Efficiently simulating quarkonium's evolution beyond the dipole approximation

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Work done in collaboration with Andrea Beraudo, Paolo Parotto and Jorge Manuel Martínez Vera.

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- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring R_{AA}, the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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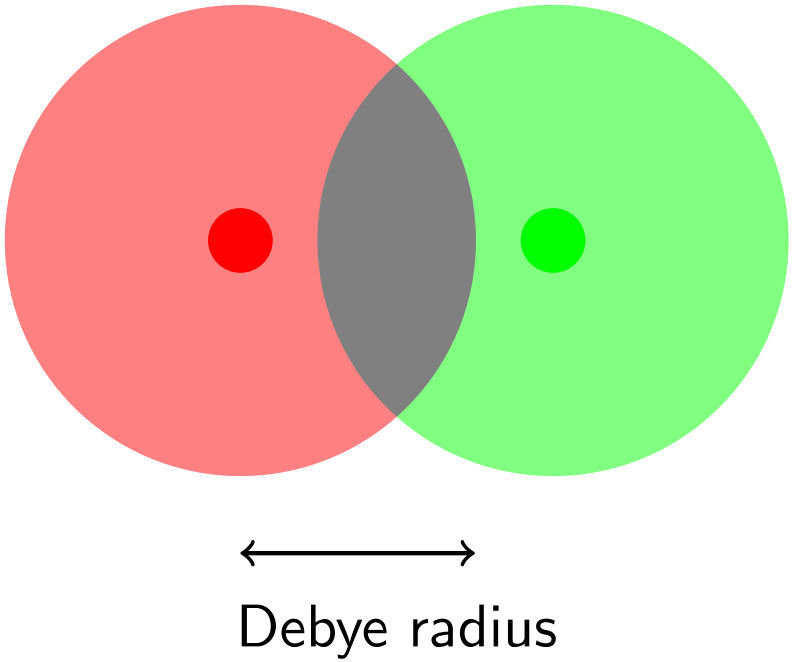
At finite temperature



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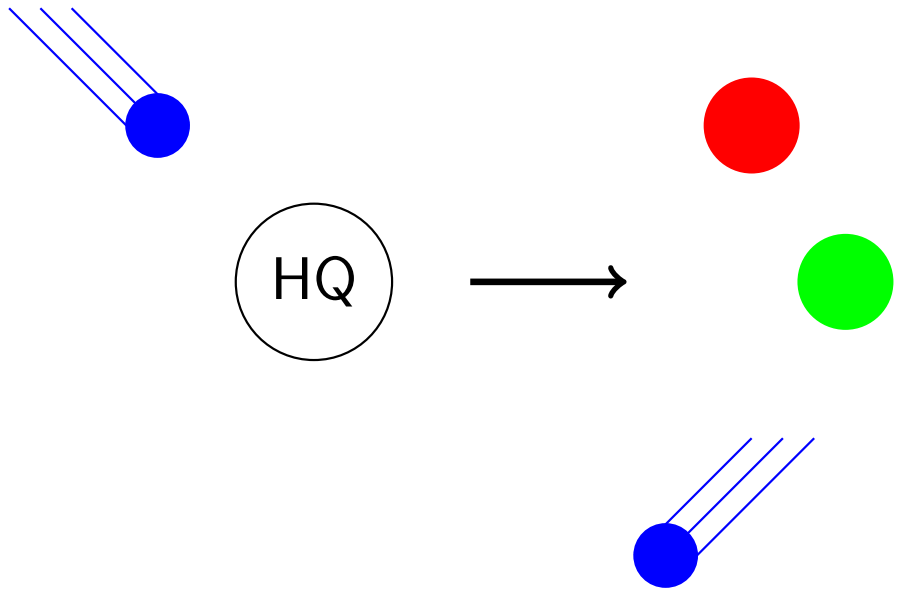
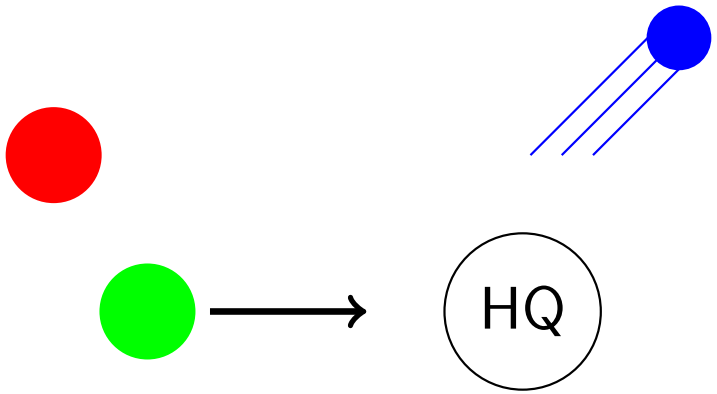


Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

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- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.

