



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MIT CENTER FOR THEORETICAL
PHYSICS – A LEINWEBER INSTITUTE

FWF Österreichischer
Wissenschaftsfonds

Jets and medium-induced radiation during the early nonequilibrium stages in heavy-ion collisions

Mainly based on

Phys.Rev.D 113 (2026) 11, 114009 [Barata, Boguslavski, FL, Sadofyev]

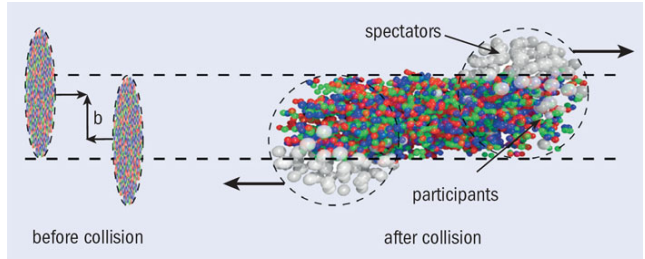
Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

August 17, 2026, Strong and Electro-Weak Matter 2026
Helsinki, Finland



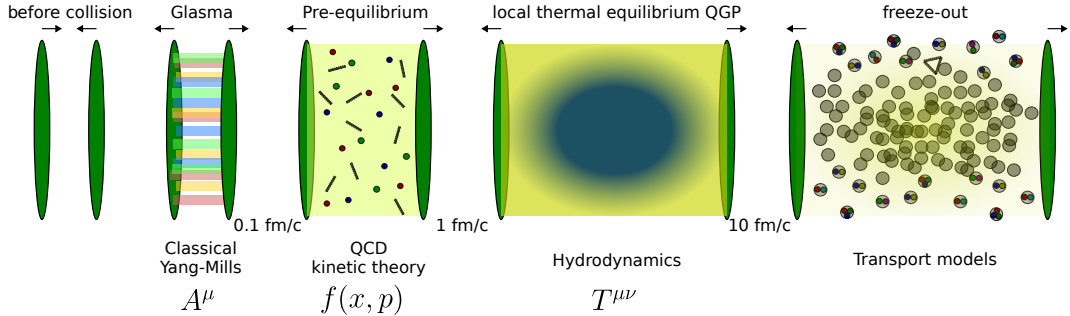
Heavy-ion collisions and the quark-gluon plasma

- Study high-temperature properties of the strong interaction (QCD)
- Collision of atomic nuclei at LHC or RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



[Alberica Toia 2013, CERN COURIER]

Time-evolution of the QGP in heavy-ion collisions

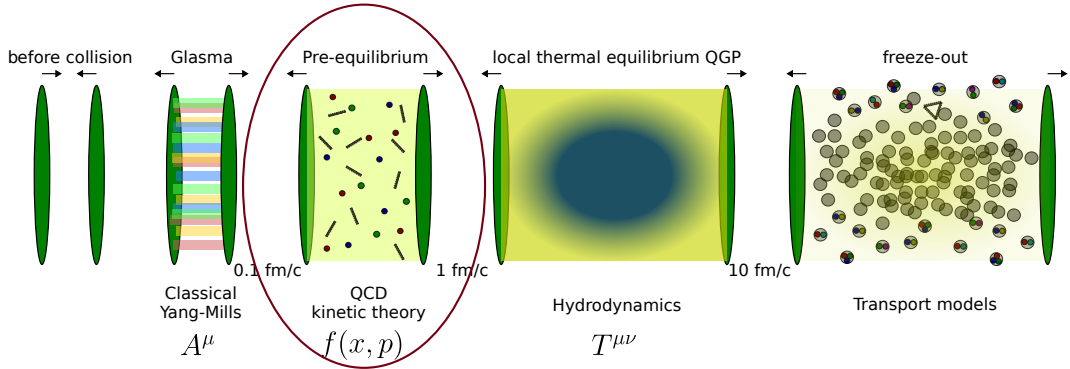


Interested in pre-equilibrium stages (“Initial stages”)

→ **QCD out of equilibrium** → Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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Effective kinetic theory (EKT) description of the QGP

How many particles/gluons

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Effective kinetic theory (EKT) description of the QGP

- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$ ↖ How many particles/gluons
- Time evolution described by **Boltzmann equation** at leading-order¹

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \underbrace{\left| \text{diagram 1} \right|^2 + \left| \text{diagram 2} \right|^2}_{\text{Collision term}}$$

The diagram shows the collision term of the Boltzmann equation at leading order. It consists of two squared terms added together, enclosed in a bracket labeled "Collision term". The first term is a red Feynman diagram representing a gluon-gluon collision, with two incoming red lines and two outgoing red lines connected by a vertical red wavy line. The second term is a red Feynman diagram representing a gluon-gluon collision, with two incoming red lines and two outgoing red lines connected by a horizontal blue box and a red wavy line.

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Collision term

- Consider 3 types of plasmas:



Isotropic
Initially overocc.



Isotropic
Initially underocc.



Bjorken
expanding

¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Thermalization in isotropic plasmas



- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

First studied in *Phys.Rev.Lett.* 133 (2014) [Kurkela, Lu], see also *Phys.Rev.D* 105 (2022) [Fu, Ghiglieri, Iqbal, Kurkela]

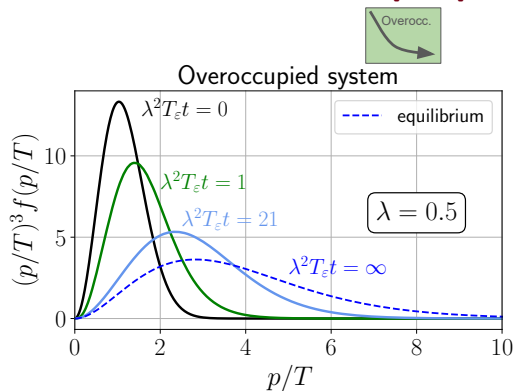
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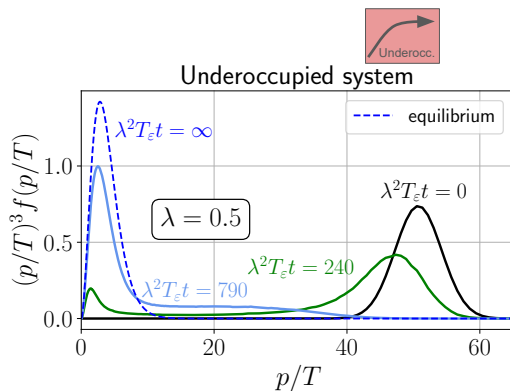
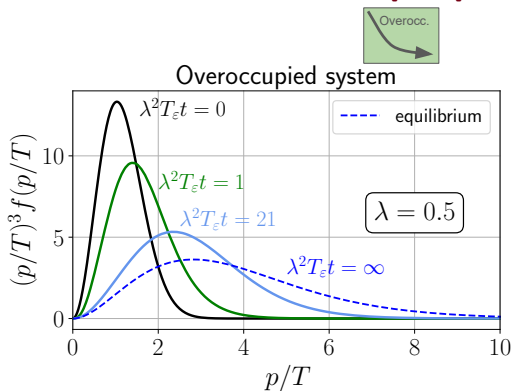
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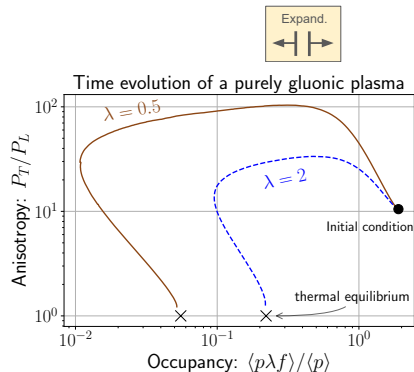
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Bottom-up thermalization in heavy-ion collisions

