# Vector meson spin alignment from local equilibrium and freeze-out geometry

**Zhong-Hua Zhang**

Fudan Univ. and Univ. of Florence

2412.19416 & 2509.20200

With Xu-Guang Huang, Xin-Li Sheng, and Francesco Becattini

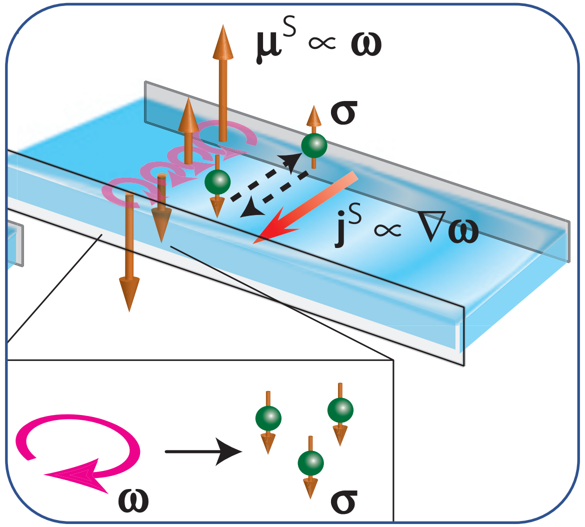
17.08.2026 @ Helsinki

17–21 Aug 2026
Helsinki, Finland

Strong and Electro-Weak Matter 2026

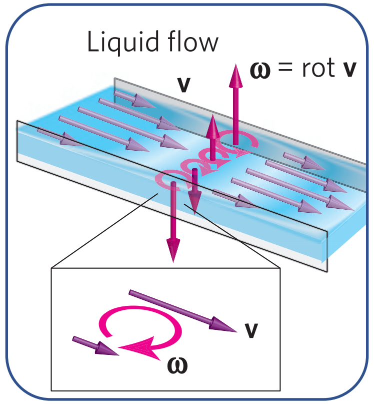
# Spin polarization in fluids

### Spin hydrodynamic generation



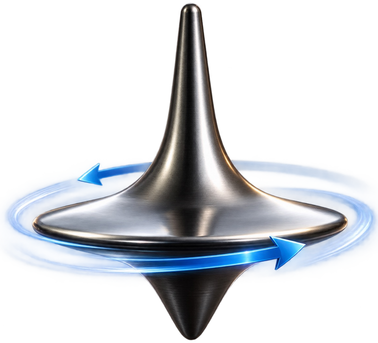

Takahashi etal Nature Physics 12, 52 (2016)

