Recent results on thermal resonances and chiral&$U(1)_A$ restoration

Angel Gómez Nicola
Universidad Complutense Madrid

OUTLINE

⋆ **Ward Identities**

- Chiral vs $U(1)_A$ restoration for S/P partners.
- Role of strangeness. $K-\kappa$ sector

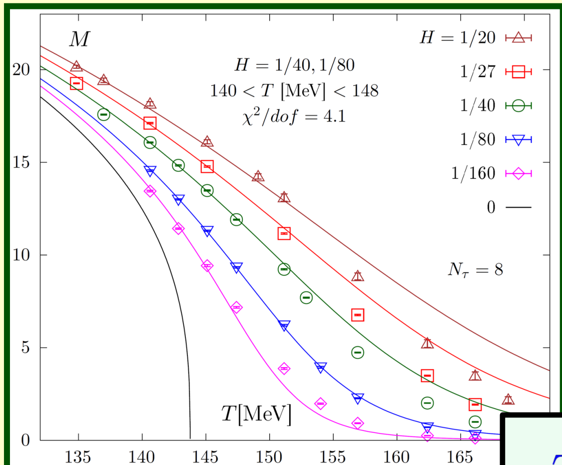
⋆ **Light thermal resonances**

- Thermal Unitarity and poles in $\pi\pi$, $K\pi$ scattering
- Scalar susceptibilities from thermal poles

SEWM 2026 Helsinki 18 August 2026

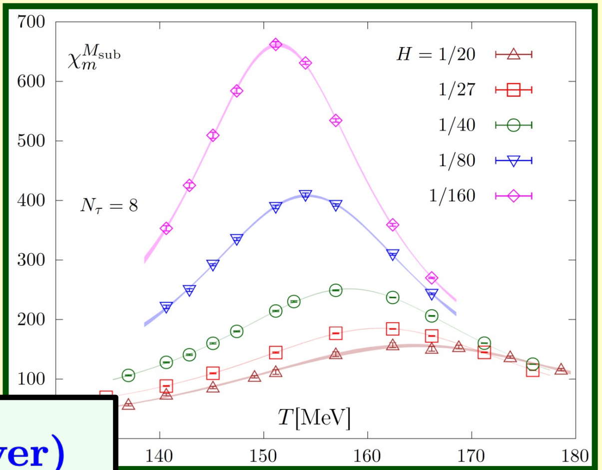
Lattice signals of Chiral and $U(1)_A$ restoration

$\langle \bar{q}q \rangle_l = \langle \bar{u}u + \bar{d}d \rangle$ **inflection point**



H.T.Ding et al PRD109 (2024)

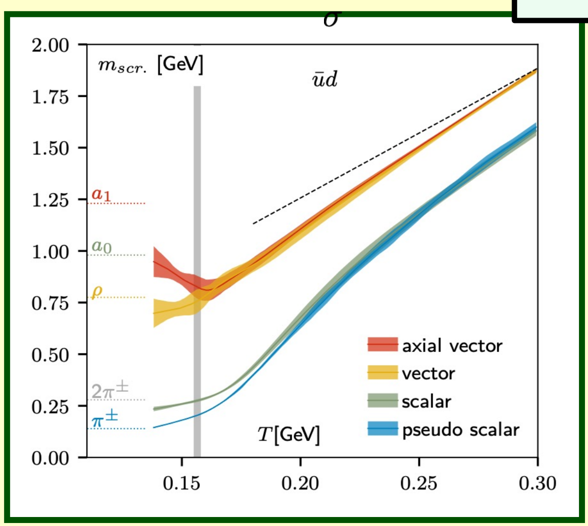
$\chi_S = \int_x \left[\langle \bar{q}q(x)\bar{q}q(0) \rangle - \langle \bar{q}q \rangle_l^2 \right] = -\frac{\partial}{\partial m_l} \langle \bar{q}q \rangle$ **peak**



H.T.Ding et al PRD109 (2024)

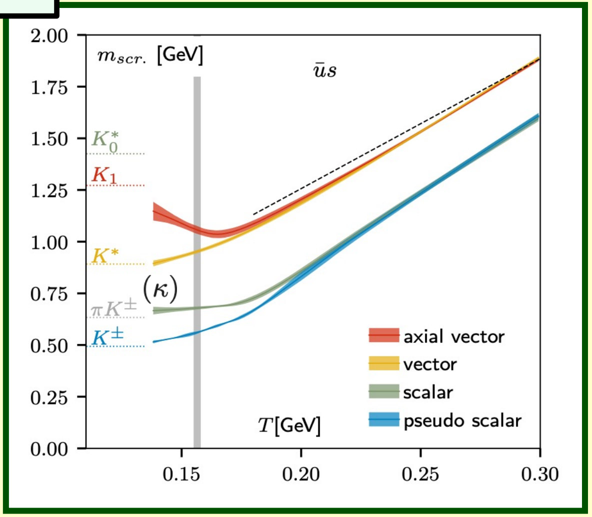
$T_{pc} \simeq 157$ **MeV (crossover)**

$T_c^{m_l=0} \simeq 134$ **MeV (2nd order)**



Degeneration of partners (screening masses)

A.Bazavov et al (Hot QCD) PRD 100 (2019)



How effective is $U(1)_A$ asymptotic restoration at T_c?

$\Rightarrow$ **Transition order and universality class, $M_{\eta'}, \chi_{top}$ reduction, $\eta - \eta'$ ideal, critical point,...**

R. D. Pisarski, F. Wilczek PRD29 (1984)
E.V.Shuryak, CNPP21 (1994)
T.D.Cohen PRD54 (1996)
S.H.Lee, T.Hatsuda PRD54 (1996)
J. I. Kapusta, D. Kharzeev, L. D. McLerran, PRD53 (1996)
T. Csorgo et al, PRL105 (2010)
A. Pelissetto, E. Vicari PRD88 (2013)
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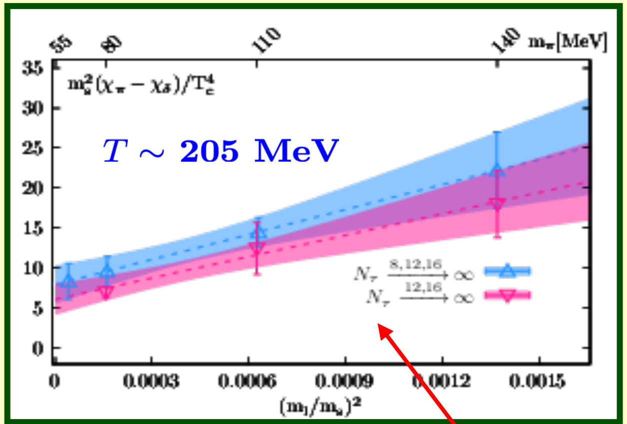
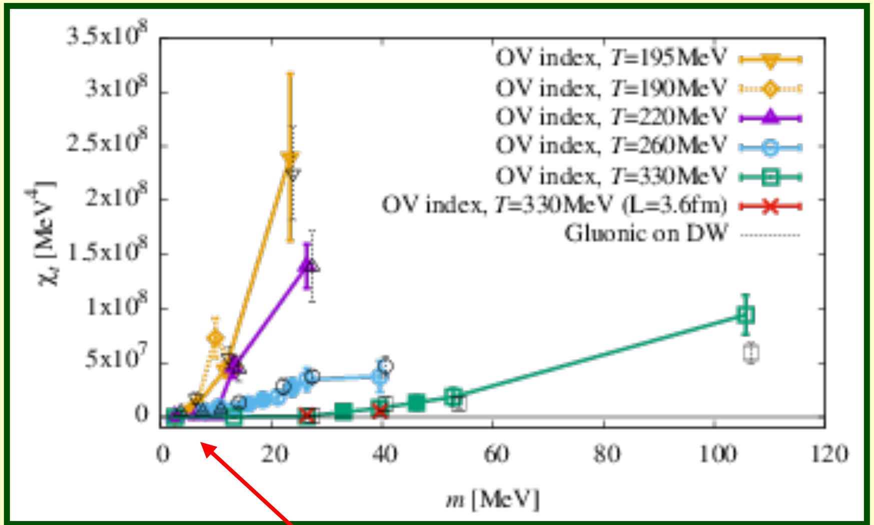
AGN, J.Ruiz de Elvira PRD97 (2018); PRD98 (2018)
AGN, J.Ruiz de Elvira, A.Vioque, JHEP 11 (2019); JHEP 08 (2023)
M. P. Lombardo, A. Trunin, IJMPA 35 (2020)



H.Ding et al PRL 126 (2021)

$N_f = 2+1$:
$U(1)_A$ broken @ T_c



S. Aoki et al, PRD (2021)

$N_f = 2$:
$U(1)_A$ strongly suppressed
not far above T_c in chiral limit

Ward Identities for chiral& $U(1)_A$ restoration

AGN, J.Ruiz de Elvira, R. Torres, PRD88 (2013)
AGN, J.Ruiz de Elvira, PRD97 (2018), PRD98 (2018), JHEP086 (2019)
AGN, J.Ruiz de Elvira, A. Vioque-Rodríguez, JHEP 11 (2019)
AGN, J.Ruiz de Elvira, A. Vioque-Rodríguez, D. Álvarez-Herrero, EPJC81 (2021)

