Open quantum system approach to the transverse momentum broadening of a colour dipole

Pietro Benzoni (Subatech, Nantes)

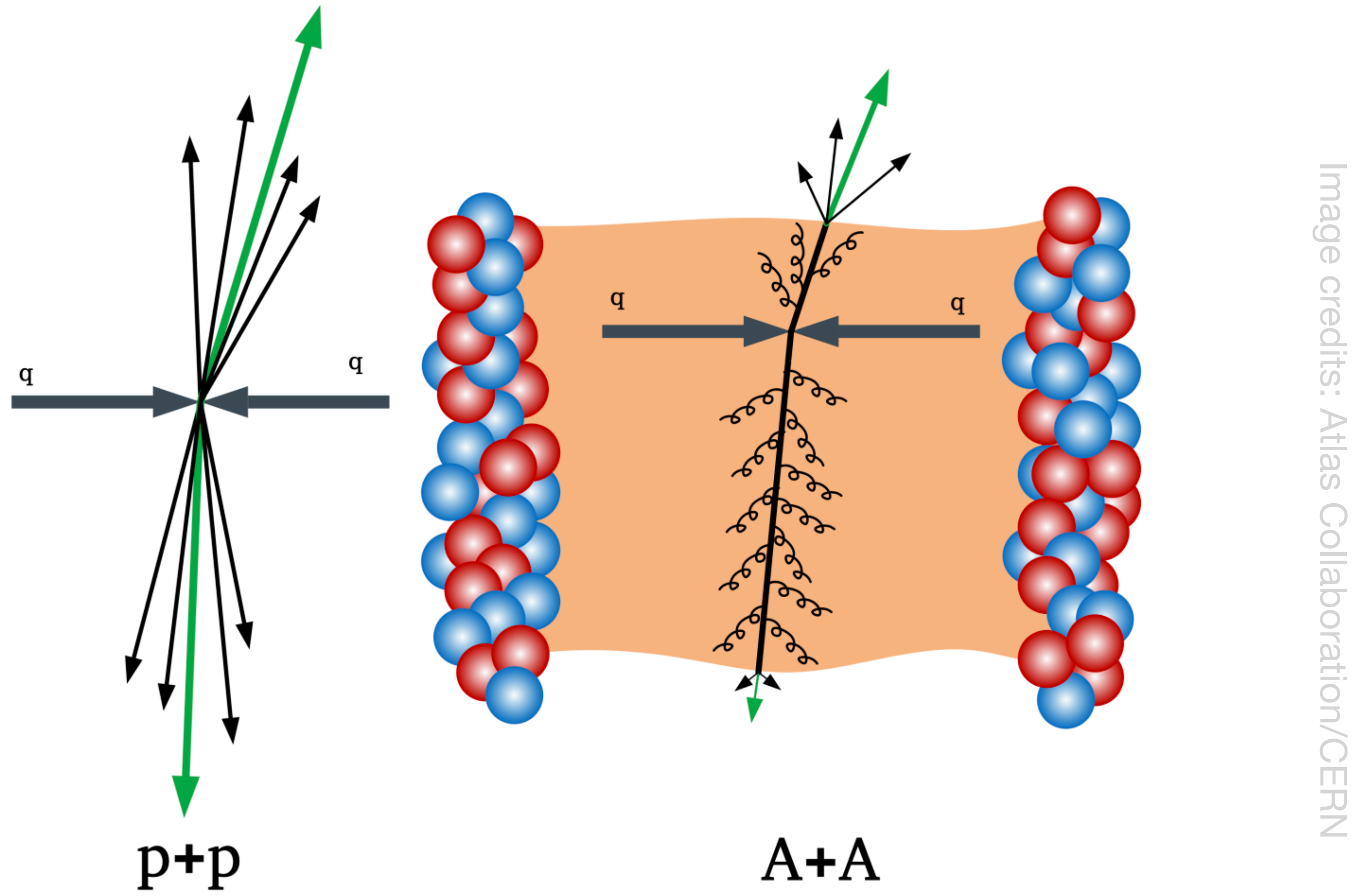
based on

F. Arleo, PB, P. Caucal, P. B. Gossiaux, 2606.07744

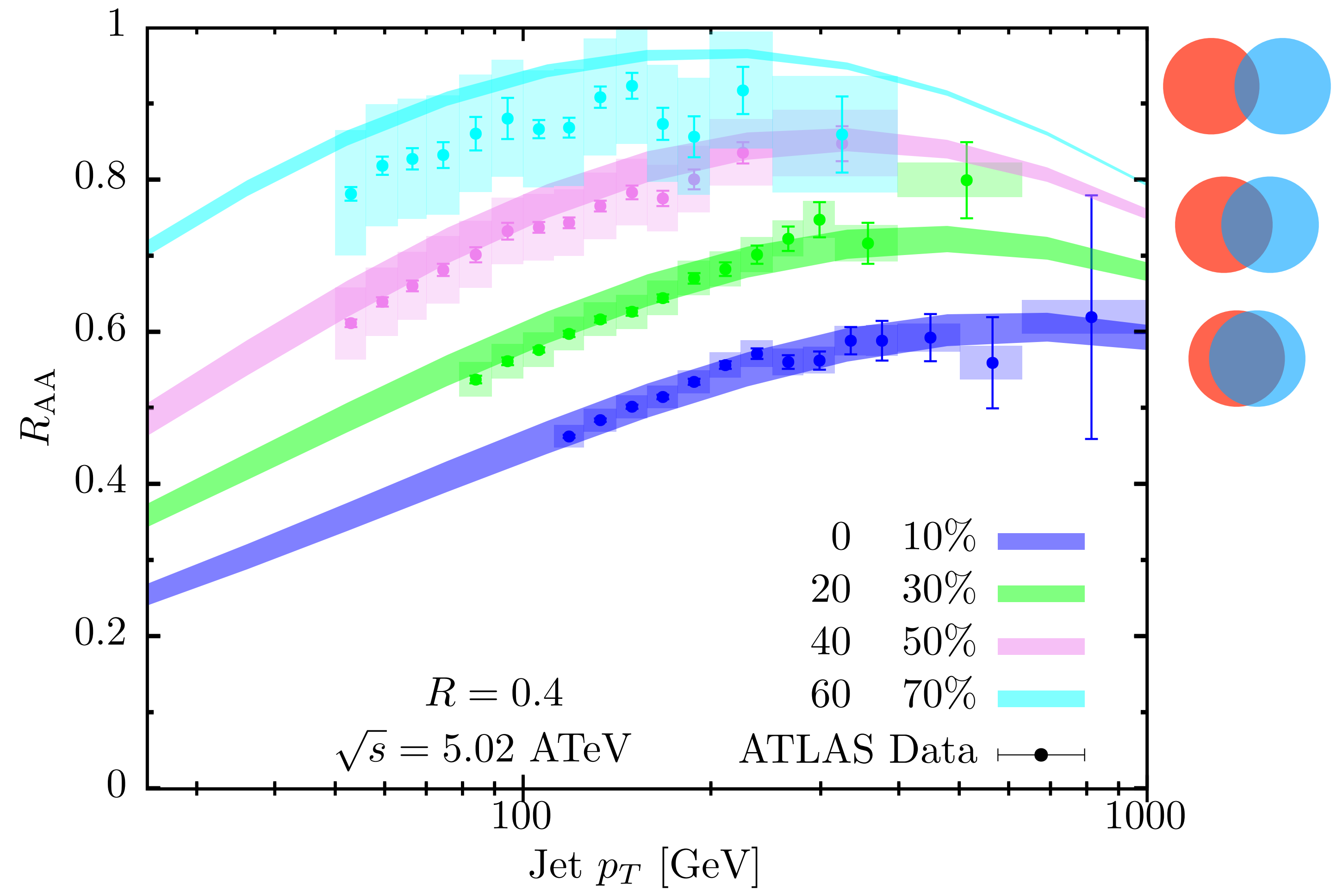
17th August 2026, Strong and Electro-Weak Matter 2026, Helsinki