### Collective motion of flow

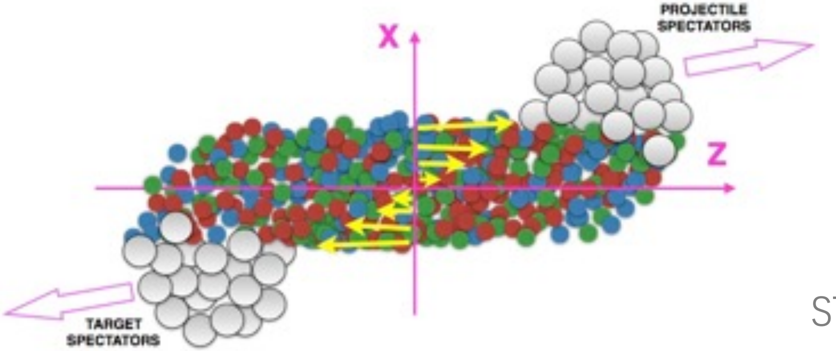



$$H_{\text{spin-rot}} = -\boldsymbol{\omega} \cdot \boldsymbol{S}$$

### Spin polarization



**Spin — A new probe in HIC to detect the transport of angular momentum**



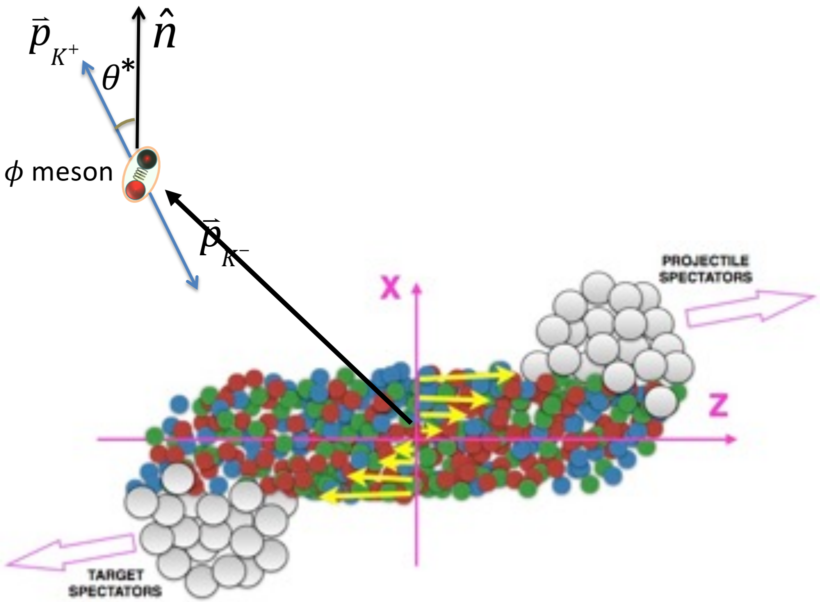

$$\omega \sim 9 \times 10^{21} s^{-1}$$

**"Most vortical fluid"**

STAR, Nature 548, 62–65 (2017)

# Spin alignment puzzle

Non-central heavy-ion collisions




$$\omega \sim 9 \times 10^{21} s^{-1}$$

"Most vortical fluid"

Quantization axis (-y)

$$P = -\langle \sigma^y \rangle \simeq \frac{\omega}{2T} \sim 10^{-2}$$

√

**Spin (vector) polarization**

Liang, Wang, PRL 94, 102301 (2005)

$$H_{\text{spin-rot}} = -\boldsymbol{\omega} \cdot \boldsymbol{\sigma}/2$$

**Spin alignment (tensor polarization)**

Liang, Wang, PLB 629, 20–26 (2005)

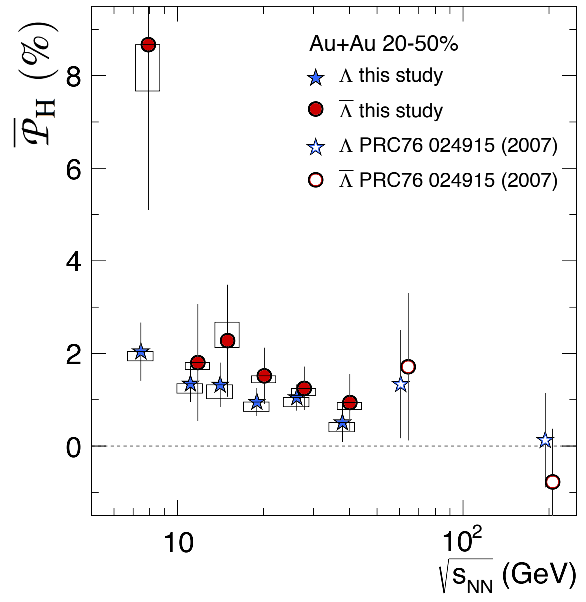
$$\delta\rho_{00} \equiv \rho_{00} - 1/3$$

$$\simeq -\frac{4}{9}\langle \sigma^y \sigma^y \rangle \sim -\left(\frac{\omega}{T}\right)^2 \sim -10^{-4}$$