- **Relate $\langle \bar{q}_i q_i \rangle$ and χ's from QCD generating functional**
- **Tested in NLO $U(3)$ ChPT**
- **Describe lattice screening masses at finite T**

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$$
\begin{array}{ccccc}
I=1 & & & & I=0 \\
\pi^a = i\bar{q}_l \gamma_5 \tau^a q_l & \overset{SU(2)_A}{\longleftrightarrow} & O(4) \text{ partners} & & \sigma_l = \bar{q}_l q_l \\
\updownarrow U(1)_A & & & & \updownarrow U(1)_A \\
\delta^a = \bar{q}_l \tau^a q_l & \overset{SU(2)_A}{\longleftrightarrow} & O(4) \text{ partners} & & \eta_l = i\bar{q}_l \gamma_5 q_l
\end{array}
$$

$I=0,1$

$$\chi_P^{ls}(T) = -2\frac{m_l}{m_s}\chi_{5,disc}(T) = -\frac{2}{m_l m_s}\chi_{top}(T)$$

$$\chi_{5,disc} \equiv \frac{1}{4}\left(\chi_P^\pi - \chi_P^{\eta_l}\right) \stackrel{O(4)\times U(1)_A}{\longrightarrow} 0$$

$$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x) A(0)\rangle \stackrel{U(1)_A}{\longrightarrow} 0$$

$$\chi_P^{ls} = \int_T dx \langle \mathcal{T} \eta_l(x) \eta_s(0)\rangle$$

$$\chi_P^O \equiv \int_T dx\, \langle O(x) O(0)\rangle \qquad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

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- **Relate $\langle \bar{q}_i q_i \rangle$ and χ's from QCD generating functional**
- **Tested in NLO $U(3)$ ChPT**
- **Describe lattice screening masses at finite T**

$I=1$		$I=0$
$\pi^a = i\bar{q}_l \gamma_5 \tau^a q_l$	$\overset{SU(2)_A}{\longleftrightarrow}$ $O(4)$ partners	$\sigma_l = \bar{q}_l q_l$
$\updownarrow U(1)_A$		$\updownarrow U(1)_A$
$\delta^a = \bar{q}_l \tau^a q_l$	$\overset{SU(2)_A}{\longleftrightarrow}$ $O(4)$ partners	$\eta_l = i\bar{q}_l \gamma_5 q_l$

$I = 0, 1$

$$\chi_P^{ls}(T) = -2\frac{m_l}{m_s}\chi_{5,disc}(T) = -\frac{2}{m_l m_s}\chi_{top}(T) \quad \Rightarrow \quad \chi_{5,disc} \xrightarrow{O(4)} 0, \quad \chi_{top} \xrightarrow{O(4)} 0$$

$$\chi_{5,disc} \equiv \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right) \xrightarrow{O(4)\times U(1)_A} 0$$

$$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x) A(0)\rangle \xrightarrow{U(1)_A} 0$$

$$\chi_P^{ls} = \int_T dx \langle \mathcal{T} \eta_l(x) \eta_s(0)\rangle \xrightarrow{O(4)} 0 \quad (*)$$

$$\chi_P^O \equiv \int_T dx \, \langle O(x) O(0)\rangle \qquad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$\langle \eta_l \eta_s \rangle \xrightarrow{SU(2)_A} \langle \delta \eta_s \rangle = 0 \text{ by parity}$$

$U(1)_A$ restoration for exact CSR
(e.g. @T_c for two massless flavours)
Quark masses & strangeness generate a gap

$I = 1/2$

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$K^a = i\bar{q}\gamma_5\lambda^a q \overset{SU(2)_A}{\underset{U(1)_A}{\longleftrightarrow}} \kappa^a = \bar{q}\lambda^a q$

$$\chi_P^K(T) = -\frac{\langle \bar{q}q\rangle_l(T) + 2\langle \bar{s}s\rangle(T)}{m_l + m_s}$$

$\Rightarrow$ **decreases (faster as $T \to T_c$ from below)**

$$\chi_S^\kappa(T) = \frac{\langle \bar{q}q\rangle_l(T) - 2\langle \bar{s}s\rangle(T)}{m_s - m_l}$$

$\Rightarrow$ **increases as $T \to T_c$ ($\langle \bar{q}q\rangle_l \downarrow$, $\langle \bar{s}s\rangle \sim$ const)**

decreases above T_c ($\langle \bar{s}s\rangle \downarrow$ dominates)

$\Rightarrow$ χ_S^κ **peaks above T_c**

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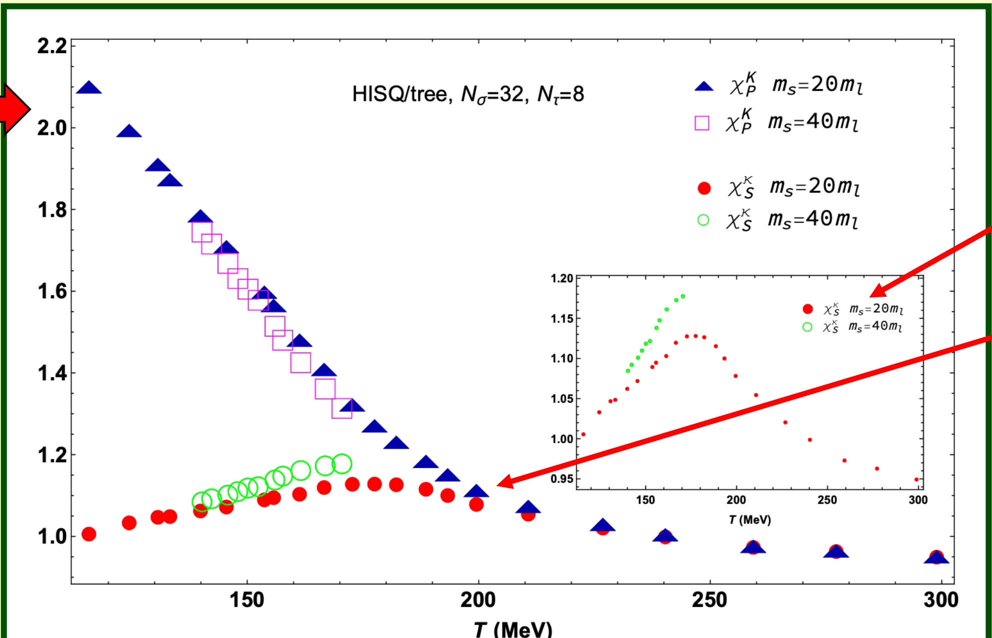
$\Rightarrow \chi_S^\kappa$ peaks above T_c

WI-reconstructed $\chi^{K,\kappa}$ from lattice condensates



A.Bazavov et al (Hot QCD) 2012-14 ($N_f = 2+1$)

$m_l/m_s \to 0^+$ enhances slope below peak from CSR

$\Rightarrow$ more efficient K/κ degeneration

$\chi_S^\kappa \to \chi_P^K$ in $O(4)\times U(1)_A$ region above χ_S^κ peak

$\Rightarrow$ suitable channel to understand strangeness in $O(4)\times U(1)_A$ restoration

Scattering and Resonances within finite-T Unitarized ChPT

AGN, F.J.LLanes-Estrada, J.R.Peláez, PLB (2002)
A.Dobado, AGN, F.J.LLanes-Estrada, J.R.Peláez, PRC (2002)

S.Ferreres, AGN, A.Vioque-Rodríguez, PRD (2019)
AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, JHEP (2023)
AGN, A.Vioque-Rodríguez, EPJST (2026)

- **IAM** $\rightarrow t_U(s;T) = \dfrac{[t_2(s)]^2}{t_2(s) - t_4(s;T)}$ $\qquad t = \underbrace{t_2}_{\mathcal{L}_2\text{tree}} + \underbrace{t_4}_{\mathcal{L}_2\text{1-loop}+\mathcal{L}_4\text{tree}} + \cdots$ ChPT PW expansion

- **Thermal resonances in $\pi\pi$ ($f_0(500)/\sigma$, $\rho(770)$), $K\pi$ ($K_0^*(700)/\kappa$, $K^*(890)$), ...**

$@s_{pole}(T) = [M_p(T) - i\Gamma_p(T)/2]^2$

- **Thermal Unitarity for meson scattering PW incl.thermal-bath processes:**

$$\mathbf{Im}\, t_U(s;T) = \begin{cases} \sigma^T(s)\, |t_U(s;T)|^2\,, & s \geq (M_a + M_b)^2 \ \textbf{(unit.cut)} \\ \tilde{\sigma}^T(s)\, |t_U(s;T)|^2\,, & 0 \leq s \leq (M_a - M_b)^2 \ \textbf{(Landau thermal cut)} \end{cases}$$

