ArXiV:2605.31194

Domain-wall-seeded nucleation on the lattice

UNIVERSITY OF HELSINKI
FACULTY OF SCIENCE

Simone Blasi, Andreas Ekstedt, Jaakko Hällfors, Kari Rummukainen

simone.blasi@desy.de
jaakko.hallfors@helsinki.fi
kari.rummukainen@helsinki.fi

First-order phase transitions in the early universe

Some extensions of the Standard Model permit first-order phase transitions at the electroweak scale. Such transitions happen via the nucleation of bubbles from the metastable phase to the stable phase. This violent process would produce a gravitational wave background that could be observed in future experiments.

Nucleation rate

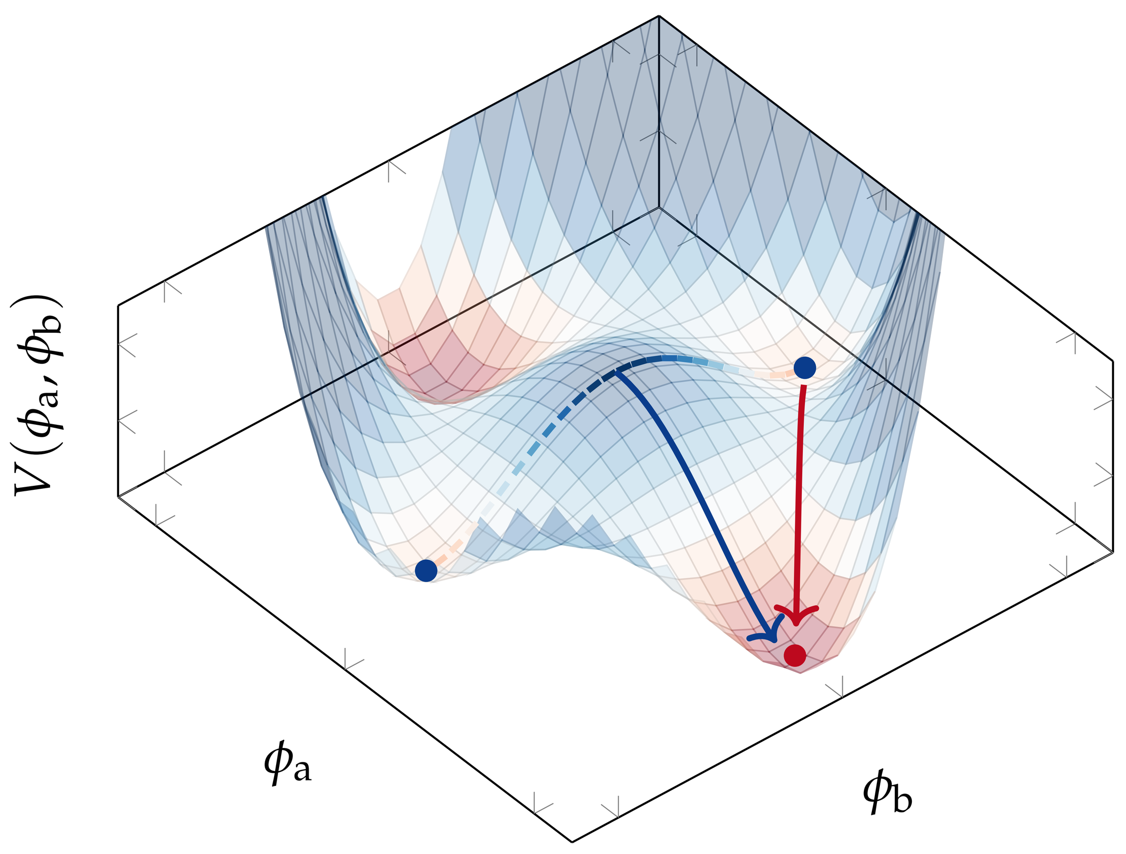
One of the quantities needed for predicting the resulting GWB is the nucleation rate, which determines how fast bubbles of the stable phase appear in the supercooled system. In the early universe, this quantity is typically computed for a spatially homogeneous metastable phase.