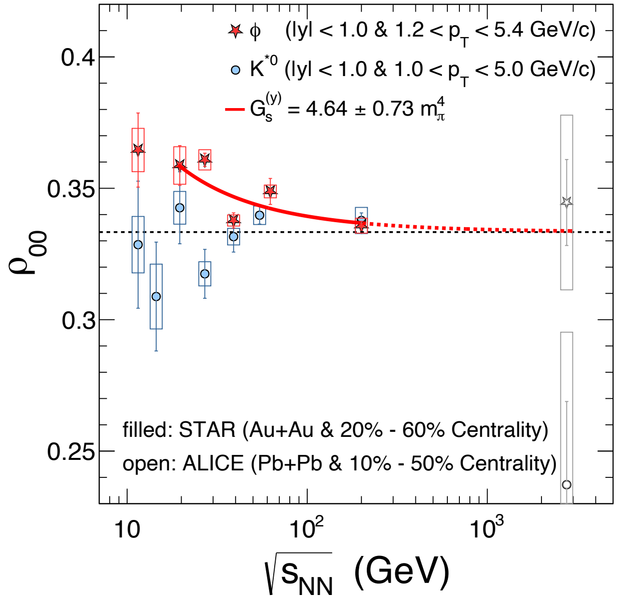
?

**Based on quark coalescence**

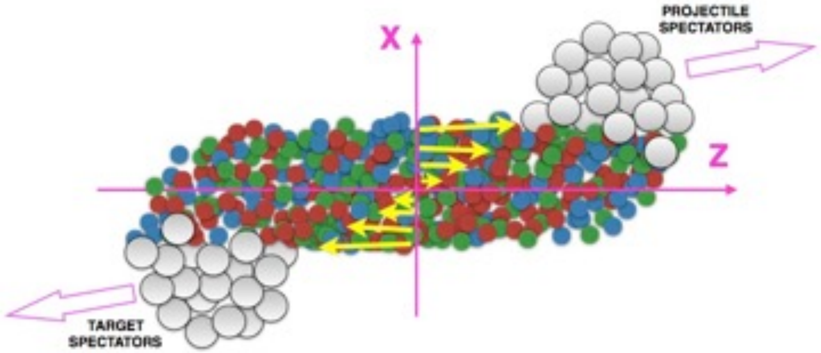
STAR, Nature 548, 62–65 (2017)




STAR, Nature 614, 244-248 (2023)



# Local equilibrium density operator




**A uniform rotation – a global equilibrium state**

$$\widehat{\rho}_{\mathrm{GE}} = \frac{1}{Z_{\mathrm{GE}}} \exp\left\{-\beta \widehat{H} + \beta \boldsymbol{\omega} \cdot \boldsymbol{\sigma}/2\right\}$$

**A local Gibbs state for massive spin-1 bosons**

Canonical stress tensor

Canonical spin tensor

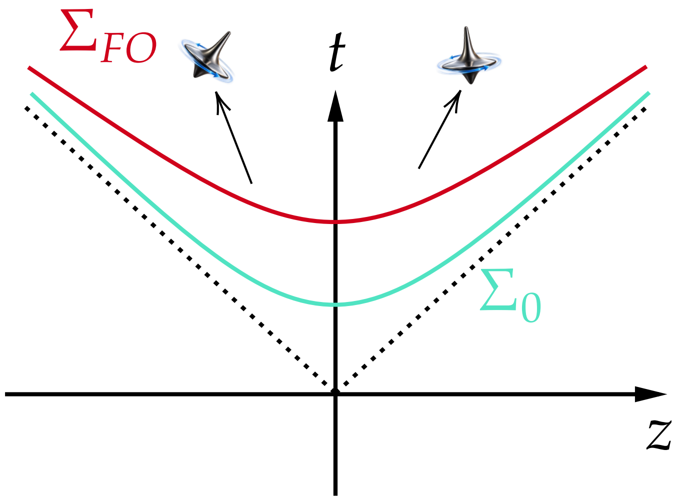
$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z_{\mathrm{LE}}} \exp\left\{-\int_{\Sigma_{\mathrm{FO}}} d\Sigma_\mu(y) \left[\widehat{T}^{\mu\nu}(y)\, \beta_\nu(y) - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\, \Omega_{\rho\sigma}(y)\right]\right\}$$

Thermal velocity

Spin potential

Zubarev etal 1979, Van Weert 1982, Becattini etal 2013

Particles emitted from $\Sigma_{FO}$ can be treated as free particles



# Spin state of vector meson

- Covariant spin operator for single particle

Spin vector $\hat{\mathcal{S}}^{\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\hat{J}_{\nu\rho}\hat{P}_{\sigma}$