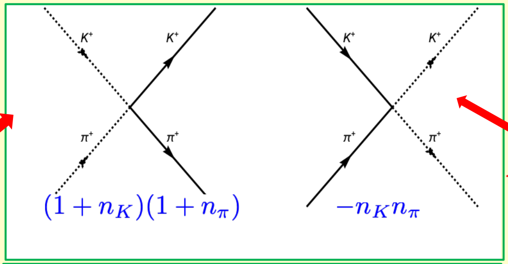
- **Thermal phase space:**

$$\sigma^T_{ab}(s) = \sigma_{ab}(s)\left[1 + n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right) + n_B\left(\frac{s-\Delta_{ab}}{2\sqrt{s}}\right)\right]$$

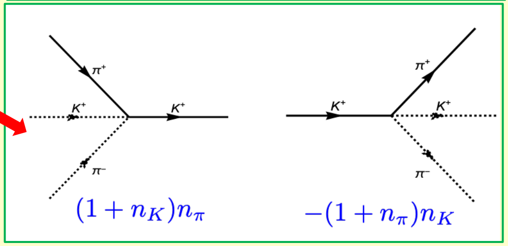
$$\tilde{\sigma}^T_{ab}(s) = \sigma_{ab}(s)\left[n_B\left(\frac{\Delta_{ab}-s}{2\sqrt{s}}\right) - n_B\left(\frac{s+\Delta_{ab}}{2\sqrt{s}}\right)\right]$$

$\Delta_{ab} = M_a^2 - M_b^2$, σ_{ab} **two-body $T=0$ phase space**

$n_i \equiv n_B(E_i) = \frac{1}{e^{E_i/T}-1}$



thermal bath particles



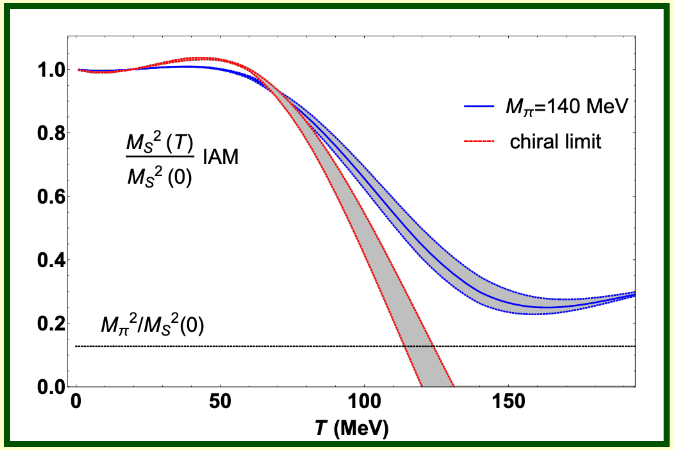
Saturating scalar susceptibilities with light thermal resonances

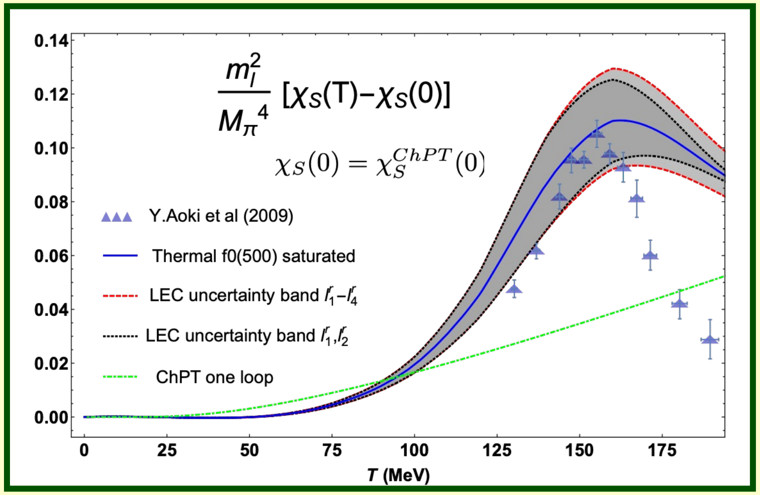
$\Rightarrow \chi_S$ **saturated by lightest $I=J=0$ state $f_0(500)(\sigma)$**

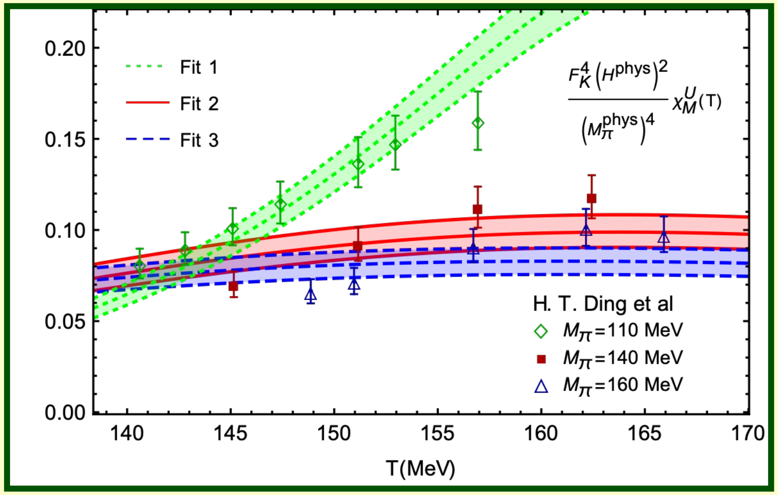
$$\chi_S(T) = G^E_\sigma(p=0) \simeq \chi_S(0)\,\frac{M^2_S(0)}{M^2_S(T)}$$

$M^2_S(T) = M^2_p(T) - \Gamma^2_p(T)/4 = \mathbf{Re}(s_p(T)) \sim \mathbf{Re}\Sigma_\sigma$

behaves as thermal mass



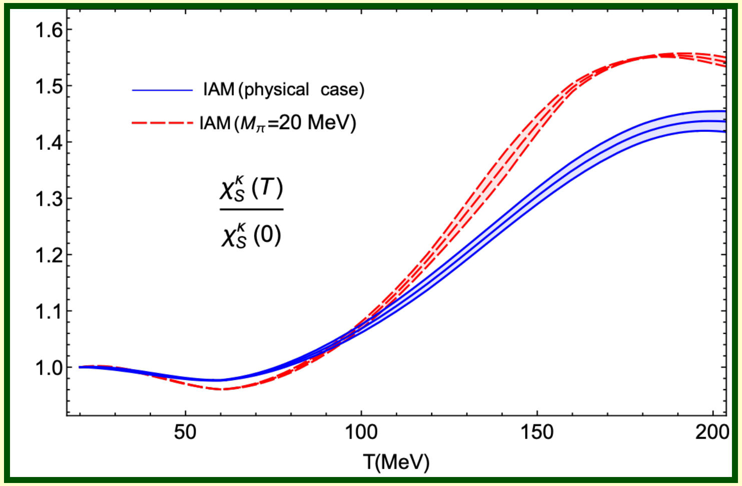


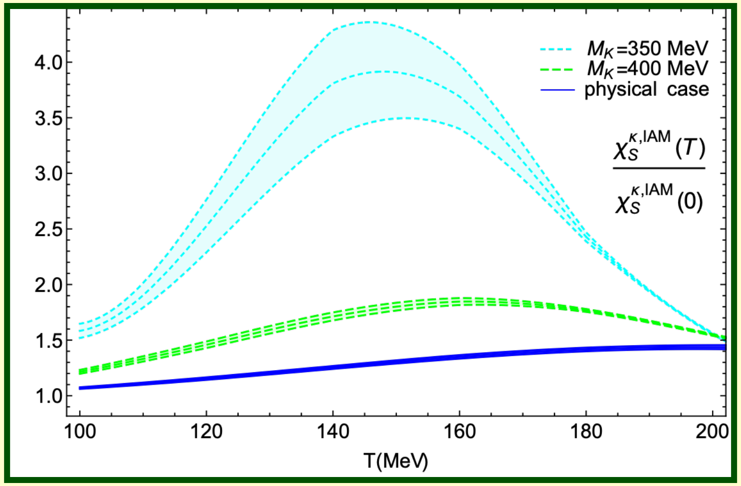


- **Reproduces expected peak $T_c \sim 158$ MeV**
- **Agrees with lattice around T_c within $T=0$ LEC uncertainties**
- **Consistent T_c reduction and χ_S growth as $M_\pi \downarrow$**

Saturating scalar susceptibilities with light thermal resonances

$\Rightarrow \chi_S^\kappa$ **saturated by** $I=1/2\ K_0^*(700)(\kappa)$ **scalar pole**





$m_l/m_s \to 0^+$: $\Rightarrow$ **larger growth below peak enhanced by chiral symmetry**
$\Rightarrow \chi_S^\kappa \to \chi_P^K$ **more efficient above peak**

$m_l/m_s \to 1$: $\chi_S^\kappa \to \chi_S$:
peak grows and moves towards T_c
($SU(3)$ **degeneration** $\sigma \leftrightarrow \kappa$)

Thermal interactions crucial to reproduce correct peak behaviour in σ, κ **channels**

