Non-Gaussian Heavy Quark Momentum Broadening

Jean F. Du Plessis

Strong and Electro-Weak Matter 2026

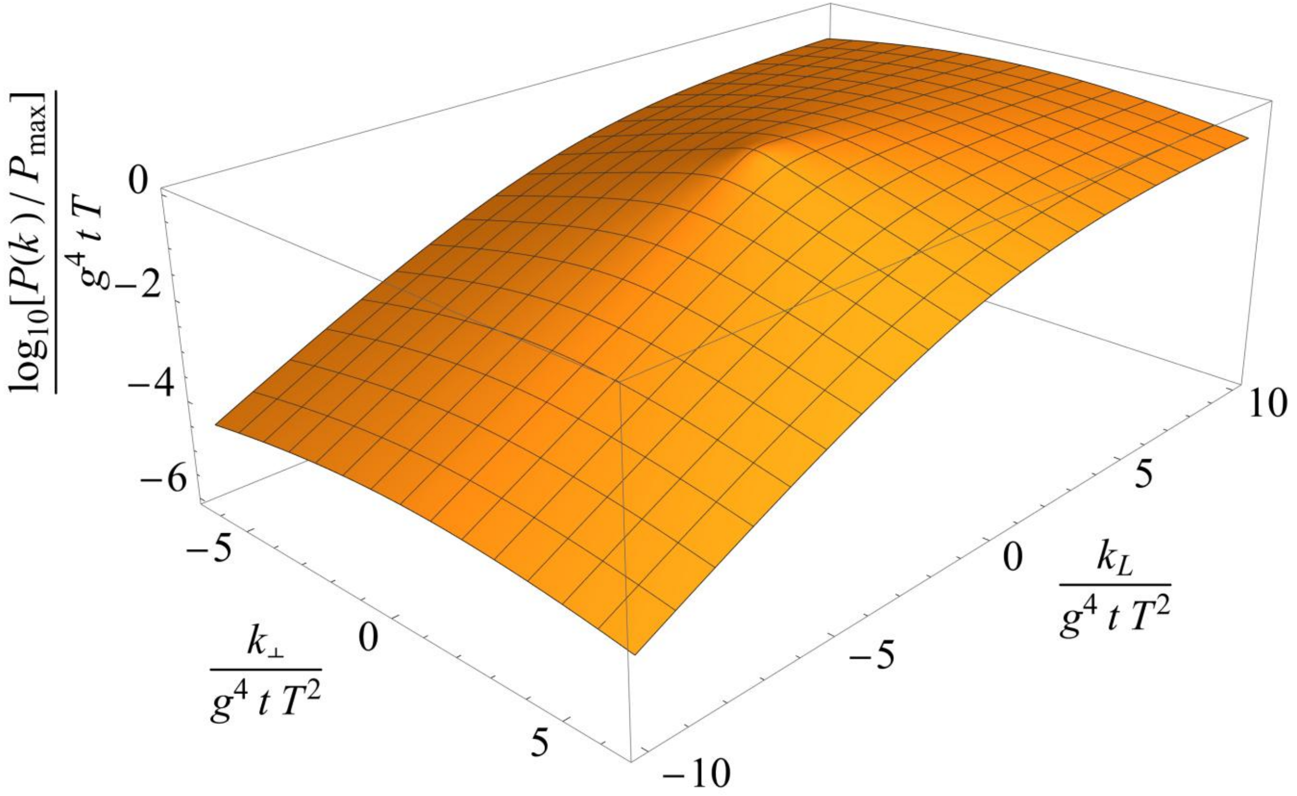
Based on 2604.21895 and ongoing work in collaboration with B. Scheihing

Heavy Quark Dynamics

- What is the probability $P_{\Delta t}(\vec{k};\vec{v})$ of a heavy quark ($M \gg T$) at velocity $\vec{v}$ transferring momentum $\vec{k}$ to the medium in a time interval Δt?
- Assume:
  - Markovian dynamics
  - Velocity approximately constant
  - Infinite homogonous finite temperature background medium
- In the Gaussian approximation

$$P_{\Delta t}\left(\vec{k};\vec{v}\right) \propto \exp\left(-\frac{(k_3 - \Delta t p_3(v)\eta_D(v))^2}{2\Delta t \kappa_L(v)} - \frac{k_1^2 + k_2^2}{2\Delta t \kappa_T(v)}\right)$$

- Equilibration forces $\kappa_L(v) = 2TE\eta_D(v)$



Failure of the Einstein Relation at $v > 0$

S. Gubser
hep-th/0612143
G. Moore and D. Teaney
hep-ph/0412346

Momentum fluctuations of heavy quarks in the gauge-string duality

Steven S. Gubser

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$$\kappa_L = 2TE\,\eta_D = \pi\sqrt{g_{YM}^2 N}\,T^3\gamma\,, \tag{3}$$

Because of the dramatic failure of the Einstein relation (3), it doesn't make sense to plug the trailing string predictions for η_D, κ_L, and κ_T into a Langevin description of a finite mass quark: it won't equilibrate to a Maxwell-Boltzman distribution, due to the largeness of κ_L as compared to η_D at highly relativistic speeds. Perhaps what is needed is a stochastic treatment of the quasi-normal modes of the trailing string, the lowest of whose frequencies were found in (71).

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How Much do Heavy Quarks Thermalize in a Heavy Ion Collision?

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Derek Teaney

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(Dated: February 2, 2008)

$\kappa_L(p)/\kappa_L(0)$) rise with the momentum of the heavy quark. It is instructive to compare the longitudinal fluctuations derived from the full Boltzmann equation to the longitudinal fluctuations expected in a Boltzmann-Langevin approach described in the next section. In the Boltzmann-Langevin approach the longitudinal fluctuations are directly related to the drag through the relation

$$\frac{d}{dt}\left\langle (\Delta p_z)^2 \right\rangle = \frac{2T}{v}\frac{dp}{dt}\,.$$

We see that the Boltzmann-Langevin prediction for the longitudinal fluctuations ($\frac{2T}{v}\frac{dp}{dt}$) underestimates the fluctuations of the full Boltzmann equation (κ_L). At large momenta the underestimate can be a factor of 2 to 4.

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$$\kappa_L^{QFT}(v > 0) \neq 2\,T\,E\eta_D^{QFT}(v > 0)$$

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Beyond the Central Limit Theorem

B. Scheihing et al.
2504.21139

- To get equilibration you need detailed balance of the probability distribution of momentum transfer of the heavy quark:

$$\mathrm{P}\left(\vec{k};\vec{v}\right)\exp\left[-\frac{\vec{v}\cdot\vec{k}}{2T}\right]=\mathrm{P}\left(-\vec{k};\vec{v}\right)\exp\left[\frac{\vec{v}\cdot\vec{k}}{2T}\right]$$

- Detailed Balance is a generic property of relativistic QFTs (assuming unitarity, time translation invariance, Markovianity and KMS symmetry)
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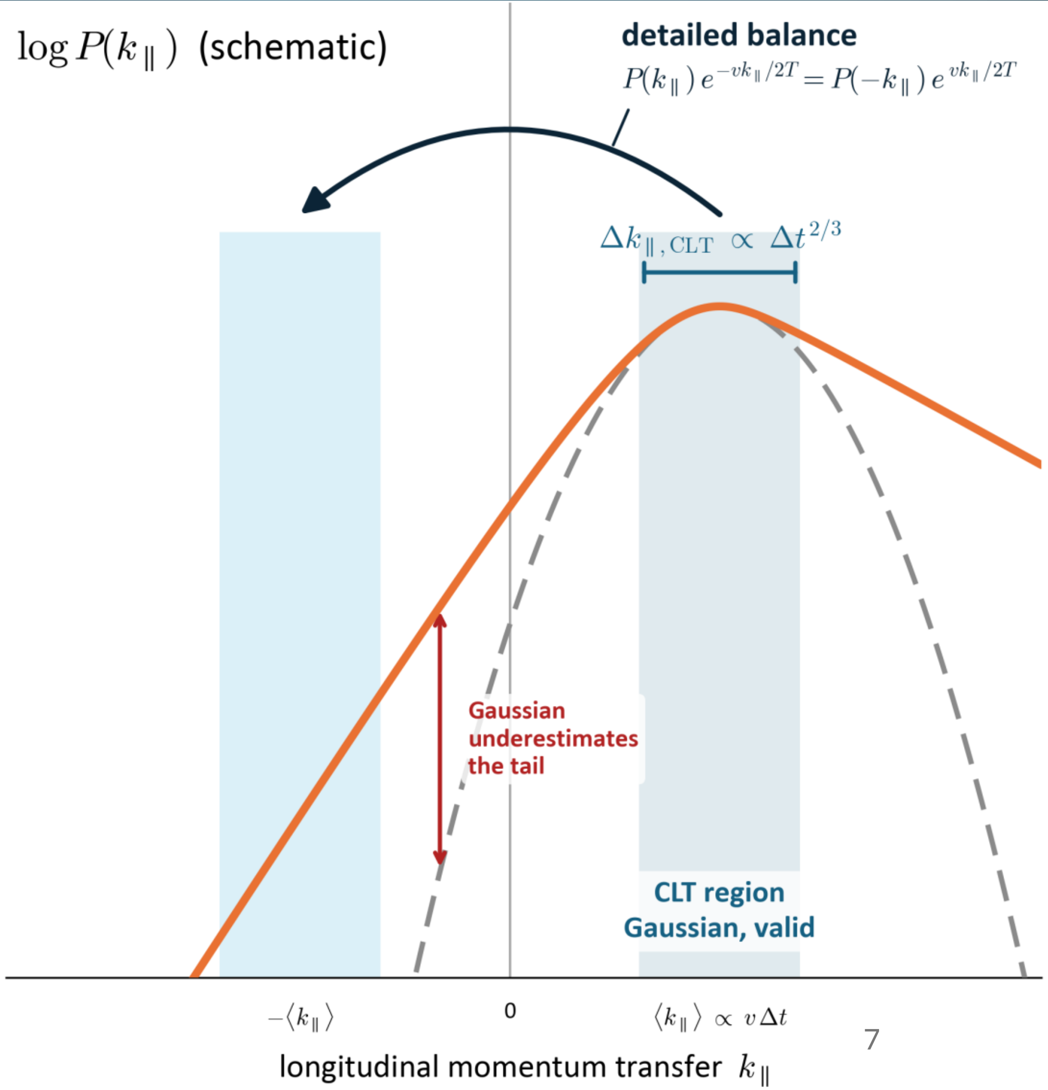
- Detailed Balance is a generic property of relativistic QFTs (assuming unitarity, time translation invariance, Markovianity and KMS symmetry)
- Only when assuming P(k;v) is **exactly** Gaussian does this force the Einstein Relation
- Central Limit Theorem only implies that the bulk of P(k;v) is **approximately** Gaussian **around its mean** at **asymptotically late times**
- Detailed Balance relates behaviour at $\vec{k}$ to $-\vec{k}$, so if the mean is at large $\vec{k}$ (as it is at late times when $v > 0$), the CLT region gets related to the tails of the distribution, generically outside where CLT applies.
- The natural language is *Large Deviation Theory*: the branch of probability theory dealing with the asymptotic behavior of the tails of sequences of probability distributions in terms of the *rate function* (see e.g. 0804.0327)

