New Method to Constrain the EoS of Cold Dense Matter

–the massive Thirring / sine-Gordon model with non-zero current density–

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Motivation

Equation of State (EoS) of cold dense matter is a central problem:

- key input for neutron star physics / gravitational wave signals
- sign problem obstructs lattice calculations
- Silver Blaze phenomenon limits analytic continuation at $T = 0$

Present a novel method to **obtain upper & lower bounds on the EoS evading the sign problem** and test it within a non-trivial QFT [1]

Theoretical interest in theories with non-zero current density

The model

In $1+1$ dimensions,

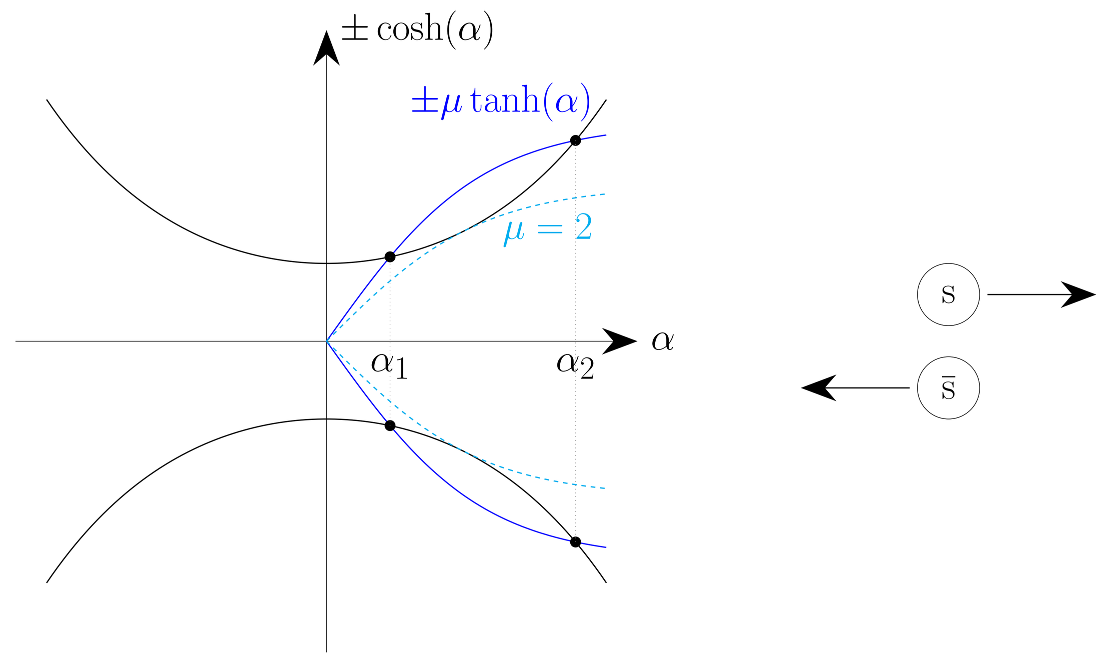
sine-Gordon (SG) model	massive Thirring (MT) model
$\frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi + \frac{m_0^2}{\beta^2}\cos(\beta\phi)$	$\bar\psi\,(\mathrm{i}\,\gamma^\mu\,\partial_\mu - M_0)\,\psi - \frac{g}{2}\,(\bar\psi\,\gamma^\mu\,\psi)^2$
topological soliton	elementary fermion
elementary boson, "breather"	lowest fermion bound state
weakly (strongly) coupled	strongly (weakly) coupled

Established results:

- duality SG/MT in refs. [3, 4]
- vacuum properties in refs. [5, 6]
- finite density in ref. [7]

