Topological Susceptibility at high T from unquenched Lattice QCD via Reweighting

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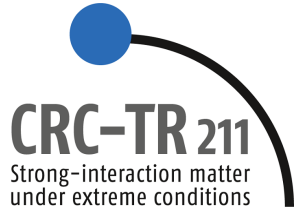

Axions & Topology

Axions & Topology

- In principle, QCD symmetries allow θ term in Lagrangian

$$\mathcal{L}_{\text{QCD}} \overset{?}{\supset} \mathcal{L}_\theta = \frac{\theta}{16\pi^2} \operatorname{Tr} F_{\mu\nu} \tilde{F}_{\mu\nu} \equiv \theta\, q$$

  with topological charge density q
- Evident vanishing of θ: Strong CP Problem
  - Potentially solved by QCD axion
- Axion mass given by topological susceptibility χ

$$m_a^2(T) \propto \chi(T), \quad \chi = \lim_{V\to\infty} \frac{\langle Q^2 \rangle}{V}, \quad Q = \int \mathrm{d}^4x\, q(x)$$

- Simulations of early-universe dynamics require knowledge of χ for $T \in [450, 1100]\,\text{MeV}$

Topology & Instantons

Instantons

- Q counts localized topological chunks: Instantons
- Inherently non-perturbative
- Come with different radii ρ
- Topological configurations rare at high T, $\chi(T) \propto T^{-7-N_f/3}$

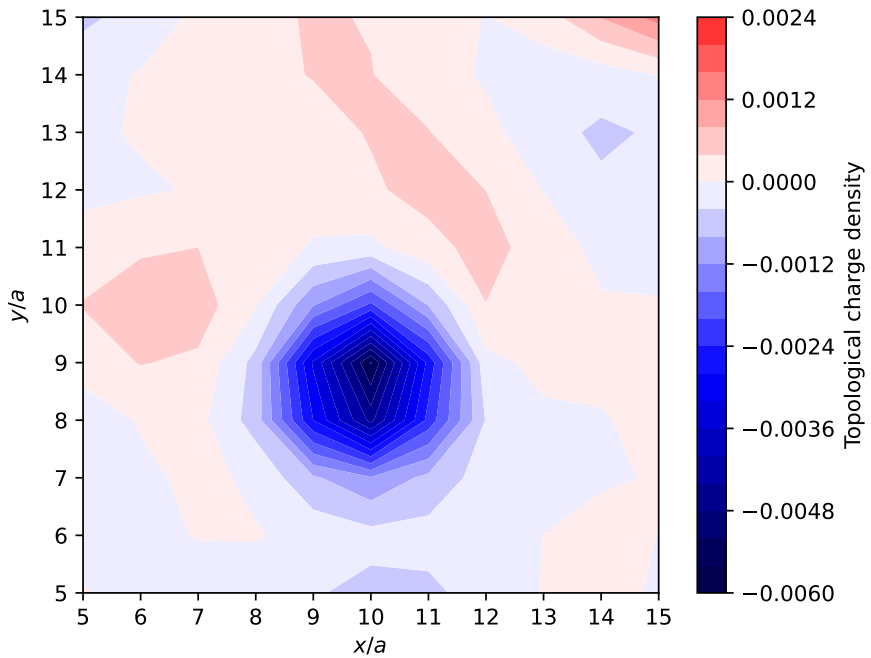

Figure: A lattice gauge configuration containing an instanton

Topological Reweighting

Reweighting

- No statistically significant sample from usual lattice algorithms
- Artificially change probability distribution using reweighting

$$\left\langle \mathcal{O} \right\rangle \propto \int \mathcal{D}U \, \mathcal{O}\left[U\right] e^{-S[U]} = \int \mathcal{D}U \underbrace{O\left[U\right] \frac{1}{w\left(\xi\left[U\right]\right)}}_{\text{Observable}} \underbrace{w\left(\xi\left[U\right]\right) e^{-S[U]}}_{\substack{\text{Probability} \\ \text{distribution}}}$$

  with reweighting function $w(\xi)$ and reweighting variable $\xi[U]$

- Sample configurations according to $w\left(\xi\left[U\right]\right) e^{-S[U]}$ and compensate for "wrong" sample in statistical analysis
- In our case: Enhance topological $Q = 1$ sector

