

Azimuthal momentum isotropization during the QGP thermalization

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in collaboration with

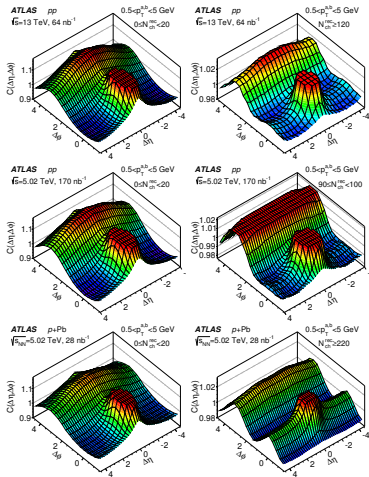
Xiaojian Du, Carlos A. Salgado and Bin Wu

based on

(July 2026). SBC, Du, Salgado, and Wu [arXiv:2607.07796]



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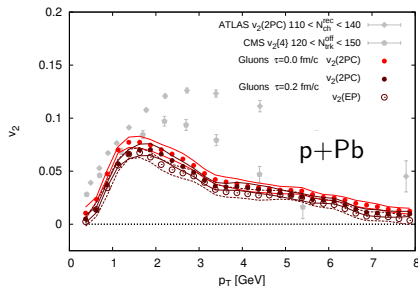


- A similar ridge to that in heavy-ion collision has been observed in small systems

Phys. Rev. C 96.2 (2017). Aaboud et al.

- Hydrodynamics might not be the cause
- Initial state mechanisms have been proposed to explain it

Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto



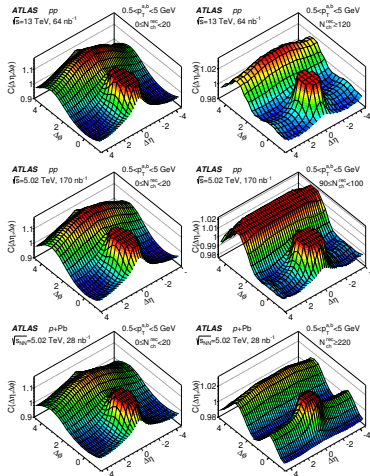
Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

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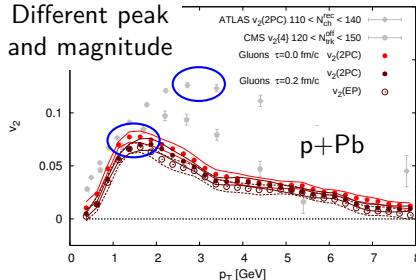
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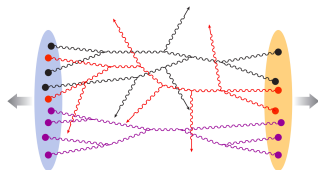


Different peak and magnitude



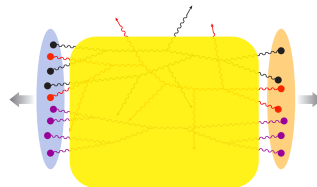
Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

CGC



Kinetic theory

Hydro (QGP)



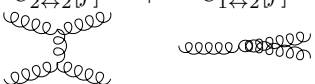
Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis, Iancu,
Jalilian-Marian, and Venugopalan

0.1 fm/c

1 fm/c

10 fm/c

- The QCD Boltzmann equation at leading order

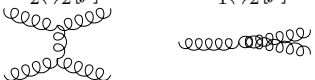
$$\left(\partial_t + \underbrace{\frac{\mathbf{p}}{p} \cdot \nabla_{\mathbf{x}}}_{\text{Expansion}} \right) f^a(\mathbf{x}; \mathbf{p}; t) = C_{2 \leftrightarrow 2}^a[f] + C_{1 \leftrightarrow 2}^a[f], \quad a = \{g, q, \bar{q}\}$$


The diagram for $C_{2 \leftrightarrow 2}^a$ shows two incoming wavy lines (gluons) and two outgoing wavy lines, representing a 2-to-2 scattering process. The diagram for $C_{1 \leftrightarrow 2}^a$ shows one incoming wavy line and two outgoing wavy lines, representing a 1-to-2 splitting process.