Initial condition², with $\lambda = g^2 N_C$:

$$f(p_\perp, p_z) = \text{“squeezed” } \frac{1}{\lambda} \times \exp(-p^2) / p$$

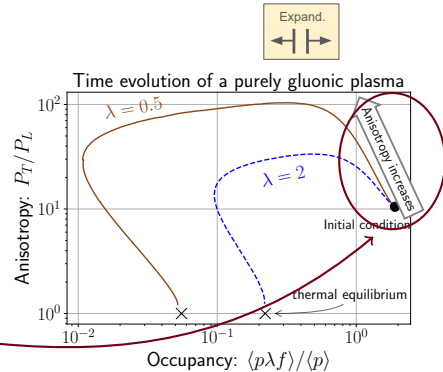


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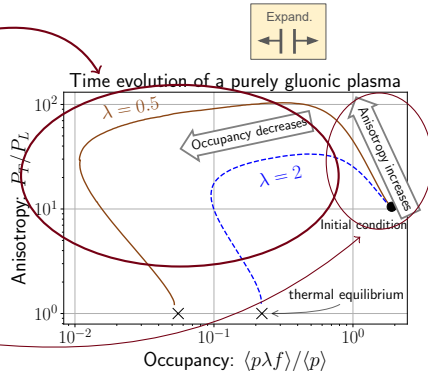


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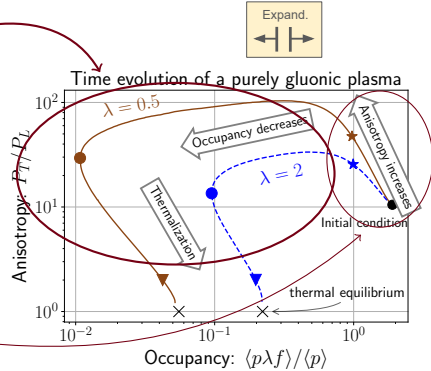
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- **Markers** represent **different stages**
- thermalizes after triangle marker

Phys.Lett.B 502 (2001) [Baier, Mueller, Schiff, Son]

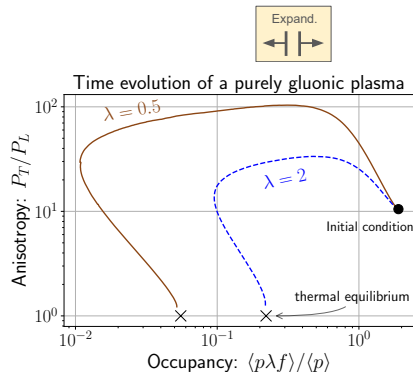
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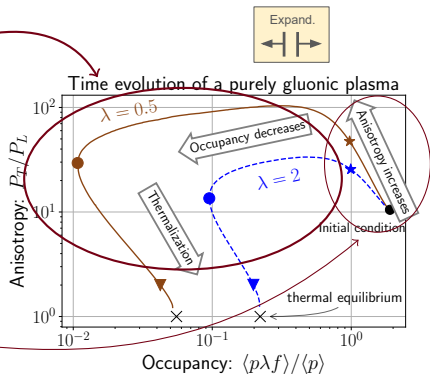
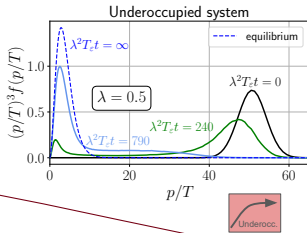
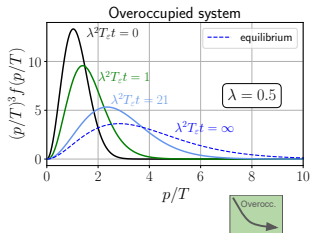


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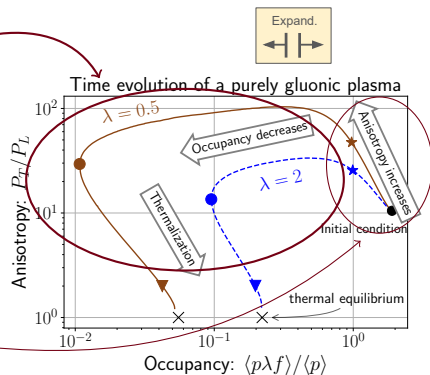
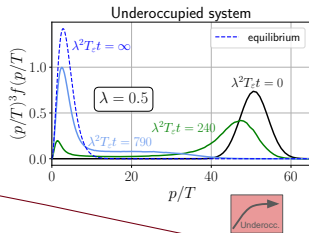
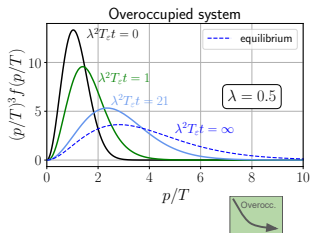


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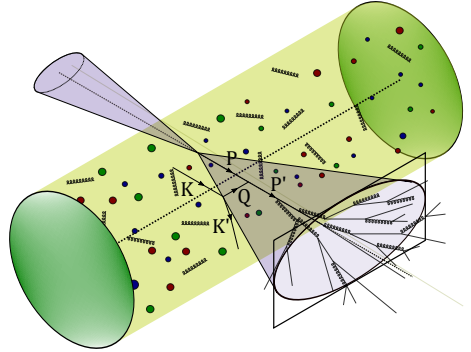
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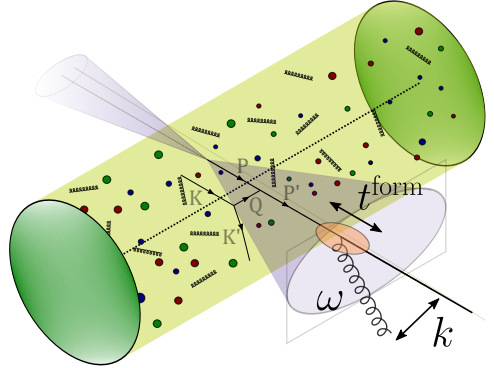
Jets in heavy-ion collisions

- **Jet** = Particle shower from **energetic parton** created in initial collision
- Imprints of **pre-equilibrium plasma**
- Many event generators use simplifying assumptions (isotropic, thermal, ...)



