Medium-induced radiation in out of equilibrium QCD plasmas

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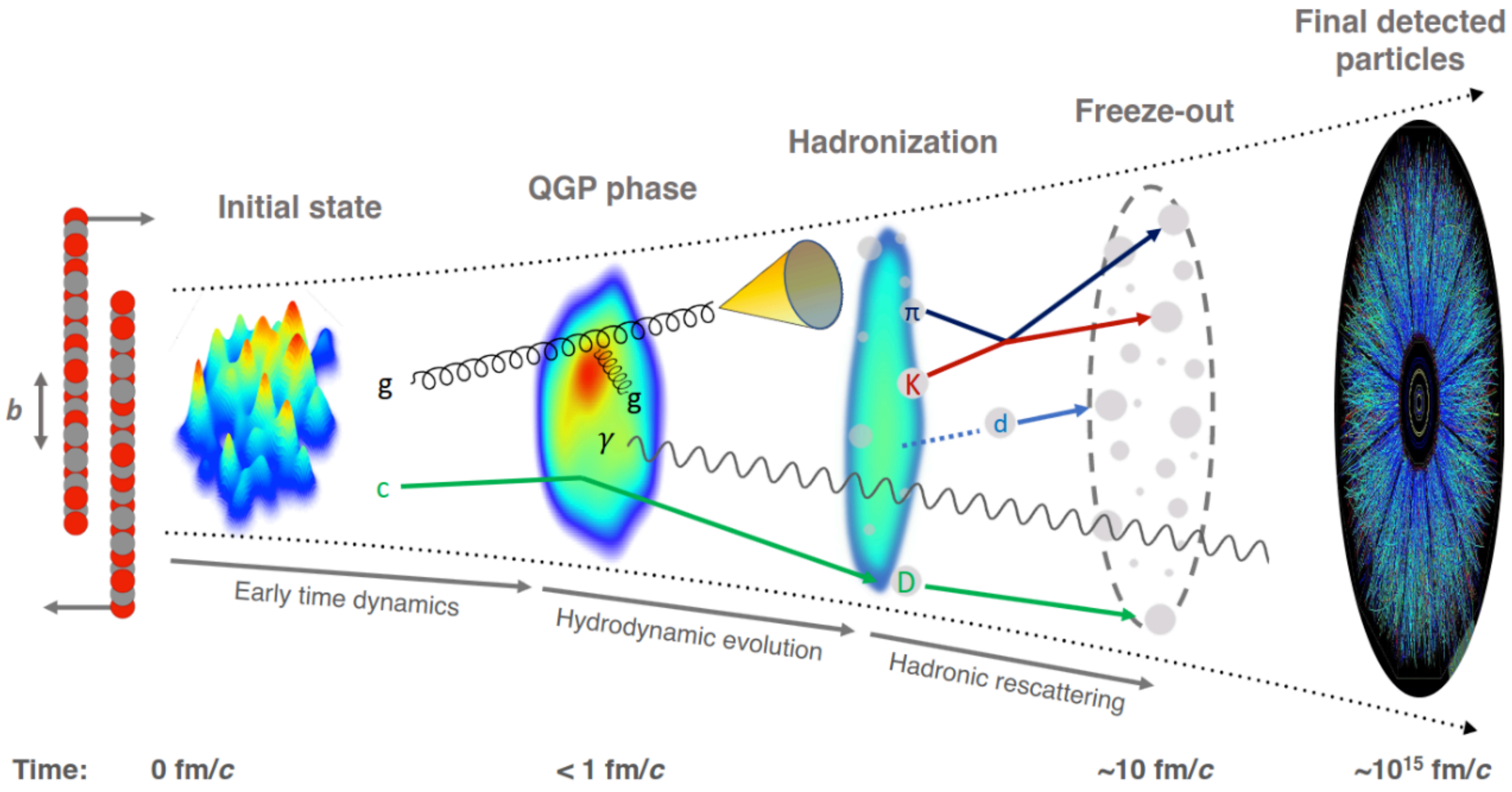
The pre-equilibrium stage in heavy-ion collisions

- Pre equilibrium stage

1. Far from local thermal equilibrium
2. Rapid longitudinal expansion
3. Strong momentum-space anisotropy
4. Evolution toward hydrodynamics[1]



$$\tau \sim 0.1 - 1 fm/c$$

[1]S. Schlichting and D. Teaney, Annu. Rev. Nucl. Part. Sci. 69 (2019) 447; [2]Figure from M. Arslandok et al., arXiv:2303.17254 (2023).

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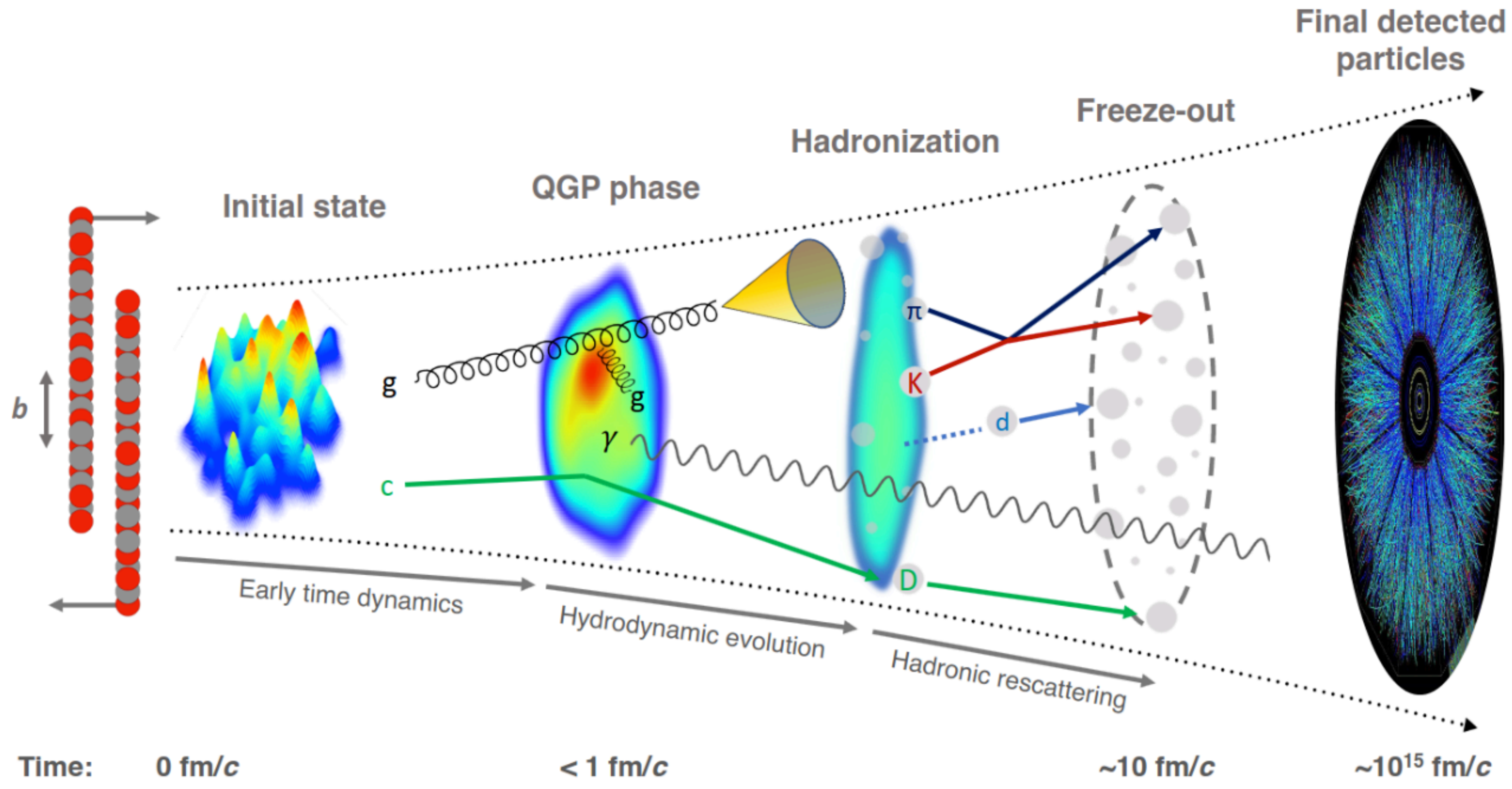

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The pre-equilibrium medium requires a microscopic description.

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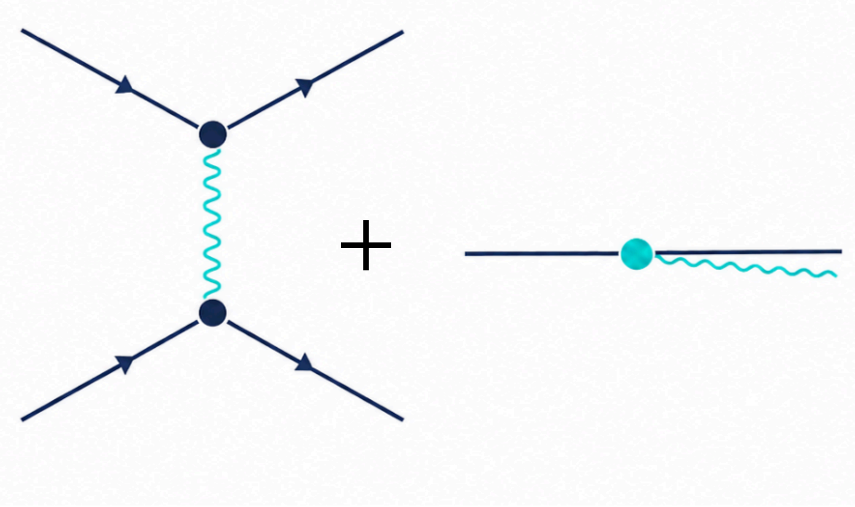
Pre-equilibrium evolution with effective kinetic theory

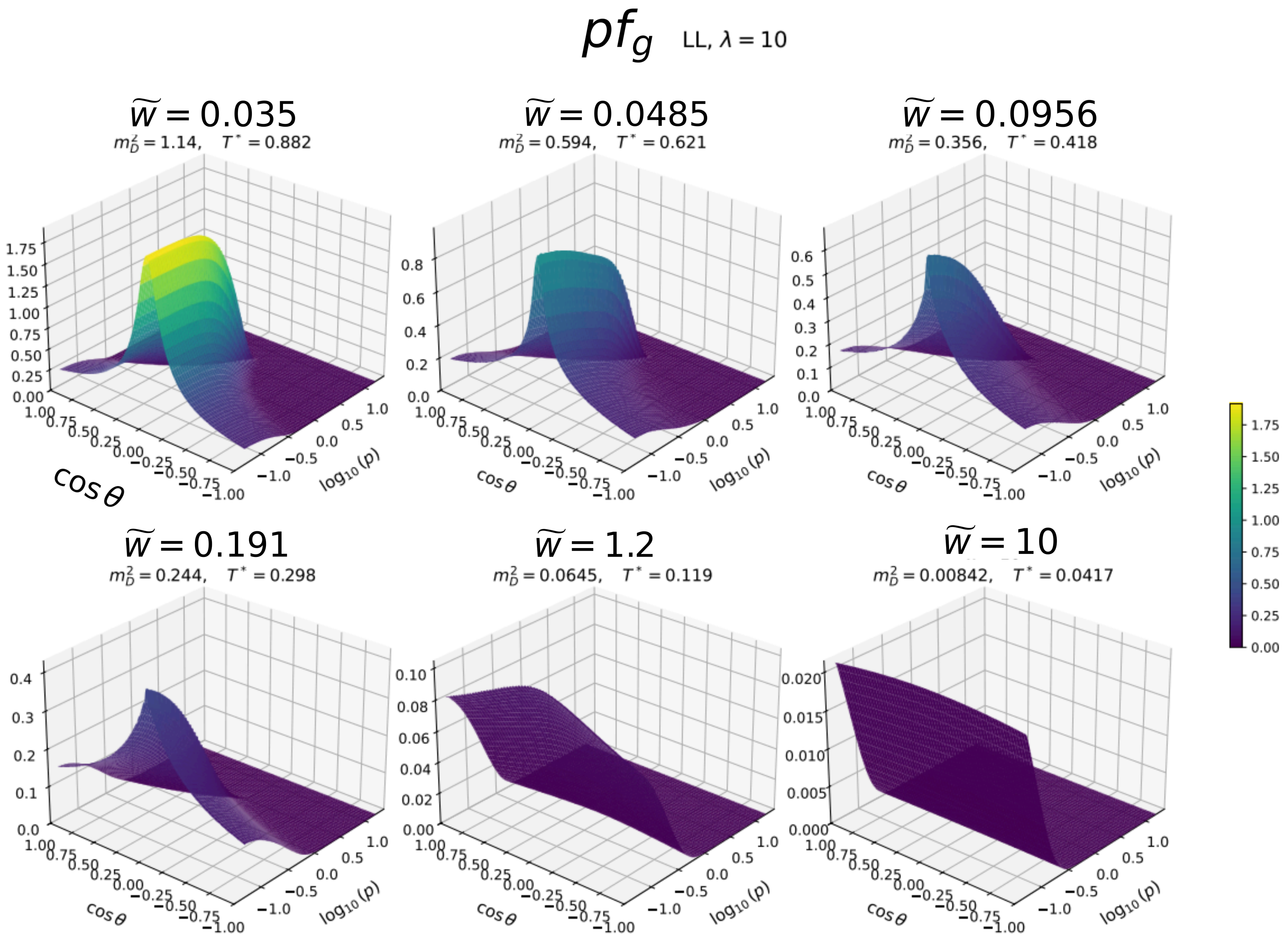
- Time evolution of medium is described by Boltzmann equation[3,4]