Fixed current

Task:
Compute $T^{\mu\nu}$ minimizing $\boxed{\epsilon - \mu\, j}$



Solution via **Bethe ansatz** for the density of states $\rho(\alpha)$

Key equations: (α … rapidity)

$$\boldsymbol{\rho(\alpha)} = \frac{\cosh(\alpha)}{2\pi} + \int_{\alpha_1}^{\alpha_2} \mathrm{d}\alpha'\,\boldsymbol{\rho(\alpha')}\,K(\alpha,\alpha')$$

$$\xrightarrow{\rho(\alpha)}\; j = 2\,m_{\mathrm{s}} \int_{\alpha_1}^{\alpha_2} \mathrm{d}\alpha\,\rho(\alpha)\,\tanh\alpha$$

$$\xrightarrow{\rho(\alpha)}\; T^{\mu\nu} = \begin{cases} 2\displaystyle\int_{\alpha_1}^{\alpha_2} \mathrm{d}\alpha\,\frac{\rho(\alpha)\,p^\mu(\alpha)\,p^\nu(\alpha)}{\cosh\alpha}\,, & \mu=\nu \\ 0\,, & \mu\neq\nu \end{cases}$$

Kernel $K(\alpha,\alpha')$ encodes all model-dependent interactions

Summary

Advantages of the novel method:

- obtain upper and lower bounds on the EoS
- EoS in terms of $\epsilon(n)$
- based on relativity, no sign problem

Its **usefulness demonstrated** within SG/MT model

Solution of the MT/SG model with non-zero current density using a Bethe ansatz

The method

General idea

Minkowski $\xleftrightarrow{\text{Wick rotation}}$ Euclidean

number density
real chemical potential ⟶ sign problem
imaginary chemical potential ⟶ no sign problem
relativity [2] — equivalent by angular rotation
current density
real Lagrange multiplier ⟶ no sign problem

Given: ground state of a theory, here: $(1+1)d$, with fixed current (density),

$$\langle\hat{j}\rangle = \begin{pmatrix}0\\ j\end{pmatrix}, \qquad \langle\hat{T}\rangle = \begin{pmatrix}T_{tt} & 0\\ 0 & T_{xx}\end{pmatrix}.$$

Use **relativity** to learn about non-zero number density properties
Zero temperature remains under boosting

Upper bound

$$\begin{pmatrix}0\\ j\end{pmatrix} \xrightarrow{\beta} \begin{pmatrix}n^{\mathrm{u}}\\ j^{\mathrm{u}}\end{pmatrix}, \qquad \epsilon(n^{\mathrm{u}}) \le T_{tt}^{\beta}(n^{\mathrm{u}}, j^{\mathrm{u}})\,, \qquad \boxed{\epsilon(n^{\mathrm{u}}) \le \frac{T_{tt} + \beta^2\,T_{xx}}{1-\beta^2}}$$

Lower bound

$$\begin{pmatrix}n^{\mathrm{l}}\\ 0\end{pmatrix} \xrightarrow{\beta} \begin{pmatrix}n\\ j\end{pmatrix}, \qquad T_{tt}(0, j) \le T_{tt}^{\beta}(n, j)\,,$$

$$\boxed{\epsilon(n^{\mathrm{l}}) \ge \frac{1-\beta^2}{1+\beta^2\,\frac{p(n^{\mathrm{l}})}{\epsilon(n^{\mathrm{l}})}}\,T_{tt} \ge \frac{1-\beta^2}{1+\beta^2}\,T_{tt}}$$