Spin tensor $\hat{\mathcal{T}}^{\mu\nu} \equiv \hat{\mathcal{S}}^{(\mu}\hat{\mathcal{S}}^{\nu)} + \frac{2}{3}\left(\eta^{\mu\nu} - \frac{1}{m^2}\hat{P}^{\mu}\hat{P}^{\nu}\right)$

- Spin state in the rest frame

$\epsilon_i^{\mu}(k)\langle\hat{\mathcal{S}}^{\mu}\rangle_k$ $\qquad$ $\epsilon_{\mu}^{i}(k)\epsilon_{\nu}^{j}(k)\langle\hat{\mathcal{T}}^{\mu\nu}\rangle_k$

Spin density matrix

$$\begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho_{01} & \rho_{00} & \rho_{0,-1} \\ \rho_{-1,1} & \rho_{-1,0} & \rho_{-1,-1} \end{pmatrix}$$

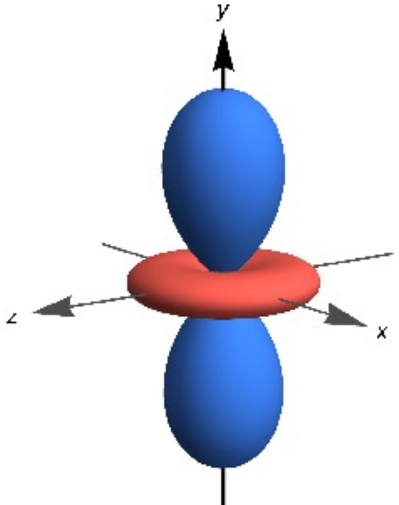
$$\rho(k) = \frac{1}{3}I + \frac{1}{2}\sum_i \mathbb{S}^i(k)\, s^i + \sum_{ij}\mathbb{T}^{ij}(k)\,\Sigma^{ij}$$

Adjoint rep. of SO(3)

Spin tensor $s^{(i}s^{j)} - \frac{\mathbf{s}^2}{3}\delta^{ij}$

Symmetric traceless tensor

Quadrupole-type observable




- Spin alignment is yy component of polarization tensor

$$\delta\rho_{00}(k) = \rho_{00}(k) - 1/3 = -\mathbb{T}^{yy}(k)$$

# Spin in (pseudo) phase space

- Wigner function of free Proca field

$$W^{\mu\nu}(x,k) = \theta(k^2)\theta(k^0)\int d^4y\, e^{ik\cdot y}\left\langle \widehat{A}^{\nu}\left(x-\frac{y}{2}\right)\widehat{A}^{\mu}\left(x+\frac{y}{2}\right)\right\rangle_{\mathrm{LE}}$$

Projected Wigner function $\Delta^{\mu\alpha}_{(k)}\Delta^{\nu\beta}_{(k)}W_{\alpha\beta}(x,k)$

$$W^{\mu\nu}_{\perp}(x,k) = (2\pi)f(x,k)\left\{-\frac{1}{3}\Delta^{\mu\nu}_{(k)} + \frac{i}{2m}\epsilon^{\mu\nu\rho\sigma}k_\rho\,\mathcal{S}_\sigma(x,k) - \mathcal{T}^{\mu\nu}(x,k)\right\}$$

Scalar distribution Polarization vector Polarization tensor

- Spin polarization in phase space

$$\mathcal{S}^{\mu}(x,k) = -\frac{i\epsilon^{\mu\nu\alpha\beta}\,k_\nu\,W_{\alpha\beta}(x,k)}{(2\pi)\,m\,f(x,k)} \qquad \mathcal{T}^{\mu\nu}(x,k) = -\frac{W^{\langle\mu\nu\rangle}_{\perp}(x,k)}{(2\pi)f(x,k)}$$