- Distribution function is related with particle spectra

$$\frac{1}{\tau} \frac{d^3 N}{d^2 \mathbf{p}_T dy d\eta} \bigg|_{\tau} \propto \int d^2 \mathbf{x}_{\perp} f(\tau; \eta; \mathbf{p}_T, p_T \sinh(y - \eta))$$

- The QCD Boltzmann equation at leading order

$$\left(\partial_t + \underbrace{\frac{\mathbf{p}}{p} \cdot \nabla_{\mathbf{x}}}_{\text{Expansion}} \right) f^a(\mathbf{x}; \mathbf{p}; t) = C_{2 \leftrightarrow 2}^a[f] + C_{1 \leftrightarrow 2}^a[f] \quad , \quad a = \{g, q, \bar{q}\}$$


The diagram for $C_{2 \leftrightarrow 2}^a$ shows two incoming gluon lines (curly) merging into two outgoing gluon lines. The diagram for $C_{1 \leftrightarrow 2}^a$ shows a single incoming gluon line merging into two outgoing gluon lines.

- Distribution function is related with particle spectra

$$\frac{1}{\tau} \frac{d^3 N}{d^2 \mathbf{p}_T dy d\eta} \bigg|_{\tau} \propto \int d^2 \mathbf{x}_{\perp} f(\tau; \eta; \mathbf{p}_T, p_T \sinh(y - \eta))$$

- Longitudinal-boost invariant expansion + homogeneity in transverse plane

$$\left(\partial_{\tau} - \frac{p_z}{\tau} \partial_{p_z} \right) f^a = C_{2 \leftrightarrow 2}^a[f] + C_{1 \leftrightarrow 2}^a[f] \quad , \quad a = \{g, q, \bar{q}\}$$

- In the weak-coupling limit $\rightarrow$ Bottom-up thermalization

Phys. Lett. B 502 (2001). Baier, Mueller, Schiff, and Son

- Effective Kinetic Theory (EKT) has been widely used in this context

JHEP 01 (2003). Arnold, Moore, and Yaffe

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. Lett. 127.12 (2021). Du and Schlichting

- Boltzmann Equation in Diffusion Approximation (BEDA) keeps most of the relevant physics.

JHEP 06 (2024). SBC, Salgado, and Wu

Phys. Lett. B 871 (2025). SBC, Du, Salgado, and Wu

	BEDA	EKT
$C_{2 \leftrightarrow 2}$	Only small angle scatterings $\Rightarrow$ Fokker-Planck + source term	Full $2 \leftrightarrow 2$ QCD dynamics @LO
$C_{1 \leftrightarrow 2}$	Deep LPM regime	Both LPM and Bethe-Heitler

- Initialize with CGC-inspired initial condition + azimuthal anisotropies

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T) \left(1 + \sum_n v_n^0(\mathbf{p}_T) \cos(n\phi) \right)$$

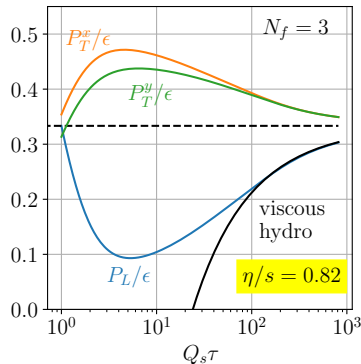
- Harmonic coefficients computed in the Event Plane

$$v_n(p_T; t) = \frac{\int dp_z d\phi \cos(n(\phi - \psi_n)) f(\mathbf{p}; t)}{\int dp_z d\phi f(\mathbf{p}; t)}$$

- Consider the anisotropy of the full system

$$f \equiv \nu_g f^g + 2N_f \nu_q f^q$$

- Azimuthal isotropization time is comparable to hydrodynamization time



- Start initializing with constant anisotropy and explore fully integrated v_n

$$v_n^0(\mathbf{p}_T) = v_n^0 \quad v_n(t) \equiv \frac{\int d^3\mathbf{p} \cos(n(\phi - \psi_n)) f(\mathbf{p}; t)}{\int d^3\mathbf{p} f(\mathbf{p}; t)}$$

- Exploring the variation due to the $C_{2 \leftrightarrow 2}$ kernel

$$\partial_\tau v_n(\tau) = \frac{\hat{q}}{4n_g} \int \frac{dp}{(2\pi)^2} \frac{d(\cos \theta)}{p^2} p^2 \left[-\frac{n^2 v_n^2 \tilde{f}/2}{p^2(1 - \cos^2 \theta)} \right.$$

