Sphaleron rate for first-order electroweak phase transitions

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Motivation: Electroweak Baryogenesis

- EWBG is a testable scenario with present and near future experiments:
  EDM, particle colliders, gravitational waves
- BSM physics is needed:
  - Needs new sources of CP violation
  - Needs a strong first order phase transition
- Standard Model has a source for baryon number violation: **Sphaleron**
  - This talk: **Sphaleron washout** lattice results
    - When is the phase transition strong enough?

Sphaleron rate Γ

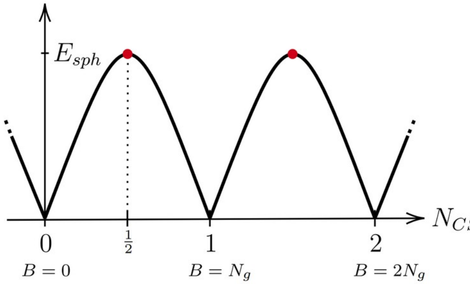
$$\Delta N_{\text{CS}} = \frac{g^2}{32\pi^2}\int_0^t dt \int d^3x \epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} = \frac{1}{3}\Delta B = \frac{1}{3}\Delta L$$

- The amount of baryon number violation can be quantified via the **sphaleron rate**: how rapidly sphaleron processes happen

$$\Gamma = \lim_{V,t\to\infty} \frac{\langle N_{\text{CS}}(t)^2\rangle}{Vt}$$

- Low temperature Higgs phase: (sphaleron rate)

$$\Gamma_{\text{brk}} \sim \alpha_W^4 T^4 e^{-E_{sph}/T}$$



- High temperature Confinement like phase: ("hot sphaleron" rate)

$$\Gamma_{\text{sym}} \sim \alpha_W^5 \log(\alpha_W^{-1}) T^4$$

Sphaleron washout

- Given enough CP violation around the expanding bubble wall sphaleron processess generate baryon number in the high T confinement like phase.
- Baryon excess swallowed by the expanding bubble: **if Higgs phase sphalerons are suppressed fast enough** the baryon number freezes out, otherwise the asymmetry will be washed out.
- When sphaleron rate is comparable to Hubble rate: baryon number frozen

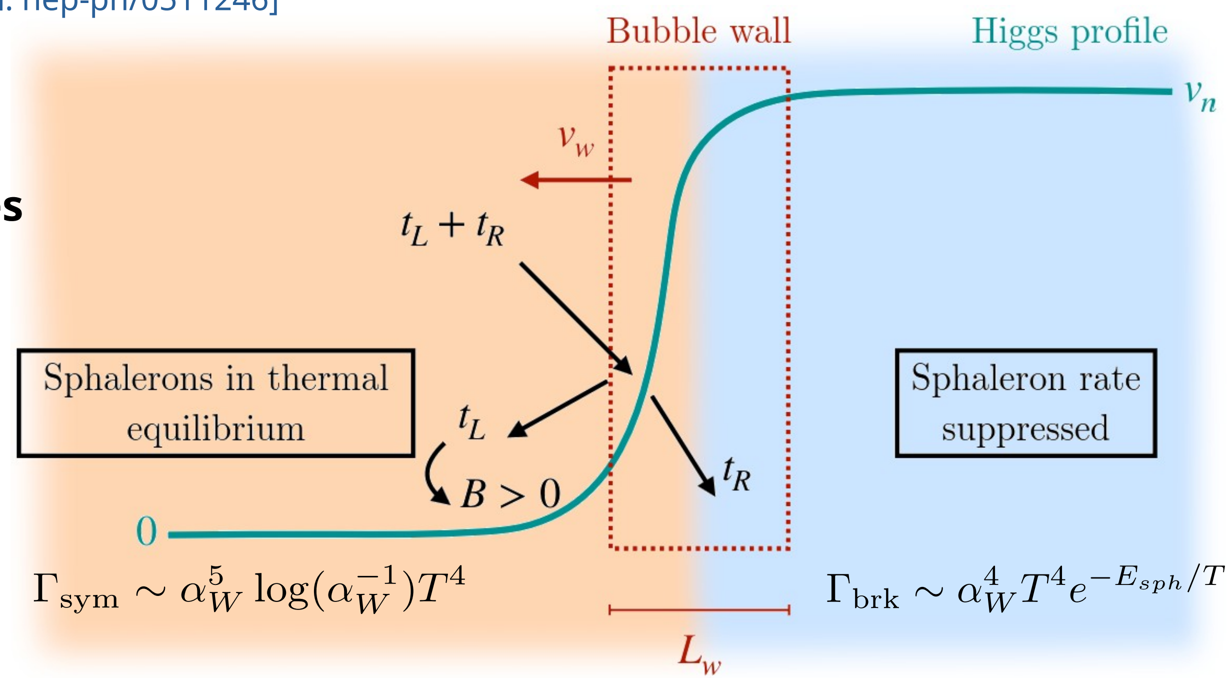
$$N_\rho \frac{\Gamma(T_f)}{T_f^3} = H(T_f)$$ [Burnier et al. hep-ph/0511246]