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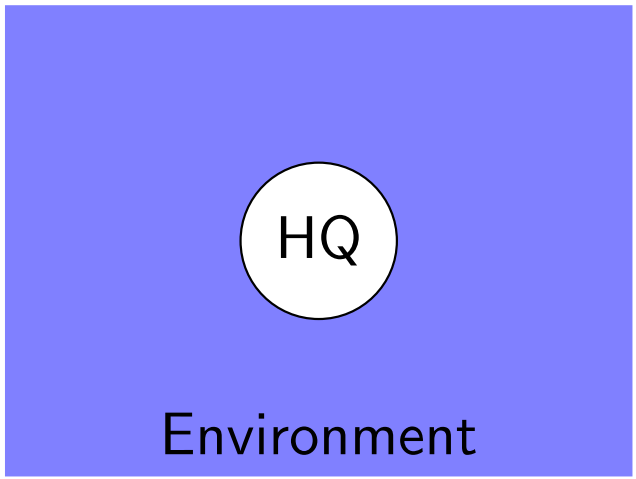

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- We can recover the Schrödinger equation and the Boltzmann equation as limits of the master equation in specific regimes.
- We need to derive the master equation from QCD. This has been done in:
  - Perturbation theory. Akamatsu (2015,2020), Blaizot and Escobedo (2017,2018).
  - Potential non-relativistic QCD (pNRQCD) in the $\frac{1}{r} \gg T$ regime. Brambilla et al. (2016,2017).
  - Schrödinger-Langevin equation (Gossiaux and Katz, 2016)

The Lindblad equation

Any master equation that is:

- Markovian
- Preserves the properties that a density matrix must fulfil (Hermitian, positive semi-definite, trace is conserve).

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In the case of quarkonium, the Markovian limit corresponds to the case in which the energy of the particles in the environment is larger than the binding energy.

The Monte-Carlo Wave Function method

The algorithm to evolve from t to $t + dt$

- With probability $1 - \langle\Psi(t)|\Gamma|\Psi(t)\rangle dt$.
  - Evolve the wave-function with $(1 - iH_{eff}\,dt)|\Psi(t)\rangle$. In our case, this implies solving a 1D Schrödinger equation because H_{eff} does not mix states with different color or angular momentum.
- With probability $\langle\Psi(t)|\Gamma_n|\Psi(t)\rangle dt$.
  - Take a quantum jump, $|\Psi(t)\rangle \to C_n|\Psi(t)\rangle$.
  - Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

Limitations

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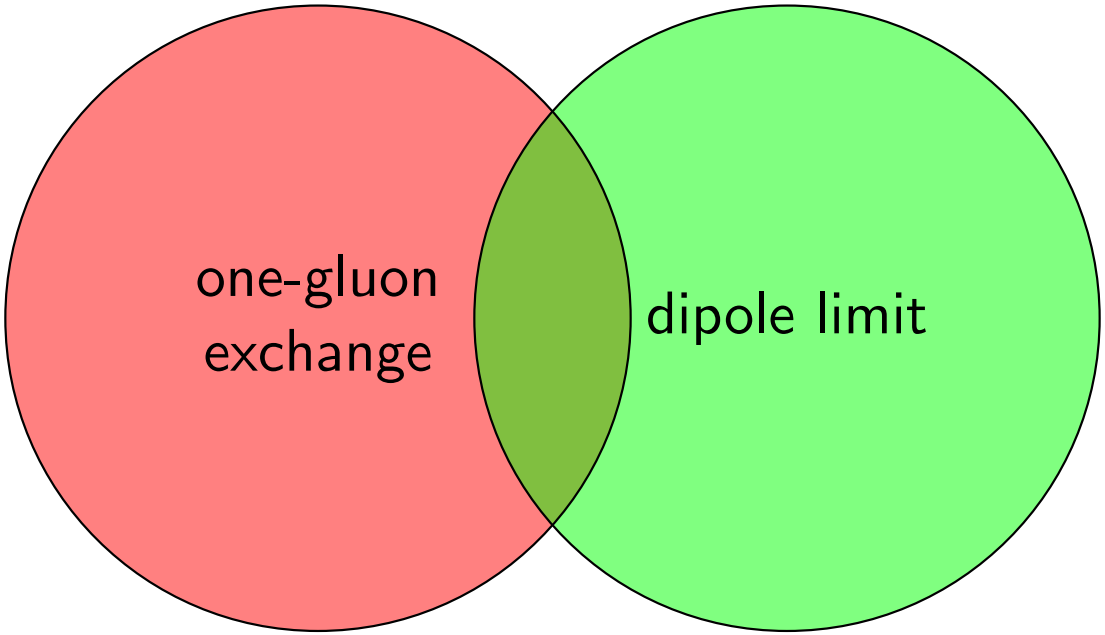
- This is a good approximation for bottomonium at the temperatures reached in current heavy-ion collisions, but not for charmonium.
- Goal: go beyond the dipole approximation in an efficient way.