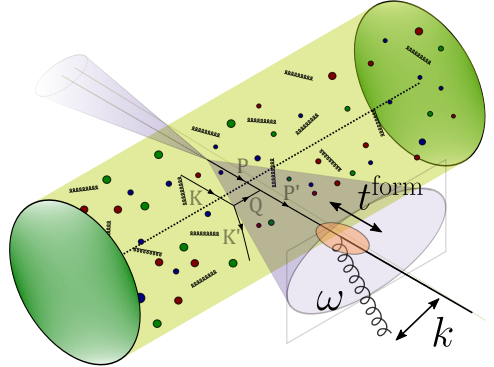
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- Describe **jet quenching** in **out-of-equilibrium plasma**!
- **Energy loss** dominated by **gluon radiation**
 - $dI/d\omega$: **Spectrum**
=Probability of emitting gluon with energy ω (relative to vacuum)

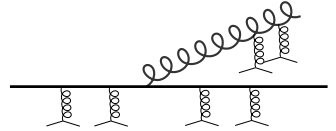


- ω : emitted gluon energy
- k : transverse momentum
- $t^{\text{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
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Spectrum $\omega \frac{dI}{d\omega} \sim$ from solution to 2D Schrödinger equation

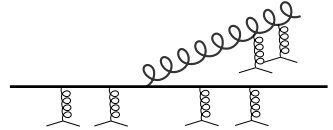


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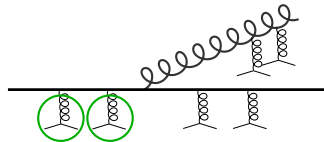
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- Input: **dipole cross section** $C(\mathbf{x}, t)$ = potential
- **Harmonic approximation** (for jets $|\mathbf{x}| \ll 1$ important!)

$$C(\mathbf{x}, t) = \frac{1}{4} \hat{q}(t) \mathbf{x}^2, \quad \hat{q} = \frac{d\langle p_{\perp}^2 \rangle}{dL} = \frac{d\langle p_{\perp}^2 \rangle}{dt} = \int d^2 q_{\perp} \, q_{\perp}^2 \frac{d\Gamma^{\text{el}}}{d^2 q_{\perp}}$$

- $\hat{q}$ quantifies **momentum broadening**, “jet quenching parameter”
- Schrödinger equation for harmonic oscillator $\rightarrow$ analytic solution!

scattering rate
 probability of receiving
 $q_{\perp}$ kick

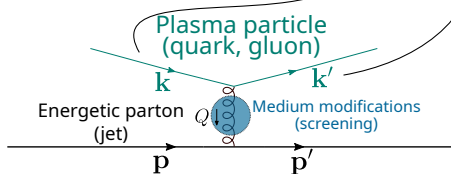


Results for $\hat{q}$: Broadening

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- Obtain **Jet quenching parameter** from kinetic theory, input $f(\mathbf{k})$,

$$\hat{q}^{ij} = \int_{q_{\perp} < \Lambda_{\perp}}^{\vec{p} \rightarrow \infty} d\Gamma \, q^i q^j |\mathcal{M}|^2 f(k)(1 + f(k'))$$



Note: Isotropic screening to prevent plasma instabilities! Phys.Lett.B 214 (1988) [Mrowczynski],
Phys.Rev.D 68 (2003) [Romatschke, Strickland]

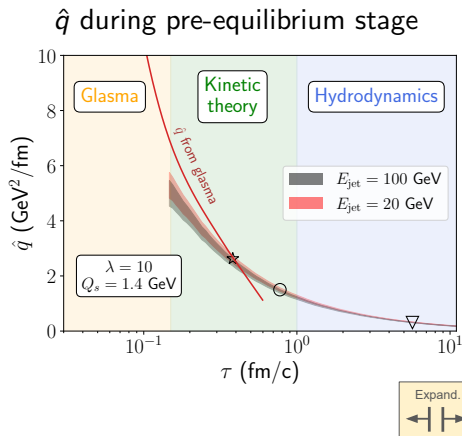
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- **Bands:** Vary cutoff and initial conditions
- Little jet energy dependence



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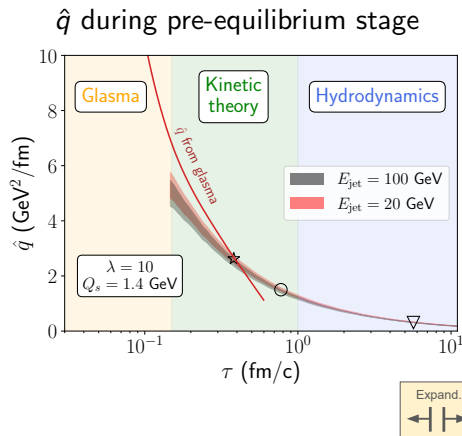
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- Supports hydrodynamic values and **large values** in **Glasma**

Glasma values from Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh]]



Problems with harmonic approximation

- Actually, harmonic approximation $C(\mathbf{x}, t) = \frac{1}{4}\hat{q}(t)\mathbf{x}^2$ not very good

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Can we do better? $\rightarrow$ Consider full kernel $C(\mathbf{q}_{\perp}, t)$ for $C(\mathbf{x}, t)$

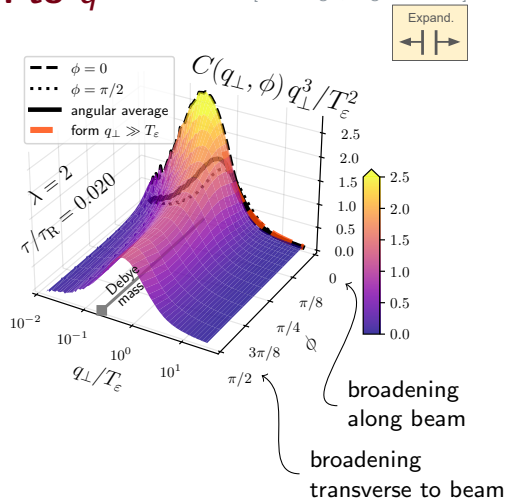
Angular dependence and contribution to $\hat{q}$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

- Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int d\Gamma_{\text{PS}} |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k} - \mathbf{q}))$$

- $C(q_\perp, \phi)$ is 2D function



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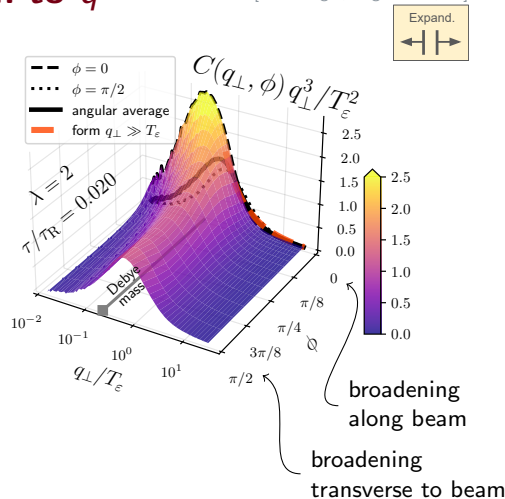
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- More differential than $\hat{q}$,

$$\hat{q}^{ij} = \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times

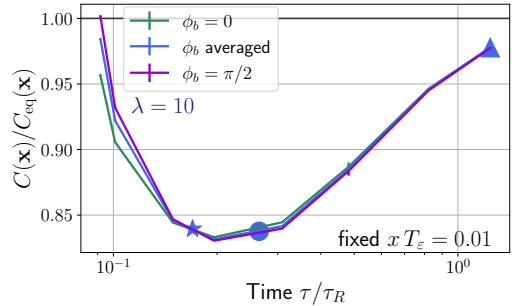


Small-distance behavior of $C(\mathbf{x})$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]



- Obtained via Fourier transform
$$C(\mathbf{x}) = \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^3} (1 - e^{i \mathbf{q}_\perp \cdot \mathbf{x}}) C(\mathbf{q}_\perp)$$
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium



→Effect of anisotropy: Poster F. Lindenbauer, We 16:00

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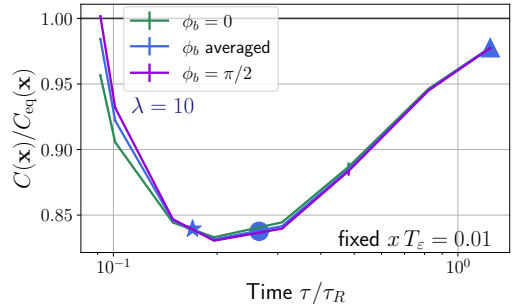
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- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium
- Take small distance behavior and calculate jet emission spectrum

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

→ Effect of anisotropy: Poster F. Lindenbauer, We 16:00



Small-distance form of $C(x)$

- Proper expansion for small distance:

$$C(\mathbf{x}, t) = \frac{1}{4} \hat{q}_0(t) \mathbf{x}^2 \log \frac{1}{\mathbf{x}^2 \mu_*^2(t)}$$

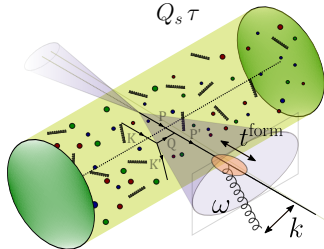
medium input,
obtained from $\hat{q}(\Lambda_\perp)$

- **Expand** perturbatively **around harmonic oscillator solution**

→ “improved opacity expansion” JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

$$C(\mathbf{x}, t) = \underbrace{\frac{\hat{q}_0(t)}{4} \mathbf{x}^2 \log \frac{Q^2}{\mu_*^2}}_{\text{harmonic approximation}} - \underbrace{\frac{\hat{q}_0(t)}{4} \mathbf{x}^2 \log(Q^2 \mathbf{x}^2)}_{\text{perturbation}}$$

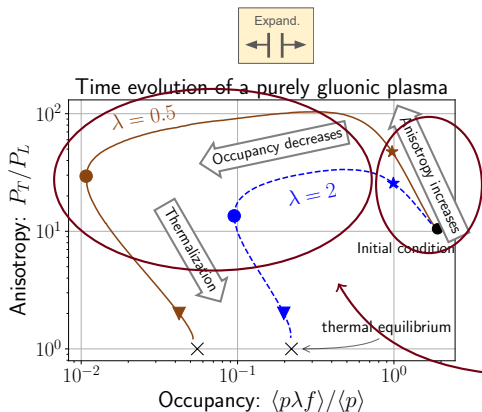
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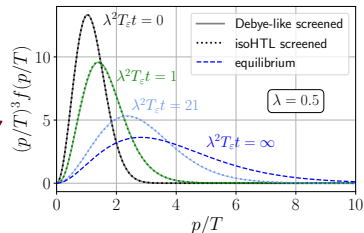
“Improved opacity expansion”

Output: Spectrum $\frac{dI}{d\omega d^2\mathbf{k}}$

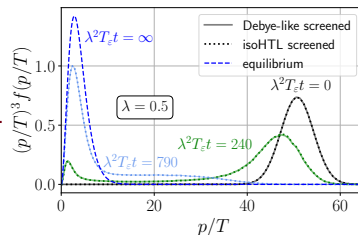
Recall: Bottom-up thermalization



Overoccupied, initial $n > n_{therm}$

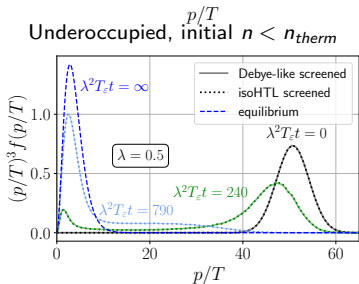
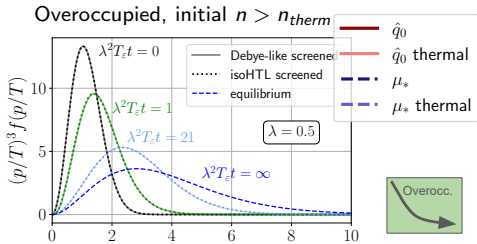
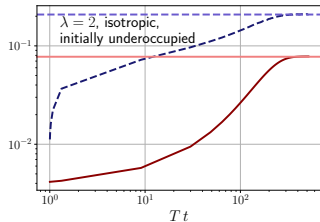
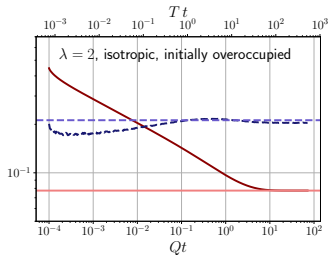