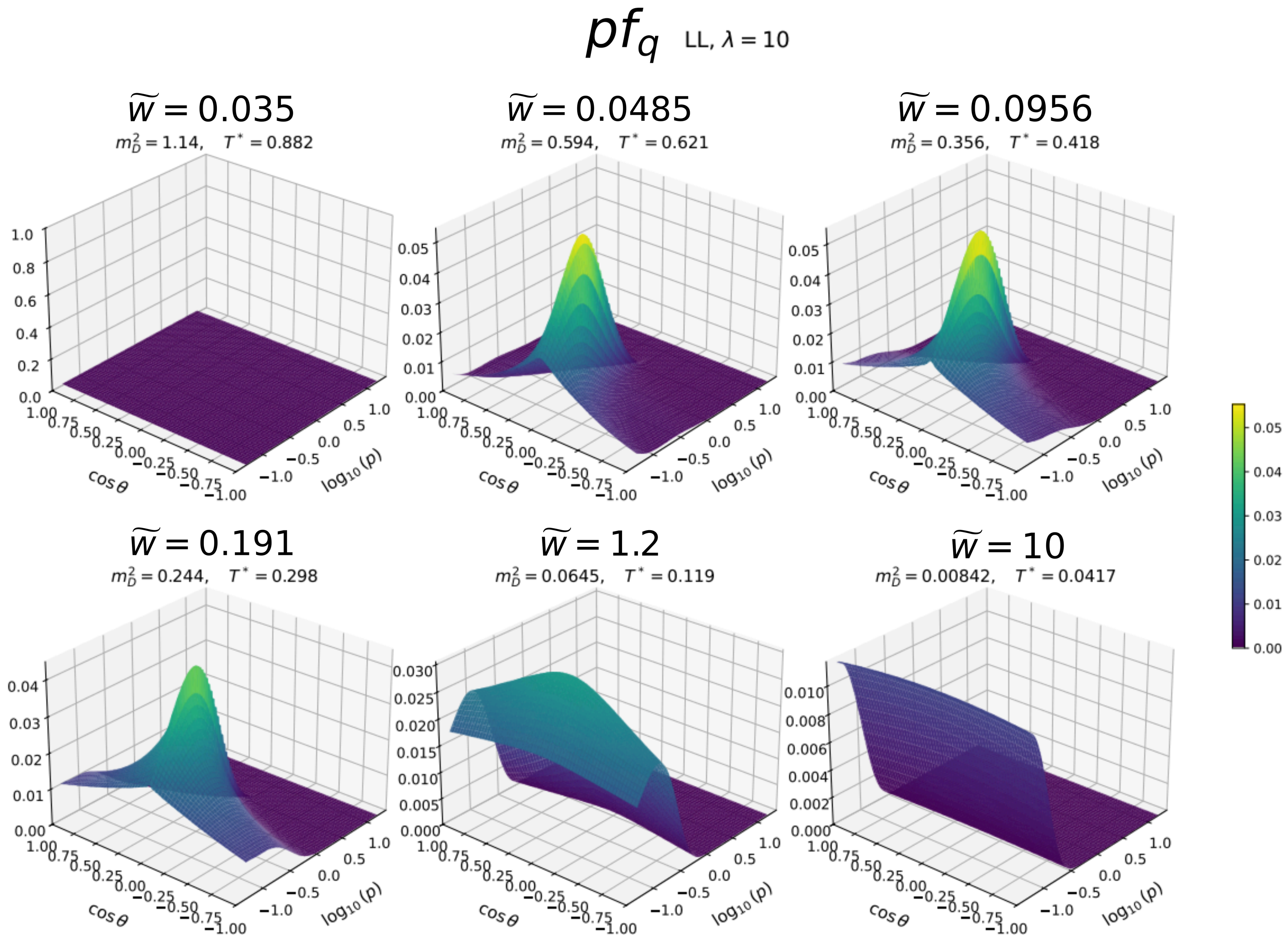
$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\mathbf{p},\tau) = -\,C_s^{2\leftrightarrow 2}[f] - C_s^{1\leftrightarrow 2}[f]\,.$$



- Parton distribution function:

$$f_s(\mathbf{p},\tau), \quad s = g, q, \bar{q}, N_f = 3$$

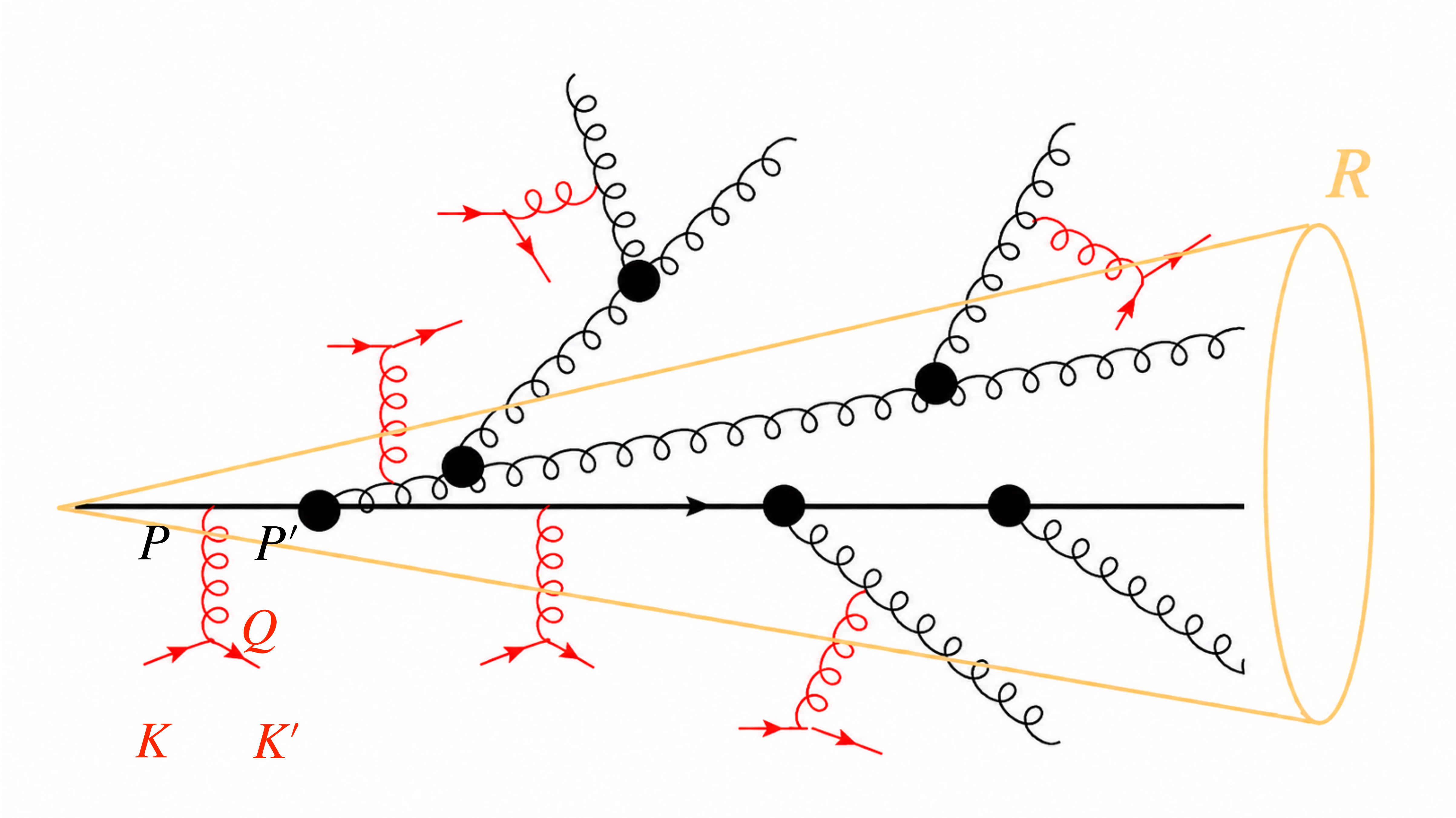




[3] P. Arnold, G. Moore and L. Yaffe, JHEP 01 (2003) 030; [4] A. Kurkela and Y. Zhu, Phys. Rev. Lett. 115 (2015) 182301.

Why study jets in the pre equilibrium plasma?

- Hard partons are produced in the initial hard scatterings.
- Begin interacting with the medium before hydrodynamization.
- Jets act as probes for medium evolution.



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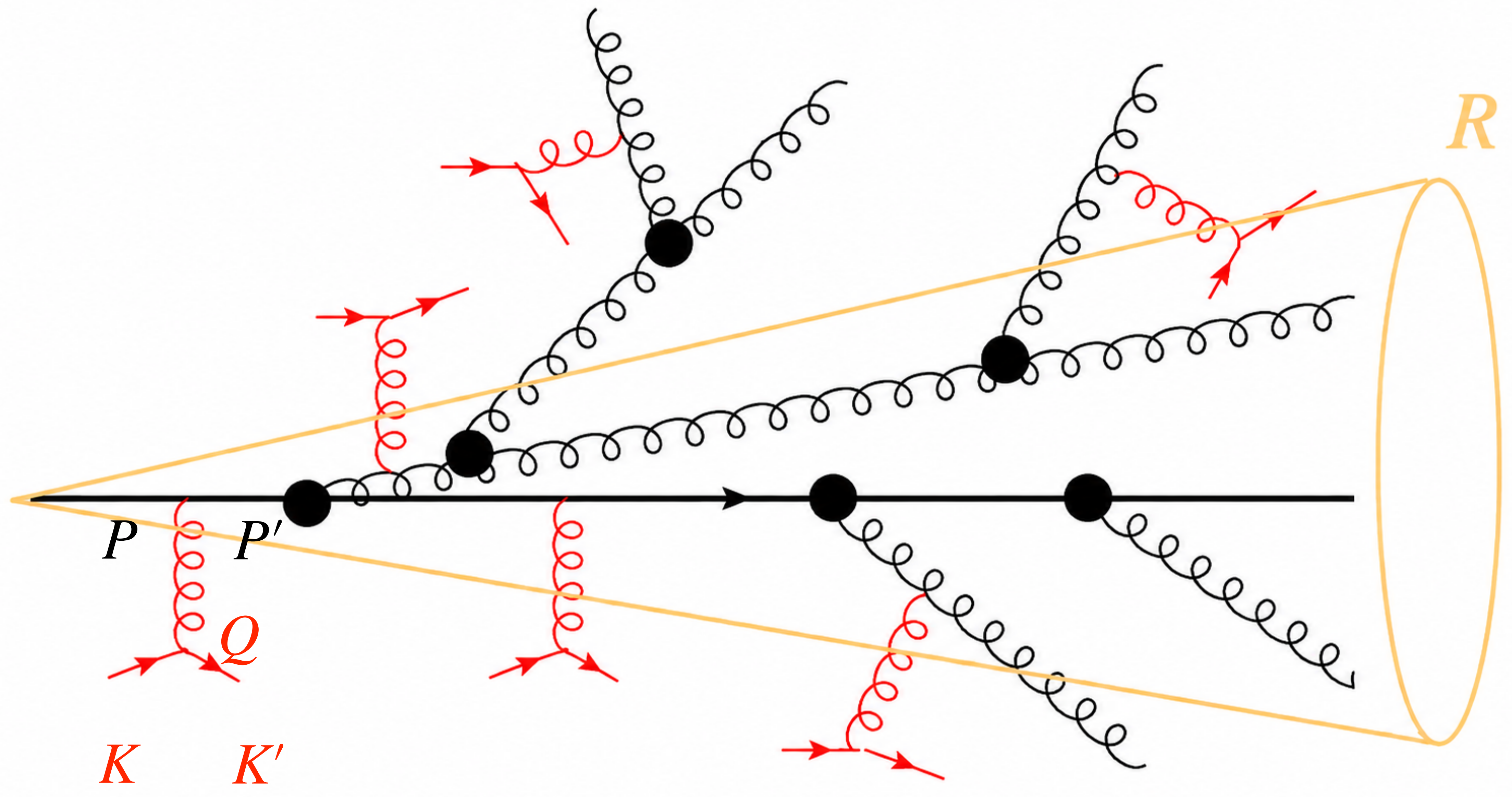