Reweighting Variable

- Q distinguishes between topological sectors but does not go smoothly between them
- Use globbiness G ($=$ peak action density) to pick up dislocations
- Dislocations: Lattice-only "baby" instantons

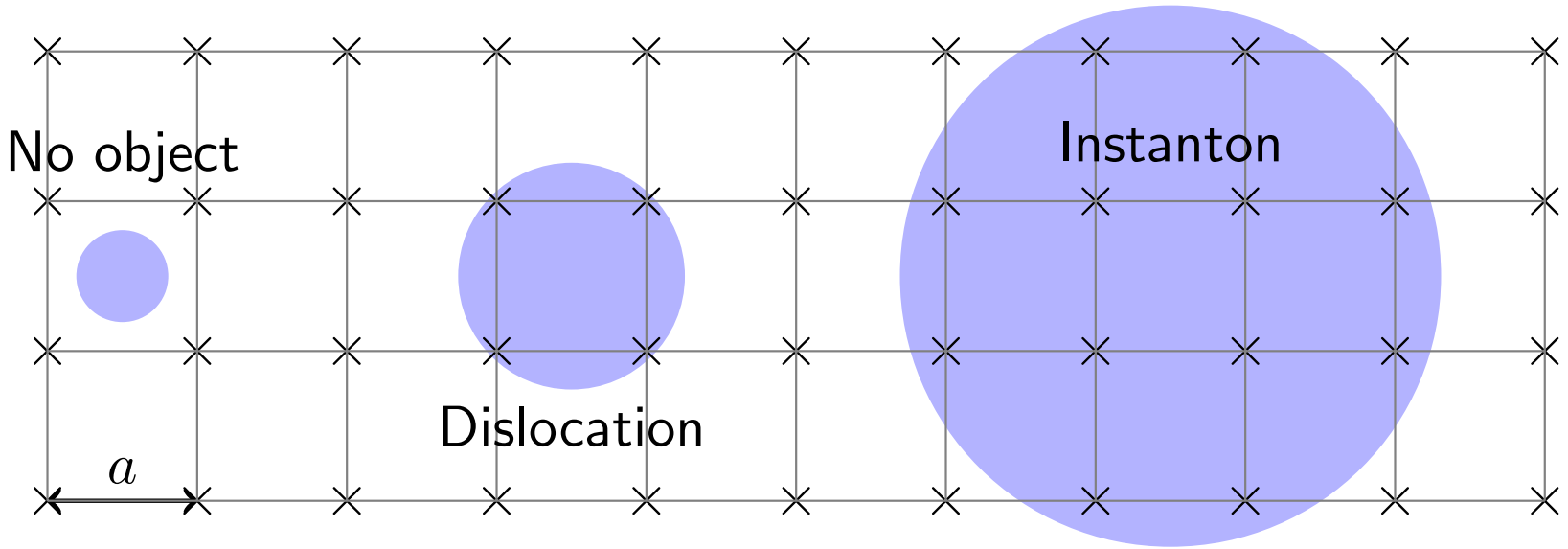

Figure: Topological objects on the lattice. Dislocations are an intermediate step when going from no topology to an instanton

Reweighting Variable

- Q and G together can act as reweighting variable
- Parametrize arc in QG plane to define angle φ as smooth reweighting variable