Symmetric traceless part — quadrupole

- Average over the hypersurface

$$\mathcal{S}^{\mu}(k) \equiv \left\langle\widehat{\mathcal{S}}^{\mu}\right\rangle_k = \frac{\int d\Sigma\cdot k\, f(x,k)\,\mathcal{S}^{\mu}(x,k)}{\int d\Sigma\cdot k\, f(x,k)} \qquad \mathcal{T}^{\mu\nu}(k) \equiv \left\langle\widehat{\mathcal{T}}^{\mu\nu}\right\rangle_k = \frac{\int d\Sigma\cdot k\, f(x,k)\,\mathcal{T}^{\mu\nu}(x,k)}{\int d\Sigma\cdot k\, f(x,k)}$$

# Evaluating Wigner function — Gradient expansion

- **Decompose density operator by gradients**

$$\widehat{\rho}_{\rm LE} = \frac{e^{\widehat{A}+\widehat{B}}}{\mathrm{Tr}\, e^{\widehat{A}+\widehat{B}}} \qquad \widehat{A}(x) = -\widehat{P}^{\mu}\beta_{\mu}(x) \qquad \widehat{B}(x) = -\int_{\Sigma_{\rm FO}} \boxed{d\Sigma_{\mu}(y)} \left[\widehat{T}^{\mu\nu}(y)\,\boxed{\Delta\beta_{\nu}(y,x)} - \frac{1}{2}\widehat{S}^{\mu\rho\sigma}(y)\,\boxed{\Omega_{\rho\sigma}(y)}\right]$$

$$\Delta\beta_{\mu}(y,x) = \beta_{\mu}(y) - \beta_{\mu}(x) = (y-x)^{\alpha}\partial_{\alpha}\beta_{\mu}(x) + \cdots \qquad \Omega_{\rho\sigma}(y) = \Omega_{\rho\sigma}(x) + (y-x)^{\alpha}\partial_{\alpha}\Omega_{\rho\sigma}(x) + \cdots$$

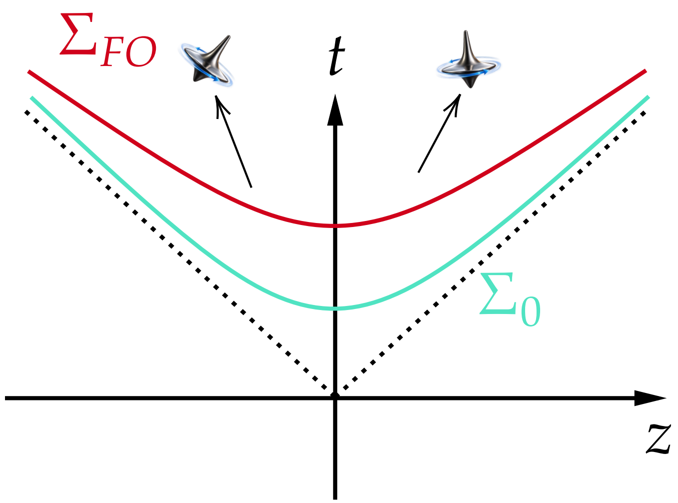
- **Dyson/Kubo series** $e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}}\mathcal{P}_{\lambda}\exp\left[\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}\right] = e^{\widehat{A}}\left(1 + \int_0^1 d\lambda\,\widehat{B}_{(\lambda)} + \cdots\right)$ with $\widehat{B}_{(\lambda)} \equiv e^{-\lambda\widehat{A}}\widehat{B}e^{\lambda\widehat{A}}$

$$W^{\mu\nu}(x,k) = \frac{\left\langle \mathcal{P}_{\lambda} e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x)}}{\left\langle \mathcal{P}_{\lambda} e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\right\rangle_{\beta(x)}} = \boxed{\left\langle \mathcal{P}_{\lambda} e^{\int_0^1 d\lambda\,\widehat{B}_{(\lambda)}}\widehat{W}^{\mu\nu}(x,k)\right\rangle_{\beta(x),c}}$$