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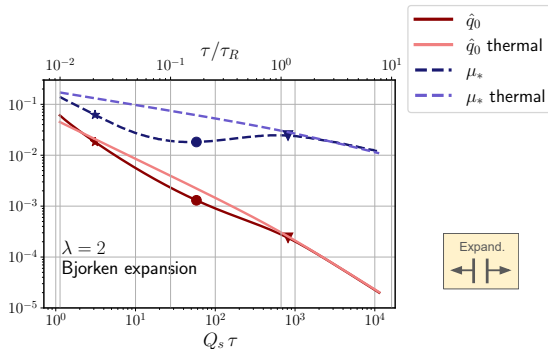
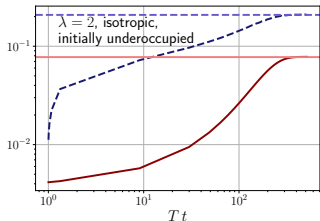
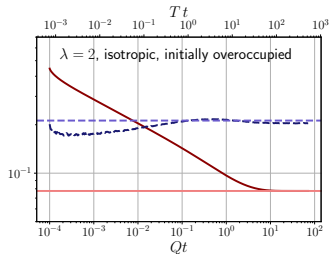
Isotropic toy models

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]



Comparison with isotropic toy models

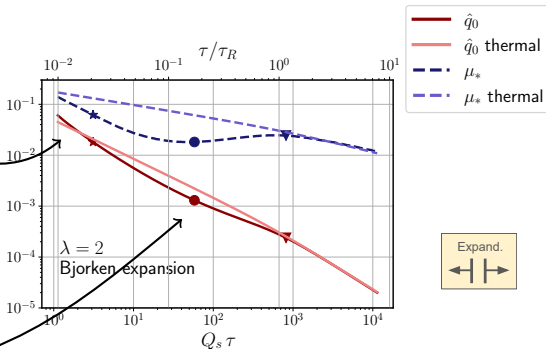
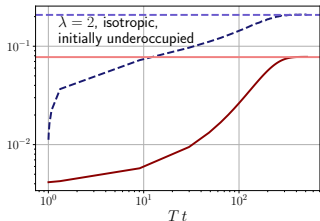
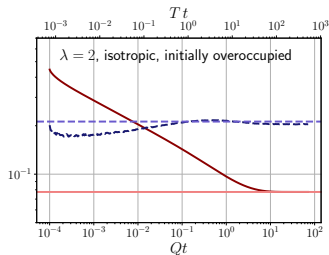
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

Comparison with isotropic toy models

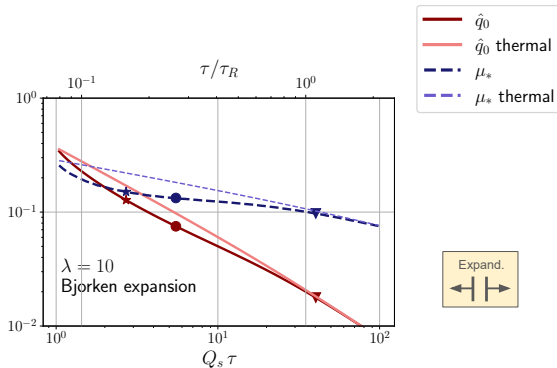
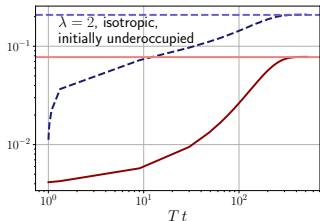
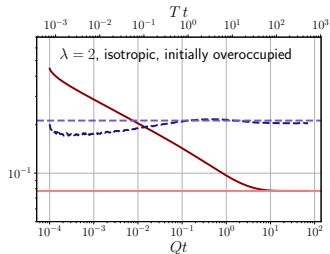
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Gluon spectrum

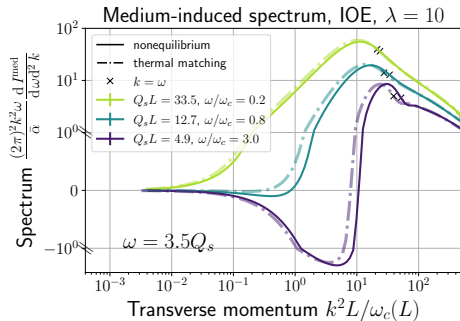
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

Expand.



- $\hat{q}_0(t)$, $\mu_*(t) \rightarrow$ spectrum $dI/d\omega$
- Consider different medium lengths (colors in plot: green, blue purple)
- Set $\hat{q}_0(t) = 0$ for $t > L$
- Form of spectrum dominated by ratio ω/ω_c ,

$$\omega_c = \int_{t_{min}}^{t_{min}+L} dt (t - t_{min}) \hat{q}(t)$$
- All information, difficult to analyze



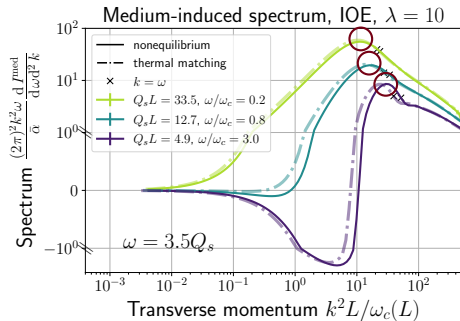
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Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



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Comparison to equilibrium

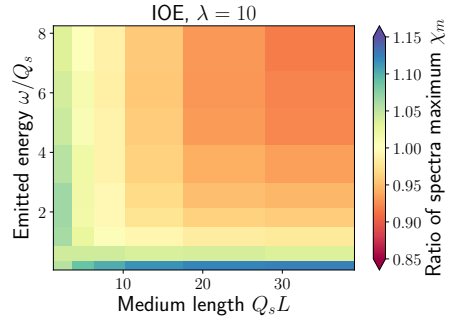
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