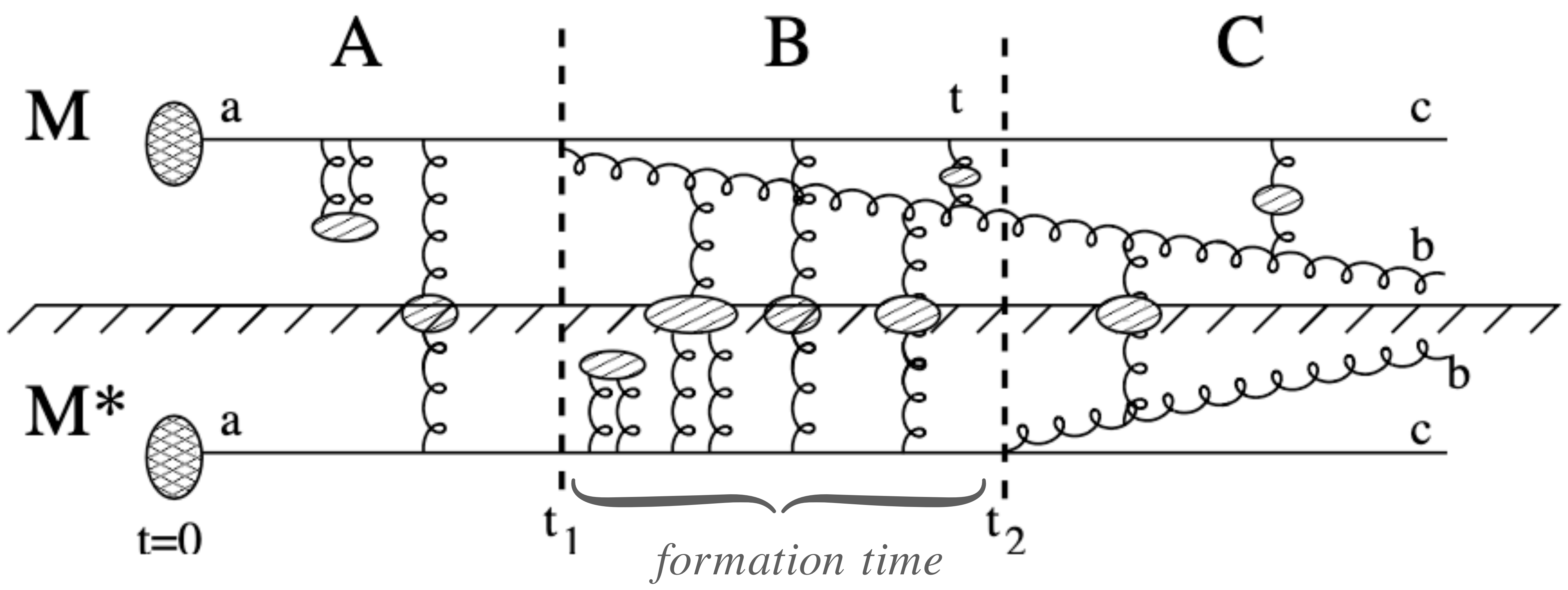
To study the medium-induced splitting rate during the pre-equilibrium stage.

[5] Figure from Mehtar-Tani, Schlichting & Soudi, JHEP 05 (2023) 091.

How the medium enters the splitting rate

- Medium-induced splitting rate[6,7]:

$$\frac{d\Gamma^{a}_{bc}(t)}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi p} \times \mathrm{Re}\int_0^t dt_1 \int_{\mathbf{q},\mathbf{p}} \frac{i\mathbf{q}}{\overline{\delta E}(\mathbf{q},t)} \tilde{\gamma}(t)\, \Gamma(t)\, K(t,\mathbf{q};t_1,\mathbf{p}) \cdot \mathbf{p}$$



$$\Gamma(t)K(\mathbf{p},t) = \int_{\mathbf{q}} C(\mathbf{q}_\perp,t) \left\{ \frac{C_b + C_c - C_a}{2} \left[K(\mathbf{p},t) - K(\mathbf{p}-\mathbf{q},t)\right] + \ldots \right\}$$

- $C(\mathbf{q}_\perp,t)$: Probability of momentum kicks $\mathbf{q}_\perp$ from medium particles.

- The evolution equation of green's function K:

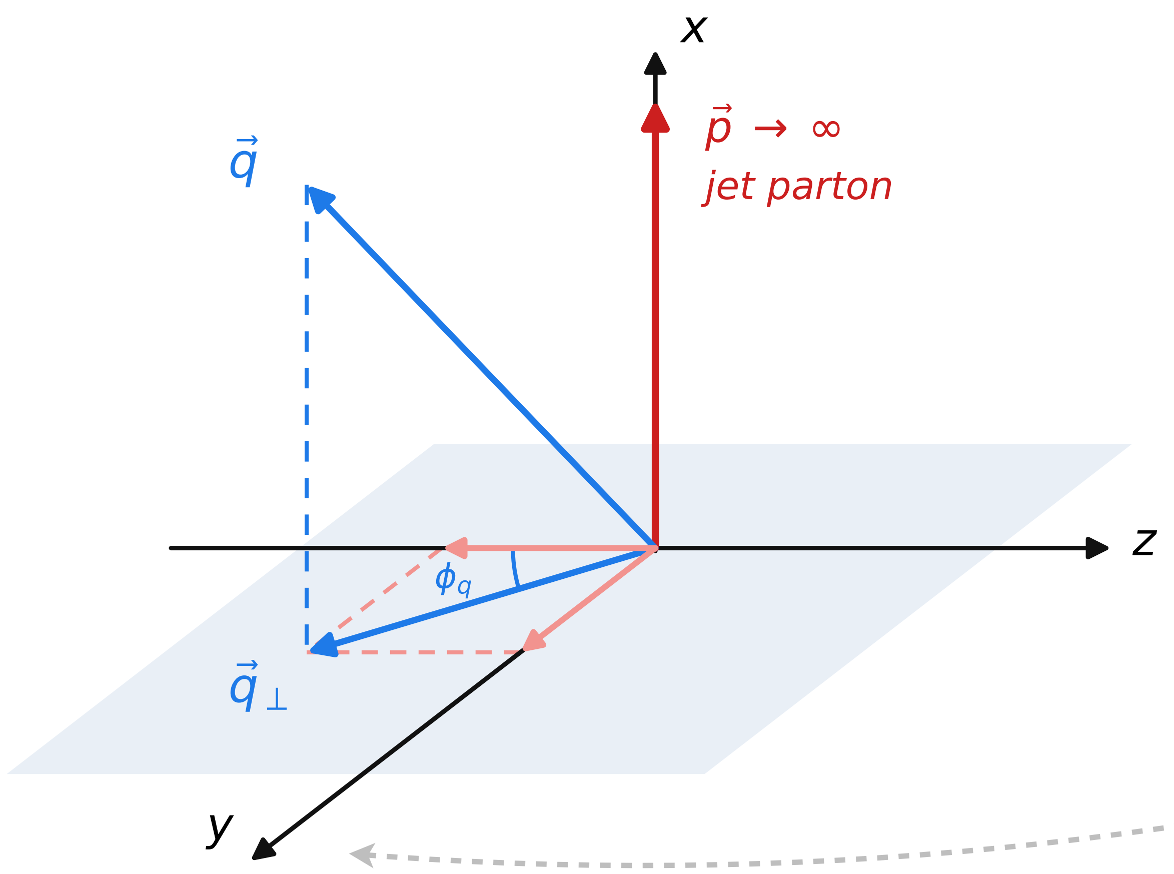
$$\left[\partial_t + i\delta E(\mathbf{q},t) + \tilde{\gamma}(t)\Gamma(t)\right] K(t,\mathbf{q};t_1,\mathbf{p}) = 0$$

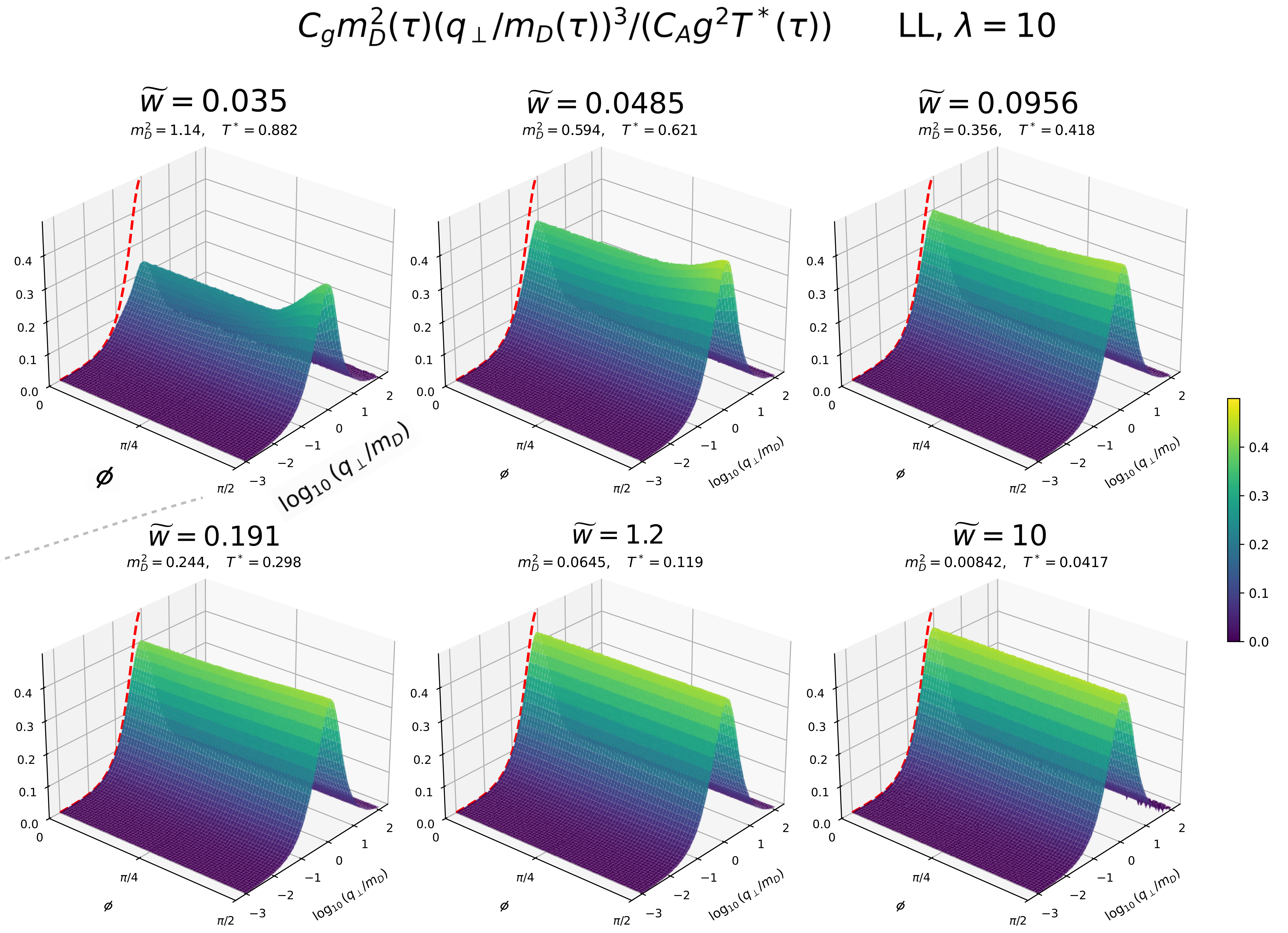
[6]S. Caron-Huot and C. Gale, Phys. Rev. C 82, 064902 (2010); [7]I. Soudi, arXiv:2409.04806 (2024); Figure from[6].