- Need to compute:

1. T_p when the transition completes

2. T_f when baryon number freezes

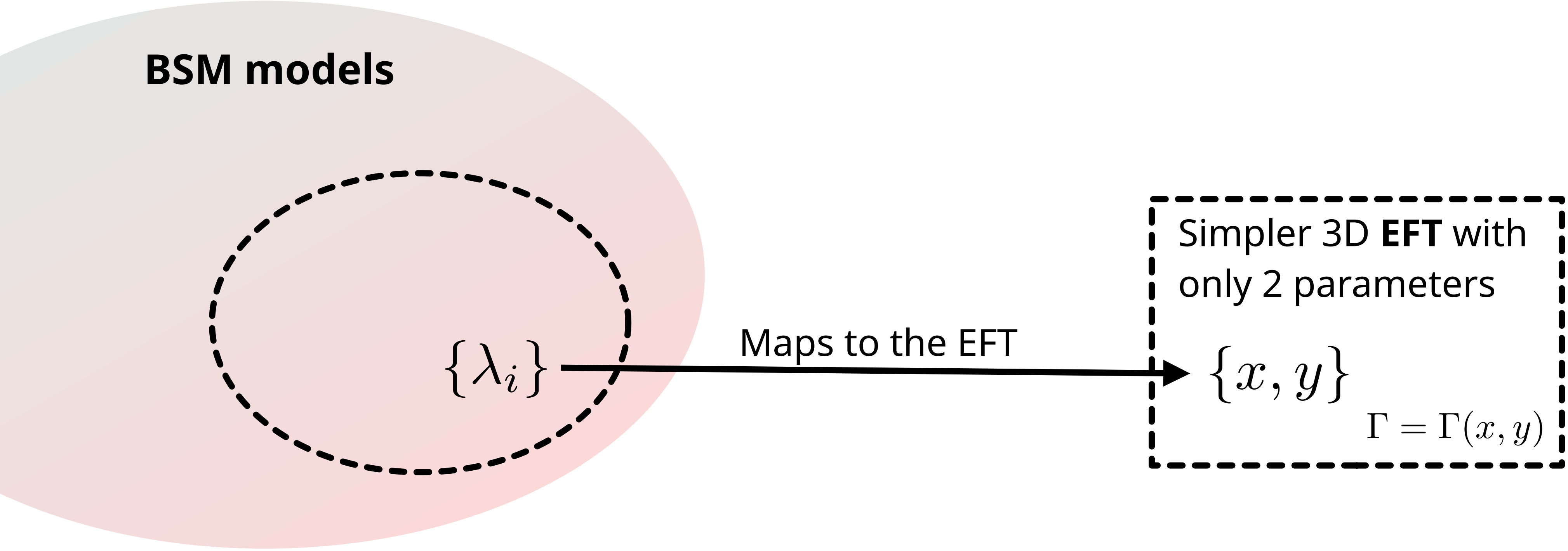
If $T_p \leq T_f \Rightarrow$ no washout



[Figure from: van de Vis et al. 2508.09989]