Jets as QGP probes

- Observe **jet quenching** effects by comparing AA and pp collisions Mehtar-Tani, Pablos, Tywoniuk (2021)



Ratio between **heavy-ion** and **protons** collisions:

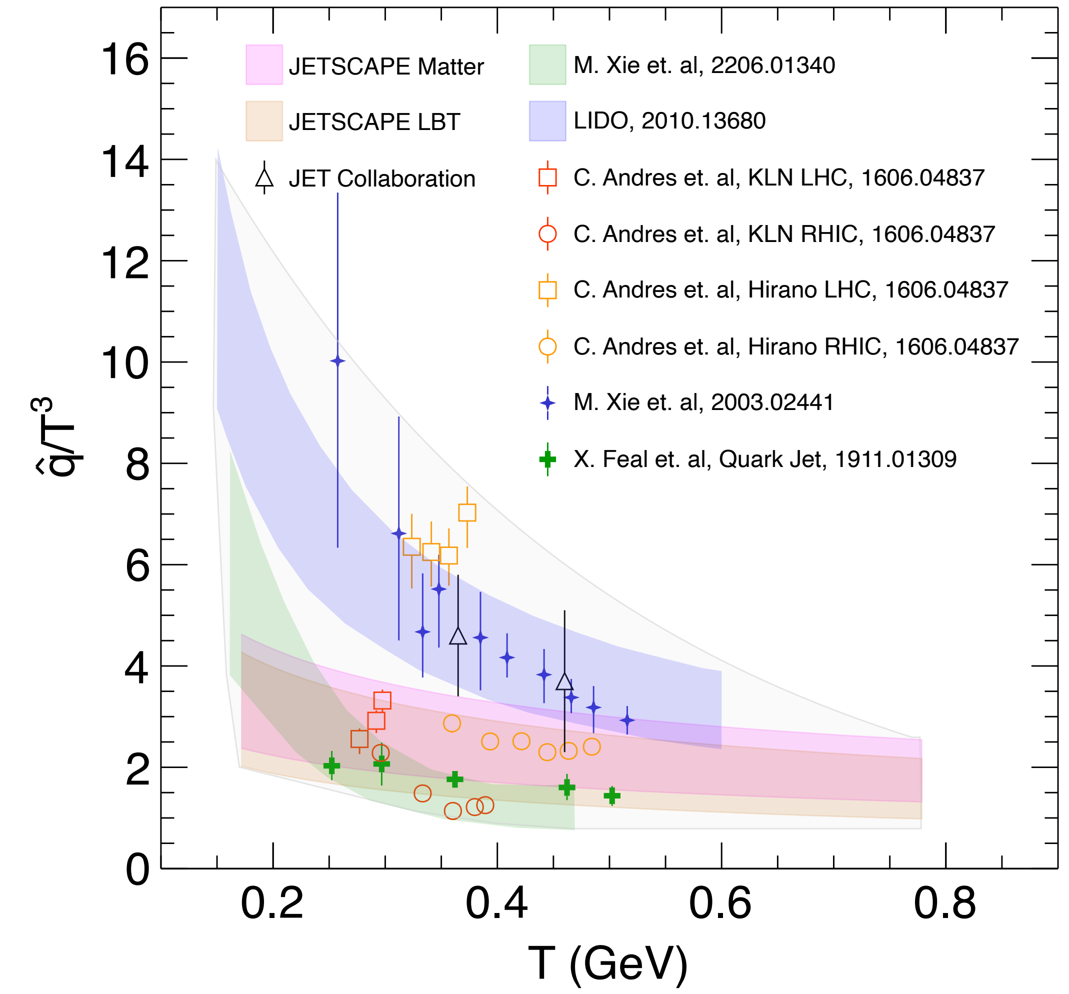


infer properties of the quark-gluon plasma (QGP) ! [Hannah's talk this morning]

→ **theoretical uncertainties**: depending on

- initial states
- QGP
- jet-medium interaction
- …

modelling



Apolinário et al. (2022)

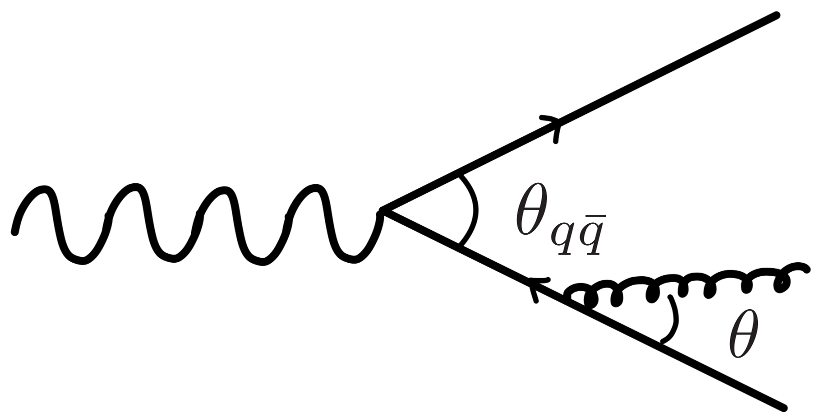
Jets as QGP probes

- Study of **jet interaction with the QCD medium (QGP)**:
  - BDMPS-Z effect: momentum broadening and **medium-induced radiation**
  - **colour (de)coherence** $\longrightarrow$ (anti-)angular ordering:

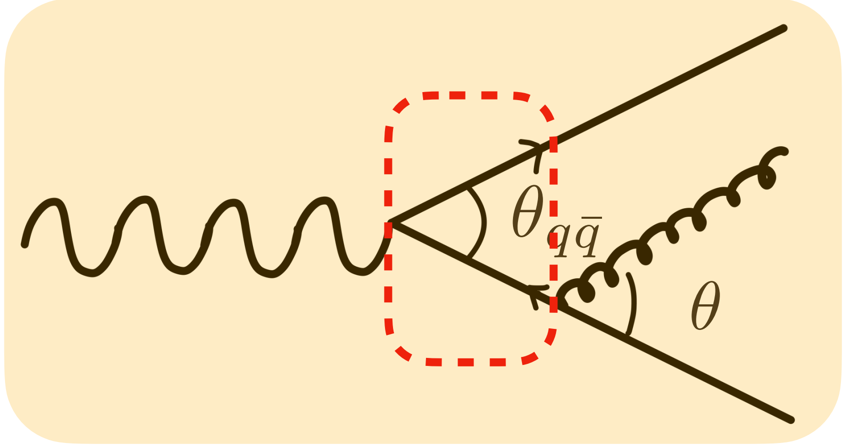
Colour coherence (vacuum)

Colour decoherence (in-medium)

Soft (low energy) gluon radiation:



$\theta < \theta_{q\bar{q}}$



otherwise, the gluon **resolves** the full $q\bar{q}$ dipole

$\longrightarrow$ **suppressed**

Dokshitzer, Khoze, Mueller, Troyan (1991)

radiation **also** for $\theta > \theta_{q\bar{q}}$

$\longrightarrow$ **enhancement** of large angle radiation

Mehtar-Tani, Salgado, Tywoniuk (2011-2013)
Casalderrey-Solana, Iancu (2011)

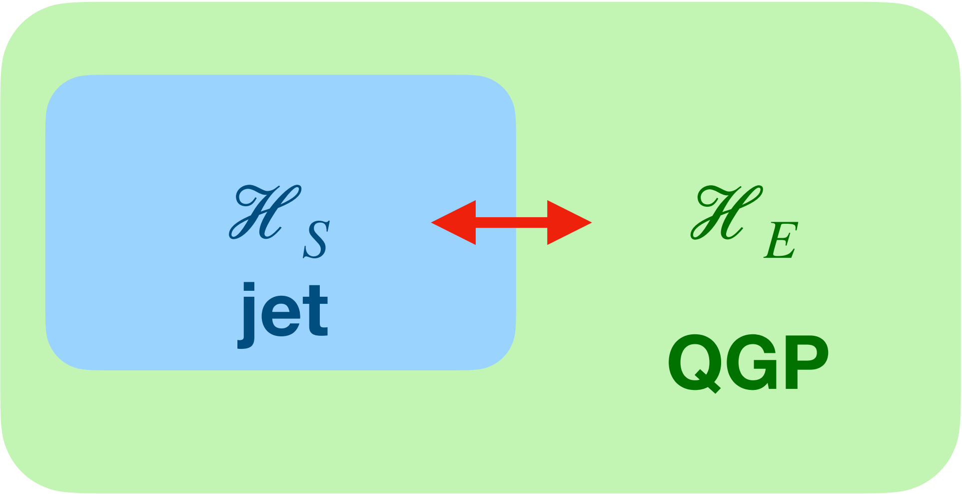
the **probability of the dipole to be resolved and decohere** plays a role in the radiation spectrum

$\longrightarrow$ let's study the colour dipole in a framework where **decoherence** is **naturally encoded**

Jets as Open Quantum Systems

- We can study these phenomena with a suitable framework: **open quantum systems**!
  - → several applications in HEP: quarkonia in QGP, neutrino physics, cosmology, …

[previous talks: Panayotis, Miguel]



$$\rho = \rho_S \otimes \rho_E$$

- Coupling with an environment modifies the subsystem and "steals" quantum correlations from it
  - → quantify **momentum broadening** and **decoherence**

e.g. spin state decoherence

$$|\psi\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$$\rho = |\psi\rangle\langle\psi|$$

probabilities

coherences

$$\rho(t=0) = \frac{1}{2} \begin{array}{c} \begin{array}{cc} |\uparrow\rangle & |\downarrow\rangle \end{array} \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{array} \begin{array}{c} \langle\uparrow| \\ \langle\downarrow| \end{array} \xrightarrow[\text{magnetic field}]{\text{stochastic}} \rho(t \to \infty) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

A simple jet as an OQS: a $q\bar{q}$ dipole

- We study the case of a **boosted $q\bar{q}$ dipole** (no radiation, *yet*):

- Learning from…
  - **Heavy quark-antiquark** (quarkonium) OQS
    Blaizot, Escobedo (2018)
  - **Single quark jet** OQS
    Barata, Blaizot, Mehtar-Tani (2023)

heavy mass → **non-relativistic** description $\xrightarrow{\text{NRQCD}}$ in 3+1 dimensions