Gaussian distribution + Wick's theorem

Connected correlations

Evaluate Wigner function upto any orders of gradient




- **Results**

**Spin alignment**

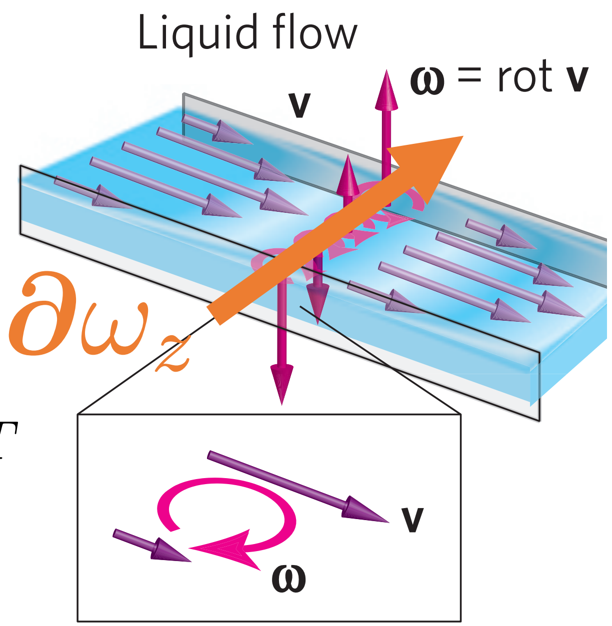
$\propto \mathcal{O}(\partial^2) \qquad \partial^2\beta\,,\ \partial\Omega\,,\ \cdots \qquad$ 2412.19416

$\propto \mathcal{O}(\partial) \times \mathrm{curvaure} \qquad$ 2509.20200

# Spin alignment by gradient of vorticity

$$\delta\rho_{00}(x,k) \simeq \left[\epsilon^y_\mu(k)\epsilon^y_\nu(k) + \frac{1}{3}\Delta^{(k)}_{\mu\nu}\right] \frac{n^{[\rho}\partial^{\mu]}\varpi_{\rho\sigma}}{6E_k}\left(2\eta^{\sigma\nu} - \hat{k}^\sigma n^\nu\right)$$

Thermal vorticity $\partial_{[\sigma}\beta_{\rho]} \sim \omega/T$




Vorticity → Spin polarization

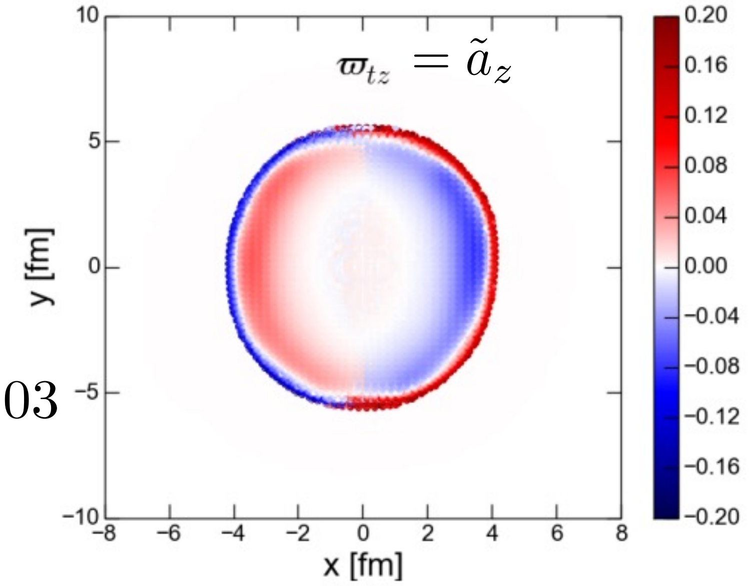
Grad of vorticity → Spin alignment

- A special case

In the rest frame of normal vector n    At the low momentum limit

$$\delta\rho_{00}(x,k) \simeq \frac{1}{6m}\left(\partial_y\tilde{a}_y - \frac{1}{3}\nabla\cdot\tilde{\boldsymbol{a}}\right)$$

