

STRONG AND ELECTRO-WEAK MATTER 2026

Biasing sphalerons non-adiabatically

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Biasing Sphalerons non-adiabatically

[arXiv:2xxx.hopefullySomeday.xxxxx] with *Martina Cataldi, Matthias Carosi, Maxime Grandjean, Wen Yin*

Baryon number, higgs walls and sphalerons

Non-local source of Electroweak baryogenesis: sonic walls

Sakharov conditions (1967)

- **Baryon number violation: Sphaleron**
- **C and CP violation: New operator**
- **Departure from thermal equilibrium: FOPT**

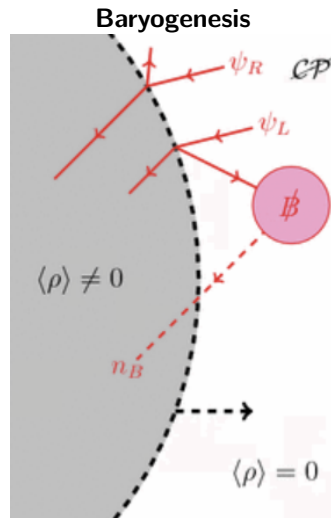
First-order electroweak phase transition (EWPT) and CP-violating sources: tops diffuse in front of the wall: μ_{eff} .

Adiabatic bias of sphalerons: $d\mu_{\text{eff}}/dx \ll 1/L_{\text{sph}}^2$

$$E(N_{CS}) = E_{\text{sph}} \sin^2(\pi N_{CS}), \quad (1)$$

$$\mathcal{L}_\theta = \frac{a}{16f_a\pi^2} \text{tr}[F_{\mu\nu}\tilde{F}^{\mu\nu}] = \mu_{\text{eff}}\dot{N}_{CS}, \quad (2)$$

$$E_{\text{eff}}(N_{CS}, \theta) = E_{\text{sph}} \sin^2(\pi N_{CS}) - \mu_{\text{eff}} N_{CS}. \quad (3)$$



Local source of Electroweak baryogenesis: fast walls $v_w \rightarrow 1$

Why non adiabatic ?

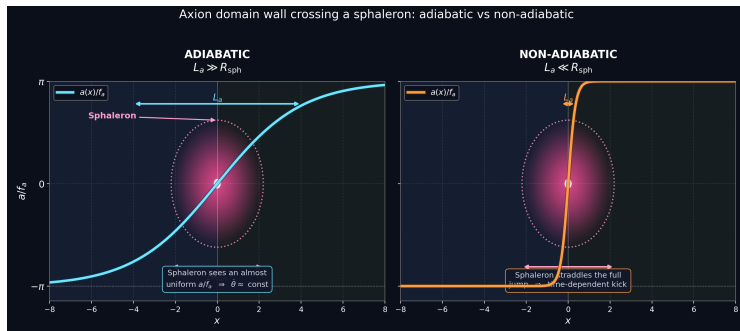


Figure: Adiabatic biasing and non-adiabatic biasing

$$L_h \sim L_a \sim 1/(\gamma_w v)$$

$$L_{\text{spha}}^{\text{sym}} \sim 1/(g^2 T), L_{\text{spha}}^{\text{broken}} \sim 1/m_W$$

$$\mu_{\text{eff}} \sim \dot{a}/f_a$$

μ_{eff} varies on scale L_a

$$L_a \sim 1/\gamma_w v \ll L_{\text{spha}}$$

How does the biasing occurs in this regime ?

Crash reminder on sphalerons

Building intuition: 2D sphalerons in Abelian Higgs model

The Abelian Toy Model (1+1D)

Lagrangian with gauge field A_μ and scalar ϕ :

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D_\mu\phi|^2 - V(|\phi|^2)$$

Large Gauge Transformations:

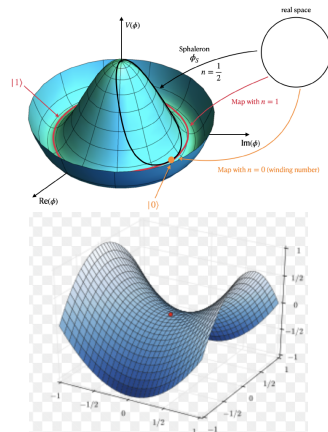
The system is invariant under:

$$\phi(x) \rightarrow e^{iU(x)}\phi(x) \quad A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu U$$

This generates a **family of degenerate vacua** $|n\rangle$.