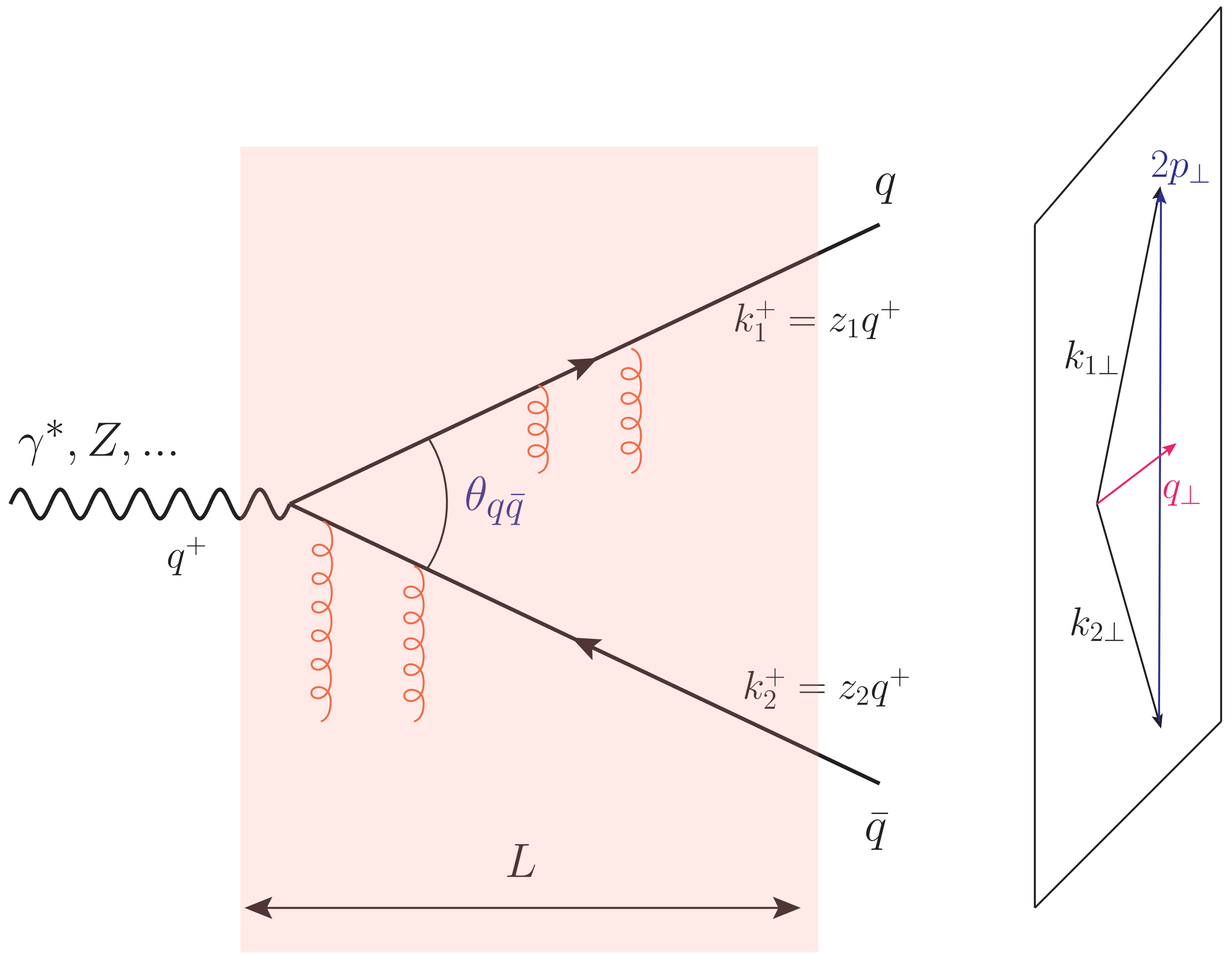
high energy → **non-relativistic** description → **in 2+1 dimensions**

- Let's consider the following kinematical configuration:

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_1^+}$$
$$q^+ = k_1^+ + k_2^+$$

$$\underbrace{E \sim q^+ \gg \boldsymbol{p}_\perp}_{\textit{boosted}} \qquad \underbrace{\boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp \sim Q_s = \sqrt{\hat{q}L}}_{\textit{back-to-back}}$$

longitudinal momentum ($E \sim q^+$); relative transverse momentum ($\boldsymbol{p}_\perp$); center-of-mass transverse momentum ($\boldsymbol{q}_\perp$)



⟶ meaningful for (e.g. LHC) phenomenology

Derivation of the master equation

Hamiltonian

kinetic term | interaction | QGP

$$H = \left(\frac{\hat{p}^2_{\perp,q}}{2k_1^+} + \frac{\hat{p}^2_{\perp,\bar{q}}}{2k_2^+}\right) \otimes \mathbb{1}_E - g \int \mathrm{d}^2\boldsymbol{x}_\perp \; \hat{n}^a(\boldsymbol{x}_\perp) \otimes A_a^-(t, \boldsymbol{x}_\perp) + \mathbb{1}_S \otimes H_{\mathrm{pl}}$$

number density: $\hat{n}^a(\boldsymbol{x}_\perp) = \underbrace{\delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,q})t^a \otimes \mathbb{1}_{\bar{q}}}_{\text{quark}} - \underbrace{\mathbb{1}_q \otimes \delta(\boldsymbol{x}_\perp - \hat{x}_{\perp,\bar{q}})\tilde{t}^a}_{\text{antiquark}}$

Partial trace

(unitary)

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i\,[H, \rho]$$

Von Neumann's equation

Partial trace over environment

$$\mathrm{Tr}_E\,(\cdot)$$

$$\rho = \underbrace{\rho_S}_{\text{system}} \otimes \underbrace{\cancel{\rho_E}}_{\text{environment}}$$

thermal equilibrium state

$$\rho_E = \frac{1}{Z} e^{-\beta H_{\mathrm{pl}}}$$

Lindblad's equation

$$\frac{\mathrm{d}\rho_S}{\mathrm{d}t} = -i\,[H_0, \rho_S] + \mathcal{D}(\rho_S)$$

(non-unitary)

interactions with the medium:

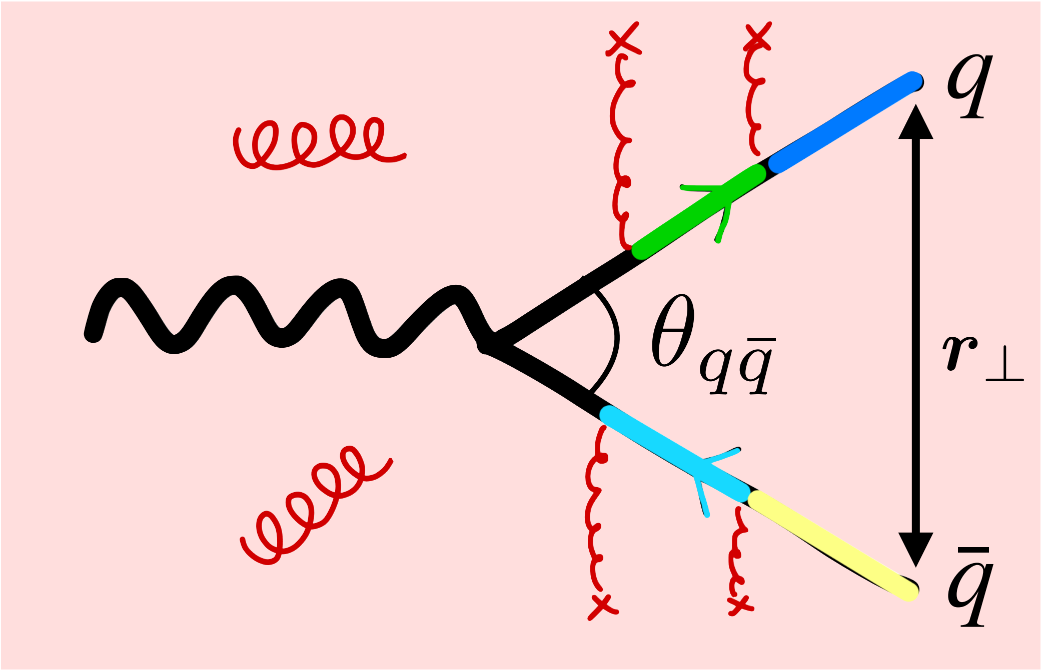
→ **imaginary potential** W describing the **collisions** with the QGP

Derivation of the master equation

Colour structure

$$\rho = \rho_s \left| s \right\rangle \left\langle s \right| + \rho_o \sum_{c=1}^{N_c^2-1} \left| o^c \right\rangle \left\langle o^c \right|$$

singlet octet



ρ_s

$\updownarrow$

ρ_o

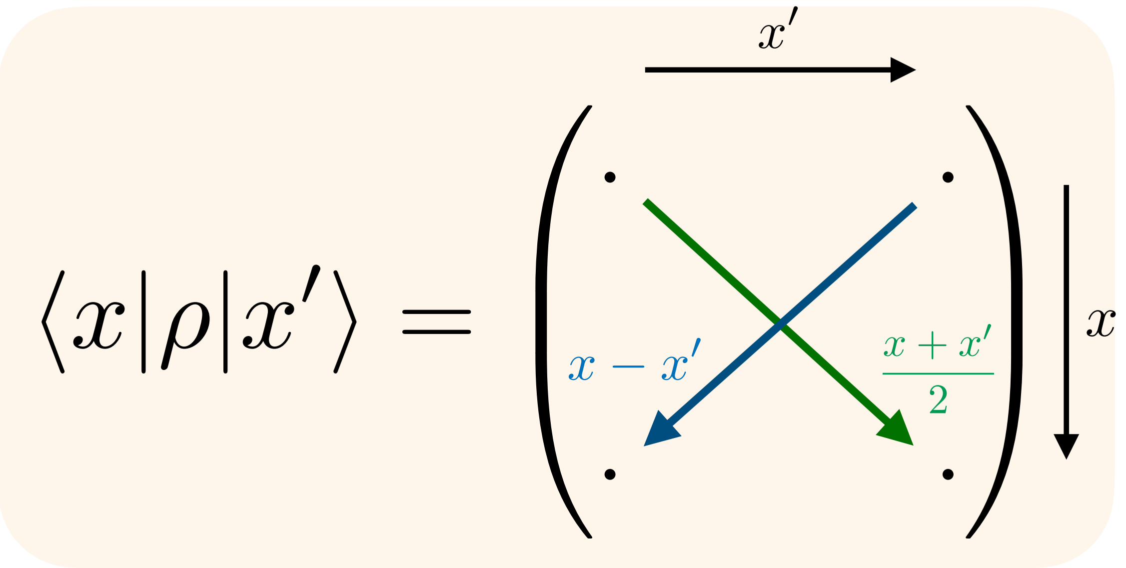
$$\theta_c = \frac{2}{\sqrt{\hat{q}L^3}}$$

resolution angle
of the medium

Wigner function

- We study the evolution of the matrix elements:

$$\langle \boldsymbol{r_{1\perp} r_{2\perp}} | \rho | \boldsymbol{r_{1\perp}}' \boldsymbol{r_{2\perp}}' \rangle = \rho(\boldsymbol{r_{1\perp} r_{2\perp}}; \boldsymbol{r_{1\perp}}' \boldsymbol{r_{2\perp}}')$$

- Introduce new variables:
  - Two-body problem: relative and center-of-mass
  - 

- Wigner function:

$$\xrightarrow{\text{partial Fourier transforms}} \boxed{\rho(\boldsymbol{r}_\perp, \boldsymbol{p}_\perp, \boldsymbol{b}_\perp, \boldsymbol{q}_\perp)}$$

relative c.o.m.

Master equation

- **Harmonic approximation** of the potential: $W \xrightarrow[\text{harm. approx.}]{\text{HTL}} \propto \hat{q}\,\hat{\mathcal{O}}\left[\boldsymbol{r}_\perp^2, \nabla^2_{\boldsymbol{p}_\perp}, \nabla^2_{\boldsymbol{q}_\perp}\right]$

jet quenching parameter

$\boldsymbol{r}_\perp$ relative position
$\boldsymbol{b}_\perp$ impact parameter
$\boldsymbol{p}_\perp$ relative
$\boldsymbol{q}_\perp$ center-of-mass
} transv. mom.

- We have a **master equation** for the Wigner function:

Colour factors:
N_c # of colours
$C_F = \frac{N_c^2 - 1}{2N_c}$

kinetic term — CoM diffusion — colour transitions

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} + \left(\frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp \cdot \nabla_{\boldsymbol{b}_\perp}\right)\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} = \frac{\hat{q}}{4C_F}\left\{\begin{pmatrix}0 & 0 \\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp} + \begin{pmatrix}-2C_F & 2C_F \\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}_\perp^2 + \begin{pmatrix}\frac{C_F}{2} & \frac{C_F}{2} \\ \frac{1}{4N_c} & \frac{C_F}{2} - \frac{1}{2N_c}\end{pmatrix}\nabla^2_{\boldsymbol{p}_\perp}\right\}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix}$$

relative diffusion

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_2^+}$$
$$q^+ = k_1^+ + k_2^+$$

symmetric splitting:
$k_1^+ = k_2^+$

quantum diffusion:

It causes *diffusion* in $\boldsymbol{p}_\perp, \boldsymbol{q}_\perp$ and *decoherence*

rescaling of variables
→ **relative diffusion**
operator **scales** as

$$\kappa = \frac{2\theta_s}{\theta_c} = \frac{Q_s}{p_\perp} \times \frac{\theta_{q\bar{q}}}{\theta_c}$$

small in $E \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp$ regime

- A “two species” Fokker-Planck -like equation: any analytical **solutions**?