EWBG needs BSM physics

- Lots of BSM models, computing the sphaleron rate (T_f) and bubble nucleation rate (T_p) is a lot of work.
- Many BSM models can be mapped to a dimensionally reduced SU(2)+Higgs 3D EFT.
  - Get sphaleron and nucleation rates model independently in the EFT using lattice methods



SU(2)+Higgs Dimensionally reduced 3D EFT

1. Start with the Standard Model or some BSM model
2. High T hierarchy of scales: integrate out heavy modes

πT (Fermions, temporal modes)

gT (temporal gauge modes)

$g^2 T$ (dimensionally reduced 3D effective theory) SU(2) + Higgs

$$L_{\rm 3D} = \frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2,$$

3. Obtain relations of starting model parameters to EFT parameters

$g_3, \lambda_3, m_3^2 =$ Functions of T and the starting 4D model's parameters

EFT with two dimensionless couplings:

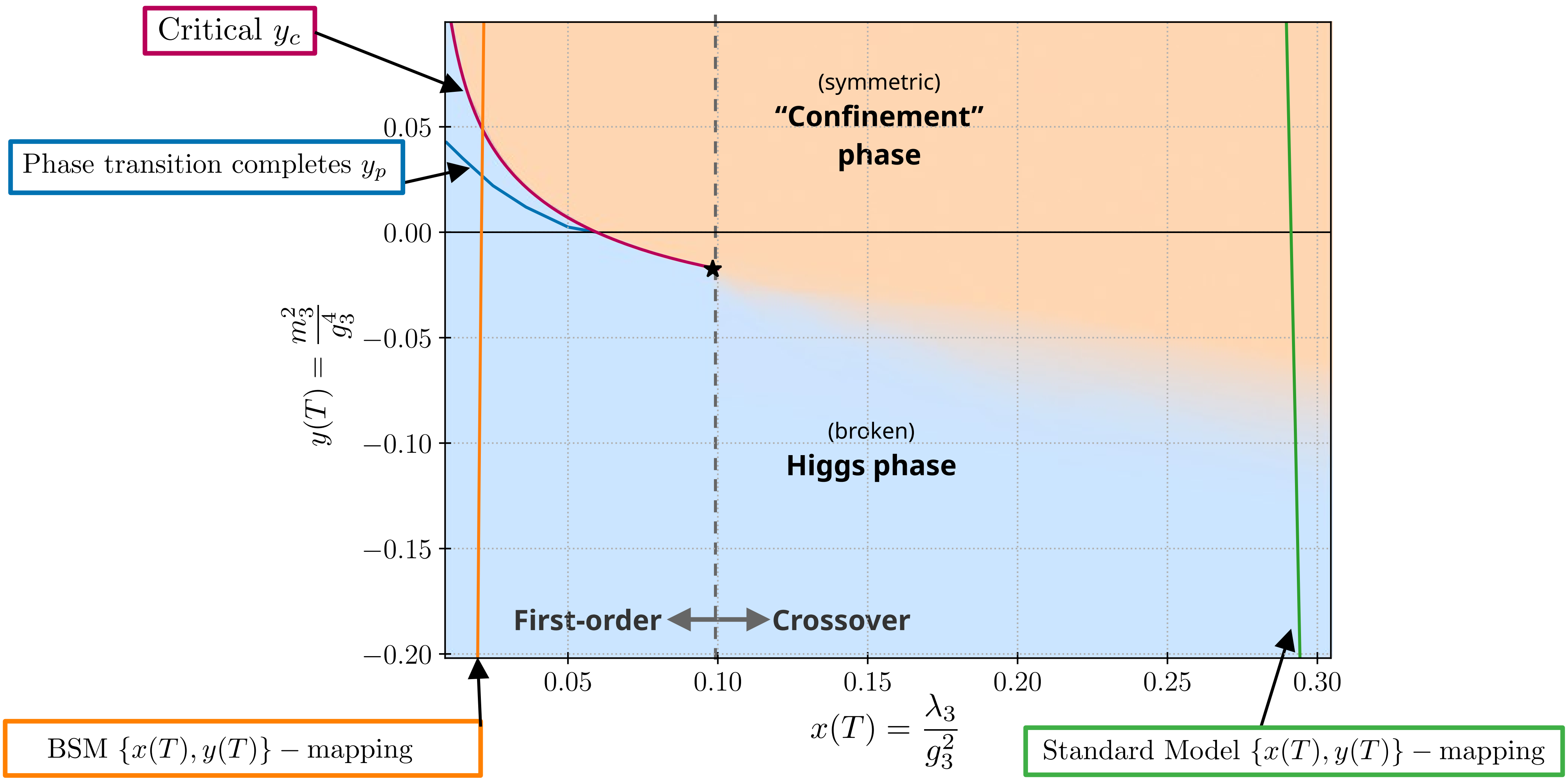
$$\boxed{x(T) = \frac{\lambda_3}{g_3^2}, \quad y(T) = \frac{m_3^2}{g_3^4}}$$