Most first-order phase transitions in nature happen through heterogeneous, or seeded nucleation. In this case the rate is enhanced by some impurity of the system. In the early universe, topological defects such as monopoles, domain walls or cosmic strings could act as nucleation sites, increasing the rate.

Toy model

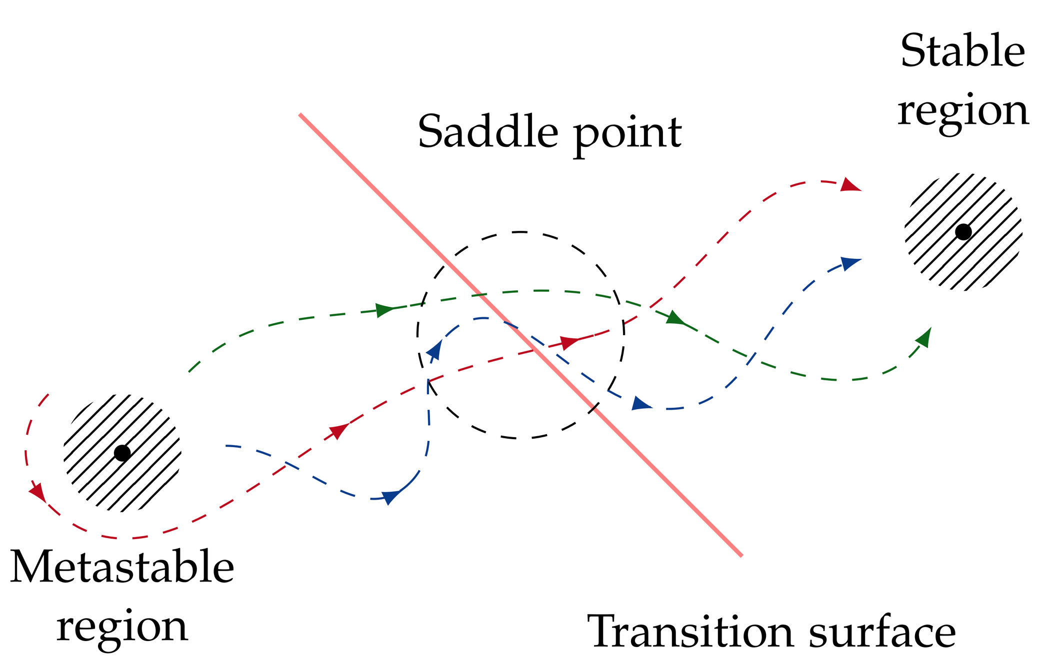
We study the problem in a two-dimensional system of two real scalar fields. Initially the system forms domain walls with both minima being metastable. Nucleation to the true minimum is then enhanced near the domain wall.

$$\mathcal{L} = \sum_{\alpha=\mathrm{a,b}} \left[\frac{1}{2}(\partial_\mu\phi_\alpha)^2 - \frac{1}{2}m_\alpha^2\phi_\alpha^2\right] + \frac{\lambda_\mathrm{a}}{24}\phi_\mathrm{a}^4 + \frac{\lambda_\mathrm{b}}{24}\phi_\mathrm{b}^4 + \frac{\mu}{4}\phi_\mathrm{a}^2\phi_\mathrm{b}^2$$