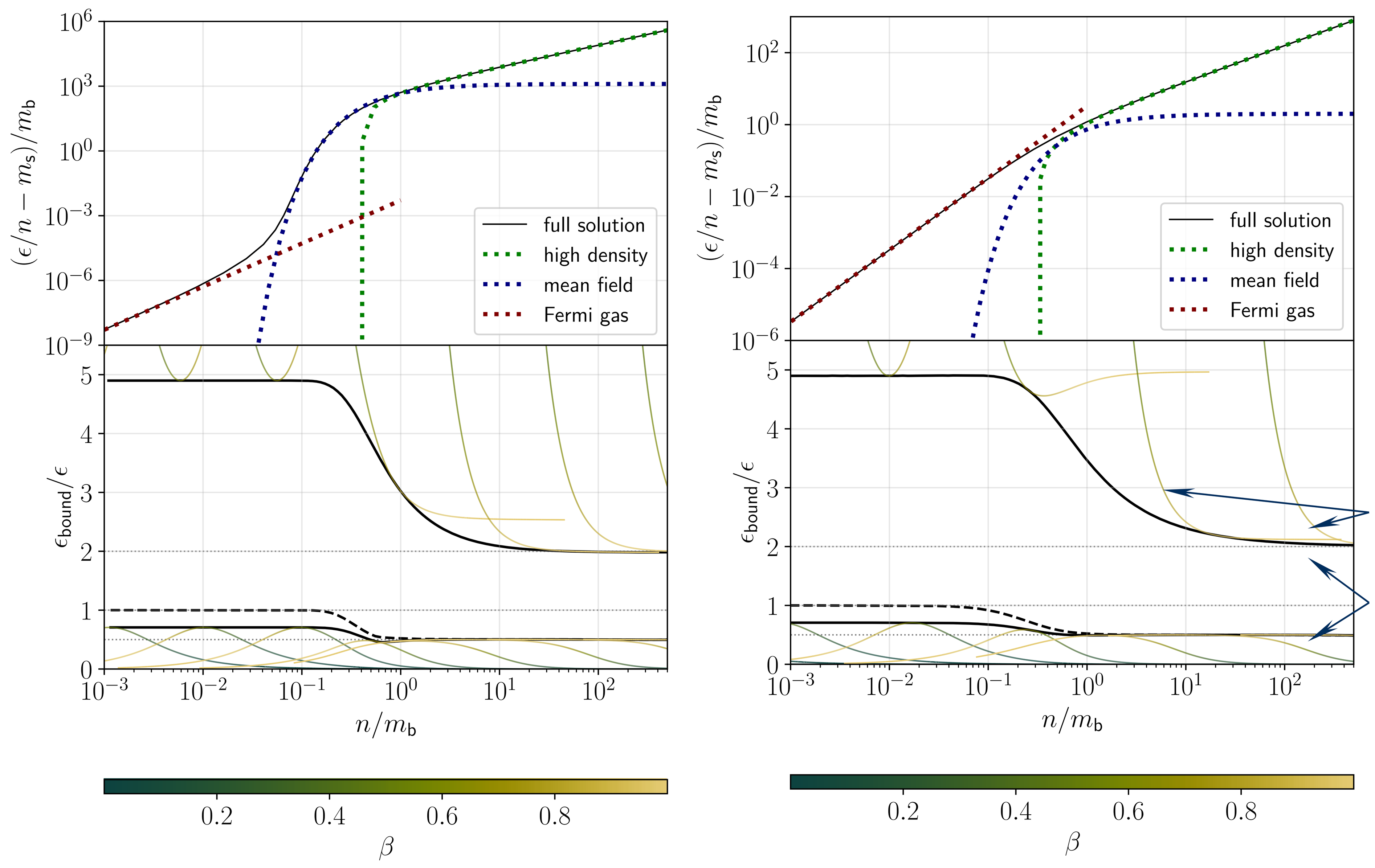
Ratio $p/\epsilon \le 1$ constrained by **causality**, fixed by any **proposed EoS model**
⟶ consistency check possible

Worst case bounds

$$3\,T_{tt}(j=n) \ge \epsilon(n) \ge \frac{1}{3}\,T_{tt}(j=n) \longrightarrow \textbf{factor of 9}\text{ between bounds}$$

- one value of j ⟶ bounds on $\epsilon(n)$ for all n
- multiple values of j ⟶ optimize bounds

Numerical results



top: interaction energy per soliton [7]
bottom: bounds on the EoS

left: weakly coupled SG
right: strongly coupled SG

bounds from a single current density

best bounds as envelope;
optimal lower bounds dashed

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References

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[3] S. R. Coleman, "The Quantum Sine-Gordon Equation as the Massive Thirring Model", (1975), Phys. Rev. D 11, 2088.

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[5] H. Bergknoff, H. B. Thacker, "Structure and Solution of the Massive Thirring Model", (1979), Phys. Rev. D 19, 3666.

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