Large-N_c behaviour of light thermal resonances

AGN, MC. Mendoza. A.Vioque-Rodríguez, in progress

- **Large-N_c crucial to understand $T_c < T < T_d$ phases**
- **$T = 0$ N_c-scaling expected to change for critical observables ($P, \langle \bar{q}q \rangle$, χ_S, ...)**

C.B. THorn, PLB99 (1981)
D.Toublan, PLB621 (2005)
U,Heinz, F.Giacosa, D.Rischke, PRD85 (2012)

T.D.Cohen, L.Y. Glozman, EPJA60 (2024)
Y.Fujimoto, K.Fukushima, Y.Hidaka, L.McLerran, PRD112 (2025)

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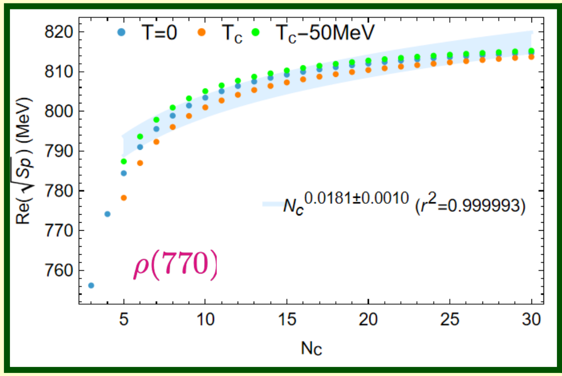
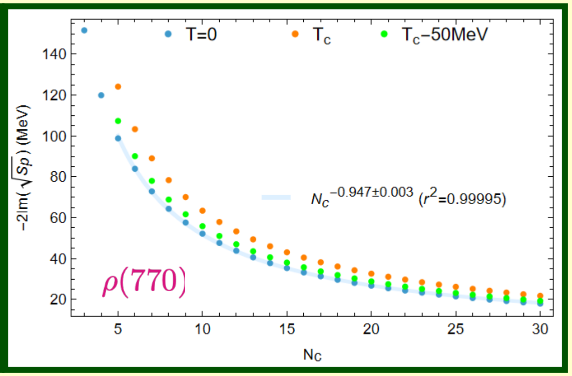
$\Rightarrow$ From large-N_c scaling of f_π and low-energy constants:

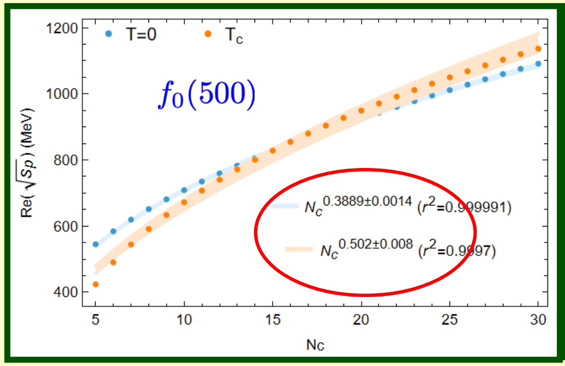


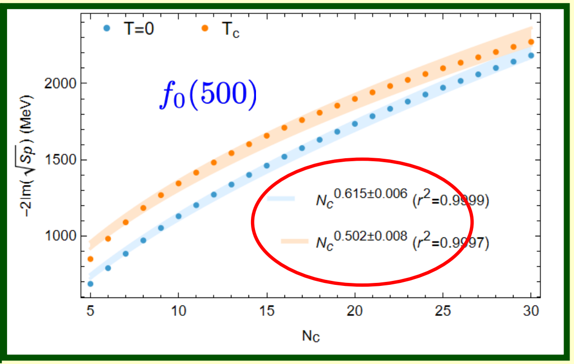


$\rho(770)$ N_c-scaling insensitive to critical region

$$M_\rho \sim N_c^0, \;\; \Gamma_\rho \sim N_c^{-1}$$

$\bar{q}q$ behaviour (T-independent)





$f_0(500)$ N_c-scaling near T_c modified by $M_S^2(T) \to 0$ and $\chi_S(T)$ growth

$$M_\sigma(T_c) \sim \Gamma_\sigma(T_c) \sim N_c^{1/2}$$

Not "ordinary" $\bar{q}q$ or glueball at any T

Soft $T_c \sim N_c^\alpha$, $|\alpha|$ <0.15 compatible with previous analyses

CONCLUSIONS

⋆ **Ward Identities powerful tools for symmetry restoration in QCD.**

⇒ $U(1)_A$ **restored at exact chiral restoration of S/P nonet.**

⇒ K/κ **alternative channel for** $O(4) \times U(1)_A$ **restoration above** χ_S^κ **peak driven by** $\langle \bar{s}s \rangle$.

⇒ **OK with lattice and ChPT.**

⋆ **Scalar thermal resonances from thermal unitarity crucial for chiral &** $U(1)_A$ **restoration**

⇒ **Saturated** χ_S **with thermal** $f_0(500)$ **reproduces** T_c **peak in agreement with lattice.**

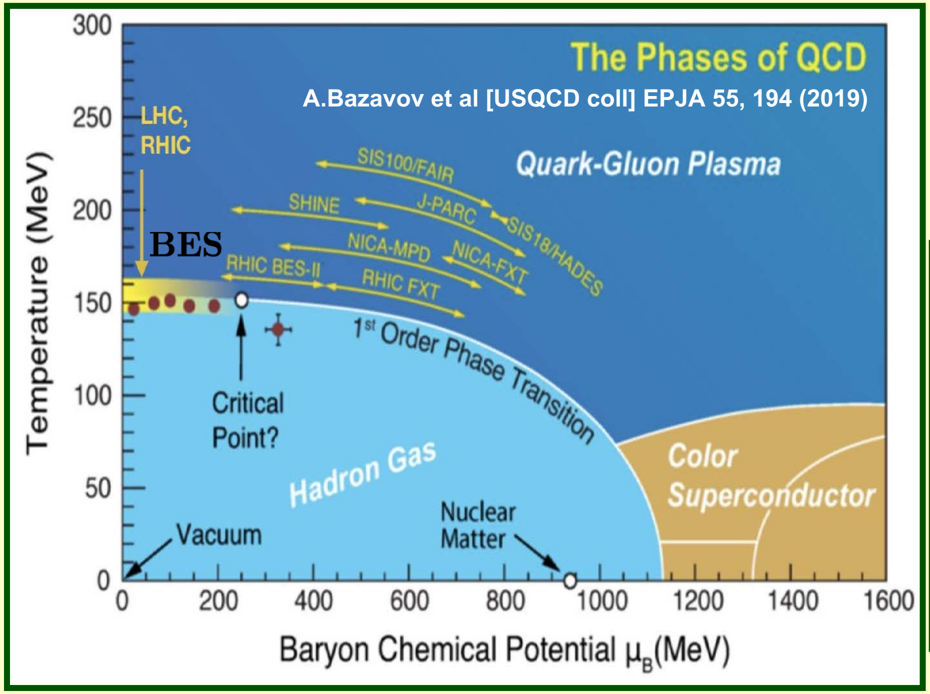
⇒ **Saturated** χ_S^κ **with thermal** $K_0^*(700)$ **peaks according to** $O(4) \times U(1)_A$ **pattern.**

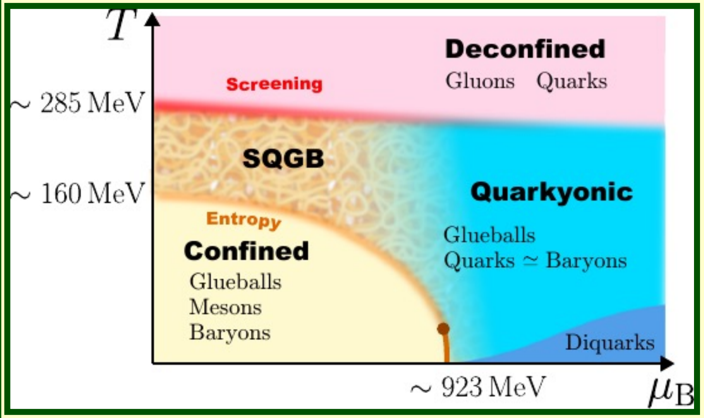
⇒ **Large-**N_c **scaling of** $f_0(500)$ **pole modified according to its critical behaviour.**

BACKUP SLIDES

QCD PHASE DIAGRAM. SOME CHALLENGES:

- $U(1)_A$ **vs chiral restoration**
- **Role of Resonances**
- $T_c < T < T_d$ **phases**
- **High density beyond BES and critical point**
- **Other dimensions:** μ_I, μ_S, μ_5, eB, ...