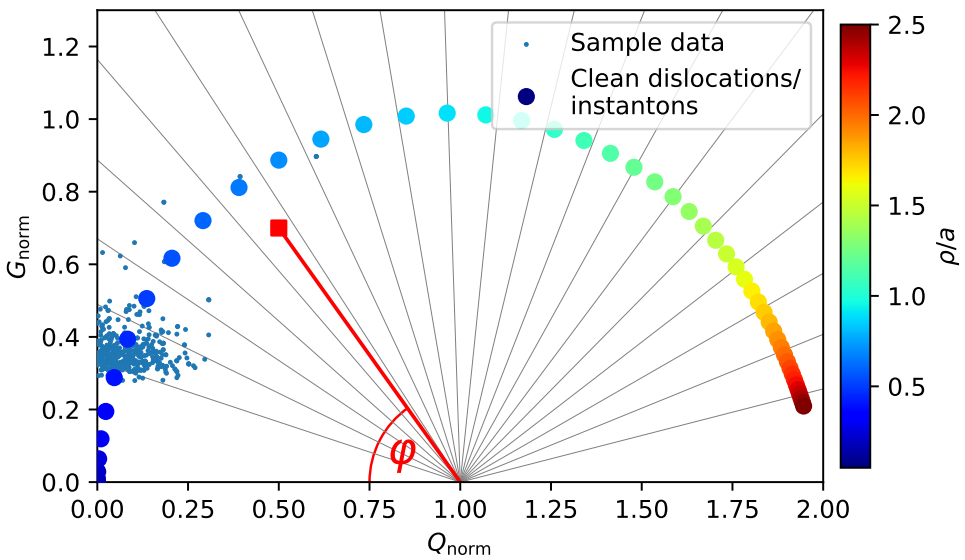

Figure: Definition of reweighting variable φ with unreweighted data

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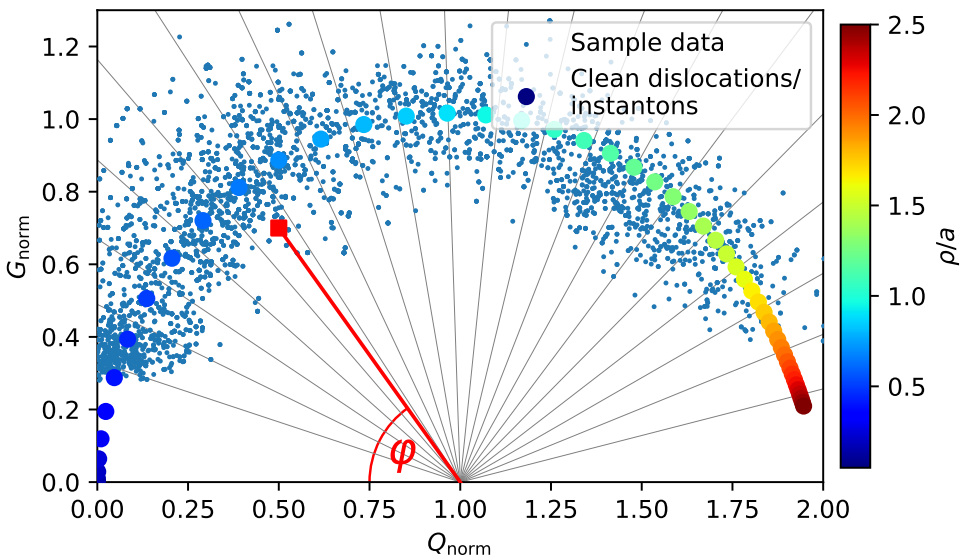

Figure: Definition of reweighting variable φ with reweighted data

Reweighting Variable

- Q and G lose sensitivity to radius for larger instantons
- Reweight in instanton radius directly as well
- Measure radius by finding smallest ball that contains topological charge of 0.5