Time evolution of Collision kernel $C(\vec{q}_\perp)$

- $C(\mathbf{q}_\perp, t)$: Transverse momentum transfer between the hard parton and the medium.

$$^{8}C(\mathbf{q}_\perp) = (2\pi)^2 \frac{d\Gamma_{\text{el}}}{d^2\mathbf{q}_\perp} = \frac{1}{2p\nu_a}\sum_{bcd}\int d\Pi_{2\to 2}\left|M^{ab}_{cd}\right|^2 f(\mathbf{k})\big(1 \pm f(\mathbf{k}-\mathbf{q})\big)$$





- peak of $C(\mathbf{q}_\perp, t)$ at $\phi = \frac{\pi}{2}$ ← Initial medium anisotropy $f_{max} = f(cos\theta = 0)$.
- As the medium becomes isotropic fast →the peak disappears quickly.
- $C(\mathbf{q}_\perp, t)$ continues to evolve with $m_D(t), T^*(t)$.

[8]F. Lindenbauer, arXiv:2512.22384 [hep-ph] (2025).

Attractor scaling toward hydrodynamics

- Scaling functions for medium quantities:

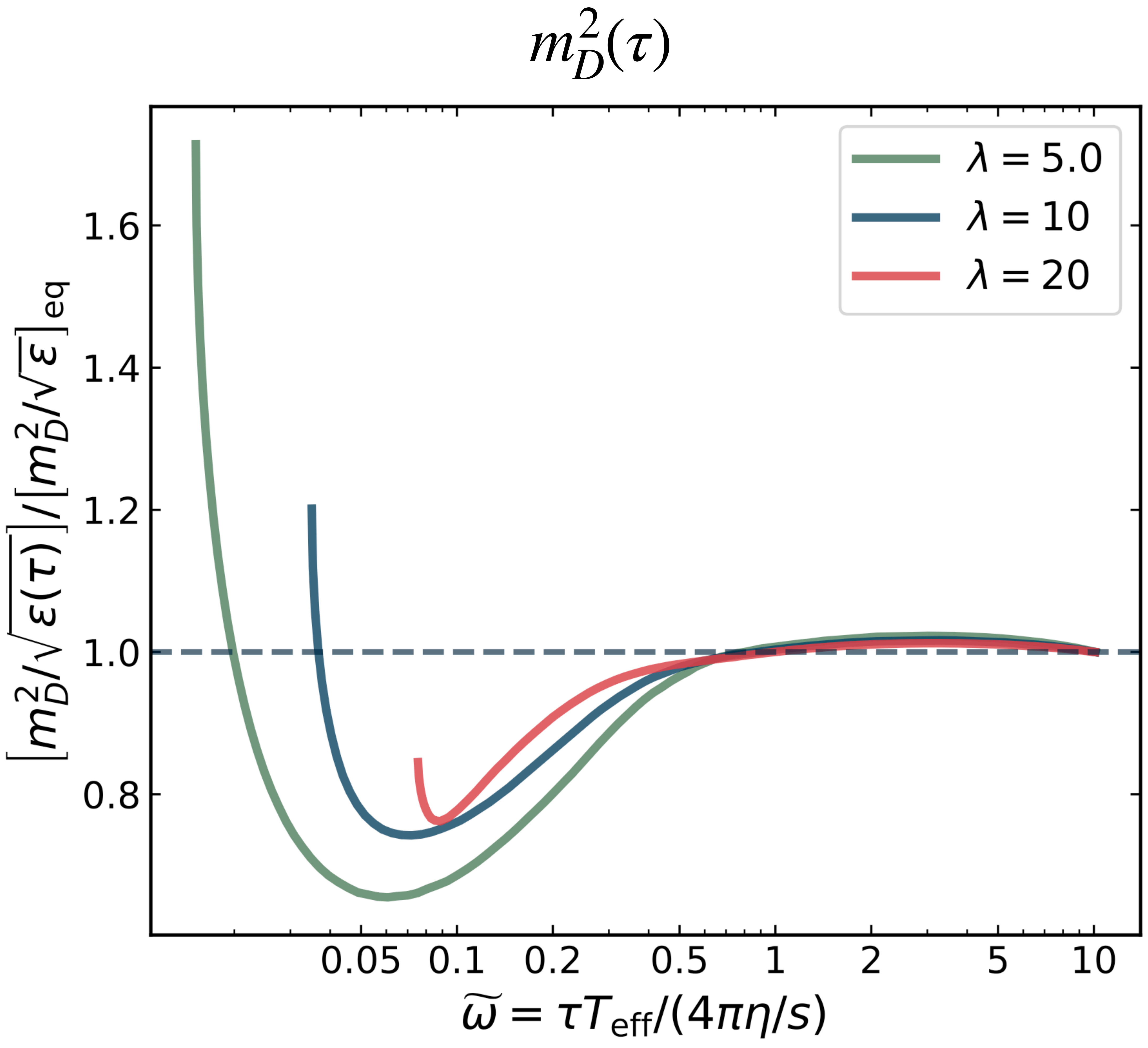
1. $\dfrac{m_D^2(\tau)}{e^{1/2}(\tau)} = \left(\dfrac{m_D^2}{e^{1/2}}\right)_{\text{eq}} \mu(\tau),$

2. $e(\tau)\tau^{4/3} = \left(e\tau^{4/3}\right)_{\text{eq}} \xi(\tau),$

3. $\dfrac{g^2 T_*(\tau)}{e^{1/4}(\tau)} = \left(\dfrac{g^2 T_*}{e^{1/4}}\right)_{\text{eq}} \mathcal{J}(\tau),$

4. $\dfrac{m_D^2(\tau)}{g^2 T_*(\tau)} \dfrac{C(\mathbf{q}_\perp, \tau)}{C_R} = \mathscr{C}\left(\dfrac{\mathbf{q}_\perp}{m_D(\tau)}, \tau\right).$

- Assuming the evolution follows the universal attractor curves. $(\lambda = g^2 N_c)$.
- Fix the physical quantities to the final-state $\left((\tau s)_{\text{eq}} \sim \dfrac{dN_{\text{ch}}}{d\eta}\right)$.



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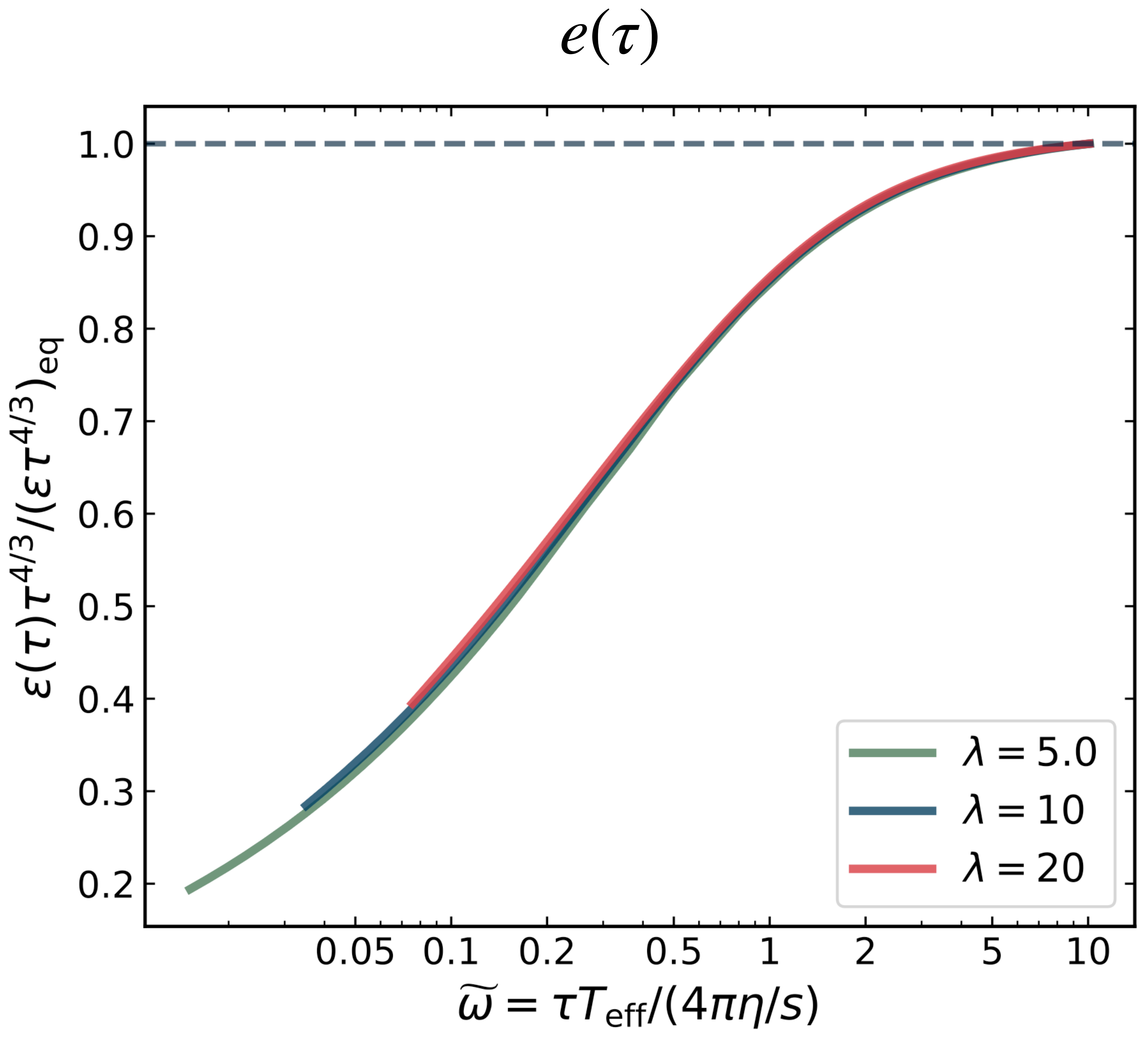