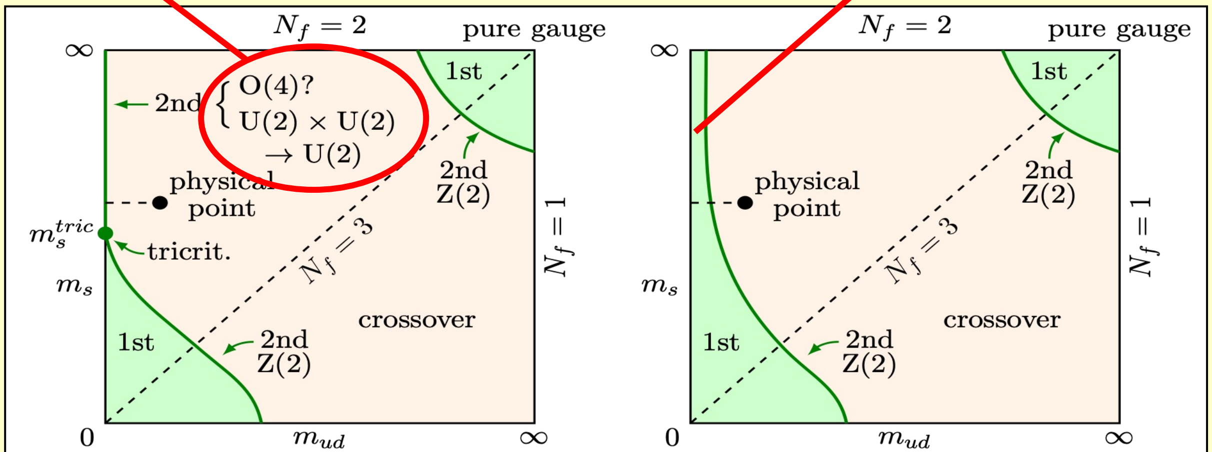
Y.Fujimoto et al PRD112 (2025)

Chiral Symmetry Restoration vs $U(1)_A$

D.J.Gross, R.D.Pisarski, L.G.Yaffe, RMP53, 43 (1981) → $U(1)_A$ asymptotic restoration via instanton suppression

Universality class depends on the **strenght** of $U(1)_A$ **breaking @T_c**

Transition order can even change if $U(1)_A$ is sufficiently restored



B.B. Brandt et al 2019

E.V.Shuryak, CNPP21, 235 (1994); T.D.Cohen PRD54 R1867 (1996); S.H.Lee, T.Hatsuda PRD54, R1871 (1996)

A. Pelissetto, E. Vicari PRD88, 105018 (2013)

Chiral Symmetry Restoration vs $U(1)_A$

Additional consequences of $U(1)_A$ restoration around T_c:

- **Reduction of $M_{\eta'}(T)$ (visible also in HIC) and $\chi_{top}(T)$**

  J. I. Kapusta, D. Kharzeev, L. D. McLerran, PRD53, 5028 (1996)

  T. Csorgo et al, PRL105, 182301 (2010)

  G. Grilli di Cortona, et al, JHEP 1601, 034 (2016)

  M. Ishii et al, PRD95, 114022 (2017)

  AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez, JHEP11, 086 (2019)

  M. P. Lombardo, A. Trunin, IJMPA 35 (2020)

  S.Aoki et al (JLQCD coll), PRD103, 074506 (2021)

- **$\eta-\eta'$ mixing ideal ($\eta \sim \eta_l$, $\eta' \sim \sqrt{2}\eta_s$)**

  M. Ishii et al PRD93, 016002 (2016)

  AGN, J.Ruiz de Elvira, JHEP1603, 186 (2016), PRD97, 074016 (2018), PRD98, 014020 (2018)

- **Affects critical point at $\mu_B \neq 0$**

  M. Mitter, B. J. Schaefer, PRD89, 054027 (2014)

Ward Identities

From QCD generating functional:

- π **SECTOR** $\rightarrow \chi_P^\pi(T) = -\frac{\langle \bar{q}q\rangle_l(T)}{m_l}$

- K **SECTOR** $\rightarrow \chi_P^K(T) = -\frac{\langle \bar{q}q\rangle_l(T) + 2\langle \bar{s}s\rangle(T)}{m_l + m_s}$

- η, A **SECTOR** $\rightarrow \eta/\eta'$ **mixing &** $U_A(1)$ **anomaly:**

$$
\begin{aligned}
\chi_P^{\eta_l}(T) &= -\frac{\langle \bar{q}q\rangle_l(T)}{m_l} - \frac{4}{m_l^2}\chi_{top}(T) \\
\chi_P^{\eta_s}(T) &= -\frac{\langle \bar{s}s\rangle(T)}{m_s} - \frac{1}{m_s^2}\chi_{top}(T) \\
\chi_P^{ls}(T) &= -\frac{m_l}{2m_s}\left[\chi_P^\pi(T) - \chi_P^{\eta_l}(T)\right] = -\frac{2}{m_l m_s}\chi_{top}(T)
\end{aligned}
$$

- κ **SECTOR** $\rightarrow \chi_S^\kappa(T) = \frac{\langle \bar{q}q\rangle_l(T) - 2\langle \bar{s}s\rangle(T)}{m_s - m_l}$

$\chi_P^O \equiv \int_T dx \, \langle O(x)O(0)\rangle$ **susceptibilities**

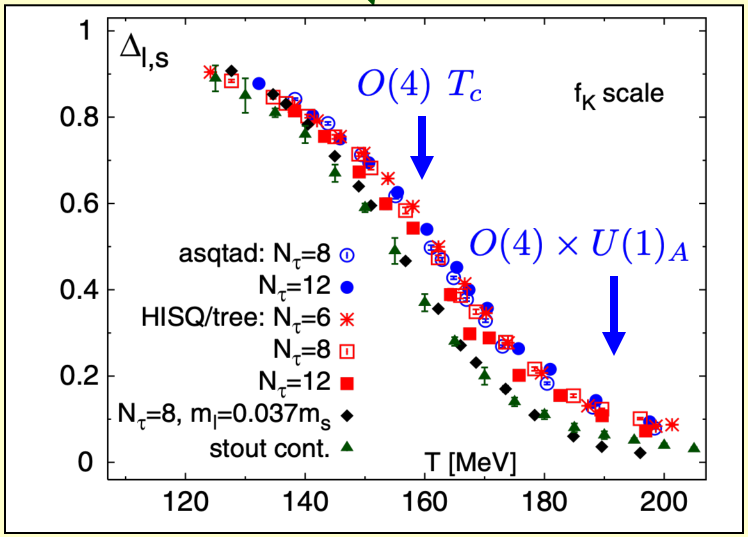
$\chi_{top} \equiv -\frac{1}{36}\int_T dx \langle \mathcal{T} A(x)A(0)\rangle$ **Topological Susceptibility** $\quad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

In addition ...

$$\boxed{\chi_S^{\kappa}(T) - \chi_P^{K}(T) = \frac{2m_s}{m_s^2 - m_l^2}\left[\langle\bar{q}q\rangle_l(T) - 2\frac{m_l}{m_s}\langle\bar{s}s\rangle(T)\right]}$$

subtracted condensate $\Delta_{l,s}$
(removes lattice finite-size divergences)

$\Rightarrow$ $N_f = 2$, consistent with $O(4)\times U(1)_A$ restoration for $m_l, \langle\bar{q}q\rangle_l \to 0^+$



$\Rightarrow$ In physical limit provides well-determined $O(4)\times U(1)_A$ breaking above T_c via $\Delta_{l,s}$ tail modulated by $\langle\bar{s}s\rangle$

Ward Identities and lattice screening masses

- M_{sc} **well measured in lattice** $\lim_{z\to\infty} K(z) \simeq e^{-M_{sc} z}$

- **With reasonable assumptions of smooth T-behaviour of residues and M_{sc}/M_{pole} in $K_{P,S}$ correlators :**

$$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2}$$

$$\Downarrow$$

$$\begin{aligned}
\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} &\sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)}{\langle\bar{q}q\rangle_l(T)}\right]^{1/2}\\
\frac{M_K^{sc}(T)}{M_K^{sc}(0)} &\sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)+2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)+2\langle\bar{s}s\rangle(T)}\right]^{1/2}\\
\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} &\sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle\bar{s}s\rangle(0)}{\langle\bar{s}s\rangle(T)}\right]^{1/2}\\
\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} &\sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle\bar{q}q\rangle_l(0)-2\langle\bar{s}s\rangle(0)}{\langle\bar{q}q\rangle_l(T)-2\langle\bar{s}s\rangle(T)}\right]^{1/2}
\end{aligned}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

Ward Identities and lattice screening masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T=0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \dots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