- Comparison to equilibrium: Maximum of spectrum plot

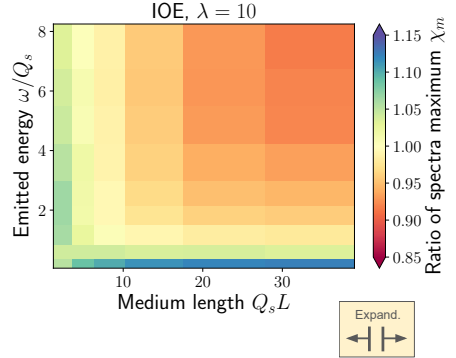
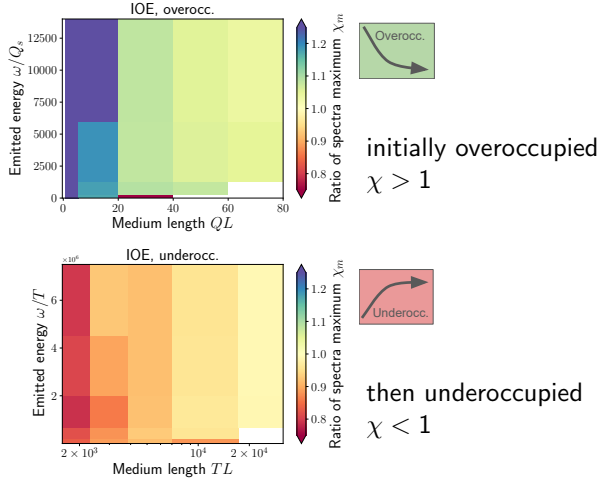
$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

- Clear systematics visible
- Does not approach 1 even for $L \rightarrow \infty$!



Comparison to equilibrium II

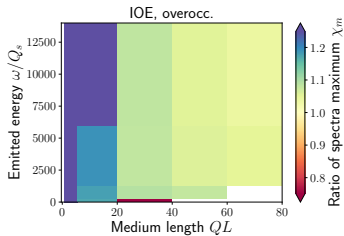
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



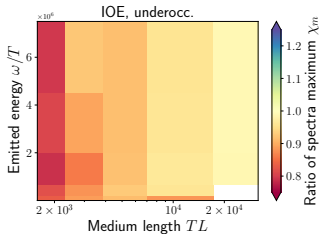
- For fixed L :
 $\chi > 1 \rightarrow \chi < 1$
- Can be explained with toy models

Comparison to equilibrium II

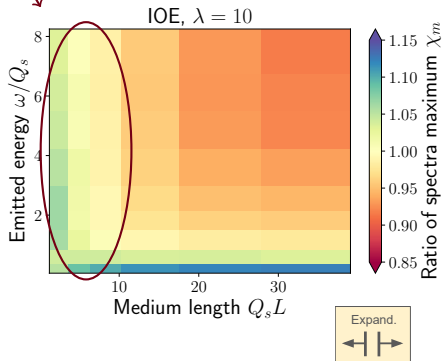
Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
 $\chi > 1$



then underoccupied
 $\chi < 1$

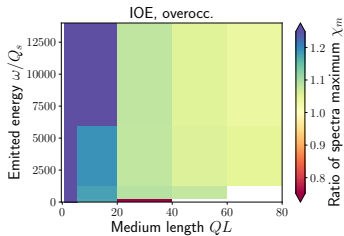


■ For fixed L:
 $\chi > 1 \rightarrow \chi < 1$

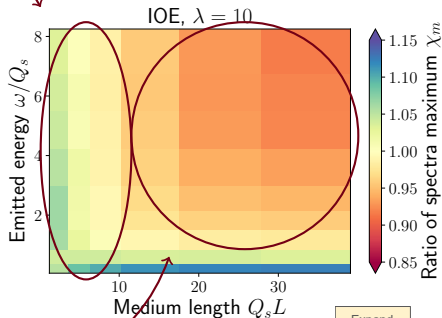
→ Can be explained with toy models

Comparison to equilibrium II

Phys.Rev.D 113 (2026) [Barata, Boguslavski, FL, Sadofyev]

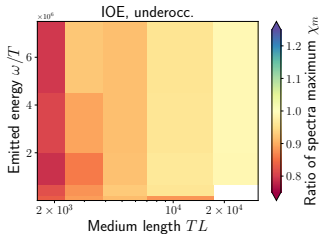


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 $\chi > 1$



■ For fixed L:
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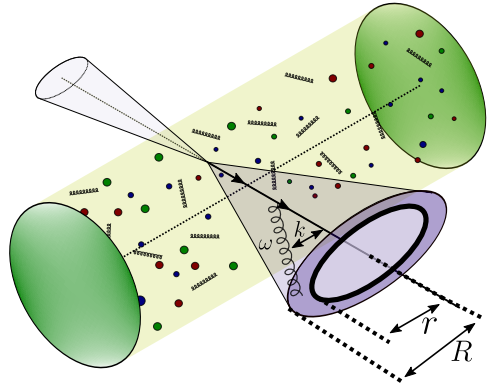
→ Can be explained with toy models



then underoccupied
 $\chi < 1$

- Energy deposited in *annulus* with radii r and R

$$\frac{d\rho(r, R)}{d\omega} = \int_{r\omega}^{R\omega} d^2\mathbf{k} \omega \frac{dI}{d\omega d^2\mathbf{k}}$$



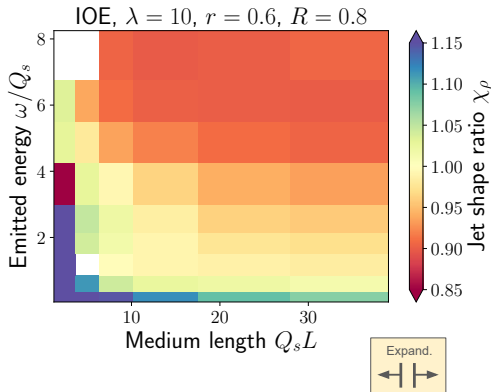
Jet shape

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

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$$\frac{d\rho(r, R)}{d\omega} = \int_{r\omega}^{R\omega} d^2\mathbf{k} \omega \frac{dI}{d\omega d^2\mathbf{k}}$$

- Qualitatively same behavior as spectra maximum



Conclusions



- Use **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
- Calculate **momentum broadening of jets** $\rightarrow \hat{q}(t)$ and $C(\mathbf{q}_\perp, t)$
- Taken as input to calculate **gluon emission spectrum**
- Clear differences to thermal medium evolution, even at **large medium lengths**

Outlook

- Prediction/effects for specific more differential observables
- General structure and scaling of spectra?
- Improve QCD kinetic theory simulations of the background screening/anisotropic splitting $\rightarrow$ See also: Poster F. Lindenbauer, We 16:00 with machine learning

Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

and ongoing work with Barrera Cabodevila, Kurkela, Mazeliauskas, Taghavi, Zhou

Thank you very much for your attention!