The Sphaleron:

- ✓ Distinct topological sectors $|n\rangle \leftrightarrow |n+1\rangle$
- ✓ Carries **non-zero energy** E_{sph}
- ✓ **Saddle point** of the energy functional
- ✓ Has one **unstable direction**



Two vacua (black) connected via saddle (red).
(R. Gannouji, 2022)

The Lagrangian, The AKY Ansatz & Saddle-Point Profile

SU(2)–Higgs system:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} - (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(|\Phi|^2)$$

Axially-symmetric hedgehog ansatz (reduces 3D to 1D): (T. Akiba, H. Kikuchi, and T. Yanagida)

$$A_i^a(\mathbf{x}) = \frac{1 - f_A(r)}{r} \epsilon_{aij} \hat{x}_j + \frac{f_B(r)}{r} (\delta_{ai} - \hat{x}_a \hat{x}_i)$$

$$\Phi(\mathbf{x}) = v_H (H(r) + iK(r) \hat{x}_a \tau^a) \xi_0$$

where $r = |\mathbf{x}|$, $\hat{x}_a = x_a/r$, $\xi_0 = (0, 1)^T$.

Boundary-value problem (radial equations):

$$f_A'' + \frac{2}{r} f_A' = \frac{(f_A^2 + f_B^2 - 1)f_A}{r^2} + (H^2 + K^2)f_A + (K^2 - H^2), \quad \text{Similar for } f_B$$

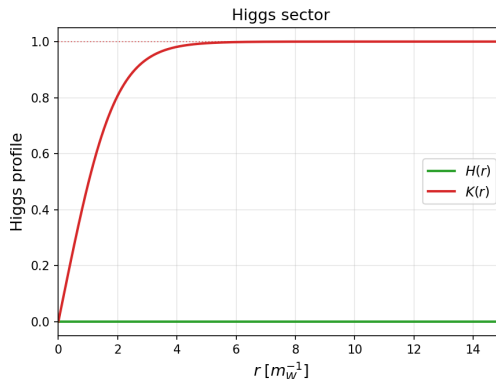
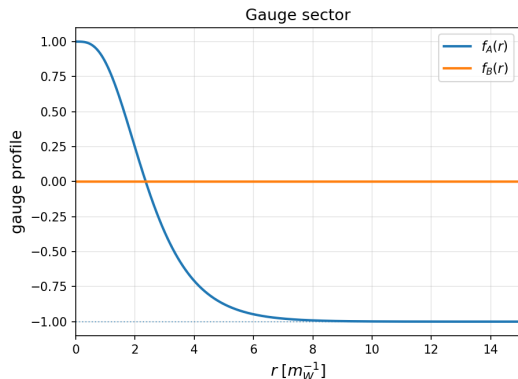
$$H'' + \frac{2}{r} H' = \frac{(f_A - 1)^2 + f_B^2}{r^2} H - \frac{2f_B}{r^2} K + \frac{\beta^2}{2} (H^2 + K^2 - 1)H \quad \text{Similar for } K$$

The AKY Ansatz & Saddle-Point Profile

Boundary conditions:

- At $r \rightarrow 0$: $f_A \rightarrow 1$, $f_B \rightarrow 0$, $H, K \rightarrow 0$ (regularity)
- At $r \rightarrow \infty$: $f_A \rightarrow \cos(2\mu)$, $f_B \rightarrow \sin(2\mu)$, $H \rightarrow \cos(\mu)$, $K \rightarrow \sin(\mu)$ (asymptotic vacuum)

Static AKY sphaleron ($\beta = 1.0$, $\mu = \pi/2$): $E_{\text{sph}} = 269.1507$, $N_{CS} = 0.5000$



Time Evolution (This is naive discretization, but we do links as well)

Take this sphaleron as initial conditions and evolve classically under the full EOM:

$$\ddot{A}_i^a = -\frac{\delta E}{\delta A_i^a}[A, \Phi] - \Gamma(\mathbf{x}) \dot{A}_i^a$$
$$\ddot{\Phi} = -\frac{\delta E}{\delta \Phi^\dagger}[A, \Phi] - \Gamma(\mathbf{x}) \dot{\Phi}$$