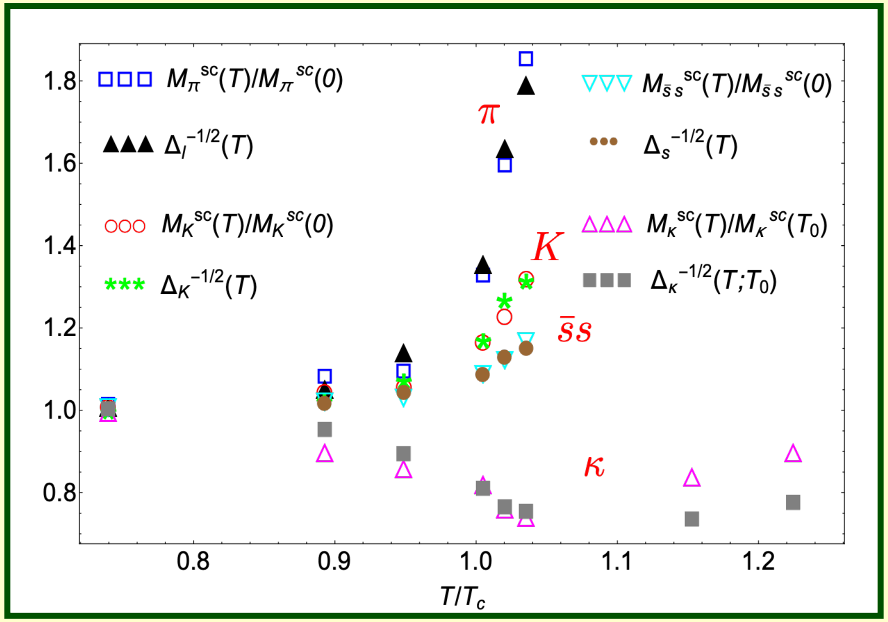
Ward Identities and lattice screening masses

Compare WI predictions for screening masses and condensates from same lattice simulation.

$$\lim_{z\to\infty} K(z) \simeq e^{-M_{sc} z}$$

$$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2}$$

(assuming T-smoothness for residues and M_{sc}/M_{pole})

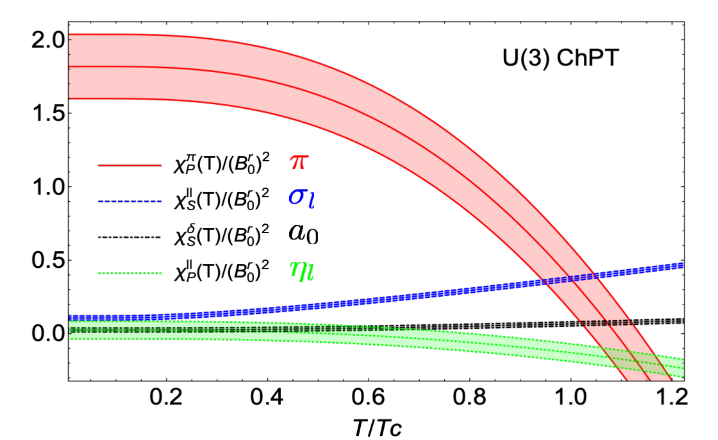


Lattice: M.Cheng et al (HotQCD) EPJC(2011); PRD(2008)

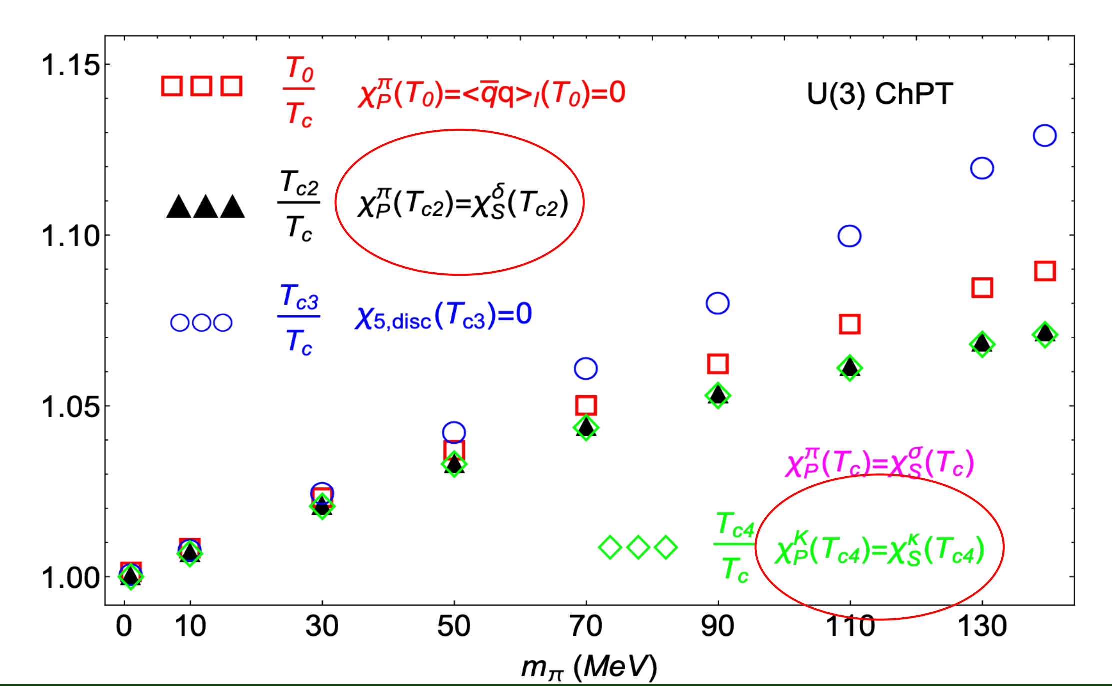
- **Δ_i subtracted condensates with two fit parameters**
- **Scaling law OK within 5% for $T < T_c$**
- **Rapid increase $M_\pi^{sc} \sim \langle \bar{q}q \rangle_l^{-1/2}$ around T_c**
- **Softer** $M_K^{sc} \sim (\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle)^{-1/2}$. **and even softer** $M_{\bar{s}s}^{sc} \sim \langle \bar{s}s \rangle^{-1/2}$
- **Minimum of M_κ^{sc} from $\langle \bar{q}q \rangle_l - 2\langle \bar{s}s \rangle$ counterpart of χ_S^κ peak**

Chiral and $U(1)_A$ partners within $U(3)$ ChPT

AGN, J.Ruiz de Elvira, PRD98, 014020 (2018)



$1/N_c \sim m_q \sim T^2 \sim p^2$ **counting**



$T_{U(1)_A} \sim 1.1 T_{chiral}$
(within ChPT uncertainties)

LECs from
X. K. Guo et al 2015

$\rightarrow O(4) \times U_A(1)$ **in chiral limit**

(**$K - \kappa$ coincidence with $\pi - \delta$ support need for $U(1)_A$ restor.**)

$\Rightarrow$ **Compatible with WIs, lattice and models (NJL Ishii et al PRD 2017)**

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

$$\boxed{\chi_S^\sigma = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^\delta + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^\pi(T) - \chi_S^\sigma(T)\right] + \frac{1}{4}\left[\chi_S^\delta(T) - \chi_P^{\eta_l}(T)\right]}$$

$\Rightarrow$ **Is the vanishing of $\chi_{5,disc}$ in conflict with χ_S^{dis} peaking at the chiral transition?**

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

$$\boxed{\chi_S^\sigma = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^\delta + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^\pi(T) - \chi_S^\sigma(T)\right] + \frac{1}{4}\left[\chi_S^\delta(T) - \chi_P^{\eta_l}(T)\right]}$$

From ChPT in the chiral limit $M_\pi \to 0^+$(IR),

$$\Rightarrow \chi_{5,disc}(T_c) \sim \mathcal{O}\left(\frac{T_c}{M_\pi}\right) \sim \chi_S^{dis}(T_c)$$

"peak" with same coeff.

$$\chi_{5,disc}(T_c) = \chi_S^{dis}(T_c) + \cancel{\frac{1}{4}\left[\chi_P^\pi(T_c) - \chi_S^\sigma(T_c)\right]} + \underbrace{\frac{1}{4}\left[\chi_S^\delta(T_c) - \chi_P^{\eta_l}(T_c)\right]}_{IR\ regular}$$

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

$$\boxed{\chi_S^\sigma = 2\chi_S^{con} + 4\chi_S^{dis} = \chi_S^\delta + 4\chi_S^{dis}} \quad (m_u = m_d)$$

On the other hand,

$$\boxed{\chi_{5,disc}(T) = \chi_S^{dis}(T) + \frac{1}{4}\left[\chi_P^\pi(T) - \chi_S^\sigma(T)\right] + \frac{1}{4}\left[\chi_S^\delta(T) - \chi_P^{\eta_l}(T)\right]}$$