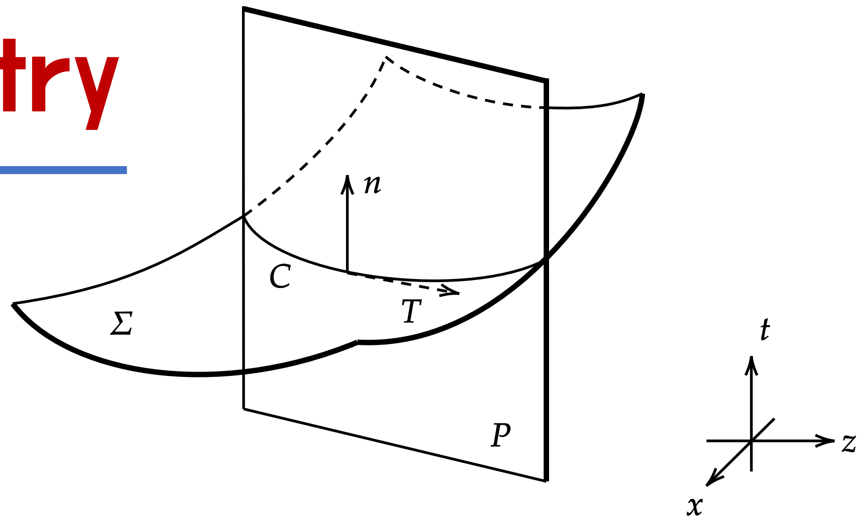
$$(\partial\tilde{a})/m \sim 0.03$$




Spin alignment detects the anisotropy of grad acceleration

Karpenko etal. Eur. Phys. J. C (2017) 77: 213

# Spin alignment by freeze-out geometry




Curvature in the tangent space $B^{\mu\nu}(x) \equiv -\Delta^{\mu\alpha}_{(n)} \frac{\partial n^\nu(x)}{\partial x^\alpha}$

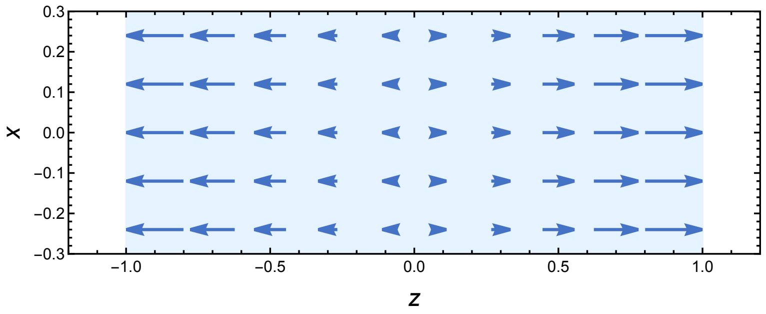
$$\mathcal{T}^{\mu\nu}(x,k) \simeq \frac{1+n_B}{6(n\cdot k)} B_{\alpha\beta}(x) \Big\{ -\xi_{\rho\sigma}\left(\eta^{\rho\sigma} + \frac{k^\rho k^\sigma}{2m^2}\right)\left[\Xi^{\alpha\mu}\Xi^{\beta\nu} - \frac{1}{3}\Delta^{\mu\nu}_{(k)}\left(\eta^{\alpha\beta} + \hat{k}^\alpha \hat{k}^\beta\right)\right]$$
$$+\,\xi_{\rho\sigma}\left[\Xi^{\alpha\rho}\Xi^{\beta(\mu}\Delta^{\nu)\sigma}_{(k)} - \frac{1}{3}\Delta^{\mu\nu}_{(k)}\Xi^{\alpha\rho}\Xi^{\beta\sigma}\right] - \delta\Omega_{\rho\sigma}\Xi^{\alpha\rho}\Xi^{\beta(\mu}\Delta^{\nu)\sigma}_{(k)} \Big\}$$

Thermal shear $\partial_{(\rho}\beta_{\sigma)}$

- **Bjorken flow** Static particle inside fluid cell $\mathcal{T}^{\mu\nu}(x,k) \simeq -\frac{c_s^2 B^{\langle\mu\nu\rangle}}{4mT_f\tau_f}$

  Bjorken PRD 27, 140 (1983)