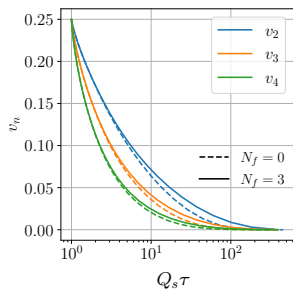
Higher n relaxes faster

$$+ \frac{1}{T_*} \left(\frac{2\tilde{f}}{p} + \partial_p \tilde{f} \right) v_n^2$$

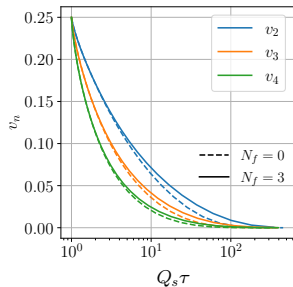
$$+ \frac{1}{T_*} \left(\frac{2\tilde{f}^2}{p} + \partial_p \tilde{f}^2 \right) \left(v_n^2 + \sum_{k,l} v_k v_l \int \frac{d\phi}{2\pi} \cos(n\phi) \cos(k\phi) \cos(l\phi) \right) \Big]$$

Produces higher n anisotropies

- For initial v_n , only v_{kn} with $k = 1, 2, 3 \dots$ are produced

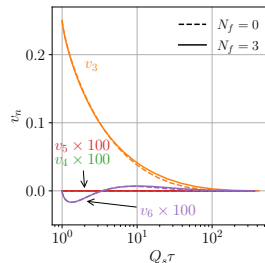
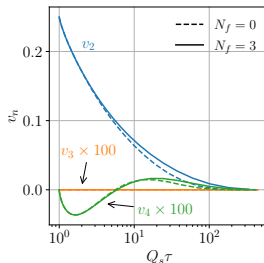


- Higher n relaxes faster
- Including quarks in the system delays the isotropization



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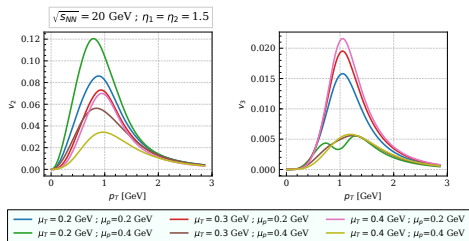
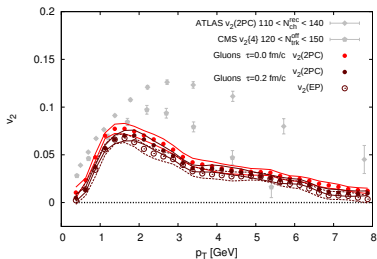
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- Based on different results from initial state interactions we model

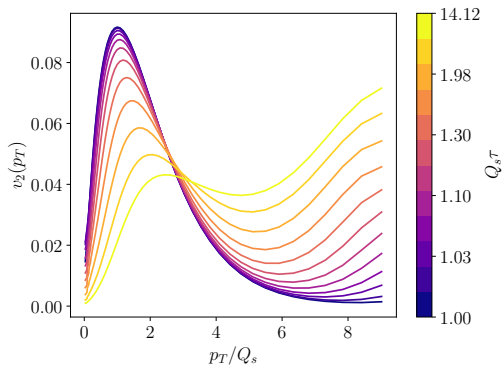
Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T) \left(1 + \sum_n v_n^0(\mathbf{p}_T) \cos(n\phi) \right), \quad v_n^0(p_T) = \tilde{v}_n \frac{p_T}{Q} e^{-p_T/Q}$$



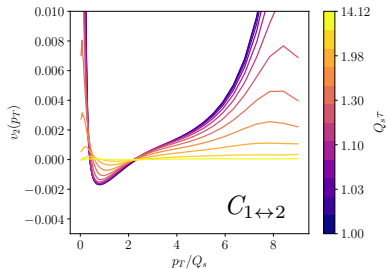
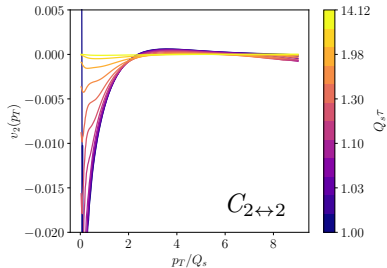
Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

Eur. Phys. J. C 79.9 (2019). Agostini, Altinoluk, and Armesto



$$v_n(p_T; t) = \frac{\int dp_z d\phi \cos(n(\phi - \psi_n)) f(\mathbf{p}; t)}{\int dp_z d\phi f(\mathbf{p}; t)}$$