Outline

Motivation

We wish to be able to efficiently simulate a larger set of possible master equations. Better description of charmonium and excited states.



We take the HTL one-gluon exchange case (Blaizot and Escobedo, 2018). However, the framework would be easily adapted to any one-gluon exchange model.

The master equation

$$\frac{d\rho_Q}{dt} = -i\left[H'_Q, \rho_Q\right] + \sum_{i=0}^{2} \int \frac{d^3q}{(2\pi)^3} \left(C_q^i \rho_Q C_q^{i\dagger} - \frac{1}{2}\{C_q^{i\dagger} C_q^i, \rho_Q\} \right),$$

where

$$H'_Q \equiv \begin{pmatrix} H'_s & 0 \\ 0 & H'_o \end{pmatrix} \equiv \begin{pmatrix} -C_F \dfrac{\alpha_s}{r} e^{-m_D r} & 0 \\ 0 & \dfrac{1}{2N_c} \dfrac{\alpha_s}{r} e^{-m_D r} \end{pmatrix}$$

Note also that the are infinitely many jump operators.

The master equation

Collapse operators

$$\frac{d\rho_Q}{dt} = -i\left[H'_Q, \rho_Q\right] + \sum_{i=0}^{2} \int \frac{d^3q}{(2\pi)^3} \left(C_q^i \rho_Q C_q^{i\dagger} - \frac{1}{2}\{C_q^{i\dagger} C_q^i, \rho_Q\} \right),$$

where

$$C_q^0 \equiv \begin{pmatrix} 0 & \frac{1}{\sqrt{2N_c}} L_q \\ \sqrt{C_F} L_q & 0 \end{pmatrix} \quad C_q^1 \equiv \sqrt{\frac{N_c^2 - 4}{4N_c}} \begin{pmatrix} 0 & 0 \\ 0 & L_q \end{pmatrix}$$

$$C_q^2 \equiv \sqrt{\frac{N_c}{4}} \begin{pmatrix} 0 & 0 \\ 0 & \overline{L}_q \end{pmatrix}$$

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where

$$L_q \equiv g\sqrt{\Delta^>(\omega = 0, q)} S_{\mathbf{q}\cdot\mathbf{r}} \quad \text{and} \quad \overline{L}_q \equiv g\sqrt{\Delta^>(\omega = 0, q)} C_{\mathbf{q}\cdot\mathbf{r}}$$

and $\Delta^> = \langle A_0 A_0 \rangle$ in the HTL approximation, $S_{\mathbf{qr}} = 2\sin\left(\frac{\mathbf{qr}}{2}\right)$ and $C_{\mathbf{qr}} = 2\cos\left(\frac{\mathbf{qr}}{2}\right)$.

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Approach

$$S_{\mathbf{qr}} = 2\sin\left(\frac{\mathbf{qr}}{2}\right) = -i\left(\mathrm{e}^{i\frac{\mathbf{qr}}{2}} - \mathrm{e}^{-i\frac{\mathbf{qr}}{2}}\right)$$

and then we can make use of the identity

$$\mathrm{e}^{i\mathbf{kr}} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} i^l j_l(kr) Y_l^m(\Omega_k) Y_l^{m*}(\Omega_r)$$

Rewriting the Lindblad equation

$$\int \frac{d^3q}{(2\pi)^3} \to \int_0^\infty dq q^2 \sum_{t=0}^{\infty} \sum_m$$

$$L_{\mathbf{q}} \to g\sqrt{\frac{2\Delta^{>}(0,q)}{\pi}} j_{2t+1}\left(\frac{qr}{2}\right) Y^m_{2t+1}(\Omega_r)$$

and

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Equivalent

Same physics. We just trade a 3-D parameter $\mathbf{q}$ by a continous parameter q and two discrite ones (t and m).

