Hydrodynamic-attractor-like behaviour in classical Yang-Mills theory

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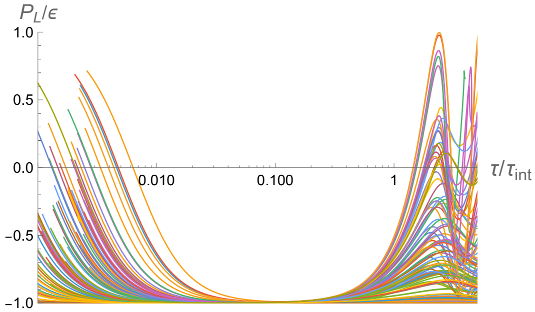
based on 260X.XXXX



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What is a hydrodynamic attractor?

Convergence to universal time evolution long before equilibrium

prototypical example:

conformal Bjorken flow, time scaled by τ_{int}

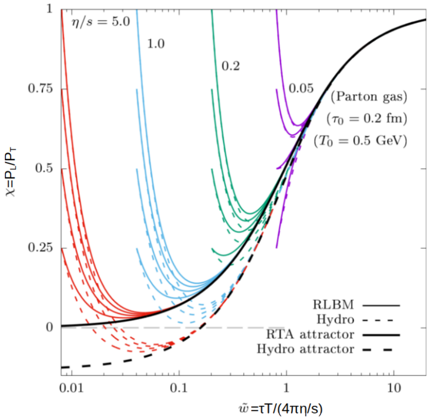
- pressure anisotropy P_L/P_T
- energy density $\epsilon\tau^{4/3}/\lim_{\tau\to\infty}\epsilon\tau^{4/3}$
- moment ratios to equilibrium $\mathcal{M}_{mn}/\mathcal{M}_{mn}^{(\text{eq})}$

mechanism:

information loss through

- equilibration at late times: $P_L/P_T \to 1$
- expansion at early times: $P_L/P_T \to \text{cst.}$ ($P_L \to 0$ in kin.theory)

… and lost information *stays* lost



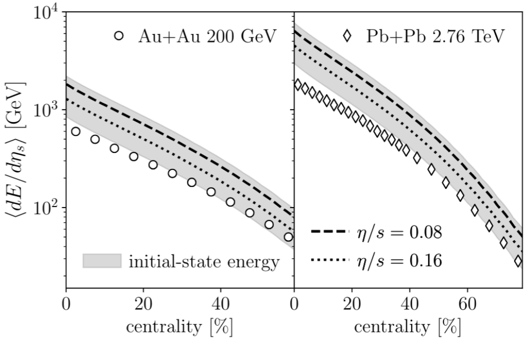
Victor Ambrus, Lorenzo Bazzanini, Alessandro Gabbana, Daniele Simeoni, Raffaele Tripiccione, Sauro Succi, Nature Computat.Sci. 2 (2022) 641-654

Practical applications in phenomenology

- even with transverse profile, at early times $\tau \ll R$, transverse dynamics can be neglected $\rightarrow$ *local* Bjorken flow attractor
- final state Multiplicity $\sim$ Entropy $\rightarrow$ attractor links multiplicity to initial energy!

Giuliano Giacalone, Aleksas Mazeliauskas, Sören Schlichting, PRL 123 (2019) 262301

$$\frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta} = \frac{4}{3}\frac{N_{\mathrm{ch}}}{S} C_\infty^{\frac{3}{4}} \left(4\pi\frac{\eta}{s}\right)^{\frac{1}{3}} \left(\frac{\pi^2}{30}\nu_{\mathrm{eff}}\right)^{\frac{1}{3}} \int \mathrm{d}^2\mathbf{x}\, [e(\mathbf{x})\tau]_0^{\frac{2}{3}}$$



Can also predict early time evolution of

- transverse energy $\mathrm{d}E_\perp/\mathrm{d}\eta$
- flow velocity u^μ
- eccentricity ϵ_n