Observables in kinetic theories

- Fundamental quantity: Distribution function $f(\mathbf{p})$

- **Energy-Momentum tensor:**

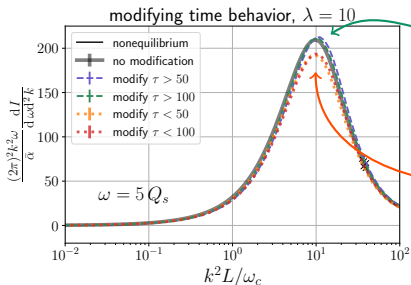
$$T^{\mu\nu} = \nu_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p}) \quad \leftarrow \text{“Moments of distribution function”}$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- How many ‘hard’ particles

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int d^3\mathbf{p} p f(\mathbf{p})^2}{\int d^3\mathbf{p} p f(\mathbf{p})} \quad \leftarrow \text{“particle number weighted with energy”}$$

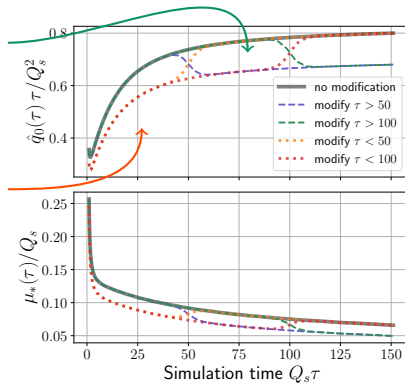
Effect of modifications

Gluon spectrum



- Only modifications at early time influence spectrum!

Input $\hat{q}_0$ and μ_*



Spectrum more explicit

IOE Spectrum is obtained via

$$\frac{dI^{(0)}}{d\omega d^2k} = \frac{2\bar{\alpha}}{\pi\omega} \text{Re} \left\{ k^{-2} \left(e^{-i\frac{k^2}{2\omega\text{Cot}(L,t_0)}} - 1 \right) + \int_{t_0}^L dt \frac{\text{Cot}(t,t_0)}{\hat{P}^2(t,t_0)} e^{-\frac{k^2}{\hat{P}^2(t,t_0)}} \right\},$$

$$\begin{aligned} \frac{dI^{(1)}}{d\omega d^2k} = & \frac{\bar{\alpha}\pi}{k^4} \text{Re} \int_{t_0}^L dt_1 \left\{ \frac{i}{2\omega} \int_{t_0}^{t_2} dt_1 \frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2} e^{-\frac{k^2}{\hat{R}_{21}^2}} \left[Q_s^2(L, t_2) I_a \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) + 2C_{12} \hat{R}_{21} k^2 I_b \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) \right] \right. \\ & \left. - \text{Cot}_{20} Q_{s0}^2(L, t_2) I_a \left(\frac{k^2}{\hat{P}^2(t_2, t_0)}, \frac{k^2}{Q_b^2} \right) + 2\hat{q}_0(t_2) C^2(t_2, L) I_b \left(\frac{ik^2}{2\omega C^2(t_2, L) \left(\text{Cot}_{20} - \frac{1}{C(t_2, L) S(L, t_2)} - \text{Cot}(t_2, L) \right)}; \frac{k^2}{Q_r^2 C^2(t_2, L)} \right) e^{-\frac{ik^2}{2\omega\text{Cot}(L,t_2)}} \right\} \end{aligned}$$

medium input,
obtained from $\hat{q}(\Lambda_{\perp})$

scale,
obtained self-consistently

And requires solving a differential equation

$$\left[\frac{d^2}{dt^2} + \Omega^2(t) \right] F(t, t_0) = 0, \quad \Omega^2(t) = -i \frac{\hat{q}_r(t)}{\omega}, \quad \hat{q}_r(t) = \hat{q}_0(t) \log \frac{Q_r^2}{\mu_*^2(t)}$$

with specific boundary conditions: $S(t_0, t_0) = \partial_t C(t, t_0)_{t=t_0} = 0$,
 $\partial_t S(t, t_0)_{t=t_0} = C(t_0, t_0) = 1$

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Introduce cutoff

$$C(x) = \underbrace{\int_{|\mathbf{q}_\perp| < \Lambda} \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp)}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots} \underbrace{(1 - e^{i\mathbf{x} \cdot \mathbf{q}_\perp})}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots} + \underbrace{\int_{|\mathbf{q}_\perp| > \Lambda} \frac{d^2 \mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp) (1 - e^{i\mathbf{x} \cdot \mathbf{q}_\perp})}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots}$$

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- **Expand** perturbatively **around harmonic oscillator solution**

→ “improved opacity expansion” JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

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$\hat{q}_0$ and μ_* from EKT

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

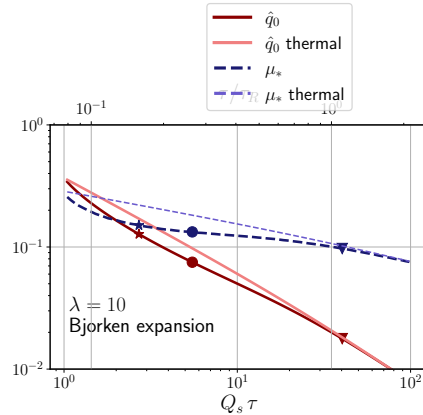
- Recall: $\hat{q}_0$, μ_* characterize small- x of $C(x)$,
- Get $\hat{q}_0$ from large-cutoff behavior,
 $\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$
- Get μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$

- Thermal:

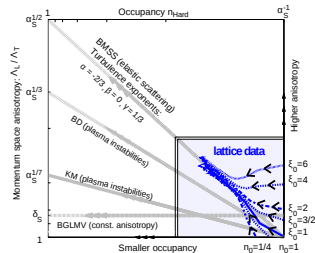
$$\hat{q}_0 = T^3 \frac{N_C C_R g^4 \zeta(3)}{2\pi^3}$$

$$\mu_*^2 = \frac{m_D^2}{4} \left(\frac{m_D}{2T} \right)^{2\zeta(2)/\zeta(3)-2} e^{2\gamma_E-2-\left(\frac{\zeta(2)}{\zeta(3)}-1\right)(1-2\gamma_E)+\frac{\sigma_+}{\pi\zeta(3)}}$$



Plasma instabilities

- Anisotropic QCD plasmas generally lead to plasma instabilities Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]
- Soft modes grow exponentially, until perturbation theory (HTL) breaks down
- Although recent progress⁴, no clear avenue for implementation in kinetic theory
- Classical statistical simulations seem to indicate that instabilities not dominant Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]