- The magnitude of the anisotropy decreases with time
- The position of the maximum is shifted to higher momenta
- Anisotropy at large p_T is produced easily because of the low occupancy in this region



• Infrared:

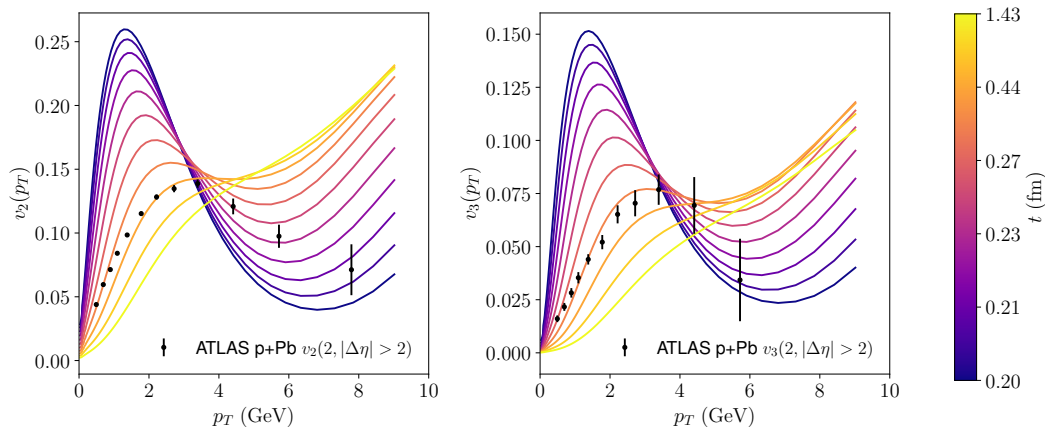
- $C_{1 \leftrightarrow 2} \rightarrow$ Increases v_2 due to **collinear radiation** of partons from the more anisotropic regions of the phase space
- $C_{2 \leftrightarrow 2} \rightarrow$ Decreases the anisotropy more efficiently than $C_{1 \leftrightarrow 2}$

• Ultraviolet:

- $C_{1 \leftrightarrow 2} \rightarrow$ Increases v_2 due to **collinear merging** of partons in the more anisotropy regions of the phase space
- $C_{2 \leftrightarrow 2} \rightarrow$ Modifies v_2 less efficiently

- Parametrization to reproduce energy density from $p + Pb$ collisions at LHC
- Experimental data mimicked at a time $t \sim 0.5$ fm

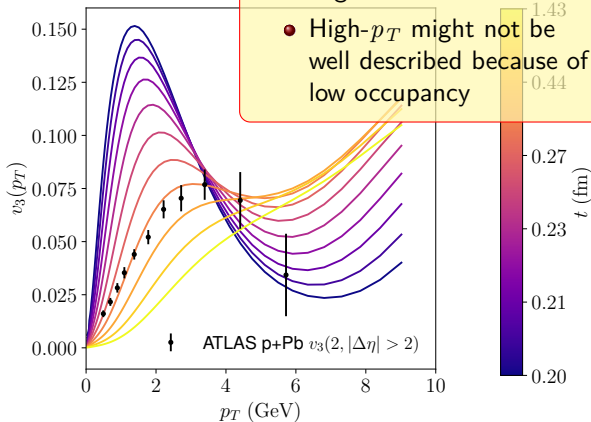
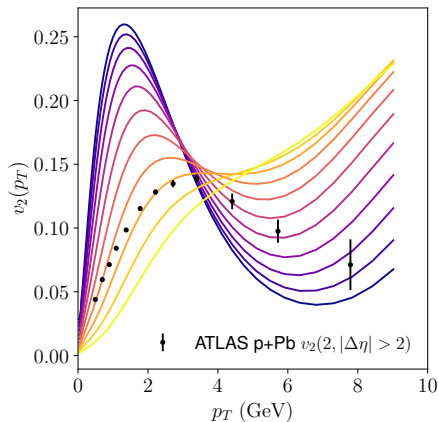
JHEP 11 (2021). Kurkela, Mazeliauskas, and Törnkvist



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Disclaimer
JHEP 11 (2021) 100, Kukuła, Mazeliauskas, and Törnkvist

- Spatial gradients are neglected
- High- p_T might not be well described because of low occupancy



- Initial anisotropies may not be fully washed out at hydrodynamization time
- Harmonic coefficients with higher n isotropize faster
- Small higher-order Fourier coefficients are generated for initial $v_n \neq 0$
- Final state interactions introduce a mechanism which shifts the maximum of the azimuthal anisotropy to higher p_T , as experimental data suggest

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Thank you for your attention!