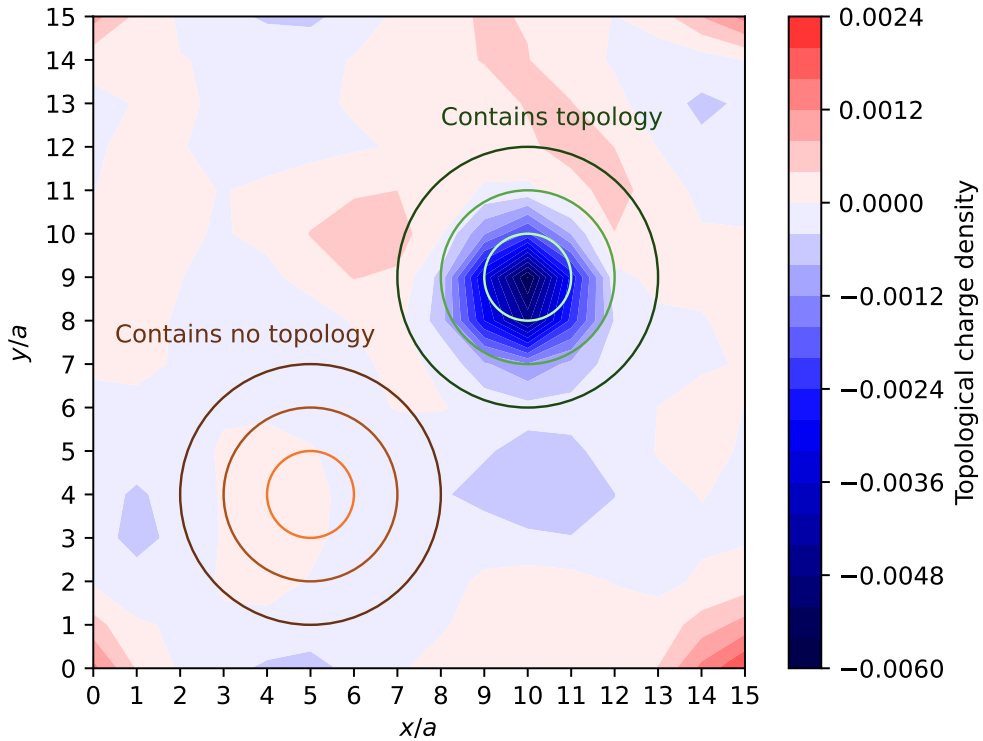

Figure: Topological charge density distribution in presence of an instanton

Technical Details

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- Reweighting function built automatically using method from Moore, Rummukainen 2001 (DOI: 10.1103/PhysRevD.63.045002)
- Three temperatures $T \in \{435, 682, 1022\}$ MeV at three lattice spacing $N_\tau = 8, 10, 12$ with $N_s/N_\tau = 3$
  - Scale setting performed in Junnarkar et al. 2023, arXiv:2308.04120
- 3+1 HISQ fermions with strange and charm quarks
  - Mass reweighting (Hasenbusch 2001, arxiv:hep-lat/0107019)
- Statistical analysis using bootstrap