Example. Plane wave expansion of Laine's potential

$$\mathrm{Im}\, V_s(r) = -\frac{g^2 C_F}{8\pi^2}\int_0^\infty dq q^2 \Delta^>(0,q)\left(1-\frac{\sin(qr)}{qr}\right) =$$

$$= -\frac{g^2 C_F}{4\pi^2}\sum_{t=0}^{\infty}(4t+3)\int_0^\infty dq q^2 \Delta^>(0,q)\left(j_{2t+1}\left(\frac{qr}{2}\right)\right)^2$$

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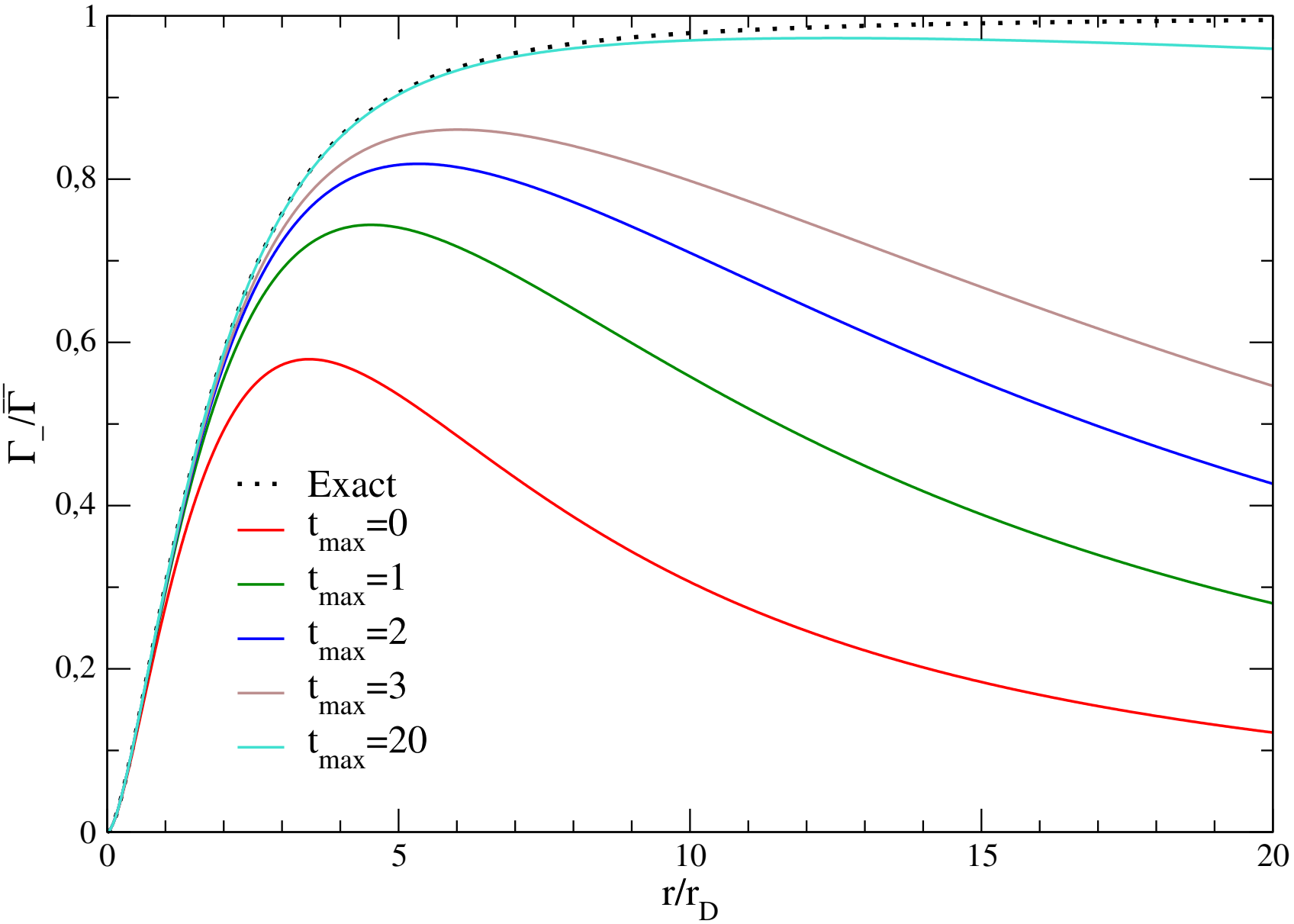
Cross-check