Master equation: colour transitions

- We take the $\mathcal{O}(1)$ in the relative diffusion term $\kappa^2 \nabla^2_{\bar{\boldsymbol{p}}_\perp}$
- We **integrate out** the center-of-mass degrees of freedom $\boldsymbol{q}_\perp$ and $\boldsymbol{b}_\perp$
  $\longrightarrow$ *quasi*-factorization (see later for the *quasi*)
- Straight-line motion and **colour transitions**:

classical rate equations

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} + \underbrace{\frac{1}{E}\boldsymbol{p}_\perp \cdot \nabla_{\boldsymbol{r}_\perp}}_{\text{kinetic term}}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix} = \underbrace{\frac{\hat{q}\boldsymbol{r}_\perp^2}{4C_F}\begin{pmatrix}-2C_F & 2C_F \\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}}_{\text{colour transitions}}\begin{pmatrix}\rho_s \\ \rho_o\end{pmatrix}$$

$$\frac{1}{E} = \frac{1}{k_1^+} + \frac{1}{k_2^+}$$ effective longitudinal momentum

- **Initial** Wigner function (colour **singlet**):

$$\rho(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; \boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0) = \underbrace{\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0)}_{\text{relative}}\underbrace{\rho_T(\boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0)}_{\text{center-of-mass}}$$

hard factor: production of the $q\bar{q}$ pair

$$\rho_R(\boldsymbol{p}_\perp, \boldsymbol{r}_\perp; 0) = H(\boldsymbol{p}_\perp)\delta(\boldsymbol{r}_\perp)$$

- Singlet **survival probability**:

$\xrightarrow{\text{large } N_c}$ $$P_s(t) = \exp\left(-\frac{\hat{q}\theta^2_{q\bar{q}}}{12}t^3\right)$$

$$t_c = \left(\frac{4}{\hat{q}\theta^2_{q\bar{q}}}\right)^{\frac{1}{3}} \qquad \theta^2_{q\bar{q}} = \frac{2\boldsymbol{p}_\perp^2}{E^2}$$

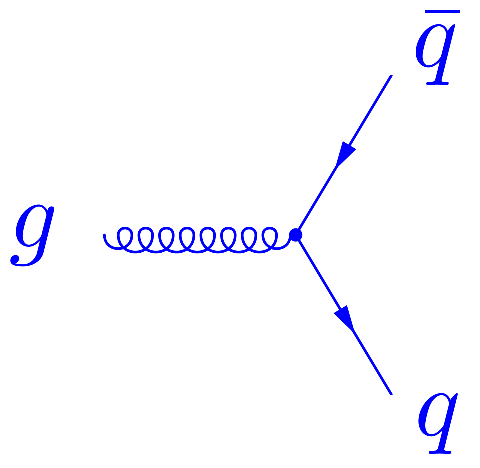
… or at $t = L$ rewritten as

$$P_s(L) = \exp\left(-\frac{\theta^2_{q\bar{q}}}{\theta^2_c}\right) \qquad \theta_c = \frac{2}{\sqrt{\hat{q}L^3}}$$

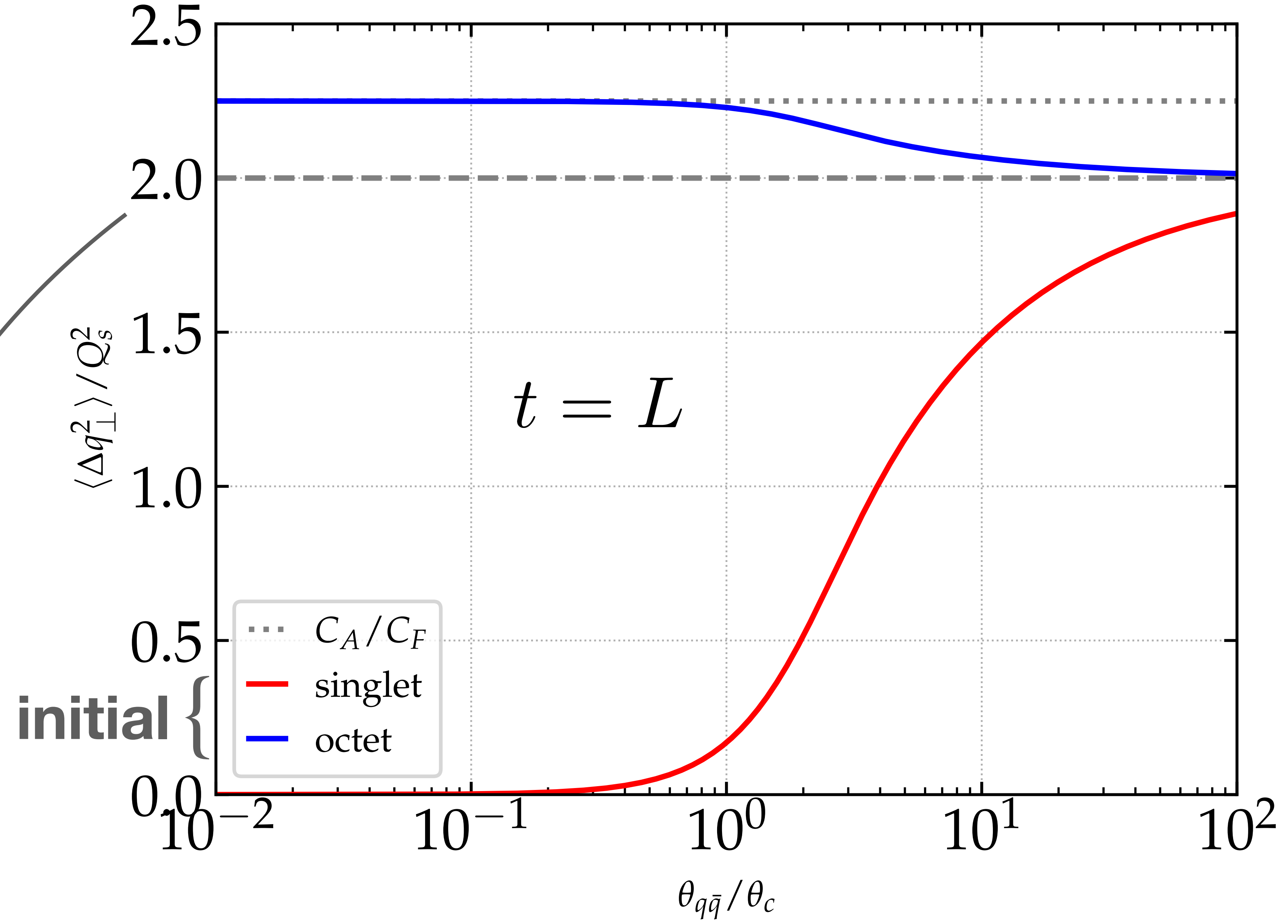
Master equation: center-of-mass diffusion