Victor Ambruș, Sören Schlichting, Clemens Werthmann, PRD 105 (2022) 014031 and PRD 107 (2023) 094013

beyond that: find new general dynamical description? Paul Romatschke, PRL 120 (2018) 1, 012301

See also: "Attractodynamics"

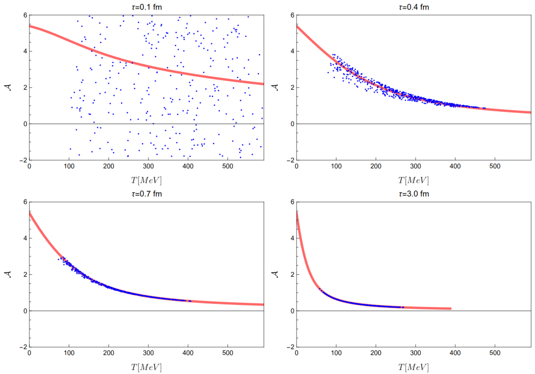
$\rightarrow$ Rachel Steinhorst, Tue 5pm

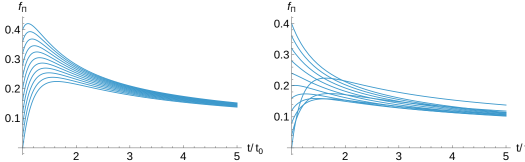
What does attractor behaviour look like more generally?

Recent examples:

- non-scaling dynamics: non-conformal Bjorken flow
  Michał Spaliński, arXiv:2508.21039
- isotropic expansion/contraction: ultracold atom attractor
  Michał Heller, Clemens Werthmann, arXiv:2507.02838

$\Rightarrow$ collapse in individual observables is specific to Bjorken flow,
more generally: attractor is a curved surface in full state space (e.g. (e, P_L, τ) for hydro)
$\Rightarrow$ need better diagnostic!



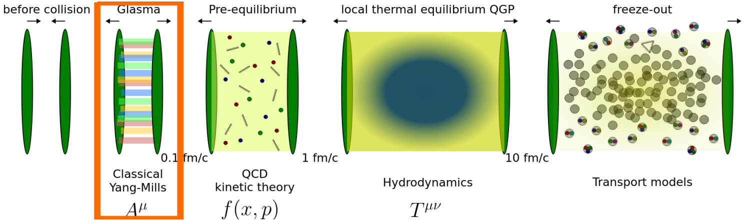


Generalize to non-equilibrating interaction

Central questions of this project

What does attractor behaviour look like in a theory without a late time equilibrium fixed point? Can information loss be reversed?

Fitting dynamical description, related to heavy ion collisions: classical Yang-Mills!



Graphic by Florian Lindenbauer

<u>Glasma:</u> phase of strong gluonic fields, quantum vertices can be neglected, wave function evolves as ensemble of independent classical trajectories

Field strength tensor $F^{\mu\nu} = \nabla^\mu A^\nu - \nabla^\mu A^\nu - ig[A^\mu, A^\nu]$

evolves according to $D_\mu F^{\mu\nu} = \nabla_\mu F^{\mu\nu} - ig[A_\mu, F^{\mu\nu}] = 0$

Simplifying assumptions

Working in $SU(2) \Rightarrow$ YM fields have 3 colors

No ensemble, but *single* classical trajectory: strong potentials instead of gluons

Bjorken-like symmetries: $A^\mu_a(\tau, x, y, \eta) = A^\mu_a(\tau)$

- longitudinal boost invariance
- transverse homogeneity
- **no** transverse isotropy
  (renders dynamics trivial)

What to expect?