[Kajantie et al. hep-ph/9508379]
Largely automated by DRalgo code
[Ekstedt et al. 2205.08815]

3D EFT in the xy-plane

- Study the EFT in its 2 parameter space on the lattice:

[Kajantie et al. hep-lat/9510020]
[Moore, Rummukainen hep-ph/0009132]



y_c and y_p curves computed on the lattice in [Gould et al., 2205.07238 refs therein]

Sphaleron rate on the lattice

- Computation of the (hot)sphaleron rate in the 3D EFT well established. confinement like phase and Higgs phase computations:
  - first-order transition region [Moore et al. Hep-ph/0009132, hep-ph/9805264, …]
  - crossover region, SM parameters [Moore hep-ph/9805264, D'Onofrio et al. 1207.0685, 1404.3565, JA et al. 2301.08626, …]

N_{cs} defined on the lattice with gradient flow cooling

$$\tilde{\Gamma}(x,y) = \underbrace{\frac{P(|N_{\rm CS} - \frac{1}{2}| < \frac{\epsilon}{2})}{\epsilon V}}_{\text{statistical}} \underbrace{\left\langle \left| \frac{\Delta N_{\rm CS}}{\Delta t} \right| \right\rangle d}_{\text{dynamical}} \,,$$

Dynamical evolution described by an effective Langevin evolution [Bödeker, arXiv:hep-ph/9801430]

$$\Gamma(x,y) = \frac{(g_3^2)^5}{\sigma_{\rm el}} \tilde{\Gamma}(x,y),$$

Sphaleron freeze-out curve in xy-plane $y_f(x)$

- Comparing the sphaleron rate to Hubble rate gives the sphaleron freeze out curve $y_f(x)$ in the xy-plane:

$$N_\rho \frac{\Gamma(T_f)}{T_f^3} = H(T_f)$$

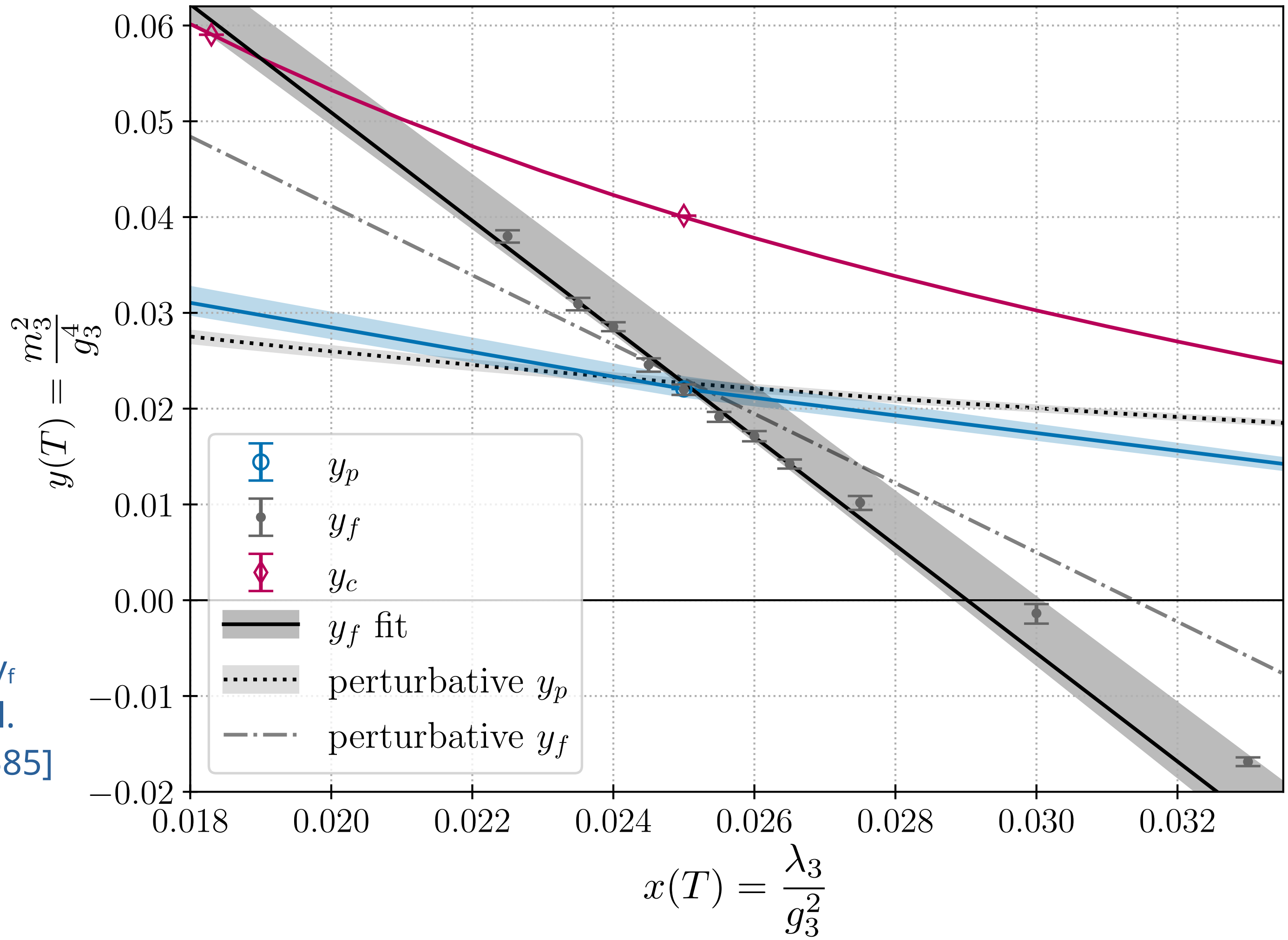
$$\rightarrow \quad \ln\Gamma(x, y_f) = \ln\tilde{\Gamma}(x, y_f) + \boxed{\ln\left(\frac{(g_3^2)^5}{\sigma_{\rm el} T_f^4}\right)} = \boxed{\ln\frac{H(T_f)}{N_\rho T_f}} \approx -38$$

Approx. constants

$\rightarrow$ $y_f(x)$ (Sphaleron freeze out curve)

$\rightarrow$ Baryon number preserved when:
$y_p(x_c) \leq y_f(x_c) \quad (\rightarrow T_p \leq T_f)$