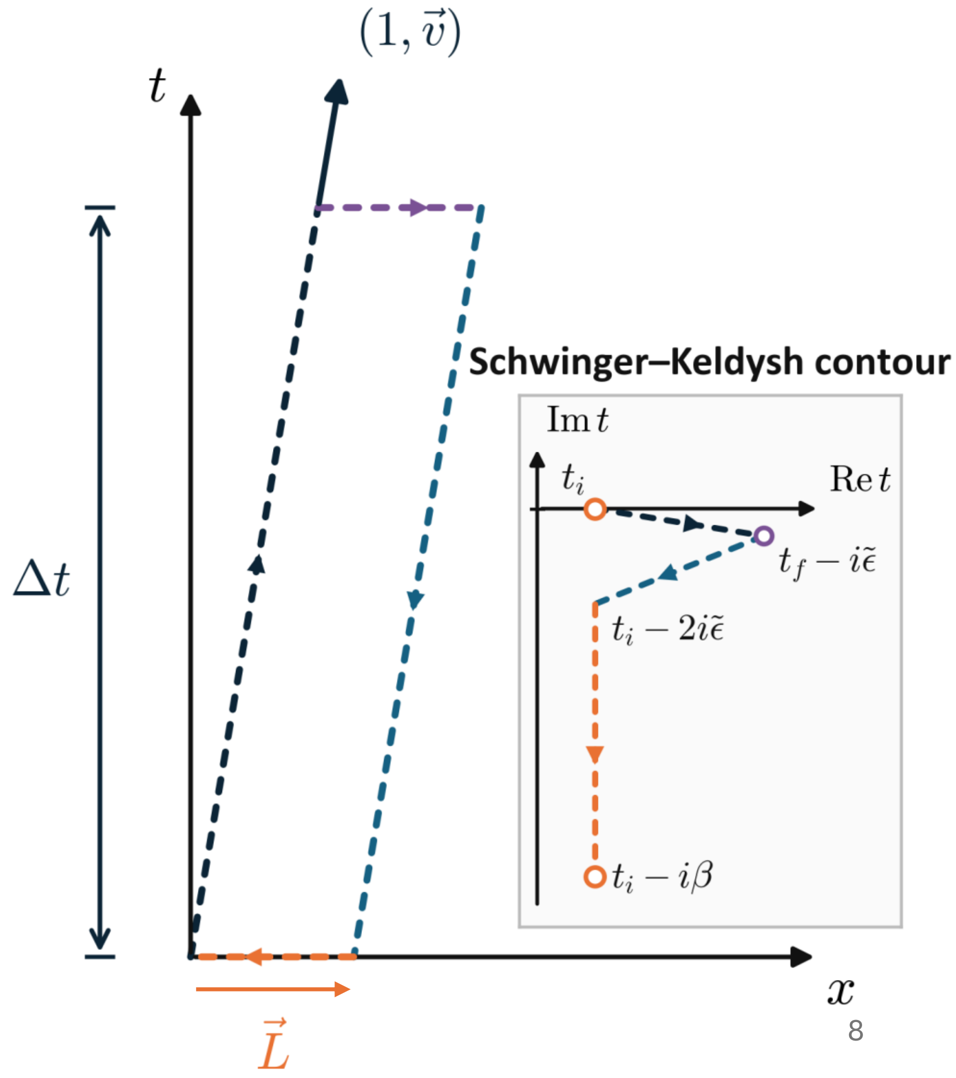


HQET, Wilson Loops and Momentum Transfer

- We want to use HQET to calculate the probability that an infinitely massive quark at velocity $\vec{v}$ loses momentum $\vec{k}$ to a static thermal background in some time interval Δt
- It can be shown that this is proportional to the Fourier transform of a Wightman ordered Wilson loop

$$P_{\Delta t}\left(\vec{k};\vec{v}\right) \propto \int d^3L\, e^{-i\vec{k}\cdot\vec{L}} \langle W_{\vec{v}}\rangle(\vec{L},\Delta t)$$

- This is in the regime of validity of the EFT when $\left|\vec{k}\right| \ll M$



Evolution Kernel and the Large Deviation Rate Function

- The state-preparation-independent part is the Markovian contribution, which enters as the extensive part of the Wilson loop

$$\ln\langle W_{\vec{v}}\rangle\left(\vec{L},\Delta t\right) = -\Delta t\; T\; S\left(\vec{L};\vec{v}\right) + \mathrm{sub-extensive}$$

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- Define the analytic continuation $K(x,v) = S(-i\, x\, \hat{v}, v\hat{v})$ in the longitudinal direction
- Detailed balance is given by $K(x,v) = K\left(-x - \frac{v}{T}, v\right)$
- Transport coefficients are Taylor coefficients of $K(x,v) \equiv -\sum_{n=1}^{\infty} \frac{\mathfrak{M}_{\mathrm{n}}(v)}{n!} x^n$

$$\eta_D p = T\mathfrak{M}_1 \qquad\qquad \kappa_L = T\mathfrak{M}_2$$

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B. Scheihing et al.
2501.06289
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- At late times

$$P\left(k_{\parallel},\Delta t\right) \propto \exp\left(-\Delta t\ T\ \tilde{S}\left(\frac{k_{\parallel}}{\Delta t\ T}, v\right)\right)$$

where the *rate function* $\tilde{S}$ is given by the Legendre transform

$$\tilde{S}(C_L, v) \equiv \left[K(x,v) - x\cdot\frac{\partial K}{\partial x}\right]_{C_L = -\frac{\partial K}{\partial x}}$$

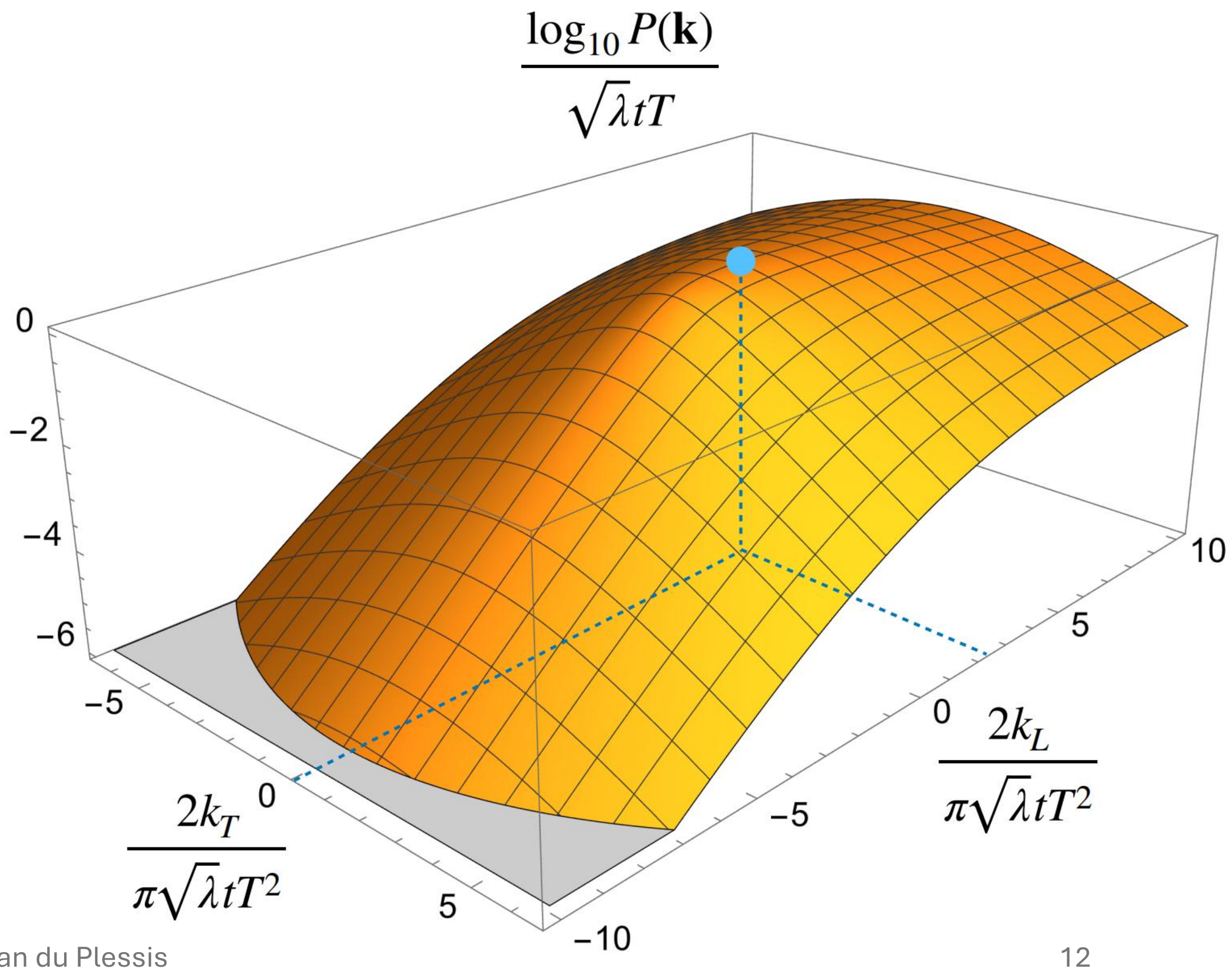
Strong Coupling

B. Scheihing et al.
2501.06289

- Full heavy quark momentum transfer probability distribution has been calculated in strongly coupled ($N_c \to \infty$) $N = 4$ SYM using holography
- Satisfies detailed balance
- Violates the Einstein Relation (as was known)
- Exact expression for

$$\tilde{S}_{\rm tot}(\mathbf{C}) = \sqrt{1+\mathbf{C}^2}\,\frac{\pi}{32}\frac{(z_+^4 - z_-^4)^2}{z_-^5(1-z_-^4)} F_1\!\left(\frac{3}{2},\frac{5}{4},1;3;-\frac{z_+^4-z_-^4}{z_-^4},\frac{z_+^4-z_-^4}{1-z_-^4}\right) + \frac{\pi}{2}|vC_3|\theta(-vC_3)$$