- no particle-like excitations $\Rightarrow$ no nonthermal fixed point
  J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan, PRD 89, 074011
- no transverse dependence $\Rightarrow$ no Nielsen-Olesen instabilities
  N. K. Nielsen, P. Olesen, Nucl. Phys. B 144, 376
- information loss from strong expansion

Evolution and Constraints

Gauge fixing:

- temporal gauge $A^\tau = 0$ does not fully fix gauge
  but: $A_\mu \to U A_\mu U^\dagger + i g^{-1} U \partial_\mu U^\dagger$
  $\Rightarrow$ no transverse gradients means no other gauge fixing has a closed form!
- at initial time, choose
  $A_2^x = A_3^x = A_3^y = 0,\ A_1^x > 0,\ A_2^y > 0$
- later find gauge d.o.f. subspace and project: for state space vector
  $X = (E^i, B^i)$, it is
  spanned by $g_a^{(b)} = f_a^{bc} X_c$

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Field strength tensor:

$$F_a^{\tau i} = \partial_\tau A_a^i + 2\delta_\eta^i \tau^{-1} A_a^\eta \qquad \text{(abelian)}$$

$$F_a^{jk} = g f_a^{bc} A_b^j A_c^k \qquad \text{(non - abelian)}$$

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Evolution equations:

$$0 = f_a^{bc}(A_b^i \partial_\tau A_c^i + \tau^2 A_b^\eta \partial_\tau A_c^\eta) = C_a$$

$$0 = \partial_\tau^2 A_a^i + \tau^{-1}\partial_\tau A_a^i + g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu A_e^i$$

$$0 = \partial_\tau^2 A_a^\eta + 3\tau^{-1}\partial_\tau A_a^\eta + g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu A_e^\eta$$

Too many dynamical equations? No: from the other equations, it follows that $\partial_\tau C_a = -C_a/\tau$

$\Rightarrow$ first equation constrains state space

Nondimensionalization:

$\tau \to \tau_{\text{scale}}\tau,\ A_a^i \to \tau_{\text{scale}}^{-1} A_a^i,\ A_a^\eta \to \tau_{\text{scale}}^{-2} A_a^\eta$

Expansion dominated regime at early times

Consider the case where τ^{-1}-terms dominate over g^2-terms: each field reacts to expansion on its own! On the level of $E^i_a = F^{\tau i}_a\,,\quad E_{L,a} = \tau F^{\tau\eta}_a\,,\quad B^i_a = \tau F^{\bar{i}\eta}_a\,,\quad B_{L,a} = F^{xy}$:

$$0 = \partial_\tau^2 A^i_a + \tau^{-1}\partial_\tau A^i_a$$

$$0 = \partial_\tau^2 A^\eta_a + 3\tau^{-1}\partial_\tau A^\eta_a$$

$$E^i_a = \partial_\tau A^i_a(\tau_0)\frac{\tau_0}{\tau} \qquad\qquad E_{L,a} = 2A^\eta_a(\tau_0) + \tau_0\partial_\tau A^\eta_a(\tau_0)$$

$$B^i_a = g\left[A^{\bar{i}}_a(\tau_0) - \tau_0\partial_\tau A^{\bar{i}}_a(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]\left\{\tau A^\eta_a(\tau_0) - \frac{1}{2}\partial_\tau A^\eta_a(\tau_0)\left[\frac{\tau_0^3}{\tau} - \tau_0\tau\right]\right\}$$

$$B_{L,a} = g\left[A^x_a(\tau_0) - \tau_0\partial_\tau A^x_a(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]\left[A^y_a(\tau_0) - \tau_0\partial_\tau A^y_a(\tau_0)\ln\left(\frac{\tau}{\tau_0}\right)\right]$$

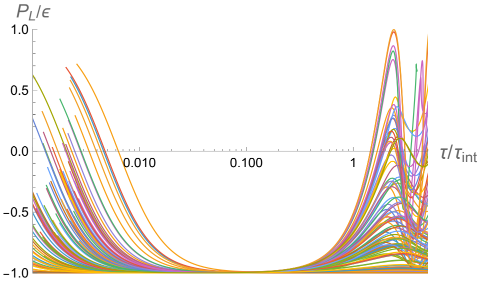
- longitudinal fields (mostly) constant
- transverse fields $\propto \tau_0/\tau$
- transv. magn. fields transition to $\propto \tau$