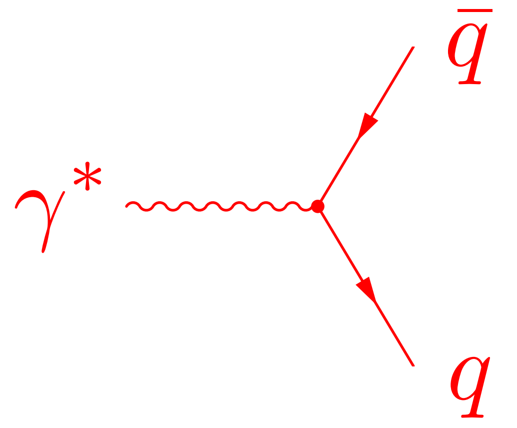
- Let's now consider the **motion of the center-of-mass** too:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} + \Big(\overbrace{\frac{1}{E}\boldsymbol{p}_\perp\cdot\nabla_{\boldsymbol{r}_\perp} + \frac{1}{q^+}\boldsymbol{q}_\perp\cdot\nabla_{\boldsymbol{b}_\perp}}^{\text{kinetic term}}\Big)\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix} = \frac{\hat{q}}{4C_F}\left\{\overbrace{\begin{pmatrix}0 & 0\\ 0 & N_c\end{pmatrix}\nabla^2_{\boldsymbol{q}_\perp}}^{\text{CoM diffusion}} + \overbrace{\begin{pmatrix}-2C_F & 2C_F\\ \frac{1}{N_c} & -\frac{1}{N_c}\end{pmatrix}\boldsymbol{r}^2_\perp}^{\text{colour transitions}}\right\}\begin{pmatrix}\rho_s\\ \rho_o\end{pmatrix}$$



$$\langle\Delta\boldsymbol{q}_\perp^2\rangle\big|_{\text{incoh.}} = \langle\Delta\boldsymbol{k_1}_\perp{}^2\rangle + \langle\Delta\boldsymbol{k_2}_\perp{}^2\rangle = 2Q_s^2$$





$$Q_s^2 = \hat{q}L$$

$$\theta^2_{q\bar{q}} = \frac{2\boldsymbol{p}^2_\perp}{E^2}$$

$$\theta_c^2 = \frac{4}{\hat{q}L^3}$$

- **Non-trivial check** of known *Wilson lines*-like calculations with OQSs
  Dominguez, Milhano, Salgado, Tywoniuk, Vila (2019)
  Isaksen, Tywoniuk (2023)
- Possible **pheno** applications: even higher Casimirs
  Cougoulic, Peigné (2017); Zakharov (2018)

Solutions: quantum decoherence

Colour decoherence

- Initial colour **singlet** state

$$|\psi(0)\rangle = \frac{|r\bar{r}\rangle + |b\bar{b}\rangle + |g\bar{g}\rangle}{\sqrt{3}} \longrightarrow \hat{\rho}(0) = \begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & |r\bar{r}\rangle\langle b\bar{b}| & & |r\bar{r}\rangle\langle g\bar{g}| \\ & 0 & & 0 & \\ |b\bar{b}\rangle\langle r\bar{r}| & & |b\bar{b}\rangle\langle b\bar{b}| & & |b\bar{b}\rangle\langle g\bar{g}| \\ & 0 & & 0 & \\ |g\bar{g}\rangle\langle r\bar{r}| & & |g\bar{g}\rangle\langle b\bar{b}| & & |g\bar{g}\rangle\langle g\bar{g}| \end{pmatrix}$$

time evolution

$t \to \infty$

$$\propto \exp\left(-\frac{\theta_{q\bar{q}}^2}{\theta_c^2(t)}\right) \rightarrow t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{1/3}$$

- Colour **decoherence**: suppression of the off-diagonal elements (coherences)
- Colour **equilibration**: total randomization of the colour of the dipole

$$\begin{pmatrix} |r\bar{r}\rangle\langle r\bar{r}| & & & & 0 & \\ & |r\bar{g}\rangle\langle r\bar{g}| & & & & \\ & & |r\bar{b}\rangle\langle r\bar{b}| & & & \\ & & & |b\bar{r}\rangle\langle b\bar{r}| & & \\ & 0 & & & \ddots & \\ & & & & & \ddots \end{pmatrix}$$