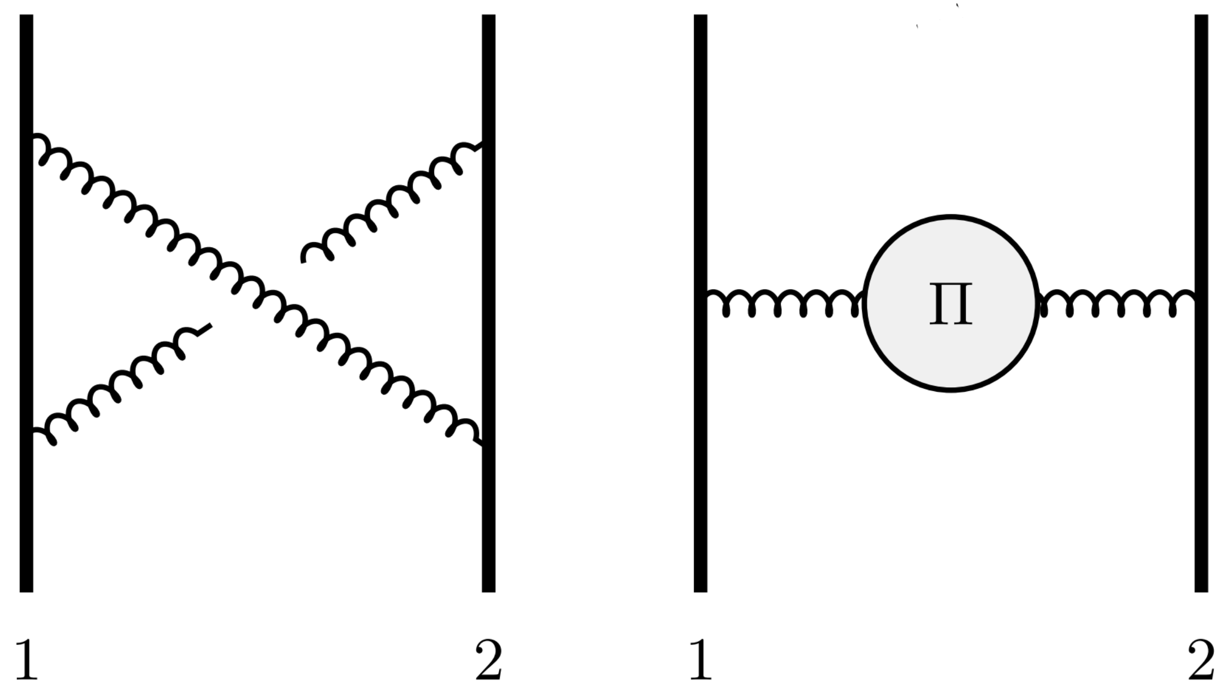
$$z_\pm^4 = \frac{1+C_\perp^2+(1-v^2)(1+C_3^2)}{2(1+C_\perp^2+C_3^2)} \pm \frac{\sqrt{C_\perp^4 + 2C_\perp^2(v^2+C_3^2(1-v^2)) + (v^2-(1-v^2)C_3^2)^2}}{2(1+C_\perp^2+C_3^2)}\,,$$



Weak Coupling Setup

JDP and B. Scheihing
2604.21895

- At leading order in Feynman gauge we only need to evaluate two classes of diagrams
- We have to consistently match perturbative "hard" ($q \sim T$) momentum transfers with HTL resummed "soft ($q \sim gT$) physics without overcounting degrees of freedom
- Following 1006.0867 this can be done to leading order by schematically computing $(\text{Perturbative}) + (\text{HTL}) - (\text{HTL, unresummed})$
- Beyond leading order there is contamination of HTL physics in the UV sector, so our final result should look at the fixed order contribution only



Weak Coupling Result

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$$
\begin{aligned}
S_{\mathrm{FO}}(\mathbf{L};\mathbf{v}) =& \frac{g^2 C_F m_D^2}{4\pi} \ln\left(\frac{m_D}{T}\right) \left[\frac{(v^2-1)\,\mathrm{arctanh}(v) + v}{2v^3}\, iL_\parallel \left(iL_\parallel + \frac{v}{T} \right) + \frac{3v^3 + (1-v^2)^2\,\mathrm{arctanh}(v) - v}{4v^3} (iL_\perp)^2 \right] \\
&+ \frac{g^2 C_F m_D^2}{4\pi} \int_0^1 d\mu \left[\mu^2\, iL_\parallel \left(iL_\parallel + \frac{v}{T} \right) + \frac{1-\mu^2}{2} (iL_\perp)^2 \right] \left[\Phi(a_E, b_E) + \frac{v^2(1-\mu^2)}{2(1-v^2\mu^2)} \Phi(a_T, b_T) \right] \\
&- \frac{g^2 C_F}{T} \int_{\mathbf{p}} \left\{ g^2 \frac{e^{i\mathbf{p}\cdot\mathbf{L}} - 1}{1 - e^{-\mathbf{p}\cdot\mathbf{v}/T}}\, \mathcal{I}_{\mathrm{Hard}} - \theta(T - |\mathbf{p}|)\, (\mathbf{p}\cdot i\mathbf{L}) \left[\mathbf{p} \cdot \left(i\mathbf{L} + \frac{\mathbf{v}}{T} \right) \right] \frac{T}{2\,\mathbf{p}\cdot\mathbf{v}}\, \mathcal{I}_{\mathrm{Match}} \right\}, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{I}_{\mathrm{Hard}} =& \frac{N_c}{2} \int_{k_0} \frac{e^{\mathbf{p}\cdot\mathbf{v}/T} - 1}{|e^{k_0/T} - 1||e^{(\mathbf{p}\cdot\mathbf{v} - k_0)/T} - 1|} \frac{\Theta(1-u^2)}{4|\mathbf{p}|} \\
&\times \left(\frac{(1-\mathbf{v}^2)^2 \left| \mathrm{sgn}(k_0) + u \frac{\mathbf{v}\cdot\mathbf{p}}{|\mathbf{p}|} \right|}{k_0^2 \left[\left(\mathrm{sgn}(k_0) + u\frac{\mathbf{v}\cdot\mathbf{p}}{|\mathbf{p}|} \right)^2 - (1-u^2)\left(\mathbf{v}^2 - \frac{(\mathbf{v}\cdot\mathbf{p})^2}{\mathbf{p}^2} \right) \right]^{3/2}} \right. \\
&- \frac{4(1-\mathbf{v}^2)}{\mathbf{p}^2 - (\mathbf{p}\cdot\mathbf{v})^2} + \frac{8 + 4N_X}{[\mathbf{p}^2 - (\mathbf{p}\cdot\mathbf{v})^2]^2} \left[\left(k_0 - |k_0| \frac{\mathbf{p}\cdot\mathbf{v}}{|\mathbf{p}|} u \right)^2 \right. \\
&\left. \left. + \frac{k_0^2}{2} \left(\mathbf{v}^2 - \frac{(\mathbf{p}\cdot\mathbf{v})^2}{\mathbf{p}^2} \right)(1-u^2) \right] \right) \\
&- 2N_f T_F \int_{k_0} \frac{e^{k_0/T}(1 - e^{-\mathbf{p}\cdot\mathbf{v}/T})}{(e^{k_0/T}+1)(e^{(k_0 - \mathbf{p}\cdot\mathbf{v})/T}+1)} \frac{\Theta(1-u^2)}{4|\mathbf{p}|} \\
&\times \left(\frac{1-\mathbf{v}^2}{\mathbf{p}^2 - (\mathbf{p}\cdot\mathbf{v})^2} - \frac{4}{[\mathbf{p}^2 - (\mathbf{p}\cdot\mathbf{v})^2]^2} \left[\left(k_0 - |k_0| \frac{\mathbf{p}\cdot\mathbf{v}}{|\mathbf{p}|} u \right)^2 \right. \right. \\
&\left. \left. + \frac{k_0^2}{2} \left(\mathbf{v}^2 - \frac{(\mathbf{p}\cdot\mathbf{v})^2}{\mathbf{p}^2} \right)(1-u^2) \right] \right)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_T(k_0 = k\eta, k) =& k^2(1-\eta^2) \\
&+ \frac{m_D^2}{2} \left[\eta^2 + \frac{\eta(1-\eta^2)}{2} \ln\left| \frac{1+\eta}{1-\eta} \right| \right] \\
\Sigma_E(k_0 = k\eta, k) =& k^2 + m_D^2 \left[1 - \frac{\eta}{2} \ln\left| \frac{1+\eta}{1-\eta} \right| \right] \\
\Gamma_T(\eta) =& \frac{\pi m_D^2}{4} \eta(1-\eta^2)\,, \\
\Gamma_E(\eta) =& \frac{\pi m_D^2}{2} \eta\,.
\end{aligned}
$$

$$
\mathcal{I}_{\mathrm{Match}} = \frac{2\Gamma_E((\mathbf{p}\cdot\mathbf{v})/|\mathbf{p}|)}{|\mathbf{p}|^4} + \frac{2\mathbf{v}^2 \left(1 - \frac{(\mathbf{p}\cdot\mathbf{v})^2}{\mathbf{p}^2\mathbf{v}^2} \right) \Gamma_T((\mathbf{p}\cdot\mathbf{v})/|\mathbf{p}|)}{|\mathbf{p}|^4 \left(1 - \frac{(\mathbf{p}\cdot\mathbf{v})^2}{\mathbf{p}^2} \right)^2}
$$

$$
a_E \equiv 1 - \frac{\eta}{2} \ln\left(\frac{1+\eta}{1-\eta} \right), \quad b_E \equiv \frac{\pi}{2}\eta\,,
$$

$$
a_T \equiv \frac{1}{2} \left[\frac{\eta^2}{1-\eta^2} + \frac{\eta}{2} \ln\left(\frac{1+\eta}{1-\eta} \right) \right], \quad b_T \equiv \frac{\pi}{4}\eta\,.
$$

$$
\Phi(a,b) \equiv \frac{1}{4} \ln(a^2 + b^2) + \frac{a}{2b} \arctan\left(\frac{b}{a} \right)
$$