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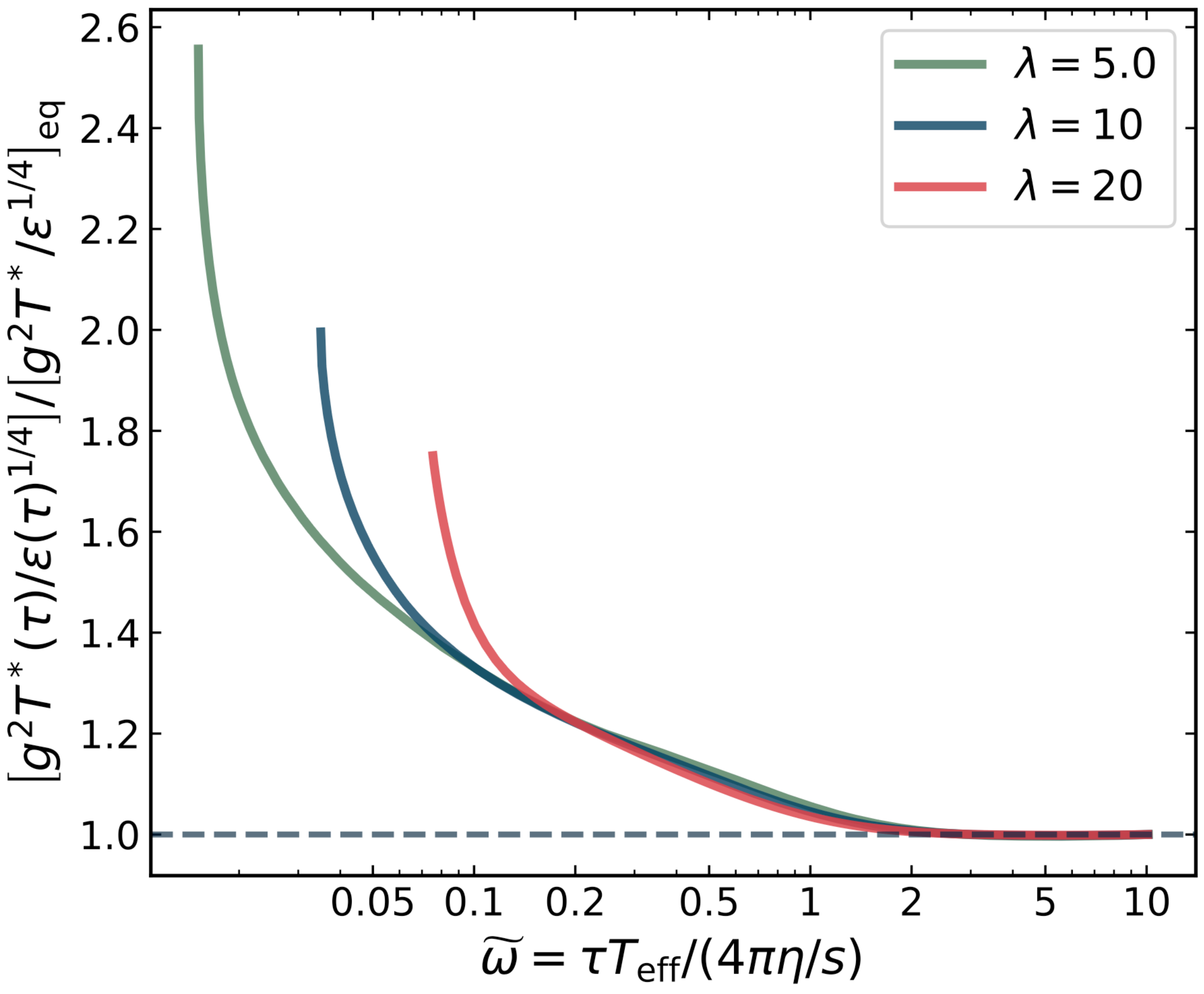
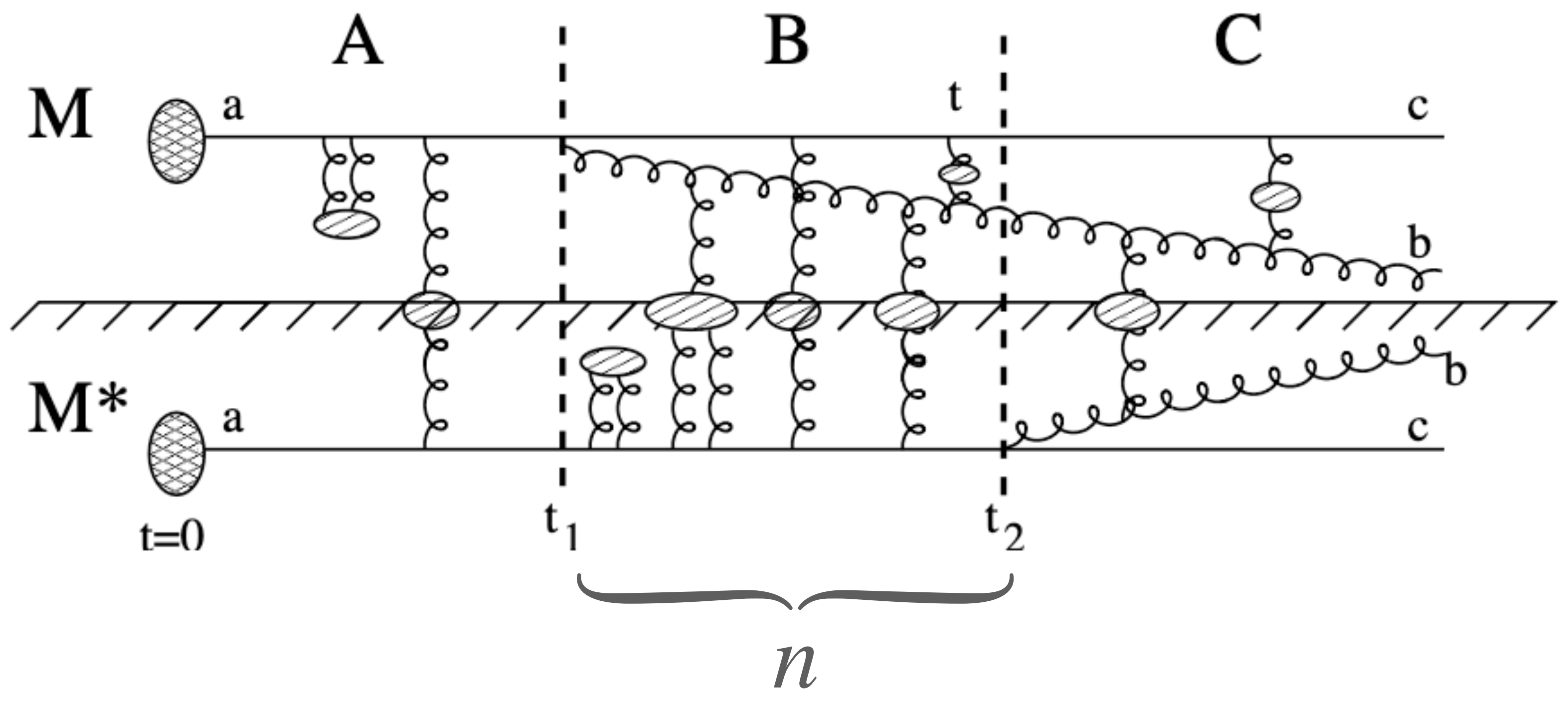
$T_*(\tau)$

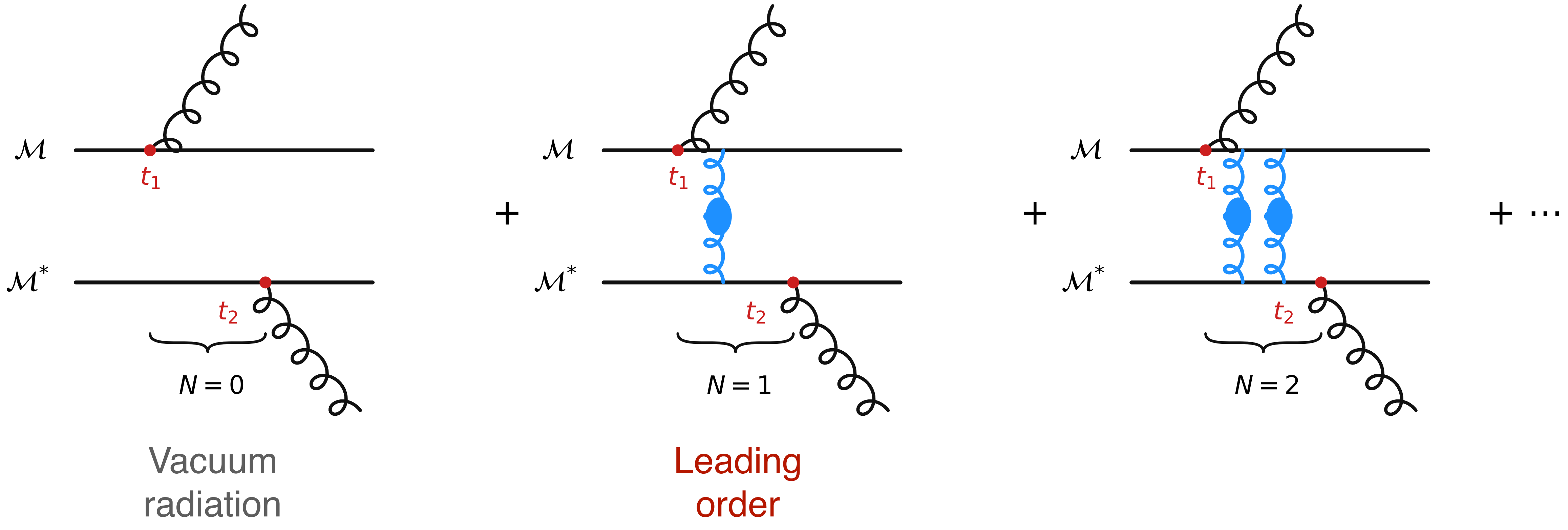


$$f_g, f_q \longrightarrow C(q_\perp, \phi, \tau) \longrightarrow \frac{d\Gamma}{dz}\,.$$



- Leading order opacity expasion:

$$\frac{d\Gamma^a_{bc}(t)}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi P} \times \mathrm{Re}\int_0^t dt_1 \int_{\mathbf{q},\mathbf{p}} \frac{i\mathbf{q}}{\overline{\delta E}(\mathbf{q},t)} \tilde{\gamma}(t)\,\Gamma(t)\,K(t,\mathbf{q};t_1=t,\mathbf{p})\cdot\mathbf{p}$$



- Initial condition:

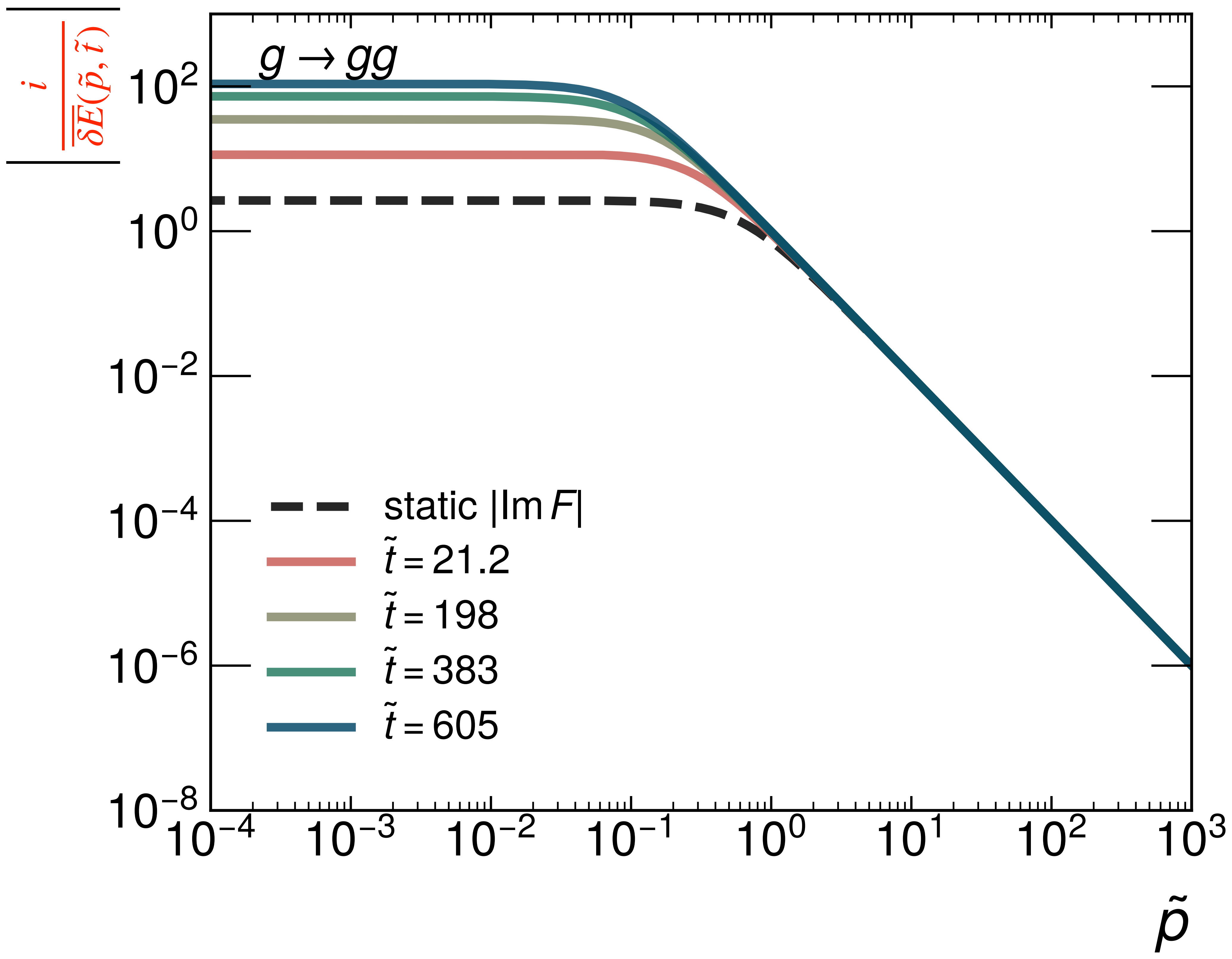
$$K(t,\mathbf{q};t_1=t,\mathbf{p}) = \frac{1}{4P^2z^2(1-z)^2}(2\pi)^2\delta^{(2)}(\mathbf{q}-\mathbf{p})$$