$e =\propto E^2 + B^2,\ P_L \propto E_T^2 + B_T^2 - E_L^2 - B_L^2$

- $E_T^2 + B_T^2 \gg E_L^2 + B_L^2$: $P_L/\epsilon = 1$
- $E_T^2 + B_T^2 \ll E_L^2 + B_L^2$: $P_L/\epsilon = -1$

Pressure anisotropy

- $P_L/\epsilon \to -1$ at early times
- interaction terms are of the form $M_a^e A_e^i \Rightarrow$ interaction timescale $\tau_{\rm int}$ is inverse square root of largest eigenvalue of $M_a^e = g^2 f_a^{bc} f_c^{de} A_{b,\mu} A_d^\mu$
- $\tau \gtrsim \tau_{\rm int}$: oscillations



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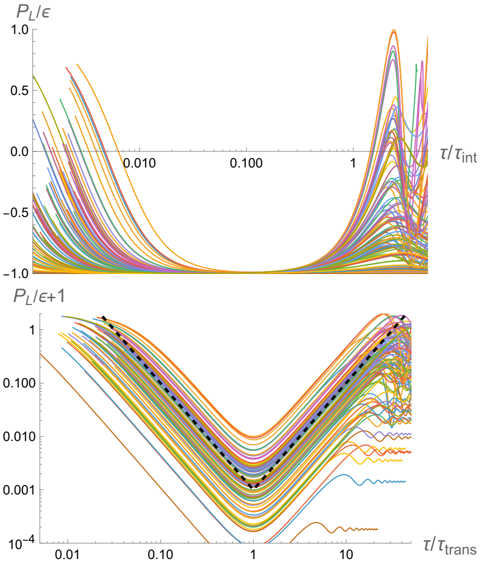


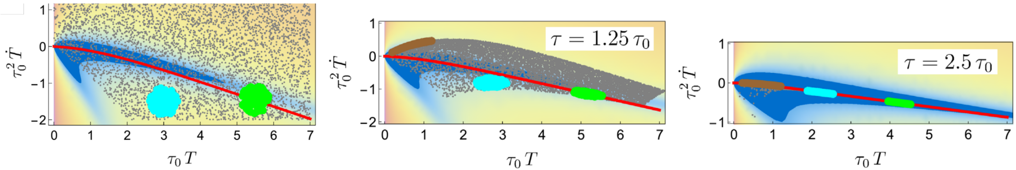
- $P_L/\epsilon + 1$ goes from $\propto \tau^{-2}$ to $\propto \tau^2$
- transition timescale

$$\tau_{\text{trans}} = \left(\frac{\tau_0^2 \sum_{i=x,y} \partial_\tau A_a^i \partial_\tau A^{i,a}(\tau_0)}{g^2 \sum_{i=x,y} f_a^{bc} A_b^i A_c^\eta f^{ade} A_d^i A_e^\eta(\tau_0)} \right)^{1/4}$$

$\Rightarrow$ **three** dynamical regimes!

but also P_L/ϵ isn't everything!

Dimensional reduction

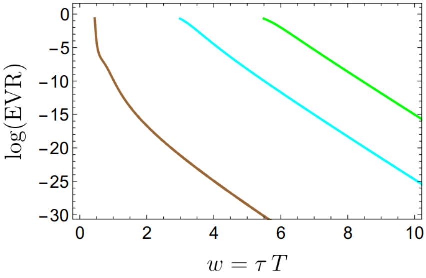


Michał P. Heller, Ro Jefferson, Michał Spaliński, Viktor Svensson, PRL 125 (2020) 13, 132301

Diagnose attractor behaviour in state space:

- track directional contraction of local ensembles in state space (E^i_a, B^i_a) (minus constraints)
- at each time, perform "principal component analysis": construct covariance matrix Σ of the ensemble spread in state space and diagonalize

$\Rightarrow$ split into directional variance components!