Can be also shown to be true using the following identities from Abramowitz and Stegun

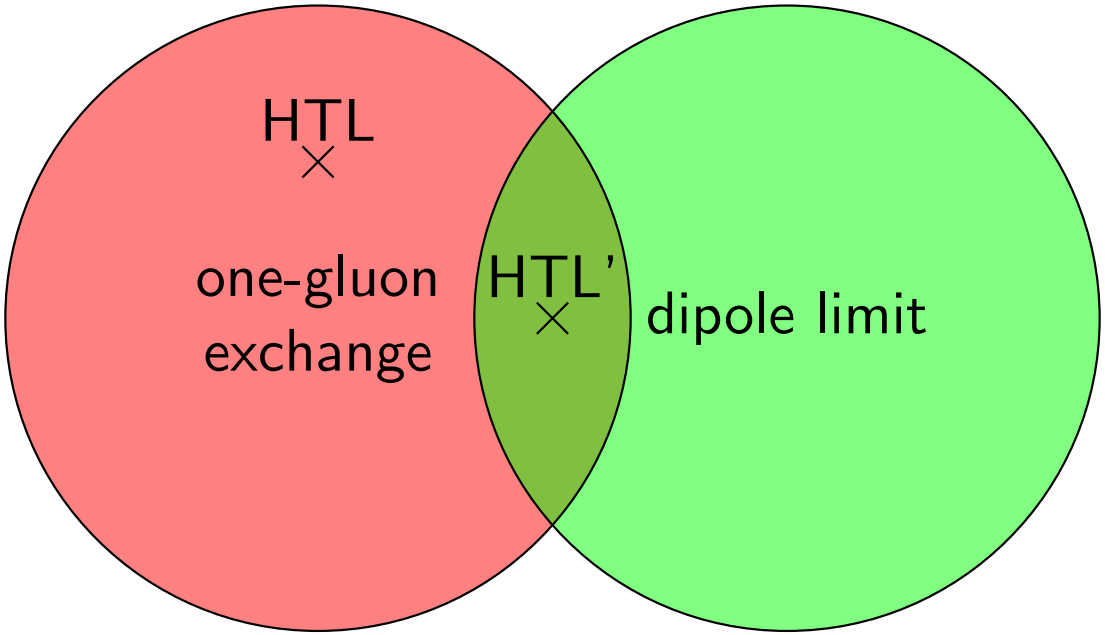
$$\sum_{n=0}^{\infty}(2n+1)(j_n(z))^2 = 1 \quad \sum_{n=0}^{\infty}(-1)^n(2n+1)(j_n(z))^2 = \frac{\sin(2z)}{2z}$$