- Splitting rate:

$$\frac{d\Gamma^a_{bc}(t)}{dk} = \frac{P^{a(0)}_{bc}(x)}{\pi p} \times \mathrm{Re} \int_0^t dt_1 \int_{\mathbf{q},\mathbf{p}} \frac{i\mathbf{q}}{\overline{\delta E}(\mathbf{q},t)} \tilde{\gamma}(t)\,\Gamma(t)\,K(t,\mathbf{q};t_1=t,\mathbf{p})\cdot\mathbf{p}$$

- Comparing to static medium:

$$\frac{i}{\overline{\delta E}(\tilde{p},\tilde{t})} = \begin{cases} \frac{i}{\delta\tilde{E}(\tilde{p})}, & \text{static} \\ \int_{\tilde{t}}^{\infty} ds\, e^{-i\int_{\tilde{t}}^{s} du\, \delta\tilde{E}(u,\tilde{p})}, & \text{expanding} \end{cases}$$



Non equilibrium expanding medium & Static medium

- Splitting rate:

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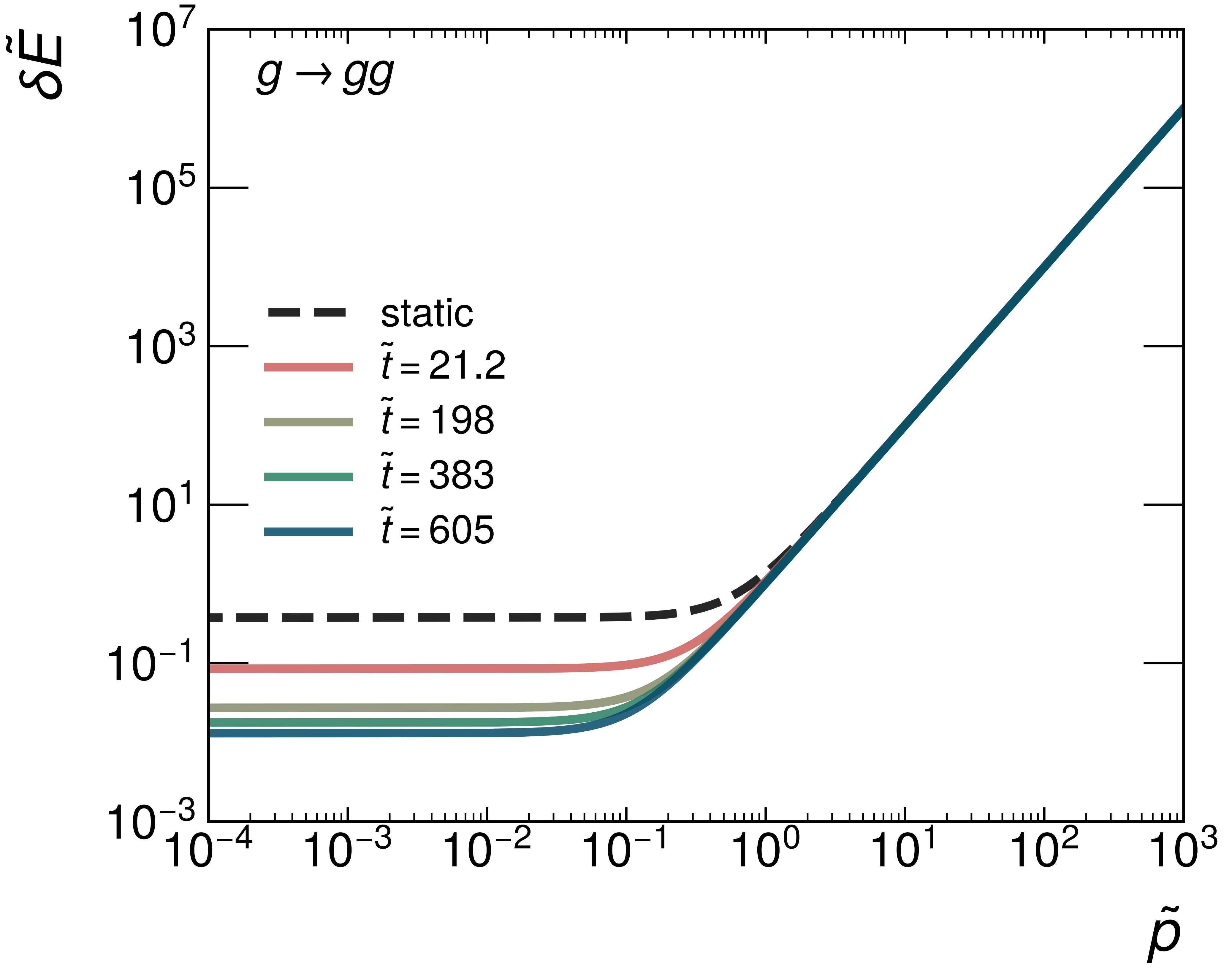
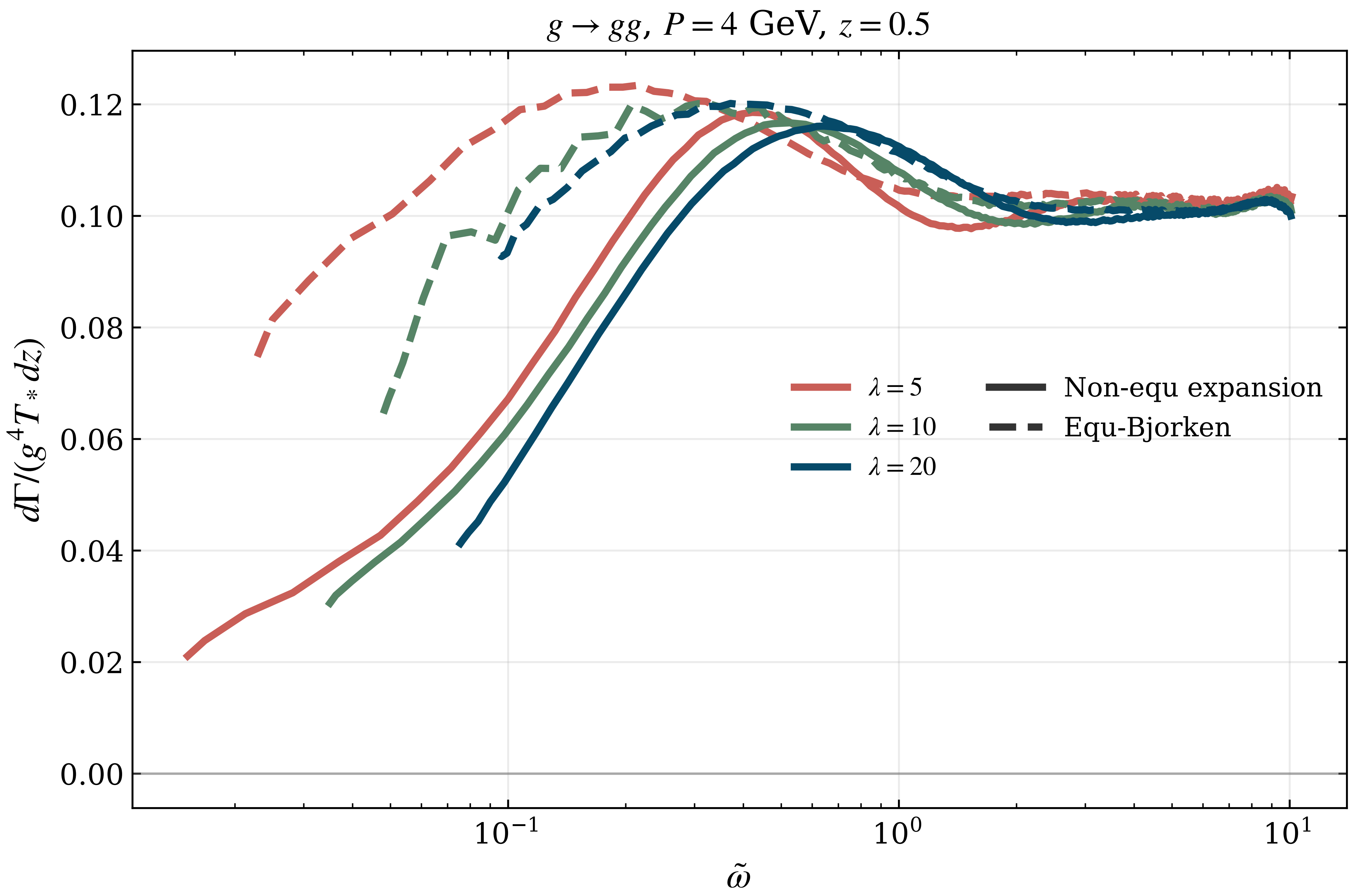
$$\delta\tilde{E}(\tilde{t},\tilde{p}) = \begin{cases} \tilde{p}^2 + (1-z)\frac{m_z^2}{m_D^2} + z\frac{m_{1-z}^2}{m_D^2} - z(1-z)\frac{m_1^2}{m_D^2}, & \text{static} \\ \tilde{p}^2 + \left[(1-z)\left(\frac{m_{z,0}}{m_{D,0}}\right)^2 + z\left(\frac{m_{1-z,0}}{m_{D,0}}\right)^2 - z(1-z)\left(\frac{m_{1,0}}{m_{D,0}}\right)^2\right]\left(\frac{m_D(\tilde{t})}{m_{D,0}}\right)^2, & \text{expanding} \end{cases}$$