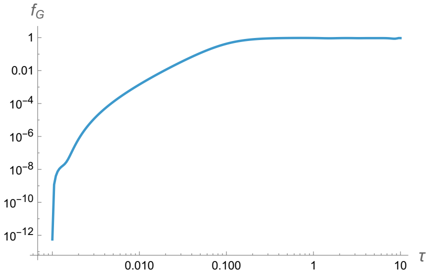


Variance in gauge d.o.f.

Eliminated gauge d.o.f. at initial time, but they can be dynamically populated again!
$\Rightarrow$ track variance fraction in gauge d.o.f.: $f_G = \frac{\mathrm{Tr}\, G\Sigma G^T}{\Sigma}$

- initially zero, but grows fast
- already at $\tau \gtrsim 0.1$, variance dominated by gauge!

$\Rightarrow$ **need** to project out gauge subspace before diagonalization (reduction by 3 d.o.f.)
problem: hard to reconcile with other constraint (reduction by another 3 d.o.f.)

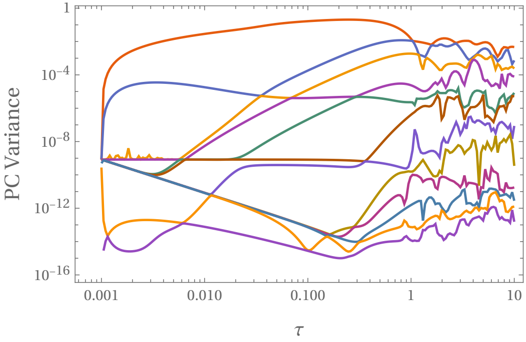


Dimensional Reduction in classical Yang-Mills theory

Consider spherical ensemble around simple initial condition for easier interpretation:

$$B_{L,3} = B^x_{T,1} = B^x_{T,2} = B^y_{T,2} = 1\,, \quad \text{others} = 0$$

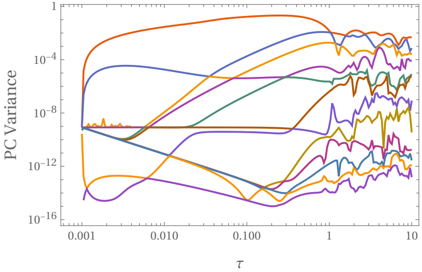
- constant variances, $\propto \tau^{-2}$ variances splitting into $\propto \tau^2$ and continued $\propto \tau^{-2}$
- late time oscillations scramble information in state space, no change in hierarchy
- no exponential growth (in contrast to Glasma: Pooja's talk just before)

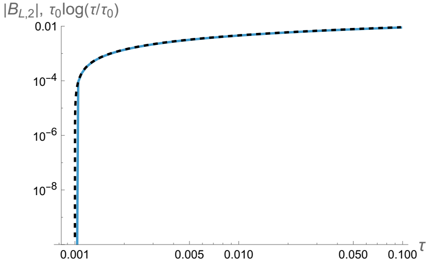


Dimensional Reduction in classical Yang-Mills theory

- quickly increasing part comes from magnetic fields, which share factors: $A^i_a(\tau_0) - \tau_0 \partial_\tau A^i_a(\tau_0) \ln\left(\frac{\tau}{\tau_0}\right)$ $\Rightarrow$ small initial A^i_a leads to fast growth
- quickly decreasing part comes from shearing deformation of correlation in quickly increasing part

$\Rightarrow$ some initial state dependence in dimensional reduction





Conclusions

- in fully classical Yang-Mills theory under Bjorken-like symmetries, attractor behaviour proceeds in three stages:
    1. expansion driven decay of transverse fields
    2. expansion driven growth of transverse magnetic (non-abelian) fields, electric fields (abelian) continue decaying
    3. interactions scramble information
- visible in both P_L/ϵ and dimensional reduction in state space
- initial state dependence can cause overshadowing dimensional reduction effects