t expansion of Laine's potential



Outline

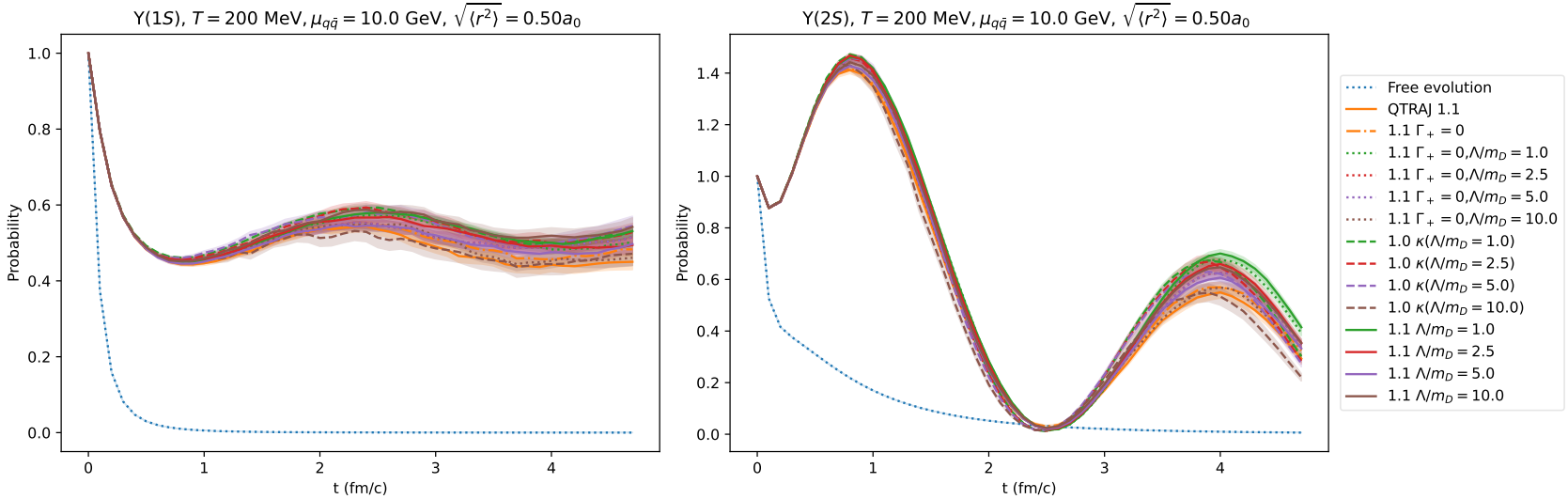
Recovering the dipole limit



We need to propose a new master equation (HTL') that can be modeled with both versions of *Qtraj*. It doesn't need to represent any physical situation. We can do it with a cut-off.

Comparison of both versions of *Qtraj*

Survival probability at constant temperature.