Weak Coupling Result

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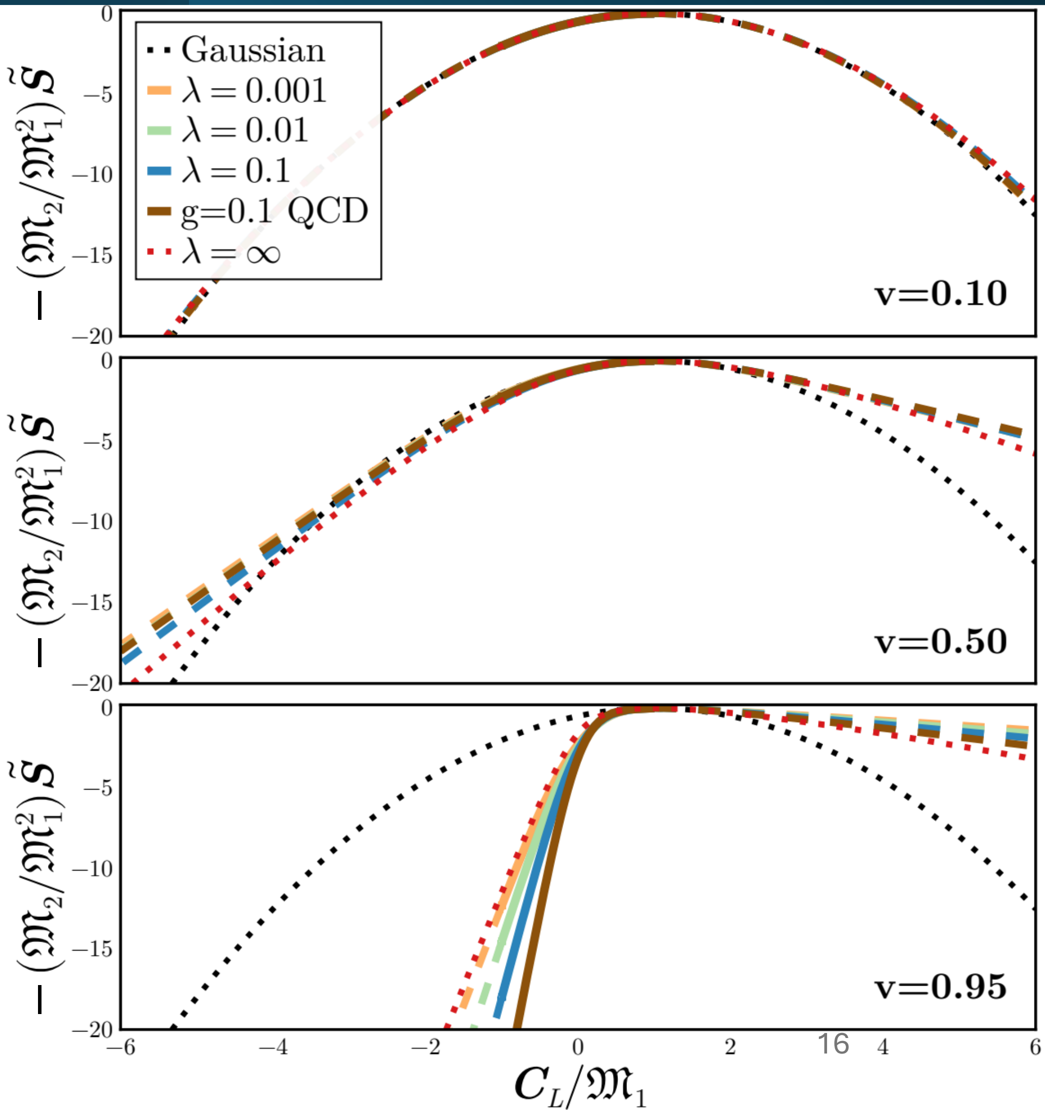

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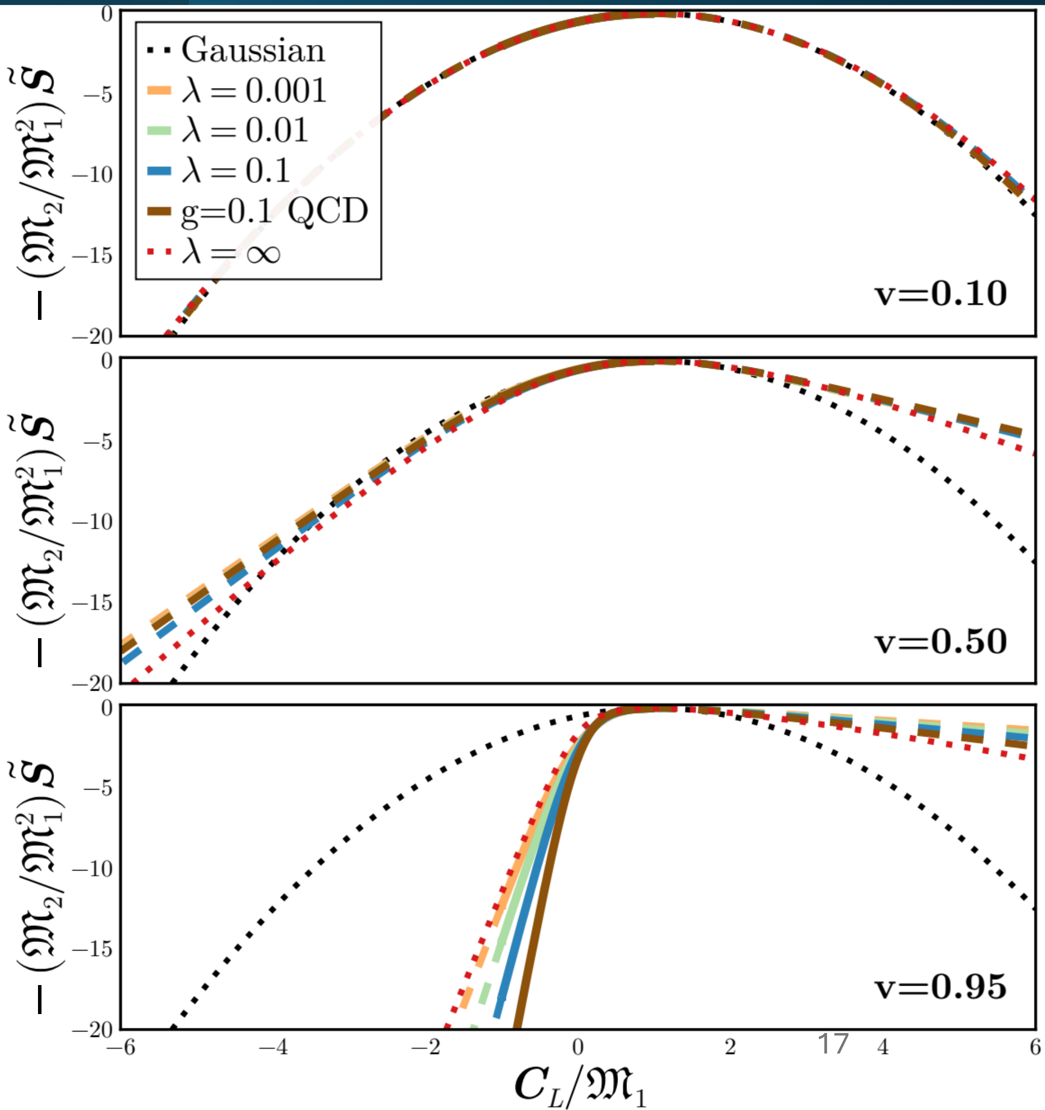
- After rescaling we see a surprising qualitative similarity between strongly and weakly coupled $\mathcal{N} = 4$ SYM as well as (massless) QCD
- Gaussian core that shrinks as $v \to 1$, and asymmetric exponential tails
- Detailed Balance relates the slopes of the tails



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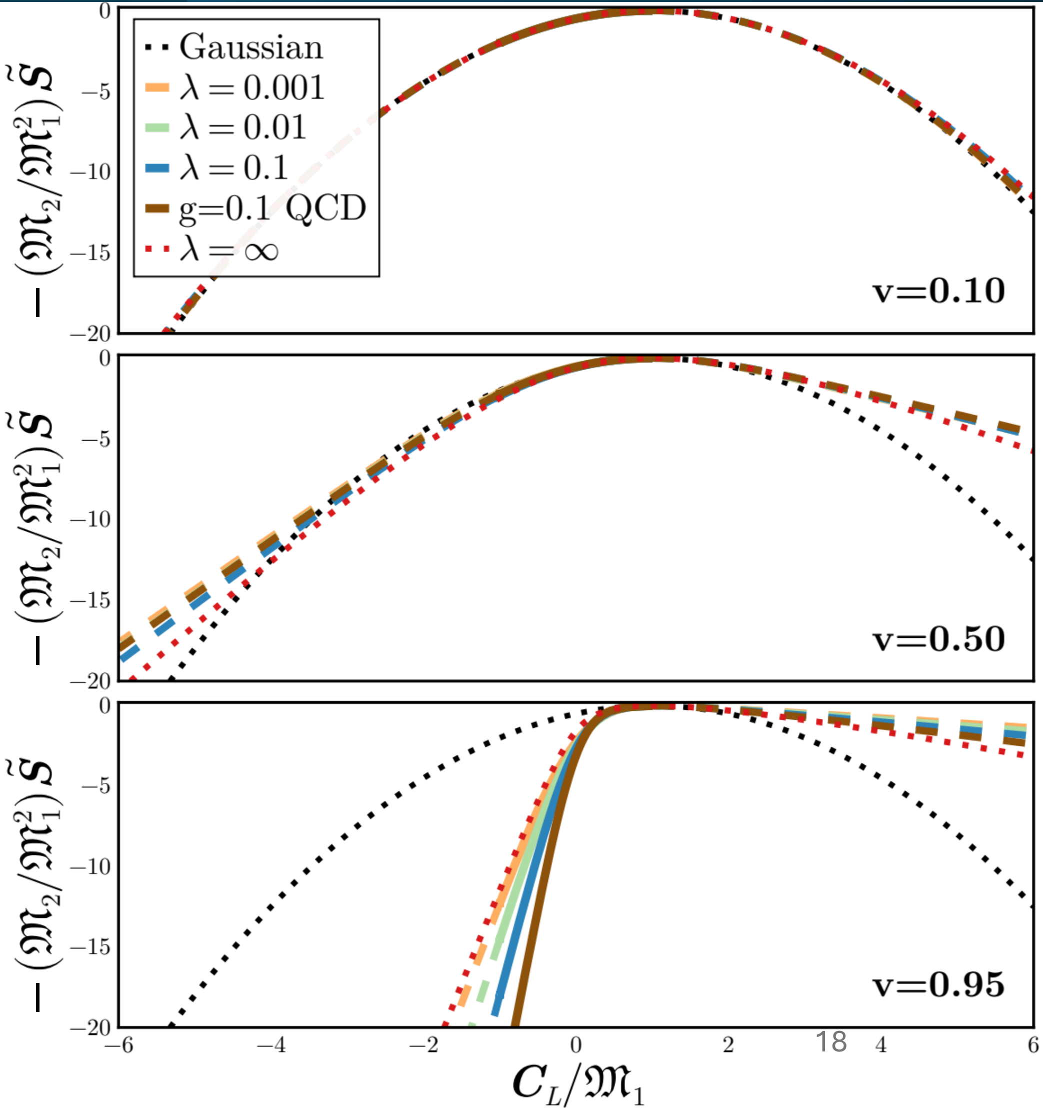
- Where did this structure come from?