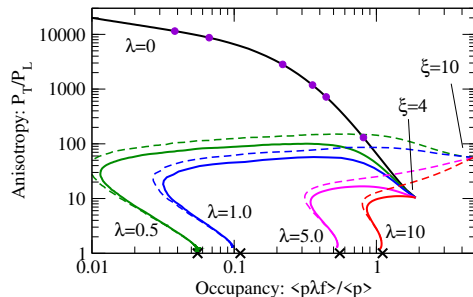
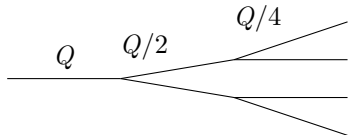
Back-up

- In the weak-coupling limit $\rightarrow$ Bottom-up thermalization

Phys. Lett. B 502 (2001). Baier, Mueller, Schiff, and Son

- There are three different stages for thermalization when considering boost-invariant expansion

- 1 System dominated by expansion
- 2 Collision and expansion contribute similarly
- 3 Democratic cascade speeds up the thermalization

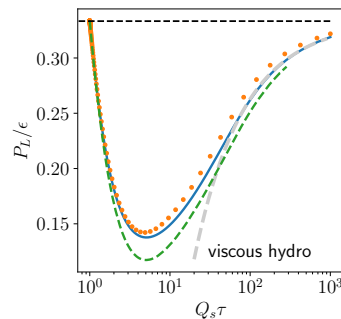
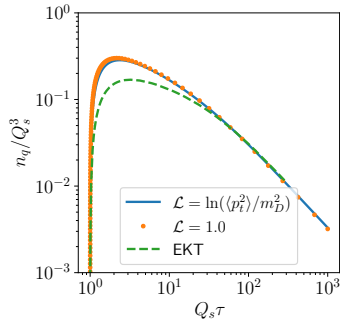
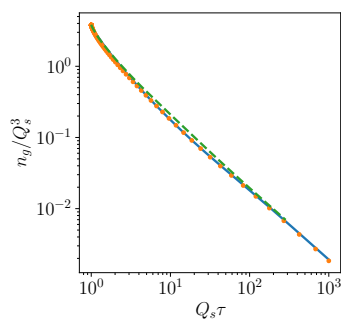


Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

- BEDA qualitatively agrees with EKT results for the same setups

Phys. Lett. B 871 (2025). SBC, Du, Salgado, and Wu

- Quantitative differences explained by differences in the collision kernels



- Evolve from CGC-like initial conditions

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. D 99.5 (2019). Kurkela and Mazeliauskas

$$f_0 = \frac{A}{4\pi N_c \alpha_s} \frac{e^{-\frac{2}{3}((p_z \xi)^2 + p_\perp^2)/Q_0^2}}{\sqrt{((p_z \xi)^2 + p_\perp^2)/Q_0^2}}, \quad F_0 = 0,$$

- $\xi \rightarrow$ Quantifies anisotropy (taken as 0 for this comparison)
- $Q_0, A \rightarrow$ Phenomenological parameters fitting CGC
- Initial time $\tau_0 = Q_s^{-1}$
- Add some initial anisotropies on top

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T) \left(1 + \sum_n v_n^0(\mathbf{p}_T) \cos(n\phi) \right)$$

- Same initial condition as before

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T) \left(1 + \sum_n v_n^0(\mathbf{p}_T) \cos(n\phi) \right), \quad v_n^0(p_T) = \tilde{v}_n \frac{p}{Q} e^{-|p|/Q}$$

- Parametrization to reproduce energy density from $p + Pb$ collisions at LHC

JHEP 11 (2021). Kurkela, Mazeliauskas, and Törnkvist

- Initial time for the kinetic stage set to $t_0 = 0.2$ fm
- Initialize anisotropy with $\tilde{v}_2 = 0.75$, $\tilde{v}_3 = 0.45$ and $\tilde{v}_{n \neq 2,3} = 0$
- Position of the maximum set to $Q \sim Q_s \approx 1.22$ GeV

- Initial $v_2 \Rightarrow$ only even harmonics are generated