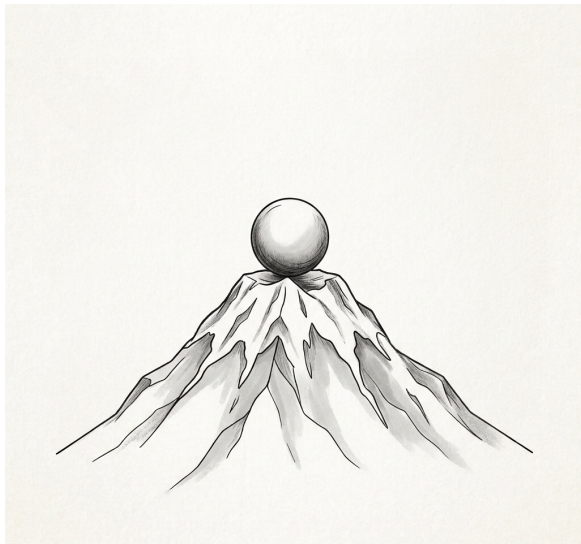
- $E[A, \Phi]$: static Yang–Mills–Higgs energy functional
- $B_i^a = \frac{1}{2}\epsilon_{ijk}F_{jk}^a$: non-Abelian magnetic field
- $\Gamma(\mathbf{x}) = \Gamma_0 + 2\text{sponge}(\mathbf{x})$: absorbing layer at the box boundary (numerical, non-physical)

Monitored observable — Chern–Simons number:

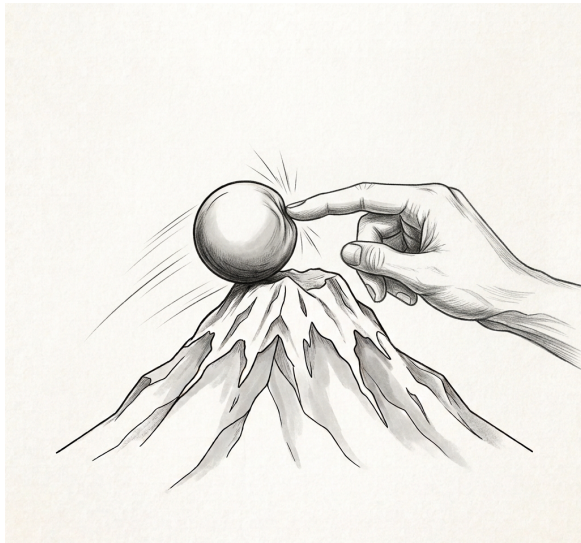
$$N_{CS}(t) = N_{CS}(0) + \int_0^t dt' \int d^3x \frac{g^2}{16\pi^2} \epsilon^{ijk} \text{Tr}[F_{ij} \dot{A}_k]$$

$N_{CS} \rightarrow 0$: sphaleron falls back | $N_{CS} \rightarrow 1$: transition completes

Top of the mountain



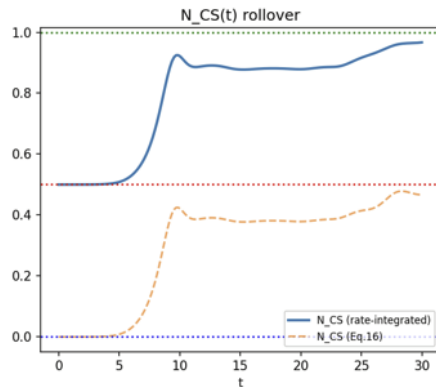
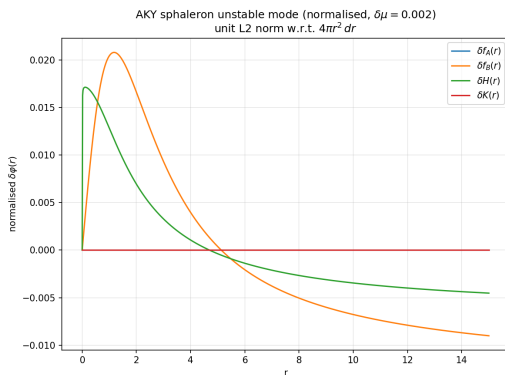
Pushing the ball off the mountain



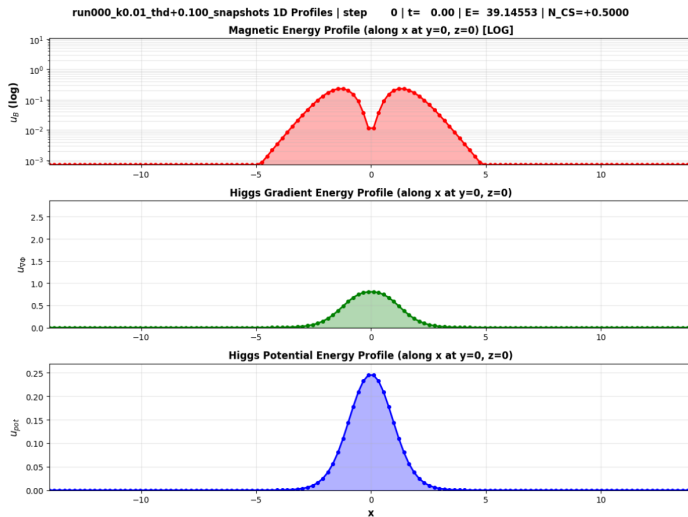
Push the sphaleron 1: Kick it ! (This is the thermal effects)