Strong Baryon number preservation condition



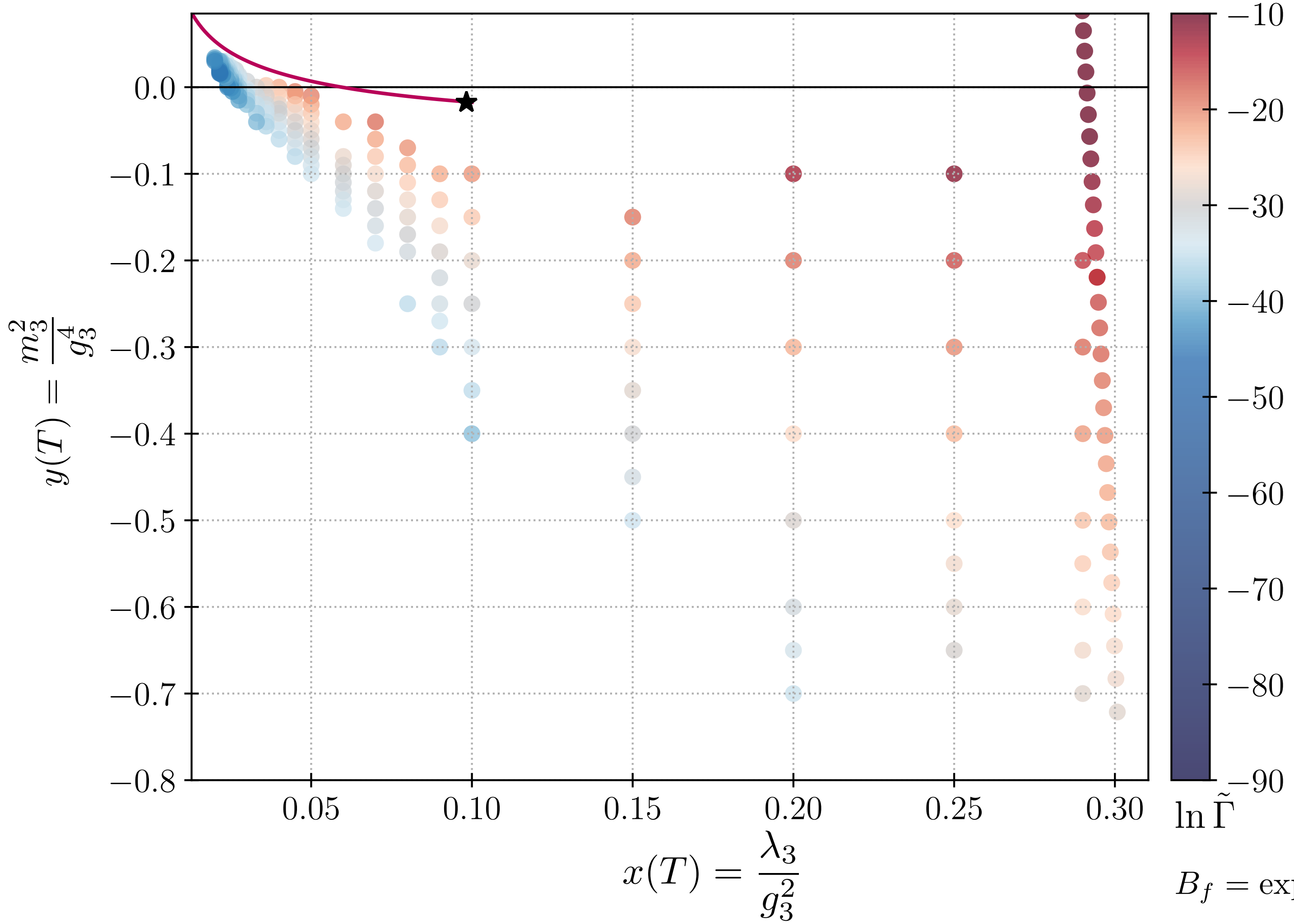
[perturbative y_f curve: Wu et al. arxiv:2506.01585]

- Sphaleron rate is smaller than the Hubble rate when the transition completes if:

$$x_c \lesssim \bar{x} \approx 0.025, \qquad v/T_c \equiv \sqrt{2\Delta\langle\phi^\dagger\phi\rangle}/T_c \gtrsim 1.33$$

none of the generated baryon number is washed out.

Less strict bounds can be made from the data



$$B_f = \exp\left(-\int_{T_f}^{T_p} \frac{\Gamma(T)}{HT^4} dT\right) B_p$$

- If you allow for partial washout you can have slightly less strong transitions… washout factor of bigger than exp(-100):