Results

Reweighting Functions

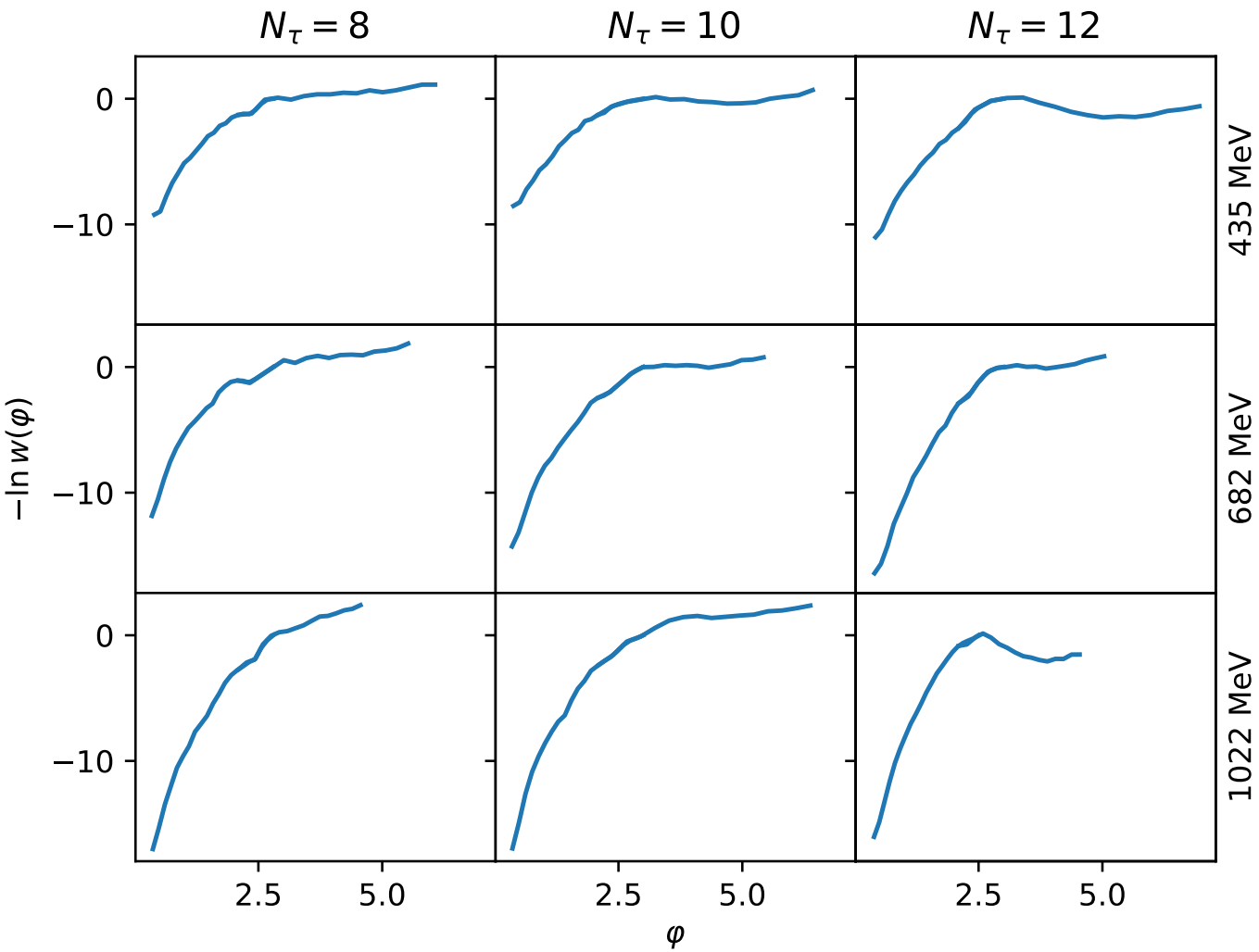

Figure: Reweighting functions for all desired temperatures and lattice spacings

Preliminary Results

- Data generation not complete
- $N_\tau = 12$, $T = 435\,\text{MeV}$ data point iffy

