

QCD corrections to the electroweak sphaleron rate

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SM interactions change baryon number

$$\partial_\mu J_B^\mu = \chi \quad \Rightarrow \quad \partial_t B \propto \Gamma_{\text{sph}} = \underbrace{\frac{1}{2} \int d^4x \langle \{\chi(x), \chi(0)\} \rangle}_{\text{hot sphaleron rate}}$$

- Important input for primordial baryon asymmetry predictions (baryogenesis, leptogenesis)

$\chi = (\alpha_w/4\pi) F\tilde{F}$ is Chern-Simons charge density / $B = \int d^3x J_B^0$ is baryon number

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What about other SM interactions?

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Effective theory for $p \sim g_w^2 T$ gauge fields

$$\vec{D} \times \vec{B} = \underbrace{D_t \vec{E}}_{\text{neglect}} + \vec{J} \quad \vec{J} = g_w \sum_s \int_p \frac{\vec{p}}{|\vec{p}|} (f_s(p) - \bar{f}_s(p))$$

s = particle species / $f_s, \bar{f}_s$ = distribution functions (adjoint rep)

- $f_s, \bar{f}_s$ obey Vlasov-Boltzmann equations $\Rightarrow$ Solve to find J

σ = isospin conductivity / ζ = stochastic noise / g_w = weak coupling

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- Next-to-leading log (NLL) approximation yields local current

$$J = \sigma E + \zeta \quad \Rightarrow \quad \Gamma_{\text{sph}} = \underbrace{(10.0 \pm 0.3)}_{\text{from lattice}} \cdot \frac{2\pi T}{3\sigma} \alpha_w^5 T^4$$

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IR divergencies enhance small-angle scattering

Linearized Vlasov-Boltzmann equation:

$$\underbrace{vD\delta f_s}_{O(g_w^2)} \pm g_w f'_F \vec{E} \vec{v} = \underbrace{C_{\text{weak}}[\delta f_s]}_{O(g_w^2 \ln g_w^{-1})}$$

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- Weak collisions are more enhanced but QCD coupling is larger
 $\Rightarrow$ **Strong interactions potentially important**

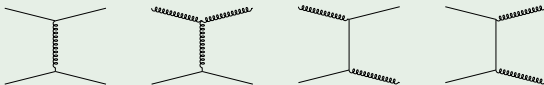
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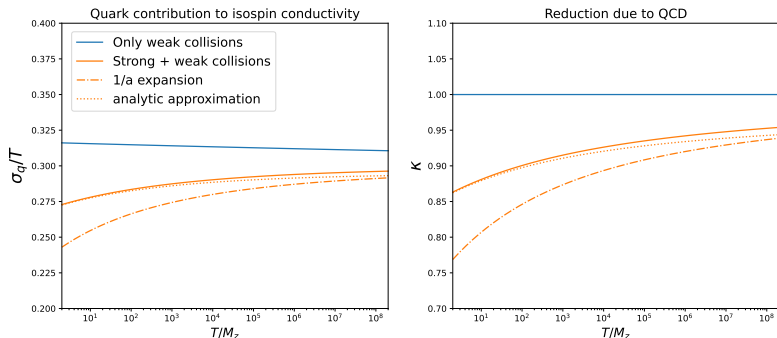
Our work: (1) Compute LL **strong collision term** (Schwinger-Keldysh)



$\Rightarrow$ Single parameter $a \sim \frac{g_w^4 \log(g_w^{-1})}{g_s^2 \log(g_s^{-1})}$ characterizes size

(2) Solve resulting differential equation

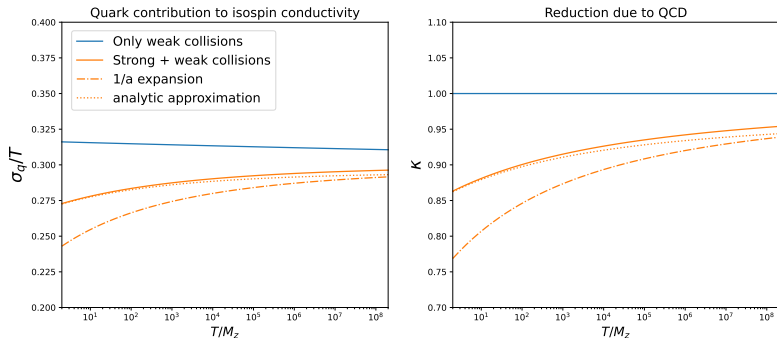
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σ_q = quark contribution to isospin conductivity / κ = ratio of with and without QCD

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■ Simple analytic approximation:

$$\kappa \equiv \frac{\sigma_q}{\sigma_{qQCD}} \approx 0.9767 \cdot \frac{a}{0.2107 + a} \quad (\text{Error} < 1.5\%)$$

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Conclusions and Outlook

$$\sigma = \underbrace{(\sigma - \sigma_q)_{QED}}_{\text{from } \ell, H, W} + \kappa \cdot \sigma_{q,QED} \quad \kappa = 0.9767 \cdot \frac{a}{0.2107 + a}$$

- We computed leading-log QCD corrections to weak-isospin conductivity
 $\Rightarrow$ **6%** larger sphaleron rate

Possible next steps:

- Next-to-leading log QCD corrections
- Masses (close to EW crossover)
- Going beyond next-to-leading log (new operators)

Backup slides