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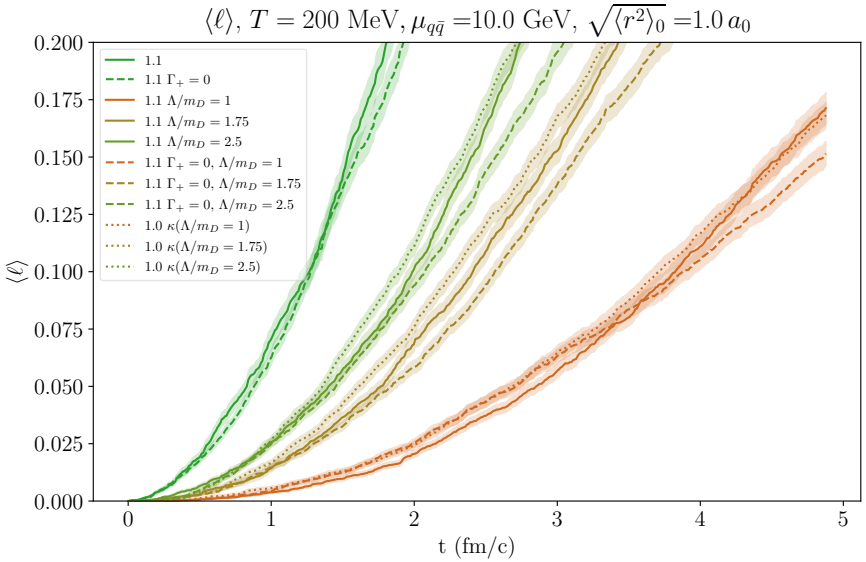
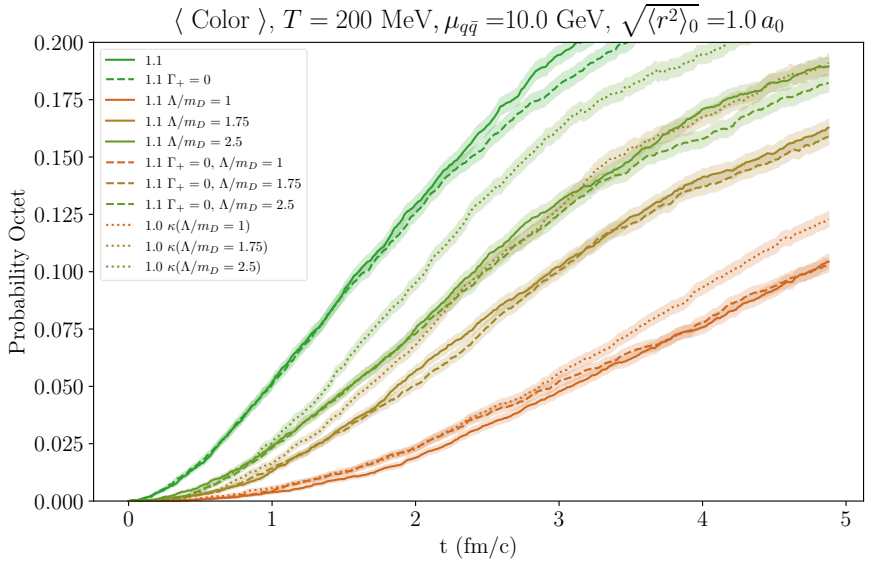
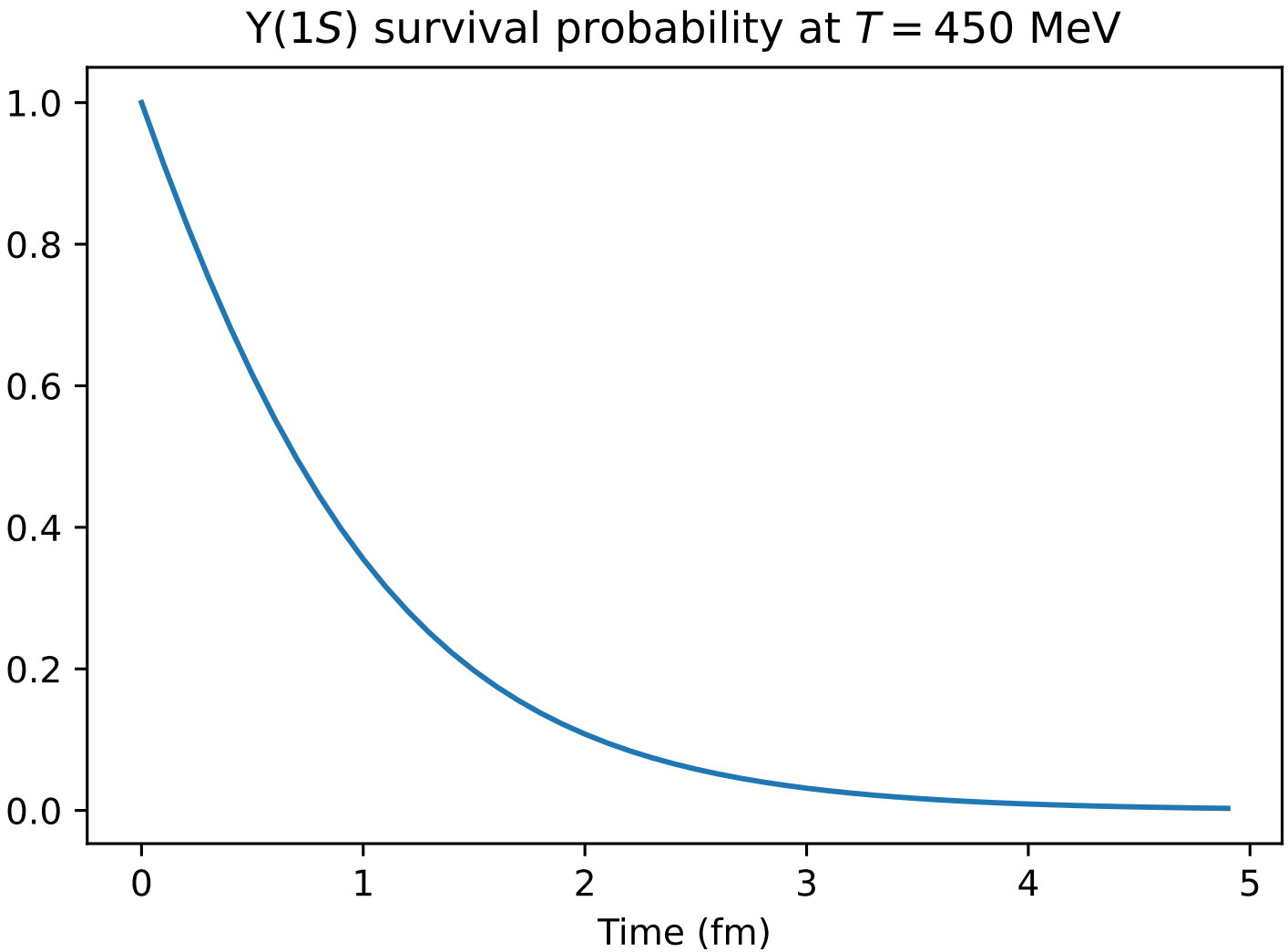




Survival probability at $T = 450\,MeV$

Preliminary results



Case in which we start with a 1S state.