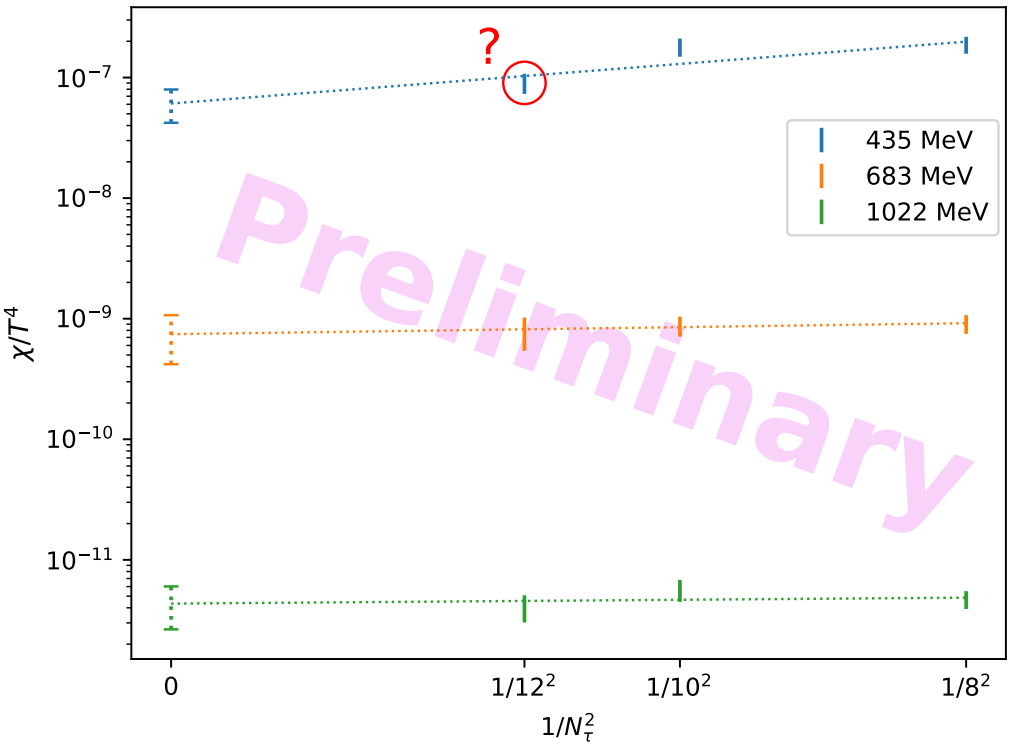

Figure: Topological susceptibility for three temperatures and lattice spacings

Summary

- Topological susceptibilty χ at high T required for axion mass
- Reweighting in topological charge, globbiness and instanton radius overcomes sampling issues
- All desired reweighting functions found
- Data generation in process
- Preliminary results existent

Appendix

Including radius in reweighting variable

- Replace Q with ρ in definition of angle

$$Q_{\text{int}} \equiv Q + k \max(0, \rho - \rho_0)$$

- Change reference point for φ for larger angles

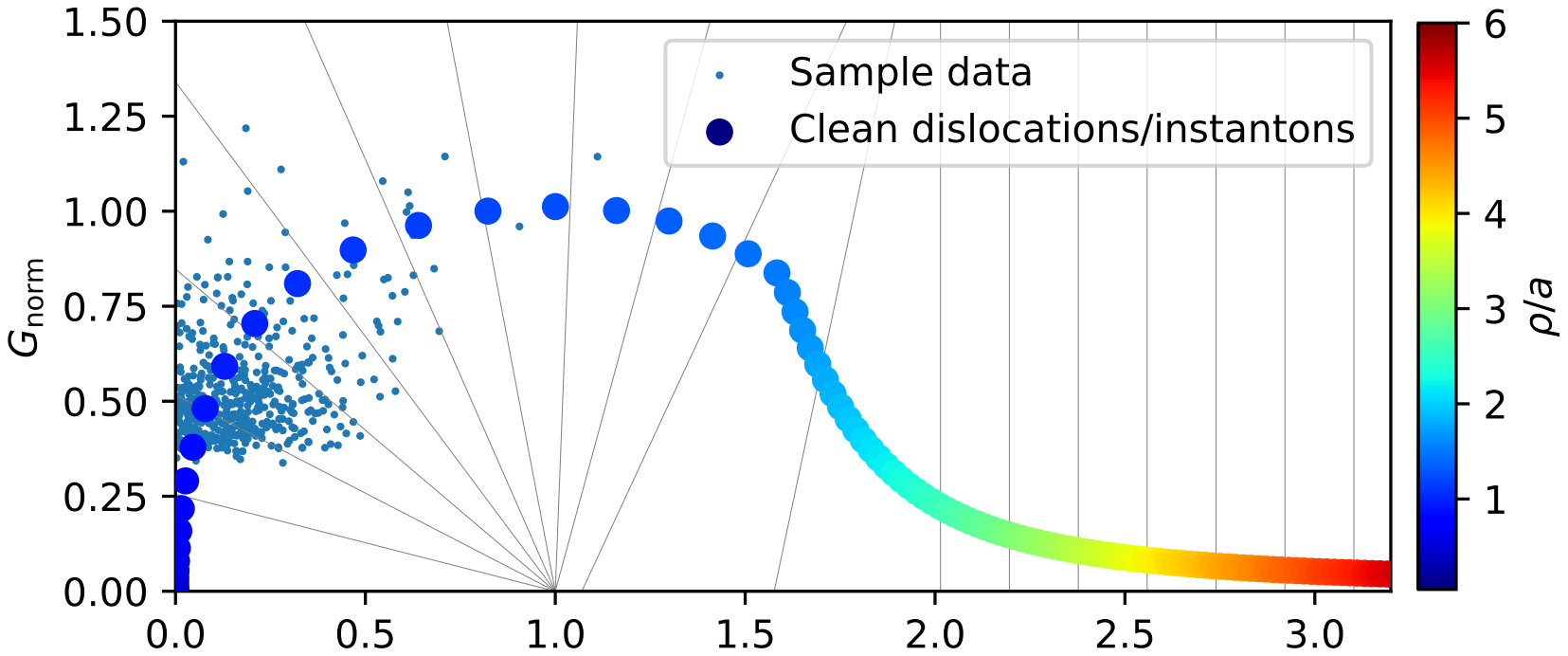

Figure: QG-plane with reweighting variable including radius

Reweighting Function

- Reweighting function $w(\varphi) \equiv \exp W(\varphi)$ needs to be defined
- Do preliminary run where it is built automatically before actual measurement
- Finish after convergence criterion met