Asymptotic Behavior

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- Recall

$$\tilde{S}(C_L, v) \equiv [K(x,v) + x \cdot C_L]_{C_L = -\frac{\partial K}{\partial x}}$$

- We saw, asymptotically

$$\tilde{S}(C_L, v) \sim \pm R_{\pm} C_L \text{ as } -\frac{\partial K}{\partial x} = C_L \rightarrow \pm\infty$$

- This happens when K has singularities at $\pm R_{\pm}$. Detailed balance relates $R_{-} = R_{+} + \frac{v}{T}$
- K having a singularity is equivalent to the asymptotic factorial growth of moments

$$\mathfrak{M}_{\mathrm{n}}(v) \sim n!\, R(v)^{-n}$$

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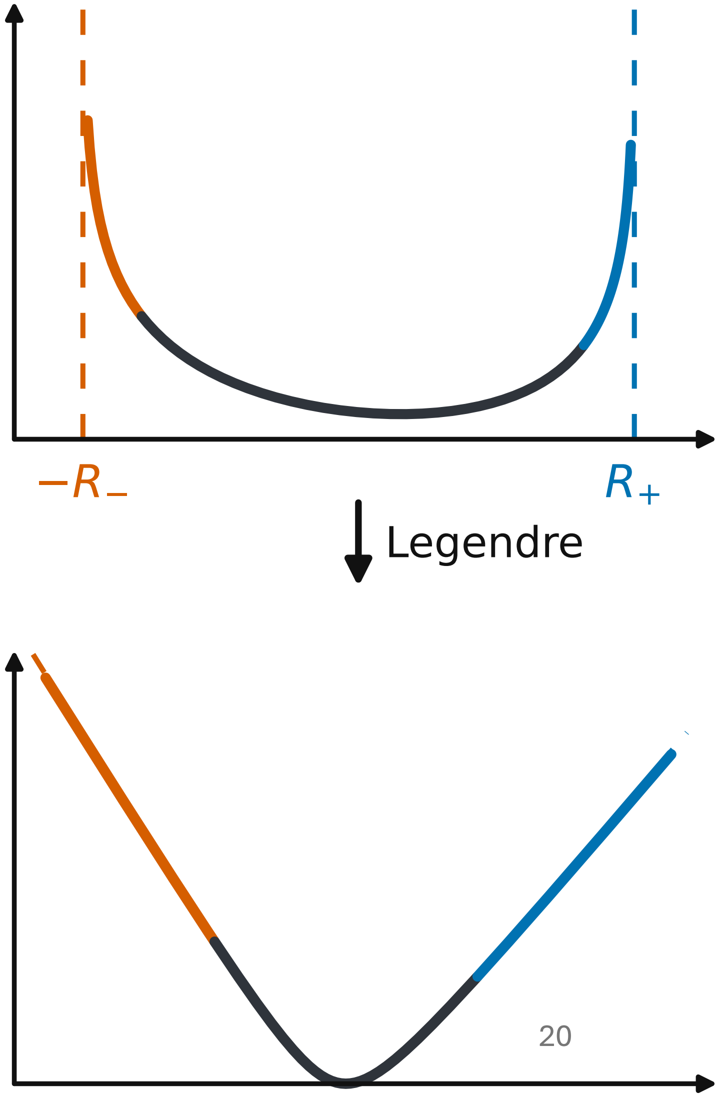

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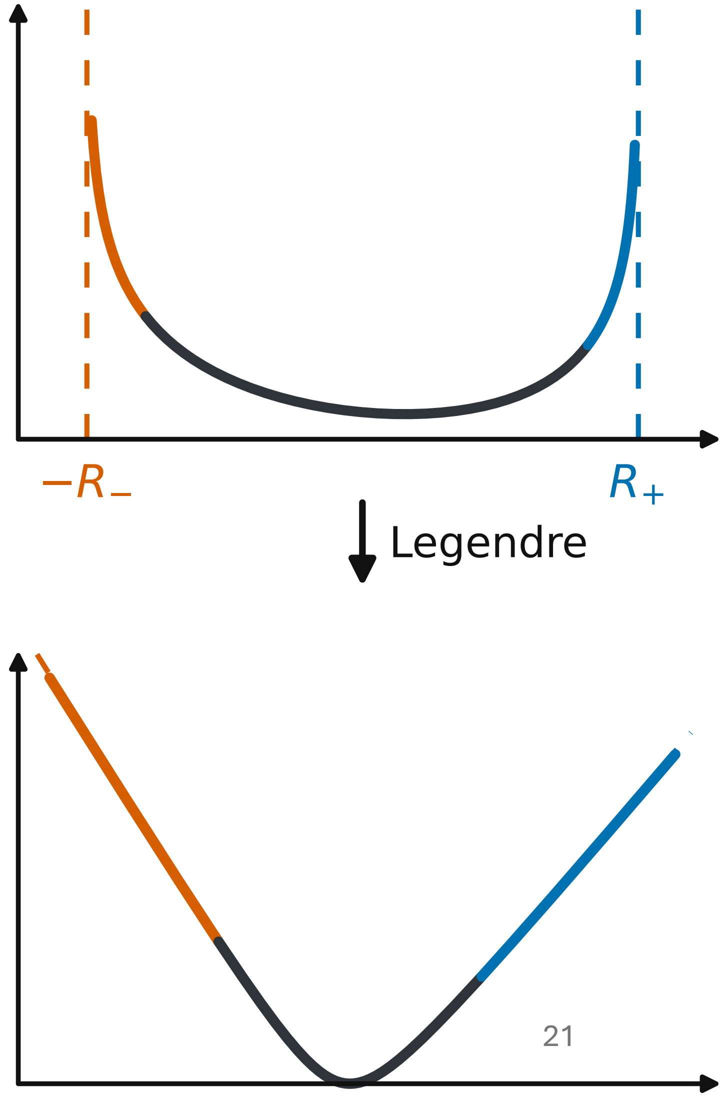
- At leading order, the tail exponent $R(v)$ is coupling-independent
- At weak coupling in the large γ limit we find $R(v) \sim 1/(4\gamma^2 T)$, and at strong coupling $R(v) \sim 1/(r_0 \gamma^{\frac{3}{2}} T)$



Tail Exponent

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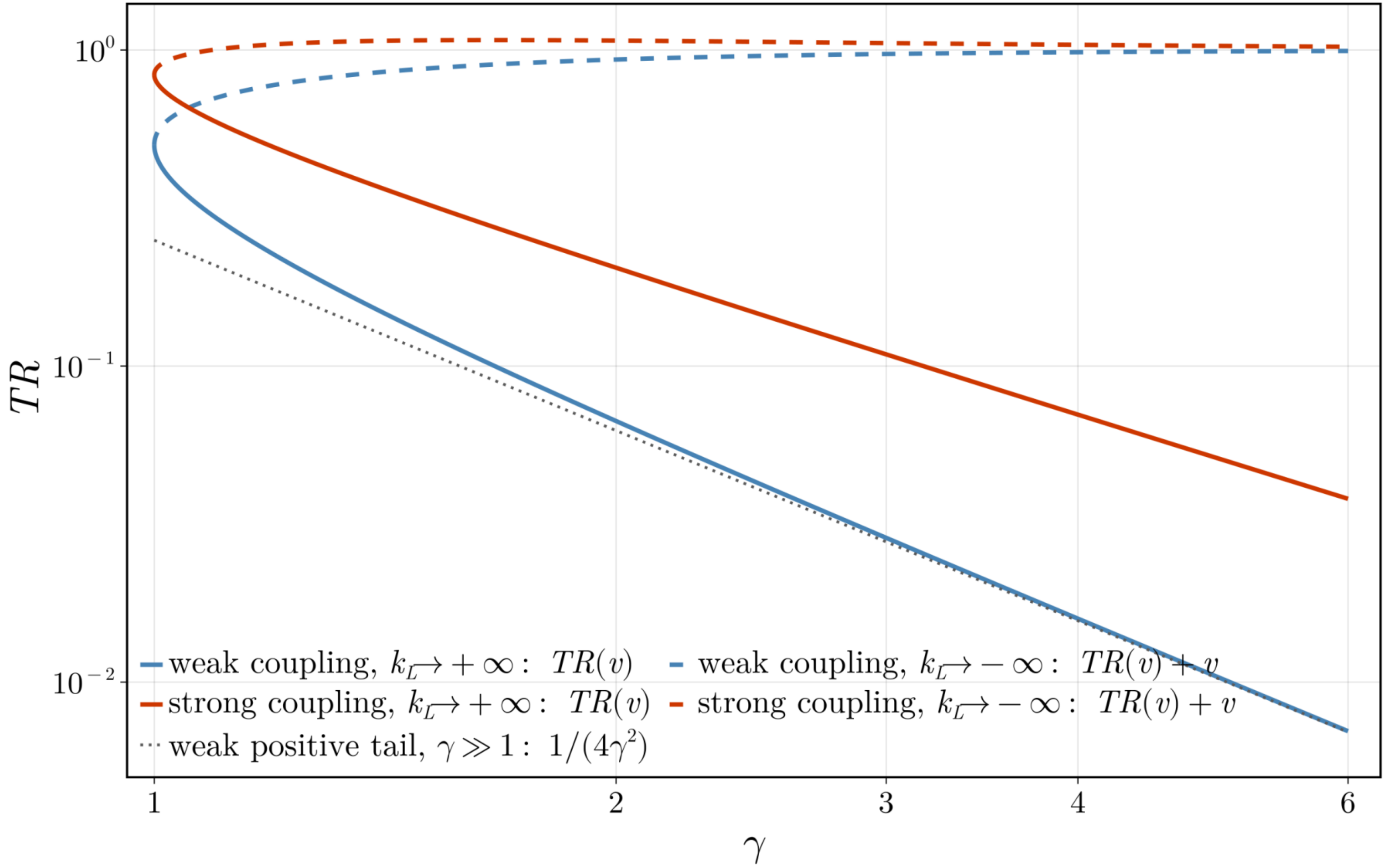