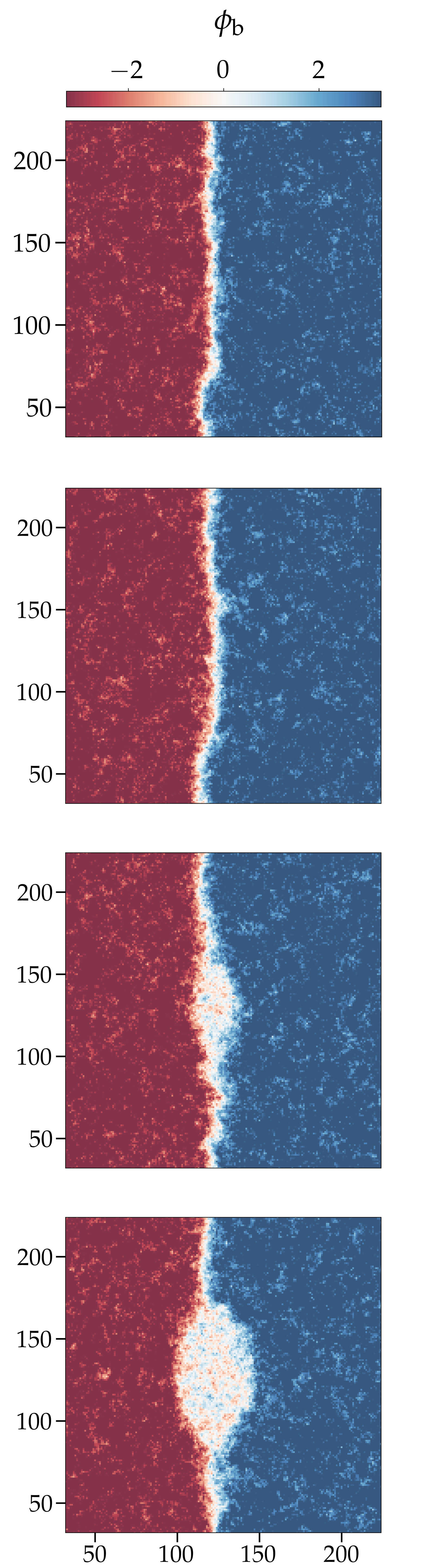


Wait-and-see

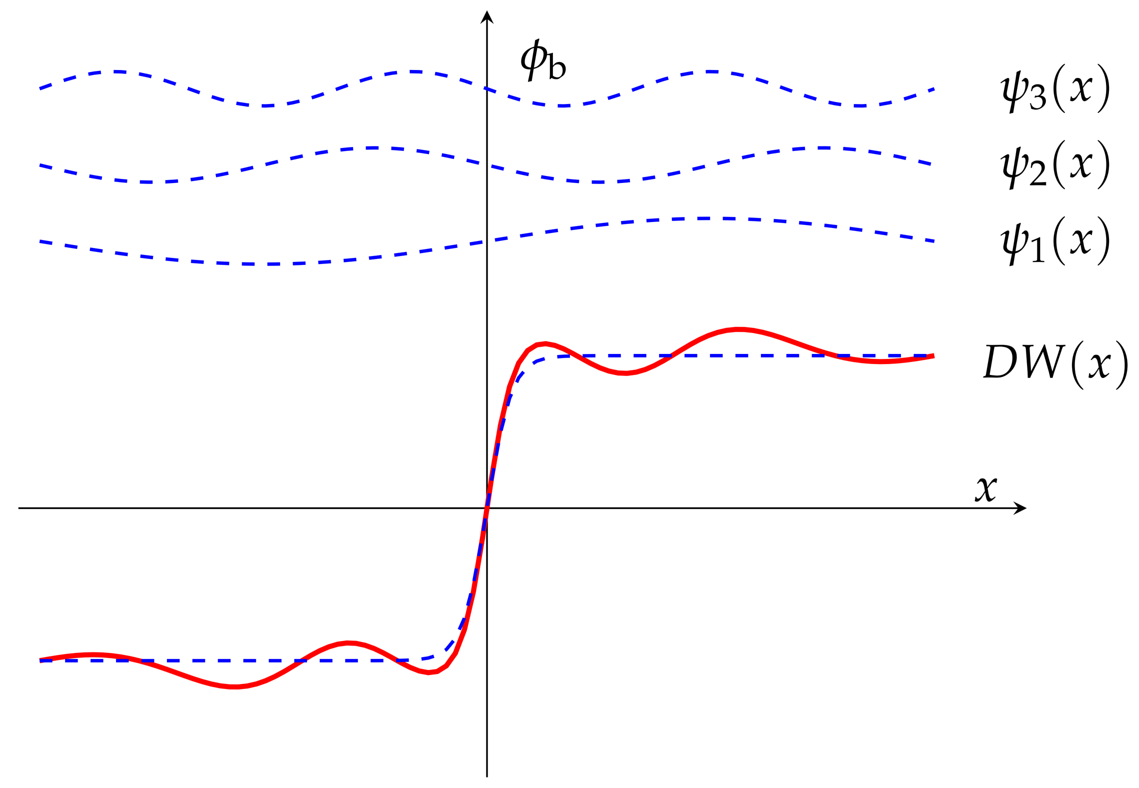
Instead of arduous saddle-point approximations, systems with large nucleation rates Γ can be simulated directly until the transition occurs.



