correlations, hydrodynamization, fluctuations
 - Including in phenomenological workflows





Thank you for your attention!

QCD kinetic theory as a parton shower

$$\left(\partial_t + \frac{\mathbf{p}}{p} \nabla_x\right) \delta f(t, \mathbf{x}, \mathbf{p}) = \delta C[n, \delta f].$$

- Spatially homogeneous case:

1. Separate “real” and “virtual” collisions: $\delta C = \delta C^r[n, \delta f] - \delta C^v[n] \delta f$.

2. Integrate the Boltzmann eq: where $\Delta(t) = \exp \left[\int^t dt' \delta C^v[n] \right]$

$$\delta f(t) = \Delta(t) \delta f_0 + \int^t dt' \frac{\Delta(t)}{\Delta(t')} \delta C^r[n, \delta f].$$

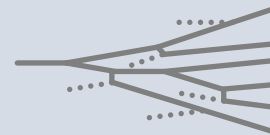
4. Iterate: $\delta f_0 = \delta(p - p_0)$.

5. Histogram the particles: $\delta f(t, x, p) = \frac{1}{N_{\text{ev}}} \frac{dN}{dpdx}$

(ii) kinetic theory $\leftrightarrow$ parton shower!

- Inhomogeneous case is trivial: free streaming between collisions.

(iii) inhomogeneous perturbations

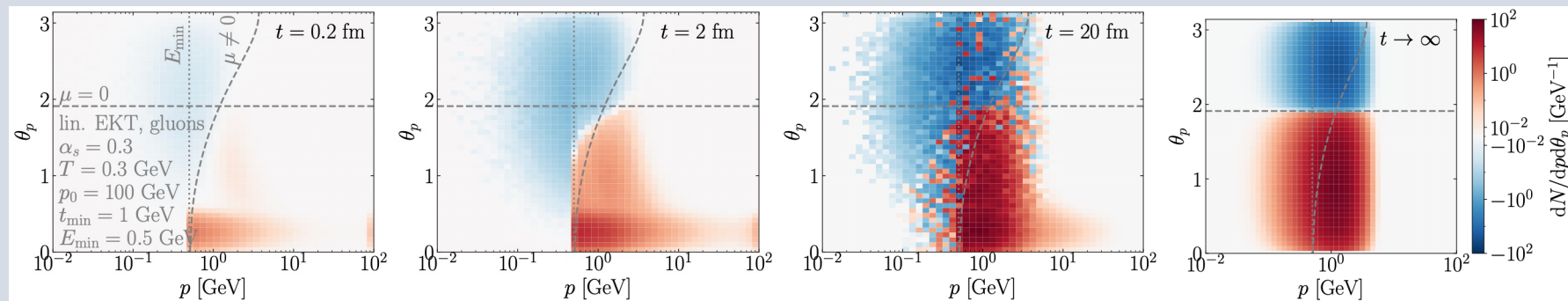


```
Initial condition:#[ID, p, x, weight]
[21 (100,0,0,100) (0,0,0,0) 1]

Output:
[[21 (1.5,0.0,0,1.5) (0.1,0.0,0,0.1) 1],
 [21 (2.8,0.0,0.0,2.8) (2.0,0.0,0,2.0) 1],
 [21 (5.0,0.0,0.0,5.0) (9.4,0.0,0.0,9.4) 1],
 [21 (1.7,1.1,0.6,1.1) (9.7,0.0,0.0,9.7) -1],
 [21 (82,-0.9,-0.2,82) (9.7,0.0,0.0,9.7) 1],
 [21 (1.0,0.4,0.2,0.9) (12.0,1.4,0.5,12.0) 1],
 [21 (2.2,0.0,0.0,2.2) (15.0,0.0,0.0,15.0) 1],
 [21 (3.3,0.0,0.0,3.3) (15.0,0.0,0.0,15.0) 1],
 [21 (1.5,0.7,0.2,1.3) (15.5,2.6,1.0,14.8) 1],
 [21 (2.1,0.9,0.3,1.8) (15.5,2.6,1.0,14.8) 1]]
```

Numerical solution to EKT

see also PDEs [Mehtar-Tani,Schlichting,Soudi]
[Brewer,Mazeliauskas,Zhou]



1. Collinear radiation of soft gluons.
 2. Broadening of soft fragments
 3. $\mu \neq 0$ equilibrium $\rightarrow \mu = 0$ equilibrium.
- (+) and (−) medium response.
 - Soft and hard medium response.
 - Equilibrium only at very late times.