Phys.Rev.D 89 (2014) [Berges, Boguslavski, Schlichting, Venugopalan]

⁴[Phys.Rev.C 105 (2023) [Hauksson, Jeon, Gale]]

Momentum broadening kernel vs. jet quenching parameter

Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int d\Gamma_{\text{PS}} |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k} - \mathbf{q}))$$

- Similar quantity, less differential,

$$\hat{q}^{ij} = \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

- $C(q_\perp)$ is **more general** than $\hat{q}$

Compare to $\hat{q}$, weighted with momentum transfer

$$\hat{q}^{ij} = \int_{q_\perp \leq \Lambda_\perp} d\Gamma q^i q^j |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k} - \mathbf{q}))$$

different integration measure

momentum cutoff needed

Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]

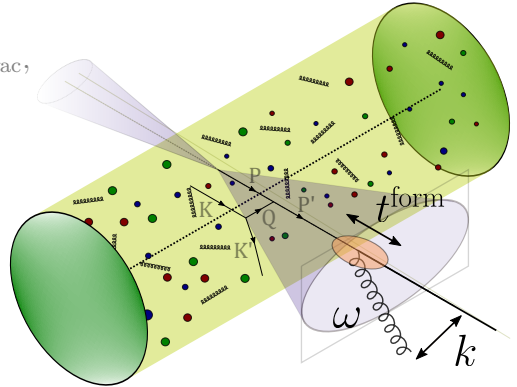
JETP Lett. 65 (1997) [Zakharov]

Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{dI}{d\omega} \sim \text{Re} \int_{t_1, t_2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\text{vac}},$$

- $dI/d\omega$: **Spectrum**
=Probability of emitting gluon
with energy ω (relative to vacuum)
- ω : emitted gluon energy
- k : transverse momentum
- $t^{\text{form}} \sim \sqrt{\omega/\hat{q}}$: formation time



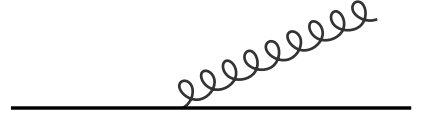
Obtaining the gluon emission spectrum

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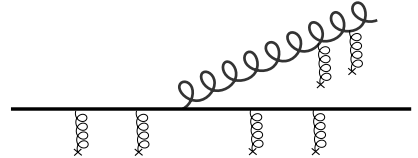
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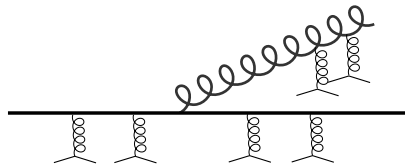
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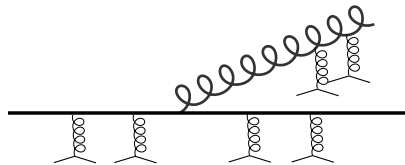
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$$(i\partial_t - \delta E + iC(\mathbf{x}, t)) G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

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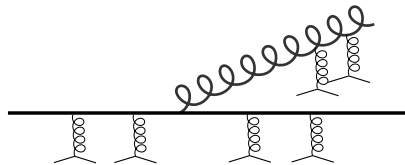
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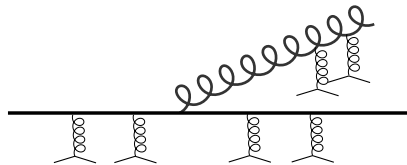
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'broadening probability'
Probability of receiving
momentum kick $\mathbf{q}_{\perp}$

Harmonic approximation for radiation

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scattering rate
probability of
receiving $q_\perp$

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kick

- $\hat{q}$ quantifies **momentum broadening**, “jet quenching parameter”
- Schrödinger equation for harmonic oscillator $\rightarrow$ analytic solution!

Small-distance form of $C(x)$

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Proper expansion for small distance:

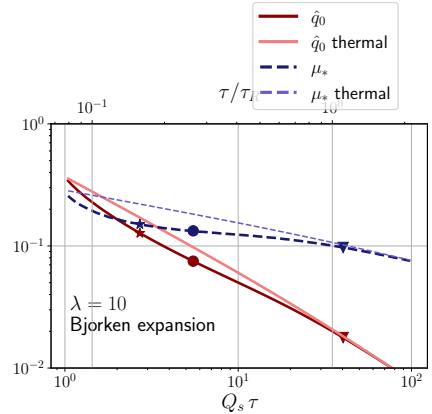
$$C(\mathbf{x}, t) = \frac{1}{4} \hat{q}_0(t) \mathbf{x}^2 \log \frac{1}{\mathbf{x}^2 \mu_*^2(t)} + \mathcal{O}(\hat{q}_0 \mathbf{x}^4 \mu_*^2)$$

- μ_* and $\hat{q}_0$ are medium-parameters uniquely determined from large-cutoff form of $\hat{q}(\Lambda_\perp)$,

$$\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$$

- Obtain μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$



Problems with $\hat{q}$ for radiation

- Recall: $\hat{q}$ appeared in small-distance expansion:

physical quantity,
enters equations
as potential

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Can we do better? $\rightarrow$ Consider full kernel $C(\mathbf{q}_\perp)$

Comparing to thermal system

Phys.Rev.D 113 (2026) 11 [Barata, Boguslavski, FL, Sadofyev]

- Recall: $\hat{q}_0, \mu_*$ obtained from large cutoff behavior of $\hat{q}$
- Compare with an evolving thermal system

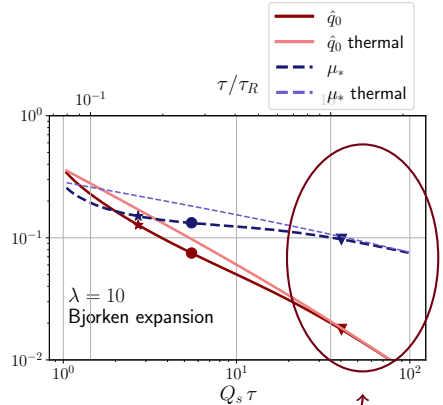
$$T_\epsilon(t) = \left(\frac{30\epsilon(t)}{\pi^2} \right)^{1/4}$$

with energy density from kinetic theory,

$$\epsilon = \int \frac{d^3\mathbf{p}}{(2\pi)^3} |\mathbf{p}| f(\mathbf{p})$$

- Thermal forms of $\hat{q}_0, \mu_*$ known⁵

⁵[Phys.Rev.D 78 (2008) [Arnold, Xiao]]



Late time
approach to thermal