$$x_c \lesssim \hat{x} \approx 0.036,$$

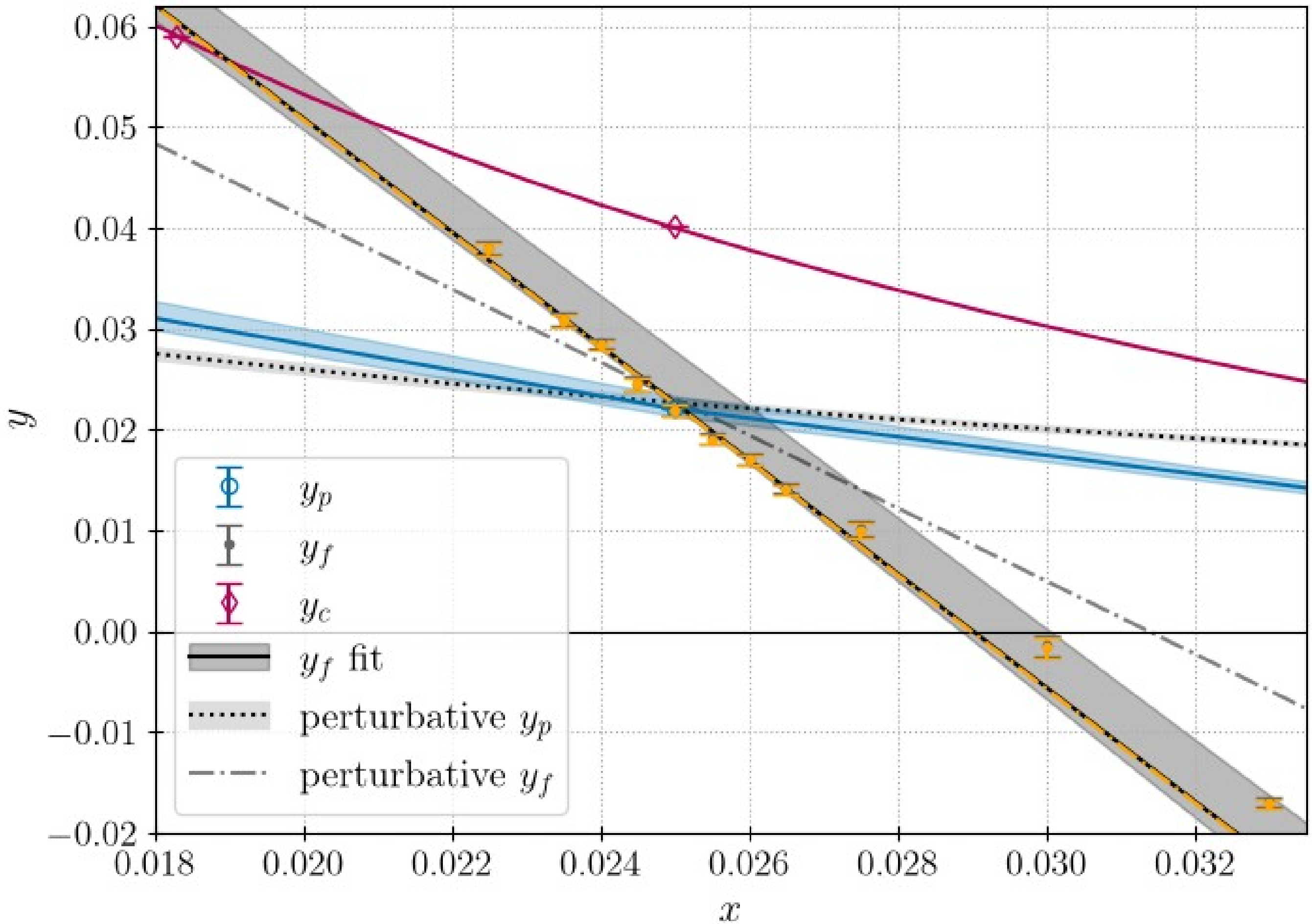
[more details: Wu et al. arxiv:2506.01585]

Summary

- BSM models that map to the SU(2)+Higgs EFT can be efficiently studied on the lattice without resulting into more complex simulations with the extra field content.
- We obtained a sphaleron freeze-out curve in the xy-plane
- Using previous determination of the critical and percolation curves we obtained a strong condition for baryon number preservation, which is slightly stricter than previous estimates $x_c \lesssim \bar{x} \approx 0.025.$