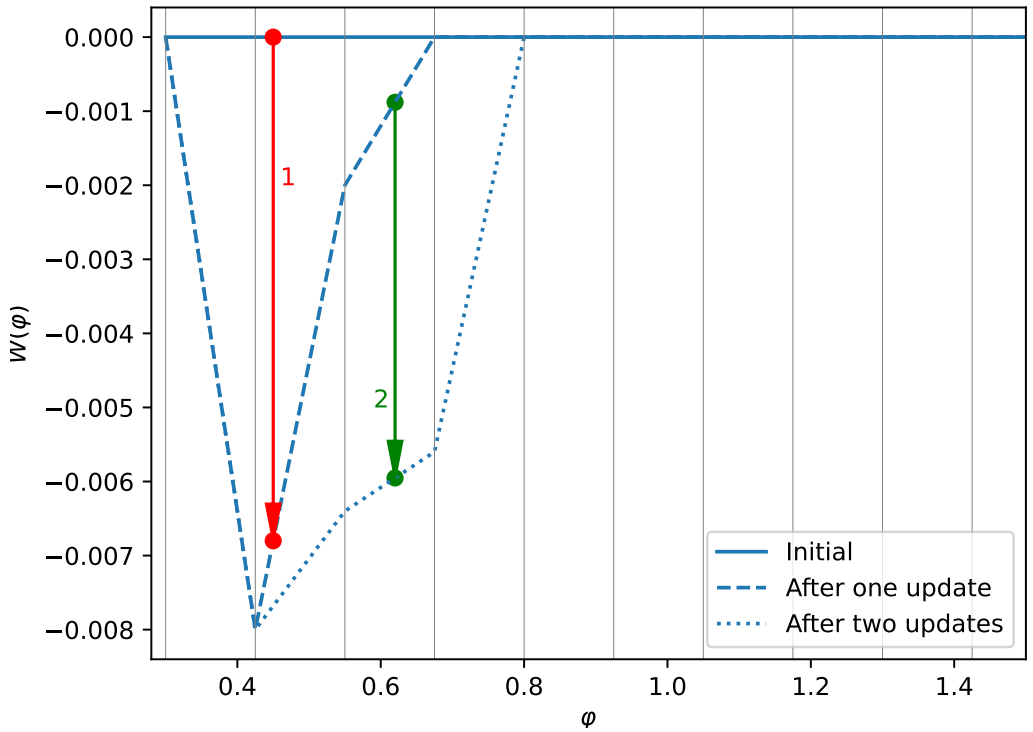

Figure: Modifying the reweighting function after each Monte Carlo update

Radii vs. lowest EVs

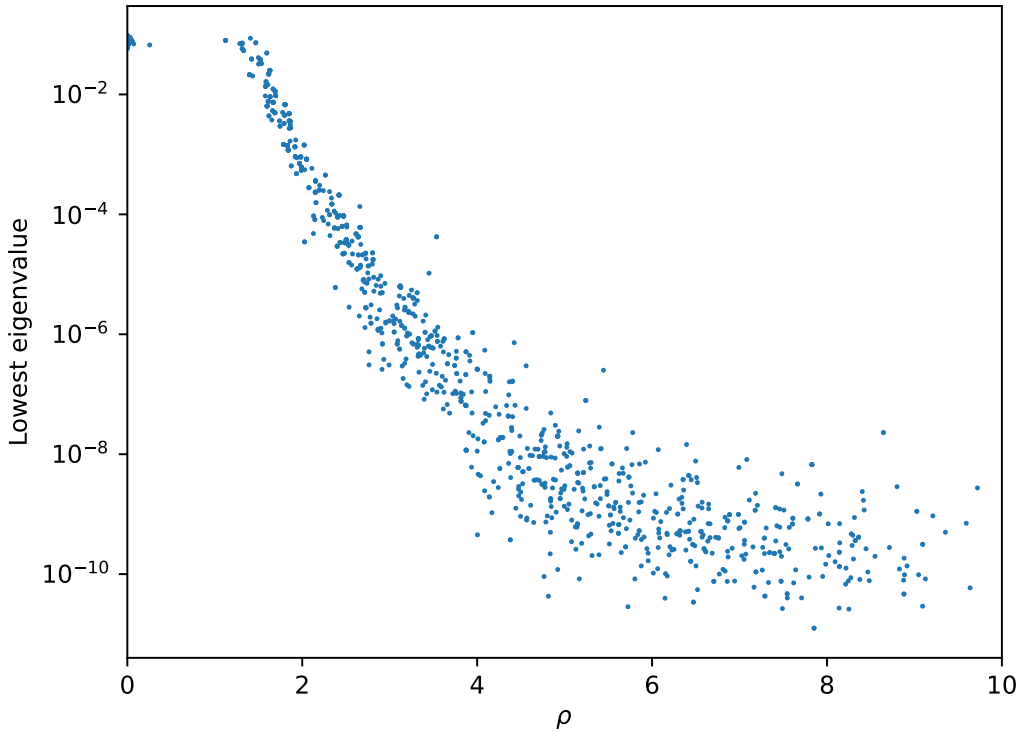