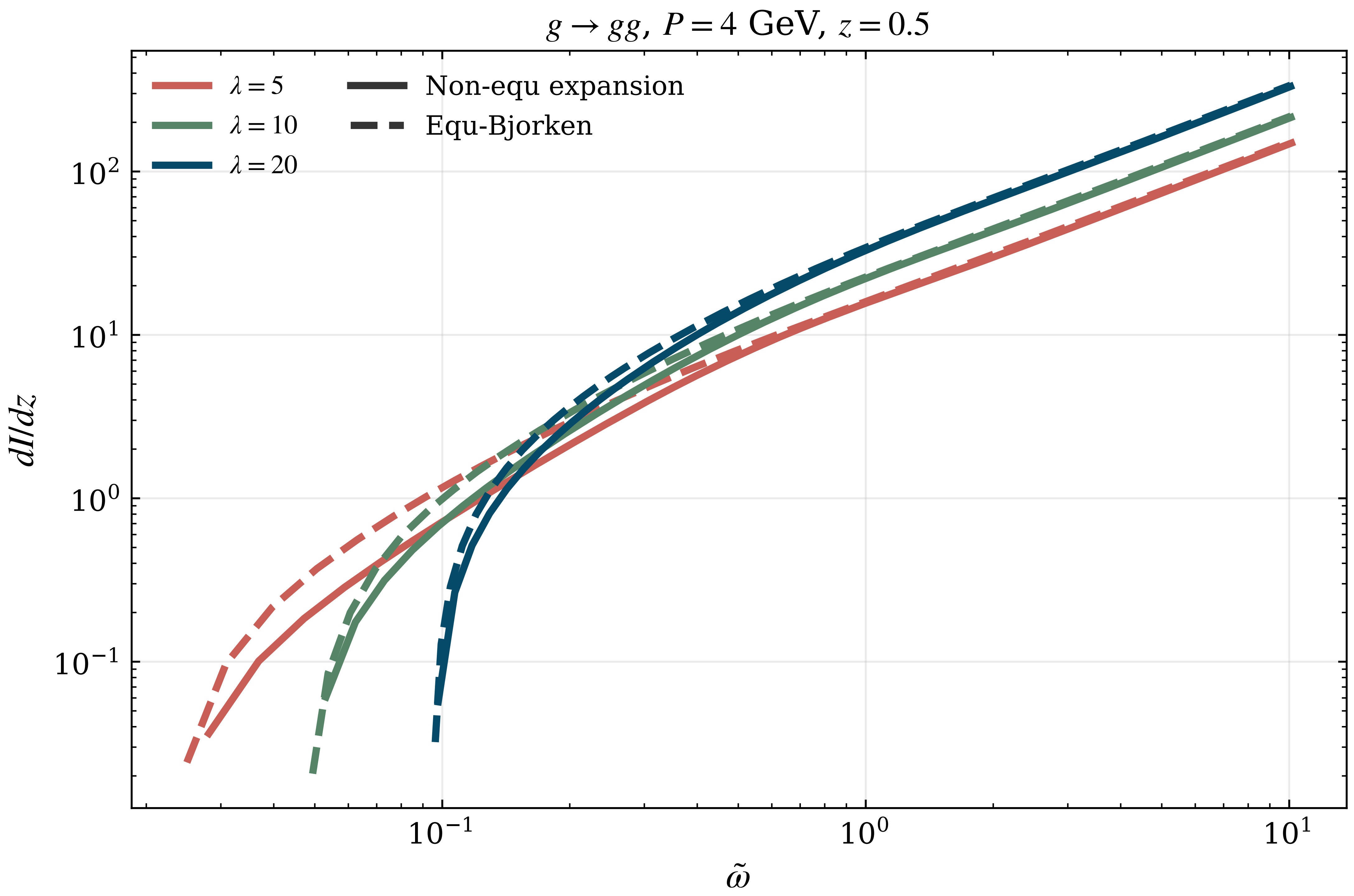


Non equilibrium expanding medium & Equilibrated Bjorken medium

- Splitting rate: $\frac{d\Gamma^a_{bc}}{dz}$

- Spectrum: $\frac{dI^a_{bc}}{dz} = \frac{2pz(1-z)}{m^2_{D,0}} \mathrm{Re} \int_{\tilde{\tau}_0}^{\infty} \frac{d\Gamma^a_{bc}}{dz} d\tilde{\tau}$





- Early-time radiation suppressed relative to the equilibrated Bjorken expansion (set $\mu(\tau) = \xi(\tau) = \mathcal{J}(\tau) = 1, \mathscr{C} = \mathscr{C}_{eq}$).

Non equilibrium expanding medium & Equilibrated Bjorken medium

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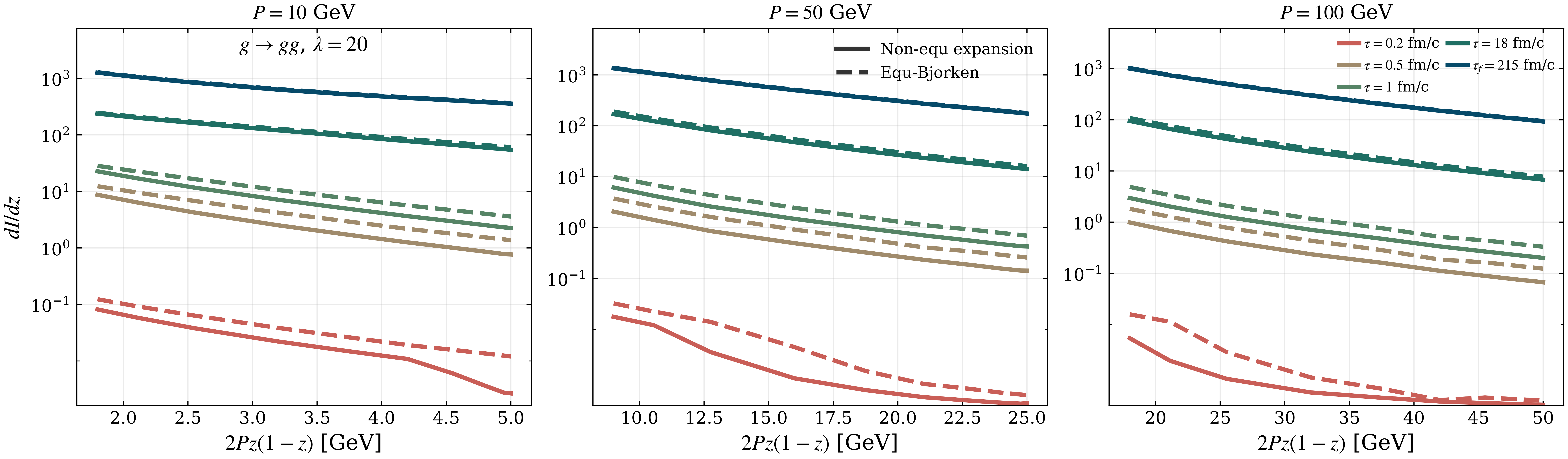


- $t_f \sim \frac{1}{\delta E} \propto 2Pz(1-z)$, small formation time → more radiation.
- Similar behavior across different jet energies.

Conclusion

- Summary:
  1. Effective kinetic theory describes the pre-equilibrium evolution of medium.
  2. The time-dependent collision kernel $C(\mathbf{q}_{\perp}, t)$ provides the input for the splitting rate.
  3. Attractor scaling connect the kinetic evolution to hydrodynamics and we can match it to final state using experimentally measured quantities.
  4. The microscopic description reveals a different splitting rate and spectrum than using equilibrated Bjorken expansion.

- Outlook:
  1. Using more physical non-equilibrium dynamic approximation to explore the radiation.

Backup

Calculation of $C(\mathbf{q}_\perp)$

- in process $a, b \to c, d$ $(p, k \to p', k'; \vec{k}' - \vec{k} = \vec{q}, p \to \infty)$:

$$C(\vec{q}_\perp) = \frac{1}{2p\nu_a} \sum_{bcd} \int d\Pi_{2\to 2} \left| M^{ab}_{cd}(\vec{p}, \vec{k}; \vec{p}+\vec{q}, \vec{k}-\vec{q}) \right|^2 F^{ab}_{cd}(\vec{p}, \vec{k}; \vec{p}+\vec{q}, \vec{k}-\vec{q})$$

$$\begin{aligned}
d\Pi_{2\to 2} &= \frac{d^3\vec{k}}{(2\pi)^3}\frac{dq_z}{2\pi}\frac{1}{8\left|\vec{k}\right|\left|\vec{k}-\vec{q}\right|\left|\vec{p}+\vec{q}\right|}(2\pi)\delta(p+k-\left|\vec{p}+\vec{q}\right|-\left|\vec{k}-\vec{q}\right|)\\
&= \frac{d^3\vec{k}}{(2\pi)^3}\frac{dq_z}{2\pi}\int\frac{d\omega}{2\pi}\frac{1}{8\left|\vec{k}\right|\left|\vec{k}-\vec{q}\right|\left|\vec{p}+\vec{q}\right|}(2\pi)\delta\left(p+\omega-\left|\vec{p}+\vec{q}\right|\right)(2\pi)\delta\left(k-\omega-\left|\vec{k}-\vec{q}\right|\right)\\
&= \frac{d^3\vec{k}}{(2\pi)^3}dq_z\frac{1}{8\left|\vec{k}\right|\left|\vec{k}-\vec{q}\right|\left|\vec{p}+\vec{q}\right|}\Theta\left(p+q_z\right)\delta\left(k-\omega-\left|\vec{k}-\vec{q}\right|\right)\\
&= \frac{d^3\vec{k}}{(2\pi)^3}dq_z\frac{1}{8\left|\vec{k}\right|\left|\vec{k}-\vec{q}\right|\left|\vec{p}+\vec{q}\right|}\Theta\left(p+q_z\right)\frac{k-q_z}{kq}\delta\left(\cos\theta_{kq}-\cos\theta^*_{kq}\right)\Theta\left(k-\frac{q+q_z}{2}\right)\Theta\left(k-q_z\right)\\
&= \frac{k^2dk\,d\cos\theta_{kq}d\phi_{kq}}{(2\pi)^3}dq_z\frac{1}{8\left|\vec{p}\right|\left|\vec{k}\right|^2q}\delta\left(\cos\theta_{kq}-\cos\theta^*_{kq}\right)\Theta\left(p+q_z\right)\Theta\left(k-\frac{q+q_z}{2}\right)\Theta\left(k-q_z\right)\\
&= \frac{dk\,d\phi_{kq}}{(2\pi)^3}dq_z\frac{1}{8\left|\vec{p}\right|q}\Theta\left(p+q_z\right)\Theta\left(k-\frac{q+q_z}{2}\right)\Theta\left(k-q_z\right),
\end{aligned}$$

$$\cos\theta^*_{kq} = \frac{q_z}{q}+\frac{q_\perp^2}{2kq}, -\infty < q_z \le \infty, \frac{q+q_z}{2} \le k \le k_{max}.$$

$$F^{ab}_{cd}(\vec{p}, \vec{k}; \vec{p}+\vec{q}, \vec{k}-\vec{q}) = f\left(\vec{k}\right)\left(1 \pm f\left(\vec{k}-\vec{q}\right)\right).$$

leading-logarithmic (LL) approximation:

$ab \leftrightarrow cd$	$\lim_{p\to\infty} \lvert\mathcal{M}^{ab}_{cd}\rvert^2/(p^2g^4)$
$q_1q_i \leftrightarrow q_1q_i$, $\bar{q}_1q_i \leftrightarrow \bar{q}_1q_i$, $q_1\bar{q}_i \leftrightarrow q_1\bar{q}_i$, $\bar{q}_1\bar{q}_i \leftrightarrow \bar{q}_1\bar{q}_i$	$4\frac{d_F^2C_F^2}{d_A}\tilde{M}_{\mathrm{screen}}$
$q_1g \leftrightarrow q_1g$, $\bar{q}_1g \leftrightarrow \bar{q}_1g$	$4d_FC_FC_A\tilde{M}_{\mathrm{screen}}$
$gg \leftrightarrow gg$	$4d_AC_A^2\tilde{M}_{\mathrm{screen}}$

$$M_{\text{Debye screening}} = \frac{4s^2}{\bar{t}^2} = \frac{16\left(k-k_z\right)^2}{q_\perp^2(q_\perp^2+m_D^2)}.\ t \to \bar{t} = t(1+\frac{m_D^2}{q_\perp^2})$$