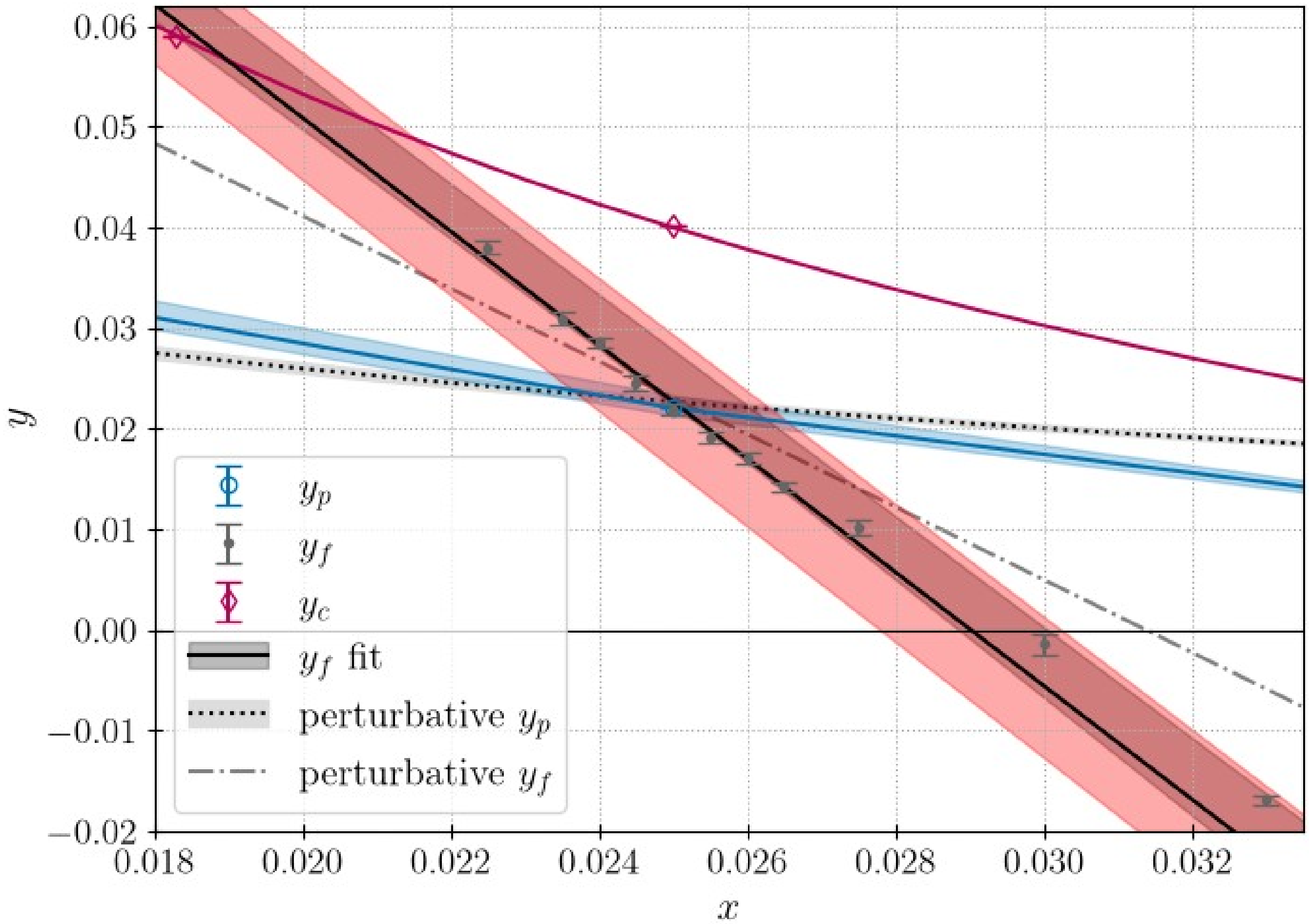
Thank you for you attention!

Uncertainties



- Assume g_3, σ_{el}, $H(T_f)$ do not change much by BSM physics.
  (Orange: decrease σ_{el} by 6%) [Bödecker, Klose arxiv:2510.20594]

Uncertainties



- Using the Langevin dynamics deep in the Higgs phase is not well justified. However, the exponential suppression from the statistical part remains the dominant contribution. (Red band: 1000% error to dynamical part.)