Figure: Scatterplot of the lowest eigenvalues vs. the measured instanton radii

Aspect ratio

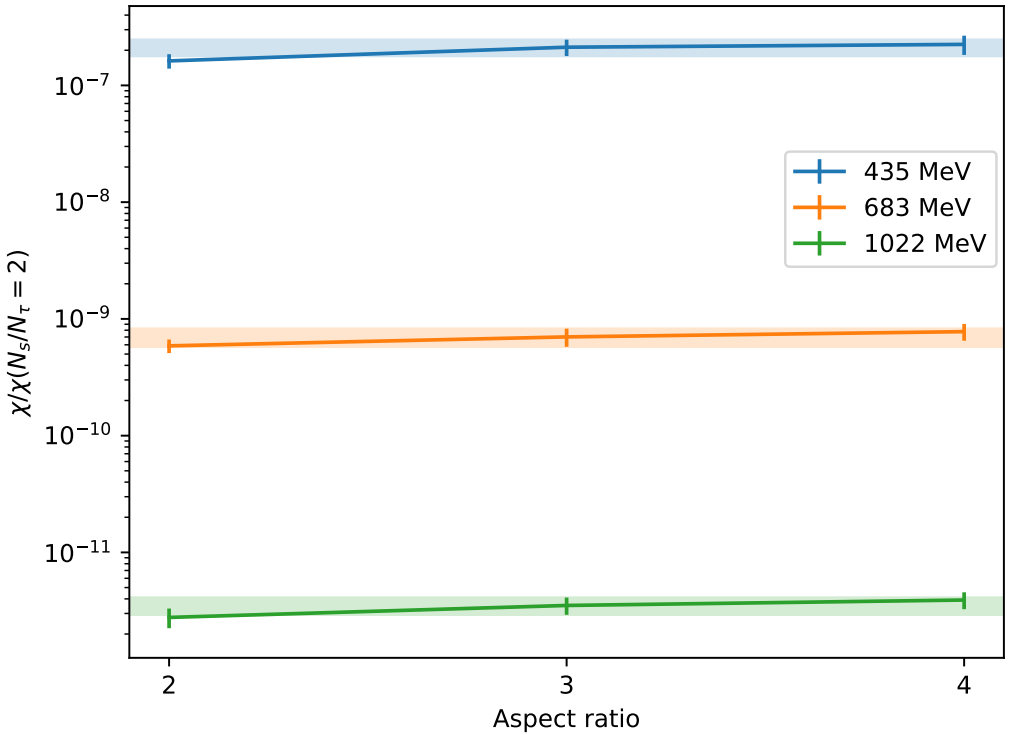

Figure: Aspect ratio dependence of $N_\tau = 8$ lattices