Probabilities with 800 trajectories at $T = 450\,MeV$

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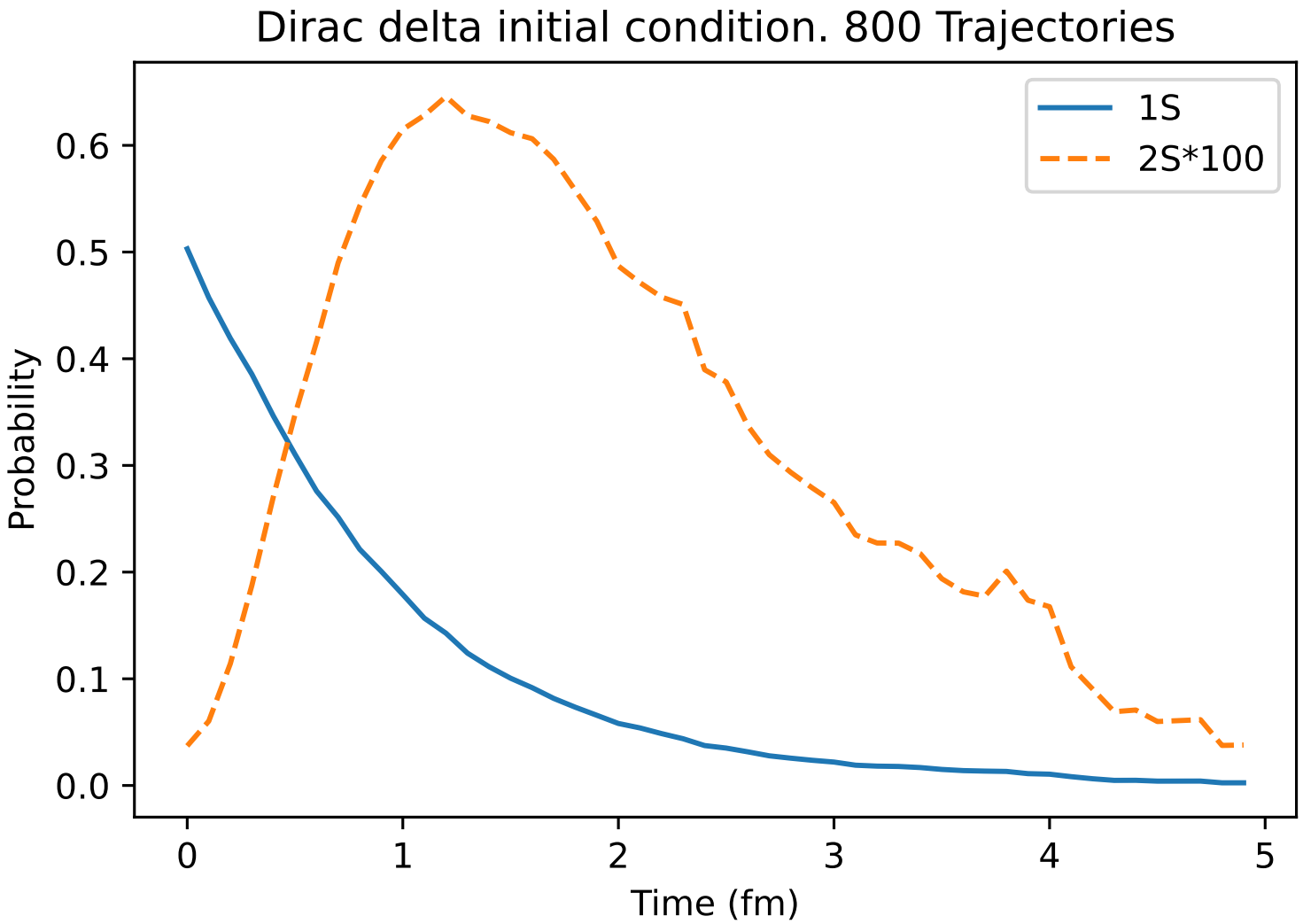

It takes around 77 minutes in my laptop using 8 kernels.

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- 1S states are *cheaper to simulate. Excited states are more expensive.*
- Results are coming. We are applying for computing time in supercomputers to make it faster.

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- We are working on phenomenological results using the new version of *Qtraj*.

Thank you for your attention

Questions?