Initial perturbation along the sphaleron unstable direction:

$$A_0 \rightarrow A_0 + \text{kick_eps} \times \xi \quad \xi(r) = \left. \frac{\partial \phi}{\partial \mu} \right|_{\mu=\pi/2} \approx \frac{\phi(r; \mu^* + \Delta\mu) - \phi(r; \mu^* - \Delta\mu)}{2 \Delta\mu},$$



The fall of the sphaleron



Push the sphaleron 2: Axionize it! (effect of the wall)

$$\mathcal{L}_\theta = \frac{a}{16\pi^2 f_a} \text{Tr} [F_{\mu\nu} \tilde{F}^{\mu\nu}] \implies \delta\mathcal{L} \propto \dot{\theta}(t) \text{Tr} [\vec{B}^a \cdot \vec{E}^a]$$

Modified equation of motion (exact, as coded):

$$\ddot{A}_i^a = \underbrace{-\frac{\delta E}{\delta A_i^a}}_{\text{Yang-Mills-Higgs}} + \underbrace{\left(-\frac{g^2}{4\pi^2}\right) \dot{\theta}(t) B_i^a[A] - \Gamma(\vec{x}) \dot{A}_i^a}_{\text{axionic torque}}$$

Physical effect

- The torque acts *only while* $\dot{\theta} \neq 0$ — i.e. while the wall crosses the sphaleron
- It tilts the energy landscape $E_{\text{sph}} \sin^2(\pi N_{CS}) \rightarrow E_{\text{sph}} \sin^2(\pi N_{CS}) - \mu_{\text{eff}} N_{CS}$ *dynamically*, not adiabatically
- Combined with the unstable-mode kick ξ , it decides whether the sphaleron rolls to $N_{CS} \rightarrow 0$ or $N_{CS} \rightarrow 1$

From N_{CS} to Baryon Number

The chiral anomaly of the baryon current:

$$\partial_\mu J_B^\mu = \frac{n_f g^2}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a = n_f \partial_\mu K^\mu, \quad n_f = 3.$$

The topological current K^μ has as its charge the *Chern–Simons number*

$$N_{CS}(t) = \int d^3x K^0 = \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} (W_{ij}^a W_{k0}^a \text{-type terms}),$$

so that integrating the anomaly over spacetime gives the **master relation**

$$\Delta B = n_f \Delta N_{CS} = 3 \Delta N_{CS}.$$

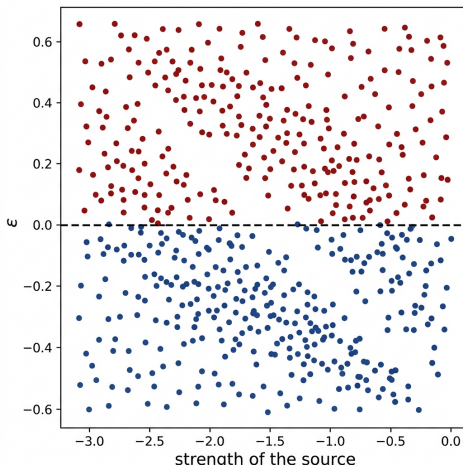
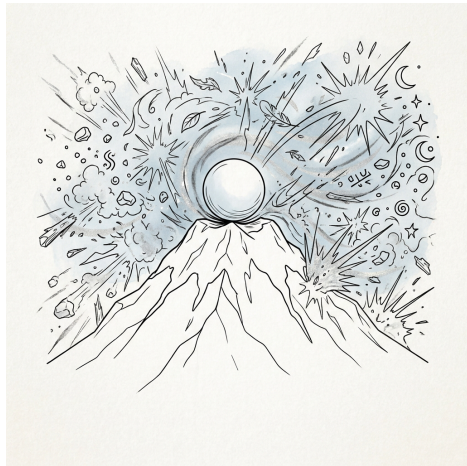
Reading the sphaleron rollover: the sphaleron sits at the top of the barrier at $N_{CS} = 1/2$. Once kicked, it must roll to one of the two neighbouring vacua:

- $N_{CS} : \frac{1}{2} \rightarrow 0 \quad \Rightarrow \quad \Delta N_{CS} = -\frac{1}{2} \quad \Rightarrow \quad \Delta B = -\frac{3}{2}$
- $N_{CS} : \frac{1}{2} \rightarrow 1 \quad \Rightarrow \quad \Delta N_{CS} = +\frac{1}{2} \quad \Rightarrow \quad \Delta B = +\frac{3}{2}$

anti-baryon number
baryon number

Statistical treatment of sphaleron fall

The ball in a thermal medium



Stochastic Treatment of the sphaleron falls

Separating the unstable mode from the thermal bath:

Generic field configuration decomposed as:

$$\phi(\mathbf{x}, t) = \phi_{\text{sph}}(\mathbf{r}) + q(t) \delta\phi_{\text{unst}}(\mathbf{r}) + \text{other modes}$$

- $\phi_{\text{sph}}(\mathbf{r})$ — static sphaleron saddle point and $\delta\phi(\mathbf{r})$ — spatial profile of unstable mode
- $q(t)$ — amplitude along unstable eigendirection (tracked explicitly)

Effective dynamics of unstable mode (Caldera-Legget, Mori-Zwanzig):

$$M\ddot{q}(t) = M\omega^2 q(t) - M \int_0^t dt' \Gamma(t-t') \dot{q}(t') + \eta(t)$$

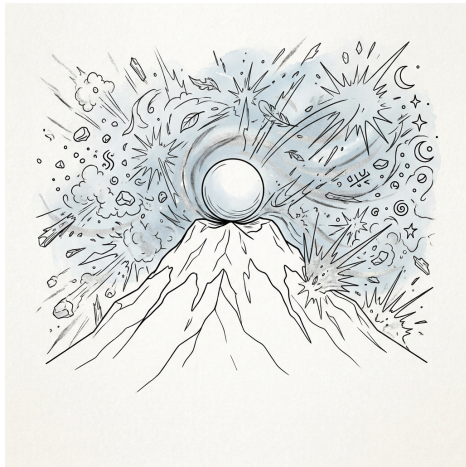
Parameters:

- $M = \|\delta\phi\|^2$ — normalization
- $M\omega^2 = |dE/d\epsilon|^2$ — curvature (unstable)
- $\langle \eta(t)\eta(t') \rangle = 2MT\Gamma(t-t')$

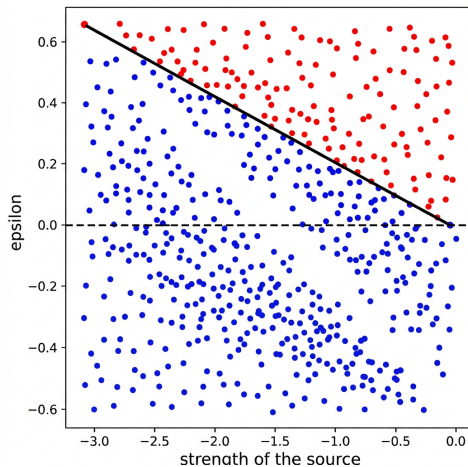
Physical terms:

- Inertia: $M\ddot{q}$ and instability: $+M\omega^2 q$
- **Memory friction:** $-M \int_0^t \Gamma$
- **Thermal noise:** $+\eta(t)$

The ball in a thermal medium and the force from the axion



The ball in a thermal medium and the force from the axion



The Fate Variable: Kick vs. Push

Adding a source.

$$M\ddot{q}(t) = M\omega^2 q(t) - M \int_0^t dt' \Gamma(t-t') \dot{q}(t') + \eta(t) + \underbrace{\xi(t)}_{\text{Axion source}}, \quad \xi(t) \propto \dot{\theta}(t),$$

where $\theta(t) = \theta_f \tanh(t/L_a)$ is the (slowly-varying, *deterministic*) axion angle sweeping through the wall.

The fate variable picks up an extra, deterministic piece:

$$c = \underbrace{\frac{s_+}{M\omega^2} \tilde{\eta}(s_+)}_{\epsilon \text{ (stochastic, thermal)}} + \underbrace{\frac{s_+}{M\omega^2} \tilde{\xi}(s_+)}_{\epsilon_\star(\theta_f) \text{ (deterministic, source-driven)}}, \quad \text{with } \tilde{\xi}(s_+) = \int_0^\infty dt e^{-s_+ t} \xi(t).$$