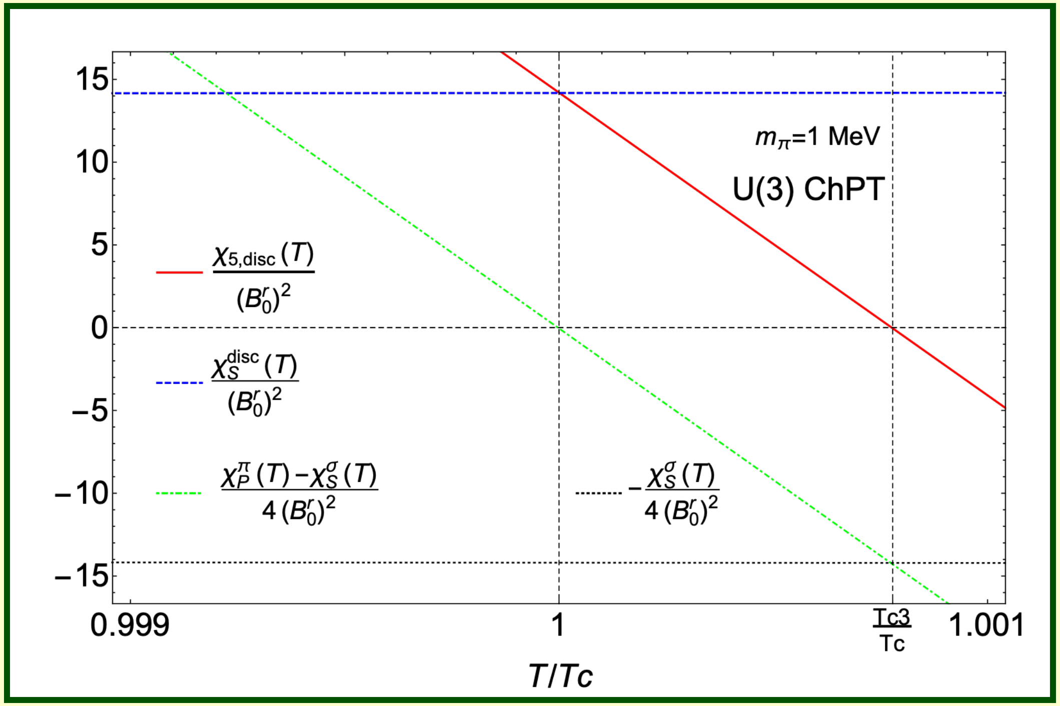
From ChPT in the chiral limit $M_\pi \to 0^+$(IR), $\Rightarrow T_{c3} = T_c + \mathcal{O}\left(M_\pi^2\right)$

$$\Rightarrow \chi_S^{disc}(T_{c3}) \sim \mathcal{O}\left(\frac{T_{c3}}{M_\pi}\right) \sim \frac{1}{4}\chi_S^\sigma(T_{c3})$$ **"peak" with same coff.**

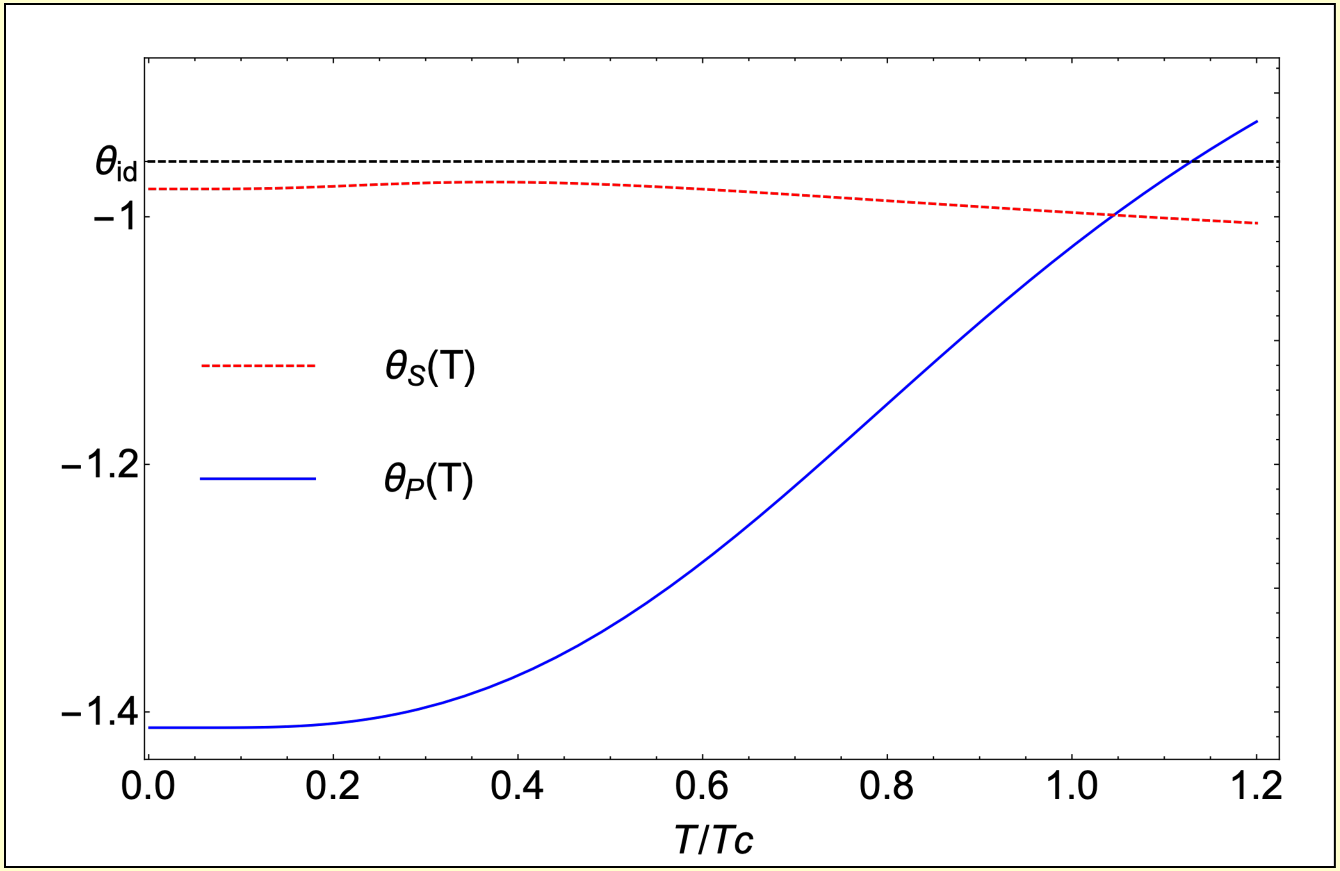
$$\chi_{5,disc}(T_{c3}) = 0 = \chi_S^{dis}(T_{c3}) + \frac{1}{4}\left[\cancel{\chi_P^\pi(T_{c3})} - \chi_S^\sigma(T_{c3})\right] + \frac{1}{4}\left[\underbrace{\chi_S^\delta(T_{c3})}_{IR\ regular} - \cancel{\chi_P^{\eta_l}(T_{c3})}\right]$$

Disconnected susceptibility and $O(4) \times U(1)_A$ restoration

Vanishing of $\chi_{5,dis}$ at $O(4) \times U(1)_A$ rest. near T_c at chiral limit compatible with χ_S^{dis} peak and both peaking at T_c



$\eta/\eta'(P)$ and $f_0(500)/f_0(980)(S)$ mixing in $U(3)$ ChPT



$$\frac{1}{2}\left[\chi^{88}_{P,S}(T)-\chi^{00}_{P,S}(T)\right]\sin\left[2\theta_{P,S}(T)\right]+\chi^{08}_{P,S}\cos\left[2\theta_{P,S}(T)\right]=0,$$

($\eta\eta'$ correlator vanishing, leading order)

Topological susceptibility in $U(3)$ ChPT

$$\epsilon_{vac}(\theta) = \epsilon_{vac}(0) + \frac{1}{2}\chi_{top}\theta^2 + \frac{1}{24}c_4\theta^4 + \ldots$$

$\sim$ **Axion mass** $\qquad$ $\sim$ **Axion coupling**

AGN, J.Ruiz de Elvira, A.Vioque-Rodríguez,
JHEP 11 (2019) 086

$$\chi_{top}^{U(3),LO} = \Sigma\frac{M_0^2\bar{m}}{M_0^2 + 6B_0\bar{m}} \qquad c_4^{U(3),LO} = -\frac{\Sigma}{\bar{m}^{[3]}}\left(\frac{M_0^2\bar{m}}{M_0^2 + 6B_0\bar{m}}\right)^4$$

$$\bar{m} = \left[\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s}\right]^{-1} \qquad \bar{m}^{[3]} = \left[\frac{1}{m_u^3} + \frac{1}{m_d^3} + \frac{1}{m_s^3}\right]^{-1}$$

$(-\Sigma)$ LO quark condensate $\qquad$ M_0 anomalous part of $M_{\eta'}$ $(m_{u,d,s} = 0)$

- **vanish** $m_q \to 0 \Rightarrow$ **well described within ChPT**
- $SU(3)$ **for** $M_0 \to \infty$, $SU(2)$ **for** $M_0, m_s \to \infty$
- **Quenched** $m_q \to \infty$: $\chi_{top}^{LO} = F^2 M_0^2/6$
  (Witten-Veneziano 1979) $\to$ **meson loops crucial**

Leutwyler,Smilga 1992: $SU(3)$ **LO**
Mao et al 2009; Bernard et al: 2012: $SU(3)$ **NLO**
Grilli et al 2016: $T \neq 0$ $SU(2)$ **NLO**

Topological susceptibility in $U(3)$ ChPT

$\rightarrow$ $T=0$ **results** $(m_u = m_d)$**:**

$\chi_{top}^{1/4}$ [MeV]	U(3)	SU(2)	SU(3)
LO	74(3)	75(3)	75(3)
NLO	74(3)	78(3)	83(2)
NNLO	81(2)		