Kinematical decoherence

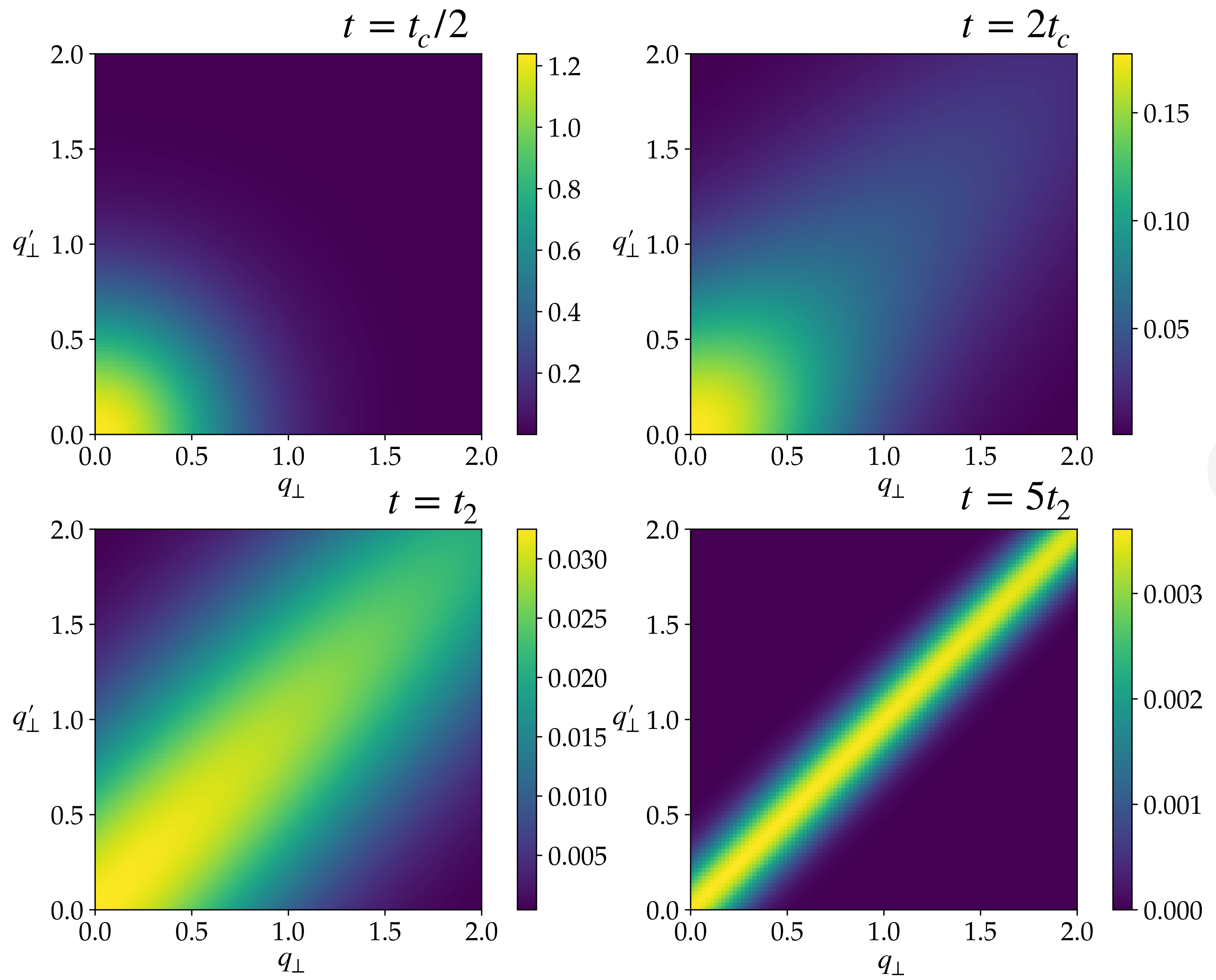
- Initial colour **singlet** state

$$\rho_T(\boldsymbol{q}_\perp, \boldsymbol{b}_\perp; 0) = \frac{1}{\pi^2} e^{-\boldsymbol{q}_\perp^2/\Lambda^2 - \Lambda^2 \boldsymbol{b}_\perp^2}$$

(Gaussian with width Λ)

Numbers:

$\Lambda = 0.5$ GeV $\quad \hat{q} = 1.5\ \text{GeV}^2/\text{fm} \quad q^+ = 50$ GeV $\quad \theta_{q\bar{q}} = 0.1$



$$t_c = \left(\frac{4}{\hat{q}\theta_{q\bar{q}}^2}\right)^{\frac{1}{3}} \sim 2.2\ \text{fm}$$

colour decoherence occurs (**inside** the medium)

$L = 4$ fm medium length

$$t_2 = \left(\frac{(q^+)^2}{\hat{q}\Lambda^2}\right)^{1/3} \sim 6.4\ \text{fm}$$

kinematical decoherence occurs (**outside** the medium)

Summary

- We have derived a **master equation** with the **OQS formalism** for a boosted $q\bar{q}$ dipole:
  - The collisions with the medium cause **colour decoherence**
    - **colour transitions** (singlet-octet)
    - **diagonalization** of the density matrix in the colour-anticolour basis
    - **colour equilibration**
  - the **broadening** of the center-of-mass transverse momentum, breaking **factorisation**

    $\rightarrow$ interest also in future application with **higher Casimirs**, e.g. gg dipole
  - **kinematical decoherence** in momentum space (if not already outside the QGP)
  - **perturbatively** solved the **full** master equation equipped of the relative transverse momentum diffusion term (for Gaussian initial states): **mild** corrections
- Work in progress to apply this framework also on radiation-related phenomena

Thank you for your attention!

Backups

Master equation: perturbative relative diffusion

- **Rescaled** master equation:

large-N_c

$$\frac{\partial}{\partial \bar{t}}\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix} + \underbrace{\left(\bar{\boldsymbol{p}}_\perp \cdot \nabla_{\bar{\boldsymbol{r}}_\perp} + \bar{\boldsymbol{q}}_\perp \cdot \nabla_{\bar{\boldsymbol{b}}_\perp}\right)}_{\text{kinetic term}}\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix}$$

$$= \frac{1}{2}\left\{\underbrace{\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\nabla^2_{\bar{\boldsymbol{q}}_\perp}}_{\text{CoM diffusion}} + \underbrace{\begin{pmatrix}-1 & 0\\ 1 & 0\end{pmatrix}\bar{\boldsymbol{r}}^2_\perp}_{\text{colour transitions}} + \underbrace{\frac{\kappa^2}{4}\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\nabla^2_{\bar{\boldsymbol{p}}_\perp}}_{\text{relative diffusion}}\right\}\begin{pmatrix}\rho_s\\ \tilde{\rho}_o\end{pmatrix}$$

$$\kappa = \frac{2\theta_s}{\theta_c} = \frac{Q_s}{p_\perp} \times \frac{\theta_{q\bar{q}}}{\theta_c}$$

- Analytical solution of the **full** equation:
  - Gaussian ansatz
  - Perturbative expansion up to $\mathcal{O}(\kappa^2)$

- Singlet **survival probability:**

in our kinematical configuration

$$E \gg \boldsymbol{p}_\perp \gg \boldsymbol{q}_\perp$$

neglecting the relative diffusion term is a **good** approximation

checked also for higher moments (variances,…)