Interpretation. The two pieces enter *additively* inside the same sign function:

$$\text{sign}(c) = \text{sign}[\epsilon + \epsilon_\star(\theta_f)] \quad \implies \quad \begin{cases} \epsilon > -\epsilon_\star(\theta_f) & \text{rolls right} \\ \epsilon < -\epsilon_\star(\theta_f) & \text{rolls left} \end{cases}$$

Statistics of Perturbations: Separatrix and Asymmetry

Tanh axion wall

$$\xi(t) = \frac{\theta_f}{2\tau_\theta} \text{sech}^2 \frac{t}{\tau_\theta} . \quad (4)$$

$$\epsilon_\star(\theta_f) \xrightarrow{s+\tau_\theta \rightarrow 0} -\theta_f \frac{s_+}{2M\omega^2} . \quad (5)$$

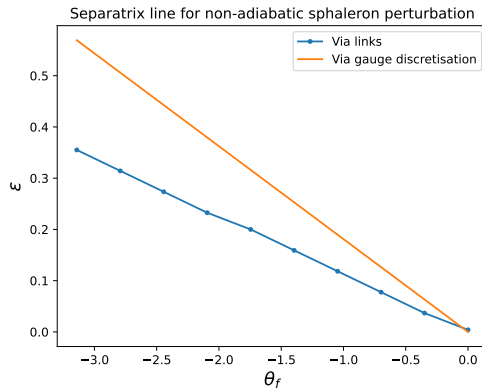
Branching probabilities:

$$X = \frac{n_{\rightarrow 1} - n_{\rightarrow 0}}{n_{\rightarrow 1} + n_{\rightarrow 0}} = \text{erf} \left(\frac{\epsilon_\star(\theta_f)}{\sqrt{2}\sigma_\epsilon} \right) , \quad \sigma_\epsilon^2 \sim \frac{T}{E_{\text{spha}}}$$

Final baryon number:

$$Y_B \sim 3Y_{\text{spha}} \times X \sim 3 \underbrace{\frac{n_{\text{spha}}}{s}}_{\sim 10^{-6}} \text{erf} \left(\frac{\epsilon_\star(\theta_f)}{\sqrt{2}\sigma_\epsilon} \right) \quad (6)$$

Boundary ϵ



Summary

- **Goal:** Electroweak baryogenesis from **fast bubble walls** ($v_w \rightarrow 1$), where the axion domain wall is **thinner** than the sphaleron ($L_a \ll L_{\text{sph}}$) — a genuinely **non-adiabatic** regime.
- **Setup:** Static AKY sphaleron solutions are evolved under the full classical EOM, kicked along their unstable direction, and **pushed** by an axionic torque $\propto \dot{\theta}(t) \vec{E} \cdot \vec{B}$ as the wall sweeps through.
- **Key result:** The sphaleron's fate ($N_{CS} \rightarrow 0$ or 1) is set by a single **fate variable**

$$c = \epsilon + \epsilon_*(\theta_f),$$

combining a random **thermal kick** ϵ and a deterministic **axion-driven push** $\epsilon_*(\theta_f)$.

- **Outcome:** This produces a net branching asymmetry

$$X = \text{erf}\left(\frac{\epsilon_*(\theta_f)}{\sqrt{2}\sigma_\epsilon}\right) \implies \Delta N_{CS} \neq 0 \Rightarrow \Delta B \neq 0.$$

- **Bottom line:** Fast, non-adiabatic walls provide a **new, local source** of baryogenesis, complementary to the standard diffusion-based mechanism of slow walls.

Back up

Back up

Definitions — Energy Densities & Topological Observables

Energy densities (SU(2)–Higgs system):

$$u_B = \frac{1}{4} \sum_{a,i,j} (F_{ij}^a)^2 \quad (\text{magnetic})$$

$$u_E = \frac{1}{2} \sum_{a,i} (\partial_t A_i^a)^2 \quad (\text{electric})$$

$$u_{\nabla\Phi} = \sum_i (D_i\Phi)^\dagger (D_i\Phi) \quad (\text{Higgs gradient})$$

$$u_{\text{pot}} = \frac{1}{4} \lambda (|\Phi|^2 - v_H^2)^2 \quad (\text{Higgs potential})$$

Topological observables:

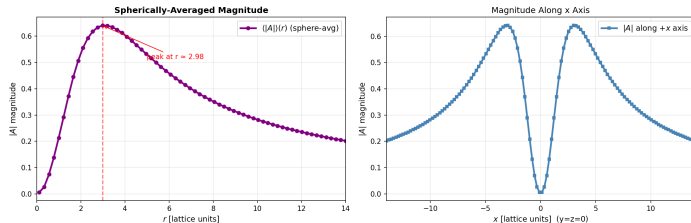
$$\rho_{CS} = \text{Chern–Simons density} \quad (\text{signed, SYMLOG})$$

$$N_{CS} = \int \rho_{CS} d^3x \quad (\text{winding / vacuum label})$$

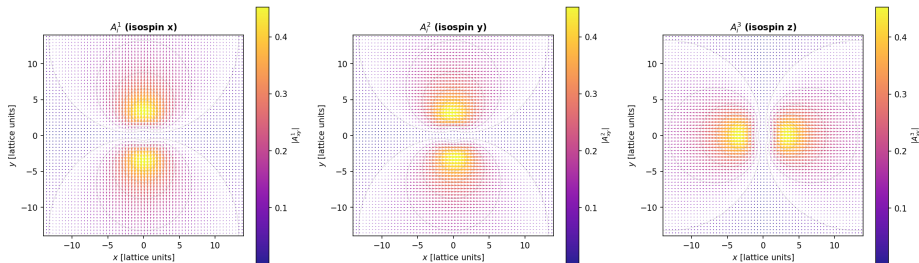
F_{ij}^a : field strength $D_i\Phi$: covariant derivative v_H : Higgs VEV λ : quartic coupling

The AKY Ansatz & Saddle-Point Profile

Static Sphaleron: Magnitude Profiles



Static Sphaleron: SU(2) Vector Field, XY-plane ($z=0.11$) [full]

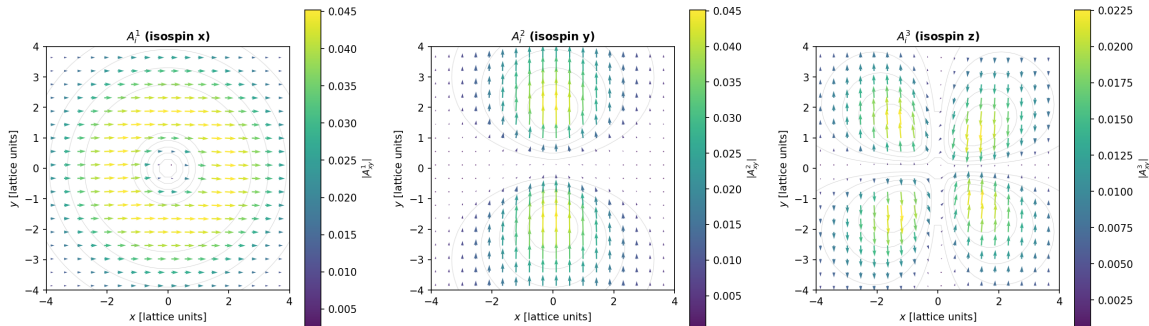


The Unstable Kick

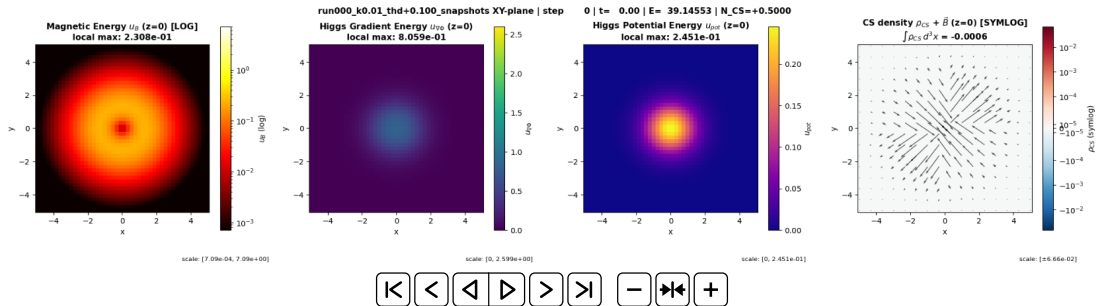
Initial perturbation along the sphaleron unstable direction:

$$A_0 \rightarrow A_0 + \text{kick_eps} \times \xi \quad \xi(r) = \left. \frac{\partial \phi}{\partial \mu} \right|_{\mu=\pi/2} \approx \frac{\phi(r; \mu^* + \Delta\mu) - \phi(r; \mu^* - \Delta\mu)}{2 \Delta\mu},$$

Unstable-Mode Kick: SU(2) Vector Field, XY-plane (z=0.11) [zoom]