A three-parameter model of non-Gaussianity

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- Large deviation theory tells us the importance of the tails
- We therefore propose:

$$K(x, v) = -\mathcal{D}(v)x\, T(xT + v) + \frac{\mathcal{L}(v)x\, T(xT + v)}{(xT - R(v)T)(xT + v + R(v)T)}$$

A three-parameter model of non-Gaussianity

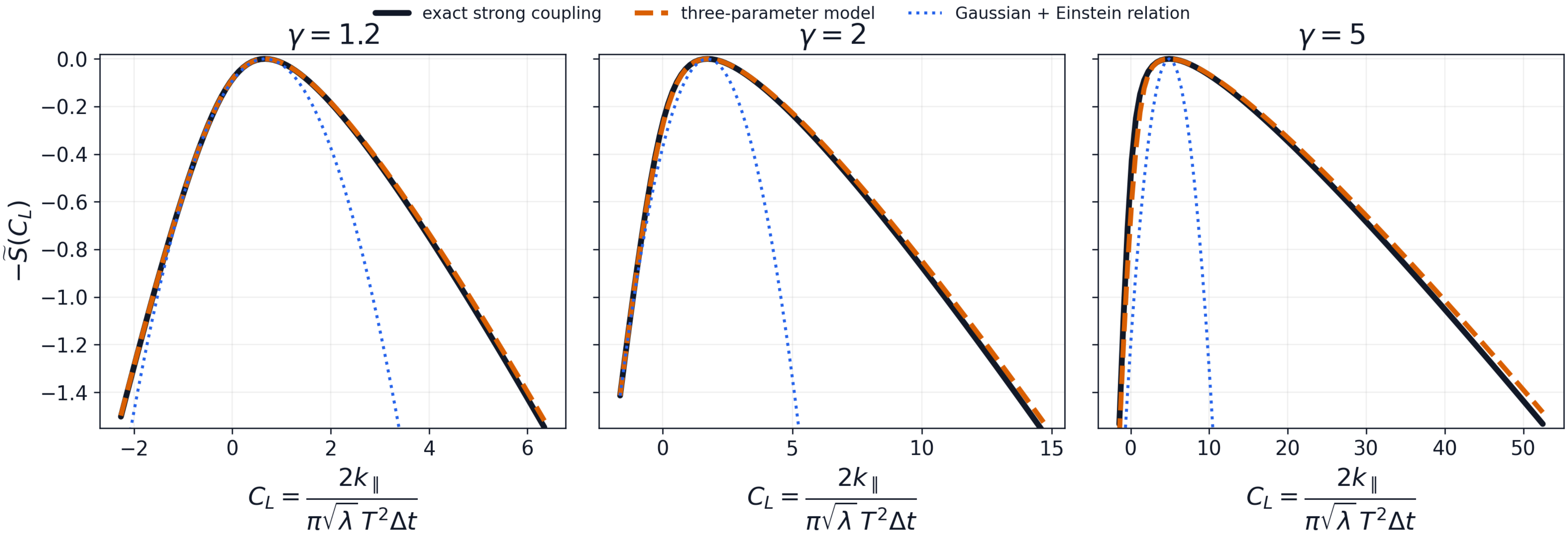
- These singularities are present at both strong and weak coupling in $\mathcal{N} = 4$ SYM, and at weak coupling in pure gauge theory as well as massless QCD
- Requiring equilibration demands $K(x, v) = K\left(-x - \frac{v}{T}, v\right)$
- Normalized Distributions have $K(0, v) = 0$
- The Central Limit Theorem tells us the importance of the drag and diffusion coefficients
- Large deviation theory tells us the importance of the tails
- We therefore propose:

$$K(x, v) = -\mathcal{D}(v)x\, T(xT + v) + \frac{\mathcal{L}(v)x\, T(xT + v)}{(xT - R(v)T)(xT + v + R(v)T)}$$

- This is completely determined by

$$T^2\mathcal{L} = \frac{1}{v^2}\left(R\, T(R\, T + v)\right)^2\left(\frac{\kappa_L}{2\, T} - E\, \eta_D\right) \quad \mathrm{T}^2\mathcal{D} = E\, \eta_D - \frac{T^2\mathcal{L}}{R\, T(R\, T + v)}$$

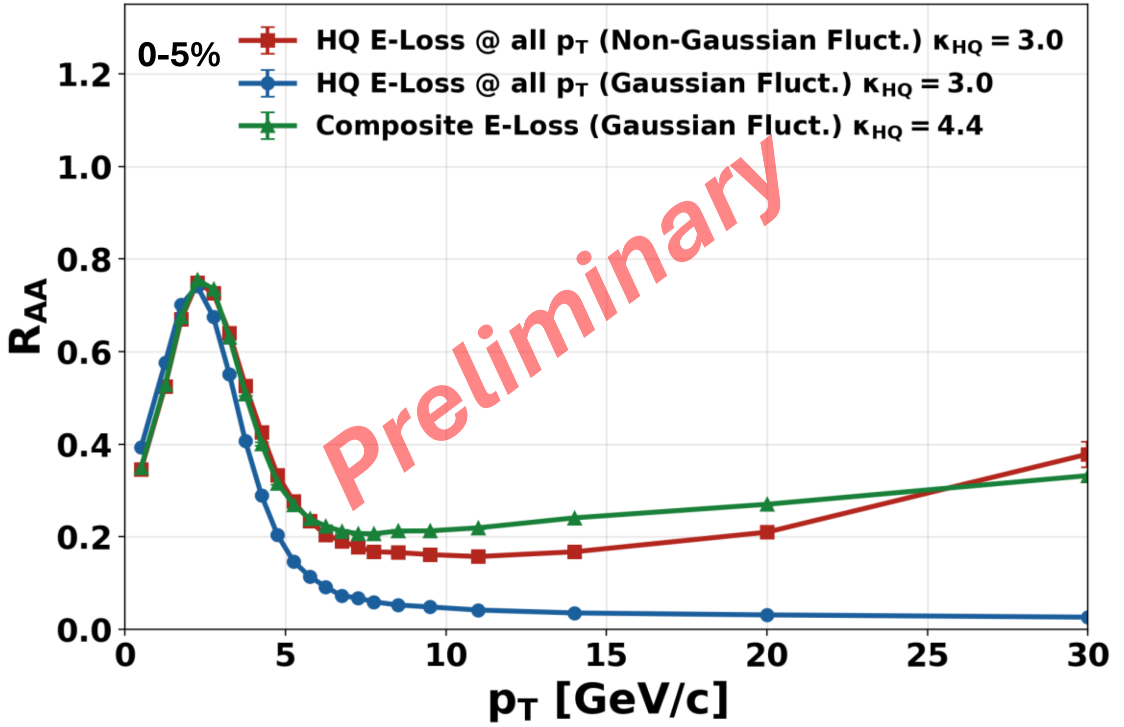
A three-parameter model of non-Gaussianity

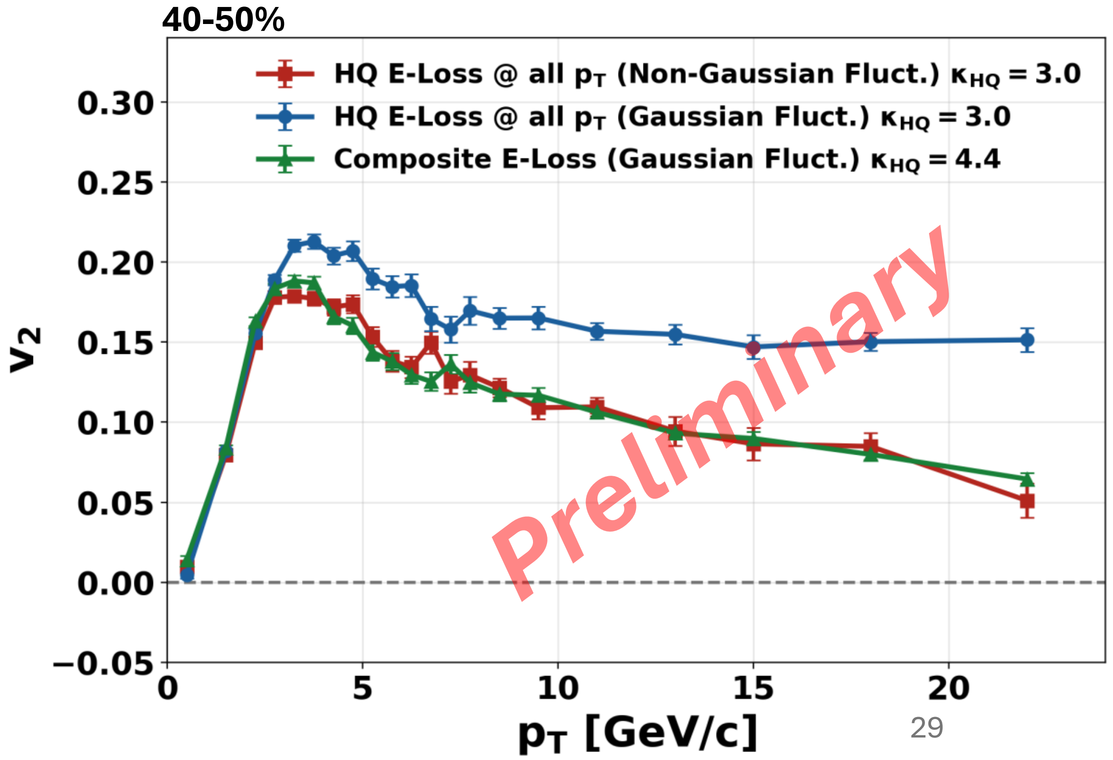


Non-Gaussian Broadening (Preliminary)