**Anisotropic freeze-out surface ➡ Tensor polarization**

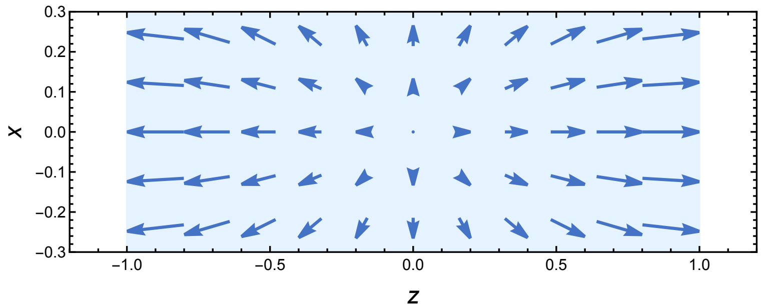



- **Gubser flow** – Bjorken longitudinal expansion + conformal **transversal** expansion

  Gubser PRD 82, 085027 (2010)

$$\delta\rho_{00} = -\frac{7q}{36m\widetilde{T}_0} \sim L^{-2} \qquad q \sim L^{-1} \qquad T \sim \tilde{T}_0 q \sim L^0$$

**Au-Au** $\sim -3\times10^{-3}$ **O-O** $\sim -10^{-2}$

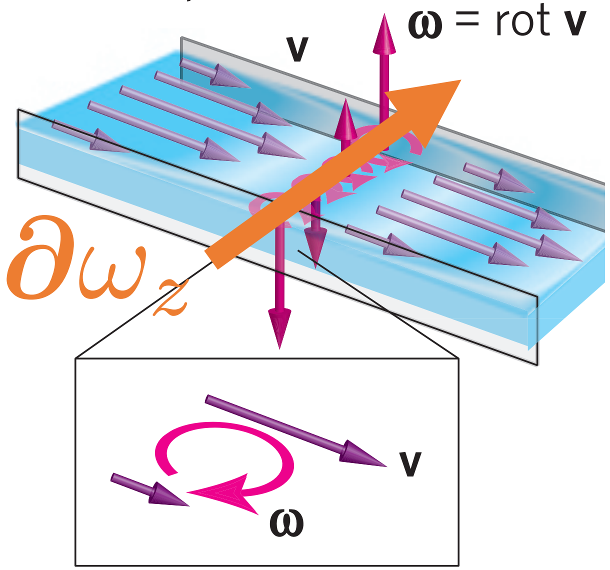


# Conclusion

Vorticity → Spin polarization

Grad of vorticity → Spin alignment

$$\delta\rho_{00}(x,k) \simeq \frac{1}{6m}\left(\partial_y \tilde{a}_y - \frac{1}{3}\nabla\cdot\tilde{\boldsymbol{a}}\right)$$



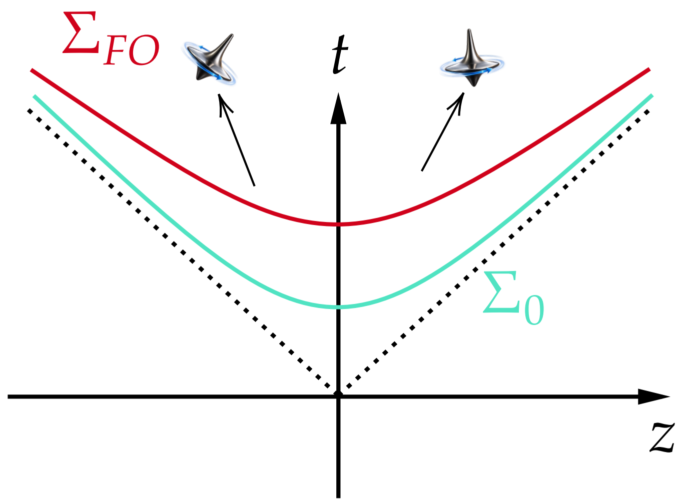

Anisotropic freeze-out surface

↓

Spin alignment

$$\delta\rho_{00} \simeq -\frac{7q}{36m\widetilde{T}_0} \sim L^{-2}$$

May provide a clean probe of this geometric effect



# Thank you!

17.08.2026 @ Helsinki

17–21 Aug 2026
Helsinki, Finland

Strong and Electro-Weak Matter 2026