XY-plane Energy Density Evolution



Backup: Numerical Implementation

Discretization & lattice

- 3D Cartesian lattice, spacing dx , box size L_{box} , radius $R_3 = |\vec{x}|$
- Fields stored as real arrays (RDTYPE); A_i^a , Φ and time-derivatives $\dot{A}_i^a$, $\dot{\Phi}$
- Spatial derivatives via finite differences $\Rightarrow F_{ij}^a = \text{field_strength}(A)$, $D_i \Phi = \text{cov_deriv_Phi}(A, P)$

Time integration

- Velocity-Verlet / leapfrog, second order in dt
- Forces from $\text{accel}(A, \dot{A}, \Phi, \dot{\Phi}, t) = (-\delta E / \delta A - \Gamma \dot{A} + \text{anomaly term}, -\delta E / \delta \Phi^\dagger - \Gamma \dot{\Phi})$
- Neumann BC (`apply_neumann`) enforced after every sub-step on all 4 fields

Absorbing sponge (numerical artifact, not physical)

- $\Gamma(\vec{x}) = \Gamma_0 + 2 \text{sponge}(\vec{x})$, $\Gamma_0 = \text{GAMMA}$
- $\text{sponge}(\vec{x}) = \frac{1}{2} \left(\frac{R_3 - R_{\text{sponge}}}{L_{\text{box}} - R_{\text{sponge}}} \right)^2$ for $R_3 > R_{\text{sponge}} = 12$, else 0
- Purpose: absorb outgoing radiation, mimic infinite volume, avoid boundary reflections

Unstable-mode kick

- $\xi = (A_+ - A_-)/(2d\mu)$ from AKY profiles at $\mu^* \pm \Delta\mu$, then L^2 -normalized

Statistics of Perturbations: Symmetry and Bias

Without external CP-violating source (symmetric case):

Sign of ϵ determines outcome:

$$\epsilon > 0 \quad \Rightarrow \quad \text{Roll LEFT : } N_{CS} : \frac{1}{2} \rightarrow 0$$

$$\epsilon < 0 \quad \Rightarrow \quad \text{Roll RIGHT : } N_{CS} : \frac{1}{2} \rightarrow 1$$

Result:

$$p(\epsilon) = p(-\epsilon) \text{ (symmetric)} \\ \Rightarrow n_L = n_R$$

No net CP violation

Gaussian distribution of kick statistics:

$$p(\epsilon) = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp\left(-\frac{\epsilon^2}{2\sigma_\epsilon^2}\right) \quad \sigma_\epsilon^2 \sim \frac{T}{E_{\text{spha}}} \sim \frac{T}{10m_W}$$

With axion torque (asymmetric case):

- CP-violating source **shifts distribution**: $p(\epsilon) \rightarrow p(\epsilon - \epsilon_{\text{source}})$
- More kicks biased in one direction (e.g., axion force pushes left)
- Outcome becomes **asymmetric**: $n_L \neq n_R$

$\Delta N_{CS} = n_L - n_R \propto \text{axion field} \quad \Rightarrow \quad \text{Baryon number generation}$
--

Thermal Bath Coupling

Fluctuation-Dissipation Relation:

$$\langle \eta(t) \eta(t') \rangle = 2MT \Gamma(t - t')$$

Noise and friction are **not independent**—they satisfy thermal equilibrium at temperature T

Spectral decomposition of bath (multiple timescales):

$$\Gamma(t - t') = \sum_{i=1}^N \frac{\gamma_i}{\tau_i} e^{-(t-t')/\tau_i}$$

- τ_i — correlation timescale of mode i
- γ_i — friction strength of mode i
- Different modes: **fast** ($\tau_i \ll 1/\omega$), **slow** ($\tau_i \gg 1/\omega$)

Key insight: Slow modes decouple! They freeze during sphaleron decay

Variance of Kicks

Suppression of slow-mode contributions:

$$\sigma_{\epsilon}^2 = \frac{T}{M\omega^4} \sum_{i=1}^N \gamma_i \frac{s_+}{1 + s_+ \tau_i}$$

where $s_+ \sim \omega$ is the decay rate and $\frac{s_+}{1 + s_+ \tau_i} \approx s_+ \tau_i$ for slow modes.

Suppression mechanism:

Fast modes ($\tau_i \ll 1/\omega$):

- Respond quickly
- Full contribution: ~ 1
- Markovian

Slow modes ($\tau_i \gg 1/\omega$):

- Too slow to respond
- Suppressed: $\sim s_+ \tau_i \ll 1$
- Frozen out

Final estimate:

$$\sigma_{\epsilon}^2 \sim \frac{T}{E_{\text{spha}}} \sim 0.01$$

Noise is **small but non-negligible**: $\sim$ temperature / sphaleron binding energy