26xx.xxxxx
JDP and B. Scheihing

- Compare **Gaussian Fluctuations** (with Einstein relation imposed on diffusion coefficient) **vs Non-Gaussian Fluctuations** (with strong coupling drag and diffusion coefficients and exponential tails)
- Non-Gaussian fluctuations mimic light-quark energy loss employed at high-p_T in composite approach
- Using **non-Gaussian fluctuations** changes system size dependence and will **shift value of κ_{HQ} extracted** via future comparison with data





Outlook and Conclusion

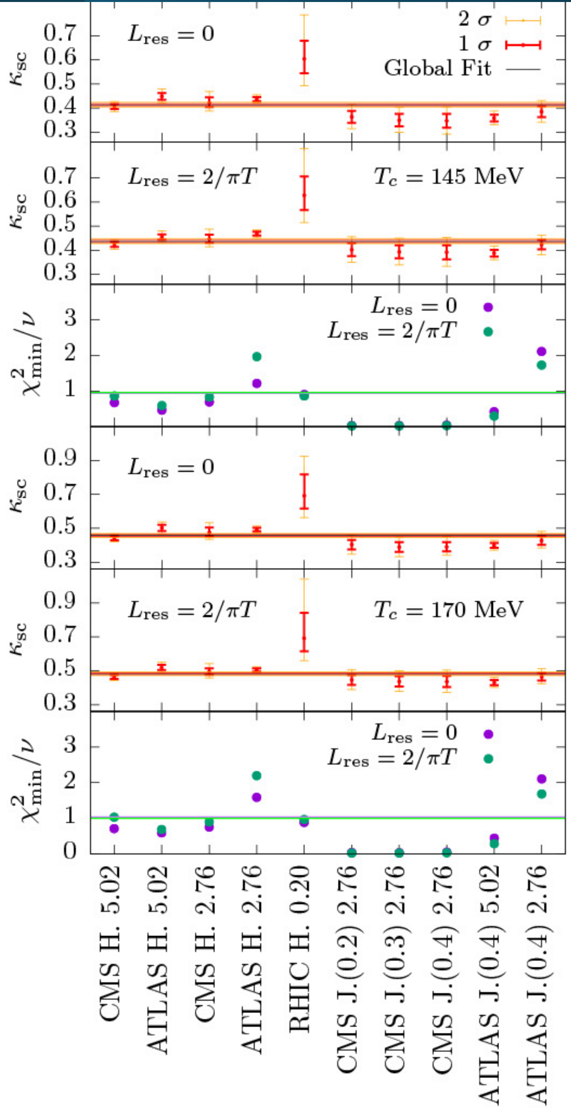
- QFT generically gives rise to heavy quark dynamics that does not satisfy the Einstein Relation at $v > 0$
- QFT generically gives rise to heavy quark dynamics that does satisfy a generalized equilibration condition, which reduces to the Einstein relation in the $v \to 0$ limit
- Strong and weak coupling seems to give rise to qualitatively similar momentum transfer distributions:
  - Gaussian core
  - Asymmetric exponential tails
- This can be understood from the analytic properties of the moment generating function K
- At equal drag, survivorship bias leads to smaller suppression when taking these non-Gaussianities into account
- This can shift the extracted spatial diffusion coefficient $2\pi T D_s(v)$, which has been calculated on the lattice at $v = 0$

Backup

Hybrid model

J. Casalderrey-Solana et al.
1405.3864 and 1808.07386

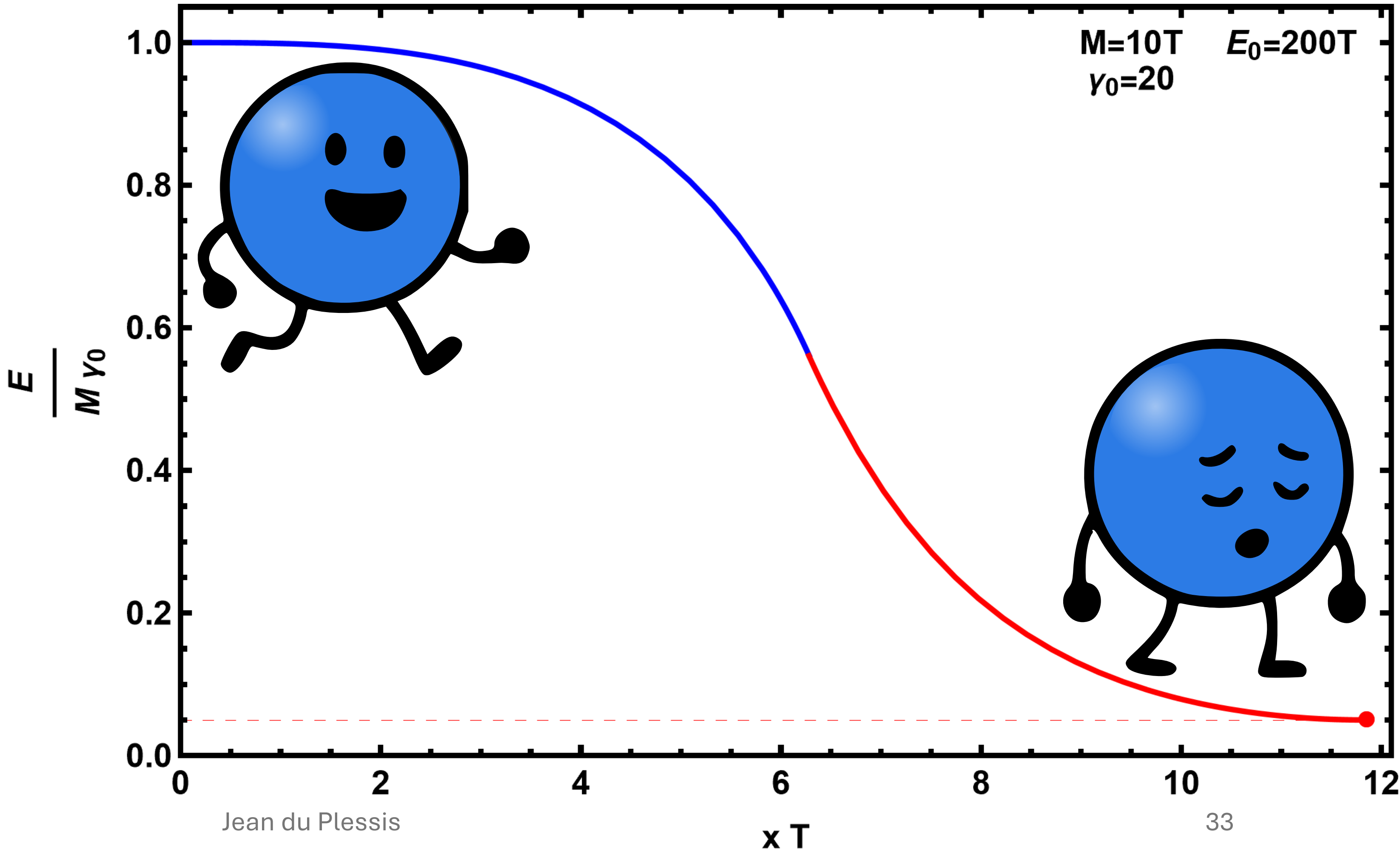
- Pythia 8 Monash 2013 tune for weak coupling parton showers
- Hydrodynamic medium
- Strongly coupled energy loss of every parton in shower
- $\kappa_{sc} = 0.404$ from global fit of hadron and jet R_{AA}; light quark stopping length in QGP 3-4 times that in N=4 SYM plasma
- Lund string hadronization in Pythia