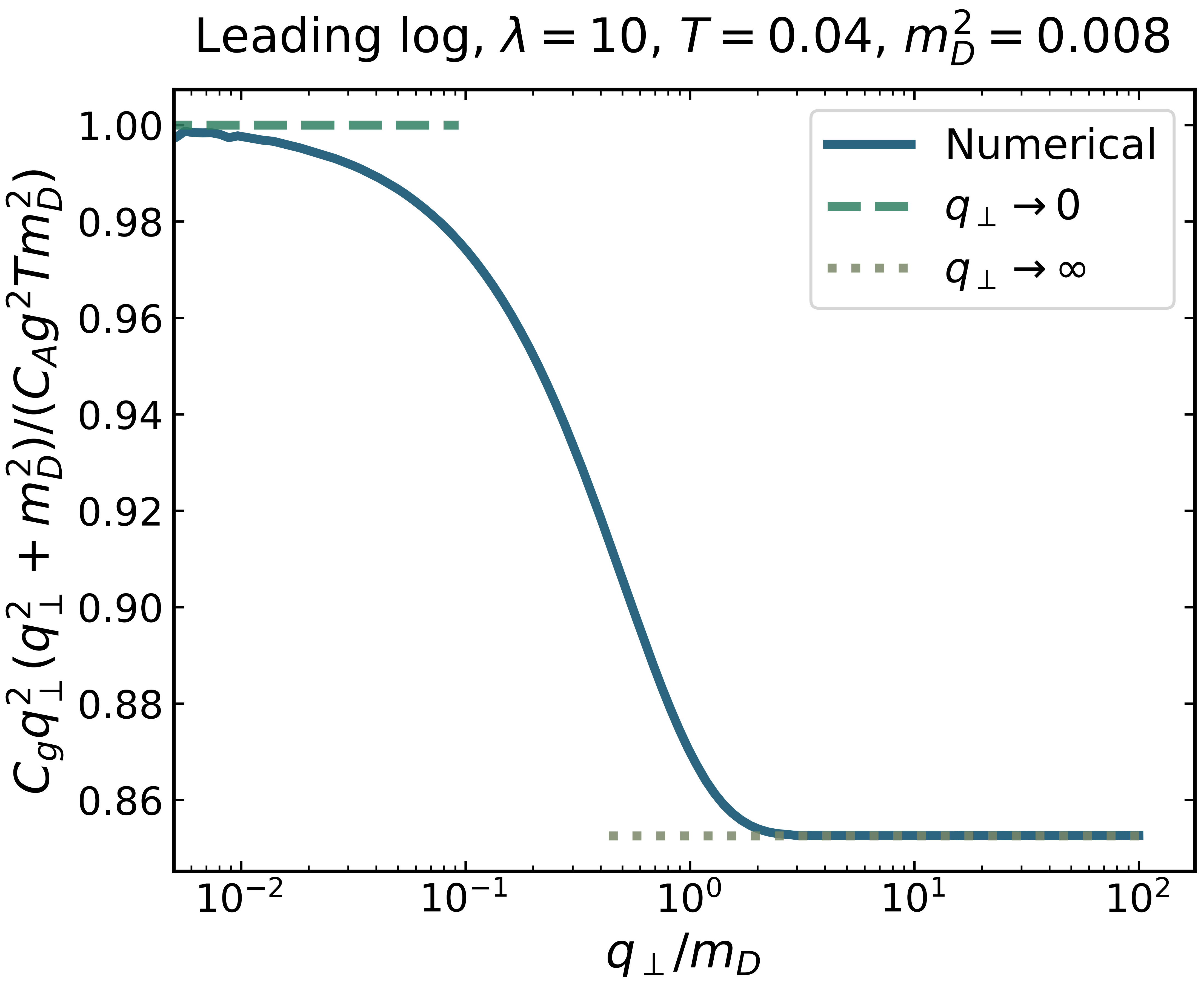
F. Lindenbauer, arXiv:2512.22384 [hep-ph] (2025).

Analytic solution of $C(\mathbf{q}_\perp)$

- For the equilibrium system,

$$f_g(p) = \frac{1}{\exp(p/T) - 1},$$
$$f_q(p) = \frac{1}{\exp((p/T) + 1}$$

$$C(\mathbf{q}_\perp) \to \begin{cases} g^2 C_R T_* \dfrac{m_D^2}{q_\perp^2 (q_\perp^2 + m_D^2)}, & q_\perp \to 0 \\ \dfrac{g^4 C_R N}{q_\perp^4}, & q_\perp \to \infty \end{cases}$$



Fix the physical quantities to the final-state

- Scaling functions for medium quantities:

1. $$\frac{m_D^2(\tau)}{e^{1/2}(\tau)} = \left(\frac{m_D^2}{e^{1/2}}\right)_{\mathrm{eq}} \mu(\tau),$$
2. $$e(\tau)\tau^{4/3} = \left(e\tau^{4/3}\right)_{\mathrm{eq}} \xi(\tau),$$
3. $$\frac{g^2 T_*(\tau)}{e^{1/4}(\tau)} = \left(\frac{g^2 T_*}{e^{1/4}}\right)_{\mathrm{eq}} \mathcal{J}(\tau),$$
4. $$\frac{m_D^2(\tau)}{g^2 T_*(\tau)} \frac{C(\mathbf{q}_\perp, \tau)}{C_R} = \mathscr{C}\left(\frac{\mathbf{q}_\perp}{m_D(\tau)}, \tau\right).$$

→

- Physical normalization from final-state multiplicity:

1. Input from final state ($0 - 5\,\%$ Pb-Pb):
$$\frac{dN_{\mathrm{ch}}}{d\eta} = 1900,\ A = 110\ \mathrm{fm}^2,\ \frac{S}{N_{\mathrm{ch}}} \simeq 6.7,\ \nu_{eff} = 40$$
2. Entropy conservation after hydrodynamization:
$$\tau s \simeq \frac{S}{N_{\mathrm{ch}}} \frac{1}{A} \frac{dN_{\mathrm{ch}}}{d\eta}$$
3. Equilibrium entropy: $s(T) = \frac{4\pi^2}{90}\nu_{\mathrm{eff}} T^3$

$$\to \left(\tau T_{\mathrm{eff}}^3\right)_{\mathrm{eq}} = 0.25669\ \mathrm{GeV}^2$$

$$\left(\frac{m_D^2}{e^{1/2}}\right)_{\mathrm{eq}} = \frac{g_{\mathrm{med}}^2\left(\frac{N_c}{3} + \frac{N_f}{6}\right)}{\left(\frac{\pi^2}{30}\nu_{\mathrm{eff}}\right)^{1/2}}.$$

$$\left(e\tau^{4/3}\right)_{\mathrm{eq}} = \frac{\pi^2}{30}\nu_{\mathrm{eff}} \left(\tau T_{\mathrm{eff}}^3\right)_{\mathrm{eq}}^{4/3},$$

$$\left(\frac{g_{\mathrm{med}}^2 T_*}{e^{1/4}}\right)_{\mathrm{eq}} = \frac{g_{\mathrm{med}}^2}{\left(\frac{\pi^2}{30}\nu_{\mathrm{eff}}\right)^{1/4}}.$$

G. Giacalone, A. Mazeliauskas, and S. Schlichting, Phys. Rev. Lett. 127, 242301 (2021), arXiv:1908.02866 [hep-ph].

Detail of calculation splitting rate

$$\frac{d\Gamma^a_{bc}(\tau)}{dk} \equiv \frac{g^2 P^{a(0)}_{bc}(z)}{\pi P} \times \mathrm{Re} \int_{\tau_0}^{\tau} d\tau_1 \int_{\mathbf{q},\mathbf{p}} \frac{i\mathbf{q}}{\delta \bar{E}(\mathbf{q},\tau)} \mathscr{C}(\tau)\, K(\tau, \mathbf{q}; \tau_1, \mathbf{p}) \cdot \mathbf{p}\,.$$

$$\frac{i}{\delta \overline{E}(\mathbf{q},\tau)} = \int_{\tau}^{\infty} d\tau' \, \exp\left[-i \int_{\tau}^{\tau'} du\, \delta E(\mathbf{q}, u)\right]$$

$$\begin{aligned}\mathcal{C}(\tau)\psi(\mathbf{p},\tau) = \int_{\mathbf{q}} C(\mathbf{q},\tau)\, \{ &\frac{C_b + C_c - C_a}{2}\, [\psi(\mathbf{p},\tau) - \psi(\mathbf{p}-\mathbf{q},\tau)] \\ &+ \frac{C_a + C_c - C_b}{2} \left[\psi(\mathbf{p},\tau) - \psi\left(\mathbf{p} + \frac{k}{p}\mathbf{q}, \tau\right)\right] \\ &+ \frac{C_a + C_b - C_c}{2} \left[\psi(\mathbf{p},\tau) - \psi\left(\mathbf{p} + \frac{p-k}{p}\mathbf{q}, \tau\right)\right].\end{aligned}$$

Evolution function:

$$\left[\partial_\tau + i\,\delta E(\mathbf{q},\tau) + \mathscr{C}(t)\right] K(\tau, \mathbf{q}; \tau_1, \mathbf{p}) = 0.$$

Initial condition:

$$K(\tau_1, \mathbf{q}; \tau_1, \mathbf{p}) = \frac{1}{4p^2 z^2 (1-z)^2} (2\pi)^2 \delta^{(2)}(\mathbf{q} - \mathbf{p})$$

Energy deference:

$$\delta E(\mathbf{p},\tau) = \frac{\mathbf{p}^2}{2Pz(1-z)} + \frac{m_z^2(\tau)}{2zP} + \frac{m_{1-z}^2(\tau)}{2(1-z)P} - \frac{m_1^2(\tau)}{2P}$$

$$\frac{1}{4p^2 z^2 (1-z)^2} \overrightarrow{\psi}(\mathbf{p}, \tau, \tau_1) = \int_{\mathbf{q}} \frac{i\mathbf{q}}{\delta \bar{E}(\mathbf{q},\tau)} \Gamma_3(t) \circ K(\tau, \mathbf{q}; \tau_1, \mathbf{p})$$

Factor out the physical scales by defining the dimensionless variables with $m^2_{D,0} = m^2_D(\tau_0)$:

$$\Delta\tilde{\tau} = \frac{m^2_{D,0}}{2Pz(1-z)} \Delta\tau, \;\; \tilde{q} = \frac{q}{m_{D,0}}, \;\; \tilde{p} = \frac{p}{m_{D,0}}$$

$$\begin{aligned}\delta\tilde{E}(\tilde{\mathbf{p}}) &= \frac{2P\, z(1-z)}{m^2_{D,0}} \delta E(\mathbf{p}) \\ &= \tilde{p}^{\,2} + (1-z)\frac{m_z^2}{m^2_{D,0}} + z\frac{m^2_{1-z}}{m^2_{D,0}} - z(1-z)\frac{m_1^2}{m^2_{D,0}}\,. \\ &= \tilde{p}^{\,2} + (1-z)\left(\frac{m_{z,0}}{m_{D,0}}\right)^2 \left(\frac{m_z(t)}{m_{z,0}}\right)^2 + z\left(\frac{m_{1-z,0}}{m_{D,0}}\right)^2 \left(\frac{m_{1-z}(t)}{m_{1-z,0}}\right)^2 - z(1-z)\left(\frac{m_{1,0}}{m_{D,0}}\right)^2 \left(\frac{m_1(t)}{m_{1,0}}\right)^2 \\ &= \tilde{p}^{\,2} + [(1-z)\left(\frac{m_{z,0}}{m_{D,0}}\right)^2 + z\left(\frac{m_{1-z,0}}{m_{D,0}}\right)^2 - z(1-z)\left(\frac{m_{1,0}}{m_{D,0}}\right)^2]\left(\frac{m_D(t)}{m_{D,0}}\right)^2.\end{aligned}$$

$$\frac{m_g^2(t)}{m_{g,0}^2} = \frac{m_q^2(t)}{m_{q,0}^2} = \frac{m_D^2(t)}{m_{D,0}^2}$$

$$\tilde{\psi}_I(\tilde{p}, \Delta\tilde{\tau} = 0, \tilde{\tau}) = \tilde{\mathbf{p}} \cdot \tilde{\Gamma}_3(\tilde{\tau}) \circ \frac{i\tilde{\mathbf{p}}}{\delta\bar{\tilde{E}}(\tilde{\mathbf{p}}, \tilde{\tau})} = \tilde{\mathbf{p}} \cdot \tilde{\Gamma}_3(\tilde{\tau}) \circ \tilde{\mathbf{p}} \int_{\tilde{\tau}}^{\infty} \tilde{d}s\, e^{-i\int_{\tilde{\tau}}^{s} du\, \delta E(u,\tilde{p})},$$

$$\frac{d\Gamma^a_{bc}}{dz} = \frac{g^4 T_* P^a_{bc}(z)}{\pi} \mathrm{Re} \int_0^{\tilde{\tau}-\tilde{\tau}_0} d\Delta\tilde{\tau} \int_{\tilde{\mathbf{p}}} e^{-i\int_0^{\Delta\tilde{\tau}} du\, \delta\tilde{E}(u,\tilde{p})} \tilde{\psi}_I(\tilde{p}, \Delta\tilde{\tau}, \tilde{\tau})$$

S. Caron-Huot and C. Gale, Phys. Rev. C 82, 064902 (2010); I. Soudi, arXiv:2409.04806 (2024).

Non equilibrium expanding medium & Equilibrated Bjorken medium

- Splitting rate: $\frac{d\Gamma^a_{bc}}{dz}$