Simulation



Semi-analytic estimate

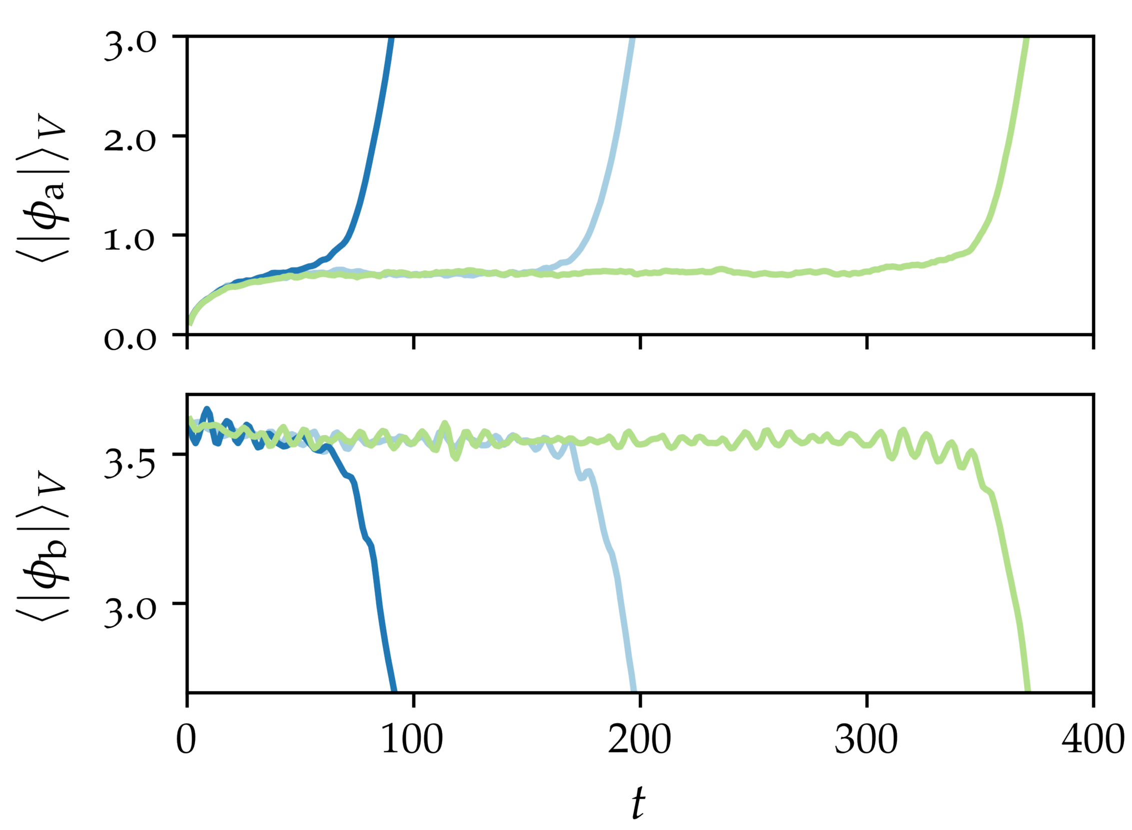


- Fields ϕ_a and ϕ_b are decomposed into modes in the x-direction, perpendicular to the DW

$\Longrightarrow$ Modes at high wave numbers become heavy and can be integrated out

$\Longrightarrow$ Seeded nucleation then reduces to homogeneous nucleation in 1D

Collecting data



1. The simulation is run until nucleation occurs
2. Time of nucleation t_i is recorded
3. `goto 1.`

$\longrightarrow$ Mean of $\{t_i\}$ gives the lifetime τ $(=\Gamma^{-1})$

Results

- Good agreement between the domain-wall-EFT and the numerical simulation
- It would be interesting to compare the results to the non-perturbative saddle-point method