Composite energy loss formula

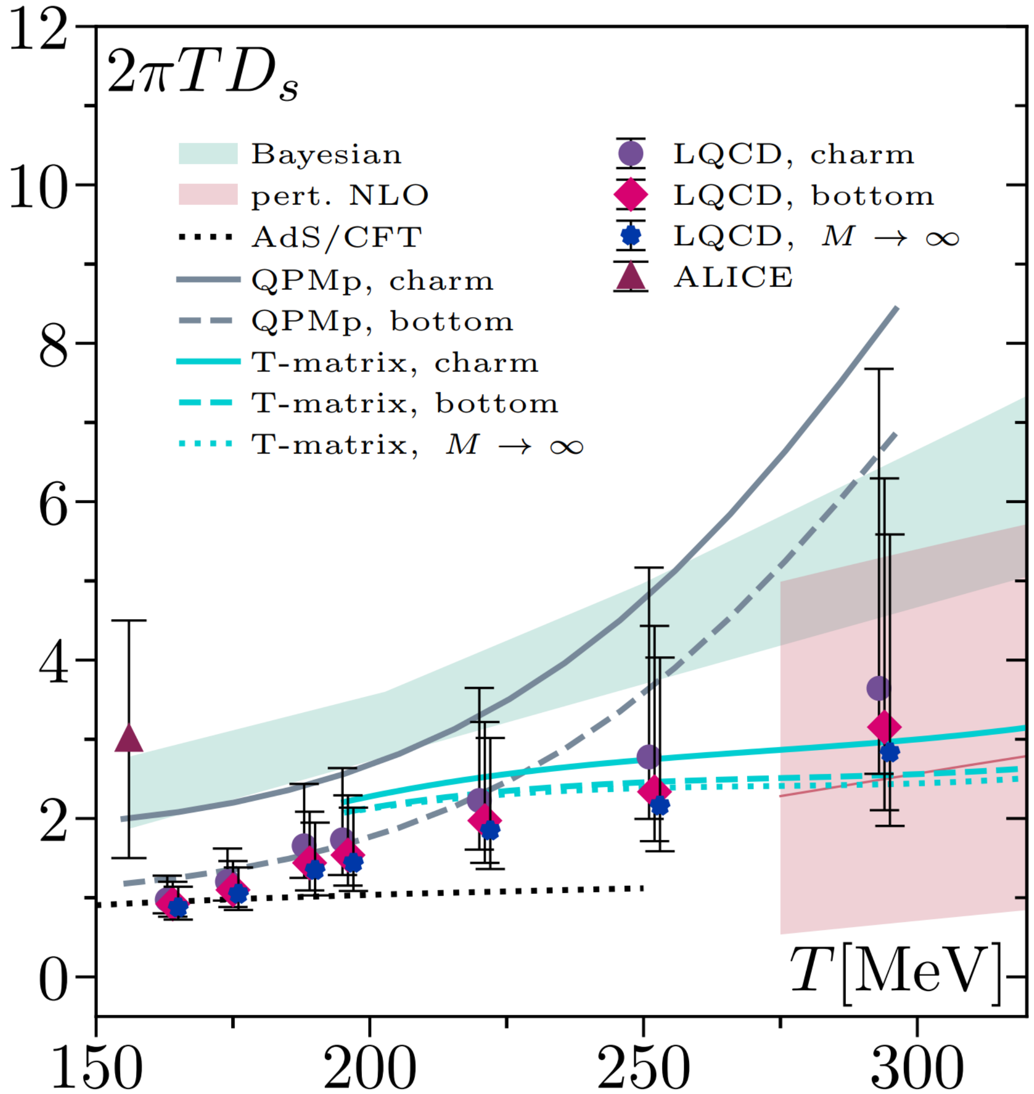
$$-\frac{dE}{dx} = \min\left(\frac{4x^2E_0}{\pi x_t^2\sqrt{x_t^2-x^2}}, \eta_D\sqrt{E^2-M^2}\right)$$

- Can uniquely match such that $E(x)$ and its first derivative are continuous due to opposite signs in second derivative
- At each step choose whether to lose energy as heavy or light quark; always choose least energy loss.
- Comes to rest at finite distance
- Starts off like massless quark, ends like heavy quark
- New parameter $\kappa_{HQ} \equiv \eta_D \frac{M}{T^2}$ added to Hybrid Model; controls slope of red curve



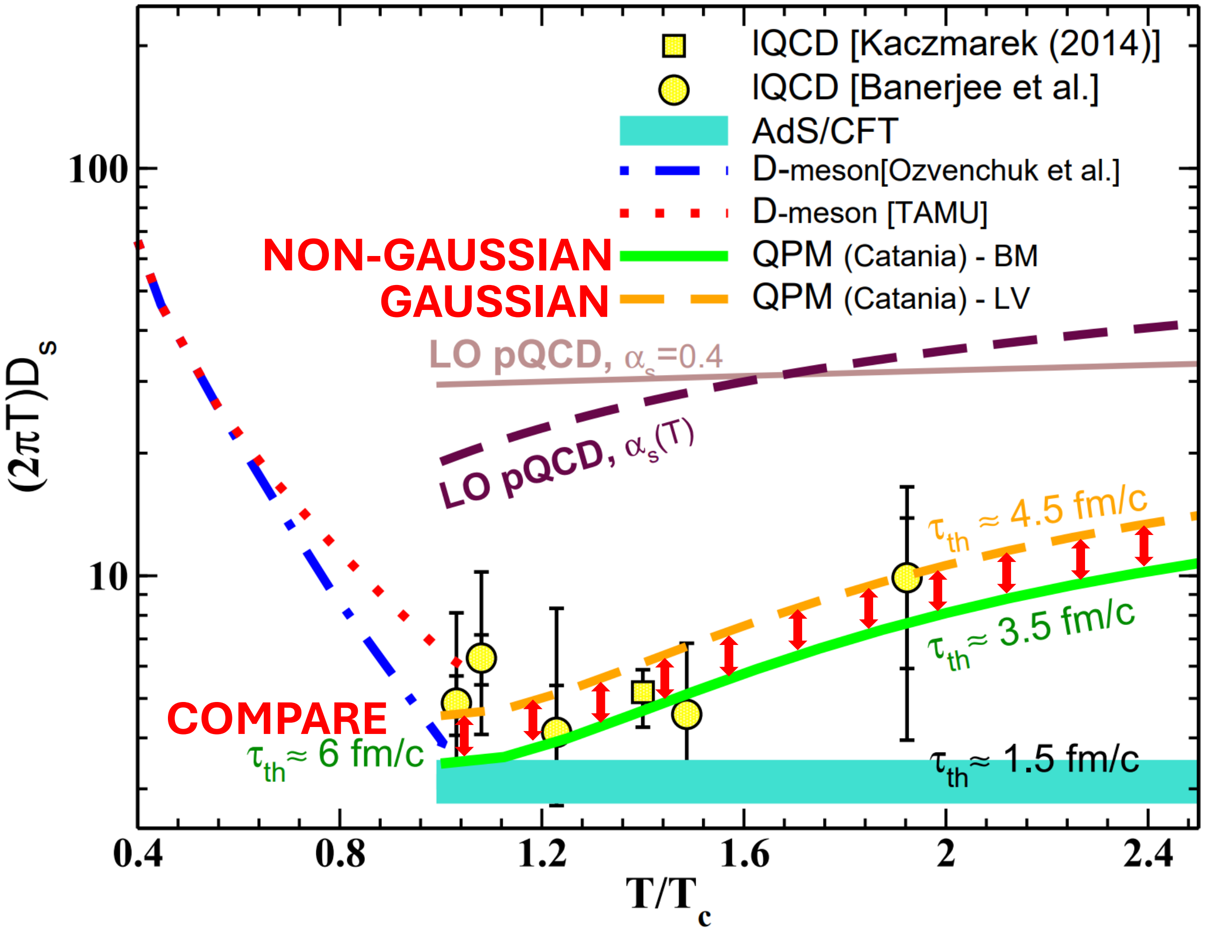
Diffusion Constant at $v = 0$ from the Lattice

Hot QCD
2506.11958



Non-Gaussian Broadening Shifts Parameter Extraction

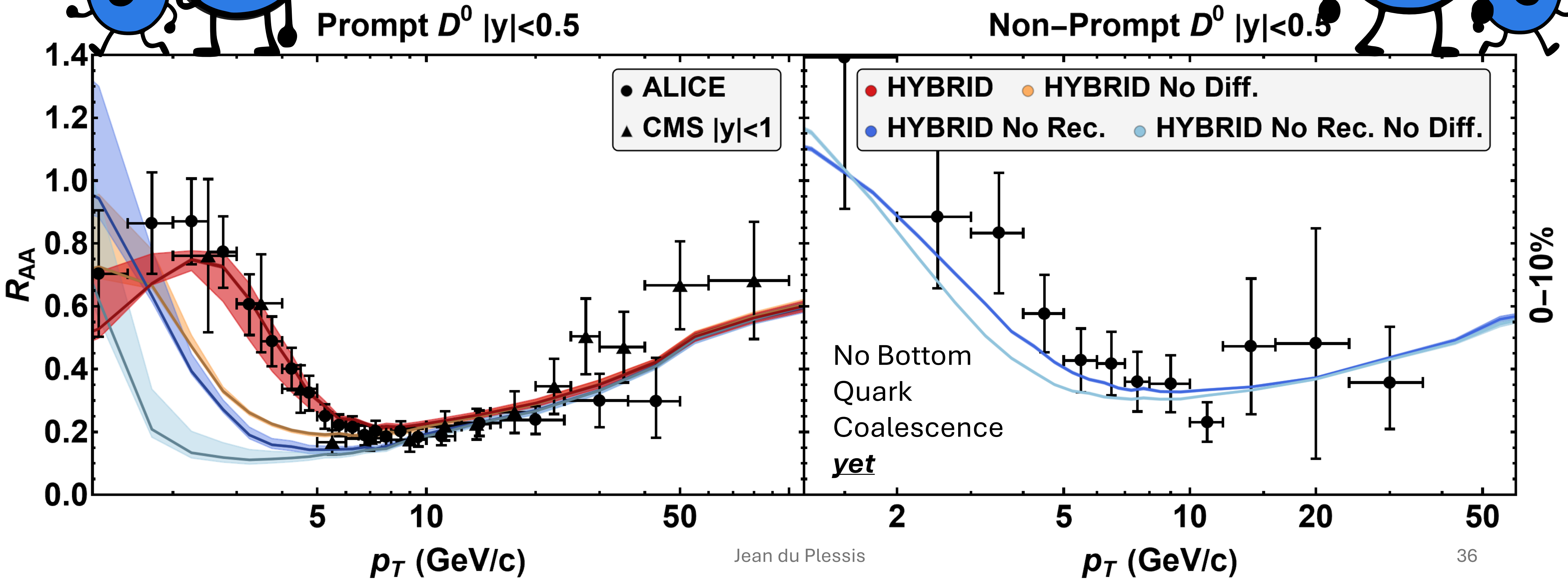
1707.05452
F. Scardina et al.



b and c Hadron R_{AA} 0-10% Cent.

2510.24847
ALICE Data: 2202.00815

Reasonable description of D^0 R_{AA} for $\boldsymbol{\kappa_{HQ} = 4.4}$, only with both recombination and diffusion



Sampling the model

- Large deviation theory guarantees that sampling a distribution with the correct rate function will give the correct late time behavior
- For numerical implementations we want a convolution-closed family of distributions that are easy to sample, such that the choice of timestep does not affect the physics in the small timestep limit
- It can be shown that sampling $G + \sum_{i=1}^{N} Z_i$ is convolution closed and has the exact proposed rate function when

$$G \sim \mathcal{N}(\mathcal{D}v\, T\Delta t, 2\mathcal{D}T\Delta t), \qquad N \sim \mathrm{Pois}(\mathcal{L}\, T\Delta t), \qquad Z \sim \begin{cases} \mathrm{Exp}(\mathrm{R\,T}) & \text{w/ prob. } \dfrac{RT+v}{2RT+v} \\ -\mathrm{Exp}(\mathrm{R\,T+v}) & \text{w/ prob. } \dfrac{RT}{2RT+v} \end{cases}$$