$\left[\chi_{top}^{latt}\right]^{1/4} = 73(9)$ **(Bonati et al 2016)**

$b_2 = \frac{c_4}{12\chi_{top}}$	U(3)	SU(2)	SU(3)
LO	-0.01737(4)	-0.02083	-0.01960
NLO	-0.018(2)	-0.029(2)	-0.025(1)
NNLO	-0.023(2)		

$b_2^{latt} = -0.0216(15)$ **(Bonati et al 2016, gluodynamics)**

$\Rightarrow$ $SU(2)$ **dominates,** $U(3)$ η' **loops and** $\eta - \eta'$ **mixing of the same order than** $SU(3)$ K, η **loops**

$\Rightarrow$ **full** $U(3)$ **in agreement with lattice within uncertainties (LEC and lattice, larger lattice uncertainty for** b_2**)**

$\Rightarrow$ **In addition, within** $U(3)$ **ChPT, explicit expressions for the leading and subleading** $1/N_c$ **contributions can be obtained:**

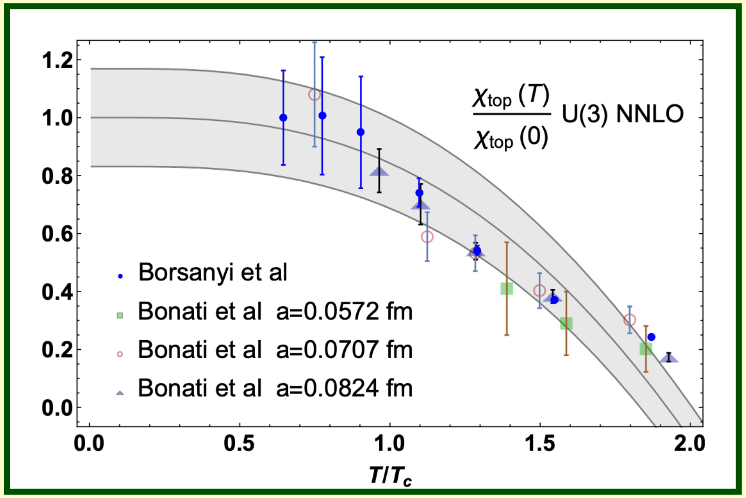
$$\chi_{top} = \underbrace{\mathcal{O}(1)}_{\text{Witten-Veneziano term}} + \mathcal{O}(N_c^{-1}) + \dots$$

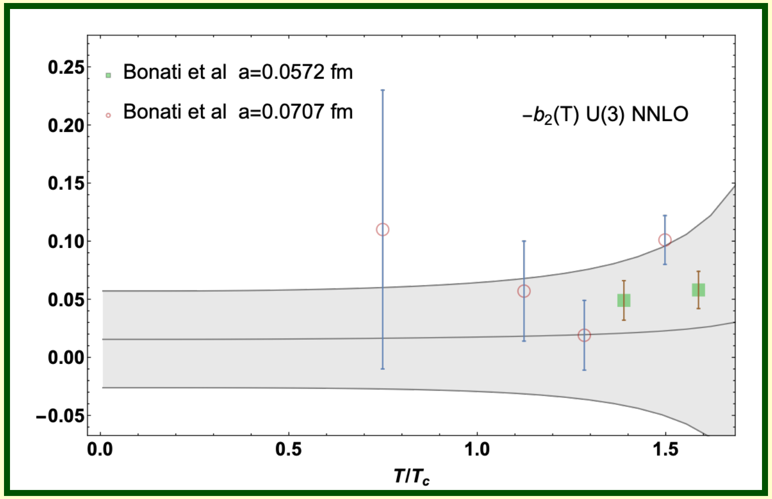
$$c_4 = \underbrace{\mathcal{O}(N_c^{-2})}_{\text{constant } \theta^4 \text{ term in } \mathcal{L}} + \mathcal{O}(N_c^{-3}) + \dots$$

OK with lattice and theo:

$\Rightarrow$ **Vicari, Panagopoulos 2009-2011**
Bonati et al 2016
Vonk, Guo, Meissner 2019

Topological susceptibility in $U(3)$ ChPT

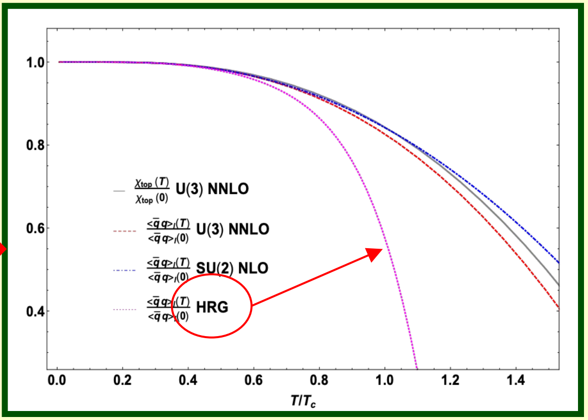




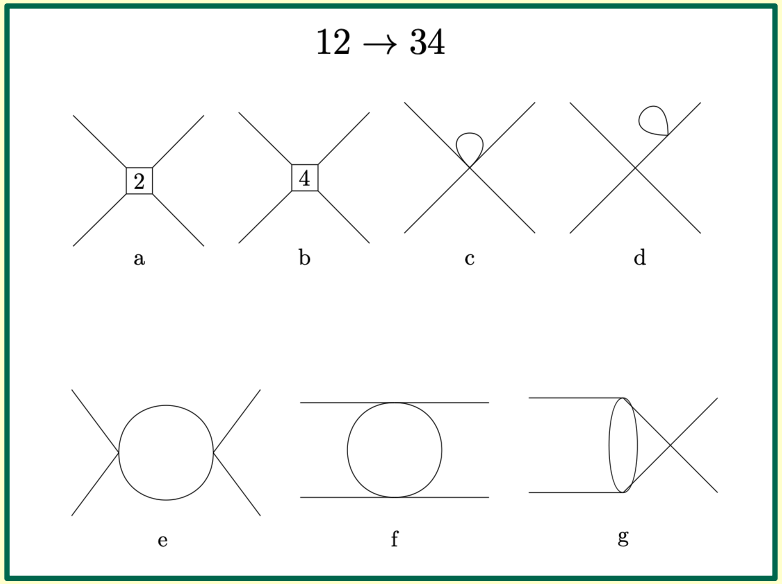
- T-dependence captured by ChPT within uncertainties (larger for c_4) far beyond the low-T applicability regime

- χ_{top} scales with T dominated by $\langle \bar{q}q \rangle_l$
- However, 2nd term in WI $\chi_{top} = -\frac{1}{4}\left[m_{ud}\langle \bar{q}q \rangle_l + m_{ud}^2 \chi^{\eta_l}\right]$ relevant near T_c

  $\rightarrow$ finite $O(4)$-$U(1)_A$ gap in physical case



Scattering and Resonances within finite-T Unitarized ChPT



- **LSZ+finite-T Green function**
- **Thermal bath breaks Lorentz covariance**

  $\to$ s, t, S^0, T^0, U^0 **on-shell independent**

  $\to$ **3 independent J_k-loop integrals**

  $\mathbf{S} = p_1 + p_2$**,** $\mathbf{T} = p_1 - p_3$**,** $\mathbf{U} = p_1 - p_4$

$$\mathcal{T}(\mathbf{S},\mathbf{T},\mathbf{U};T) = \underset{\textbf{(a)}}{\mathcal{T}_2(s,t,u)} + \underset{\textbf{(b)}}{\mathcal{T}_4^{tree}(s,t,u)} + \underset{\textbf{(c-g)}}{\mathcal{T}_4^{F}(\mathbf{S},\mathbf{T},\mathbf{U};T)} + \underset{\textbf{(e-g)}}{\mathcal{T}_4^{J}(\mathbf{S},\mathbf{T},\mathbf{U};T)}$$

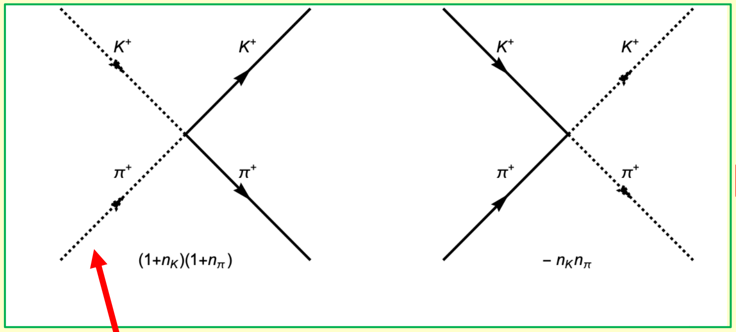
$$F_{\beta a}(T) = T \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{1}{q^2 - M_a^2} \quad \textbf{tadpole-like}$$

$$J_k^{ab}(Q_0, |\vec{Q}|; T) = T \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{q_0^k}{[q^2 - M_a^2]\,[(q-Q)^2 - M_b^2]} \qquad (k = 0,1,2)$$

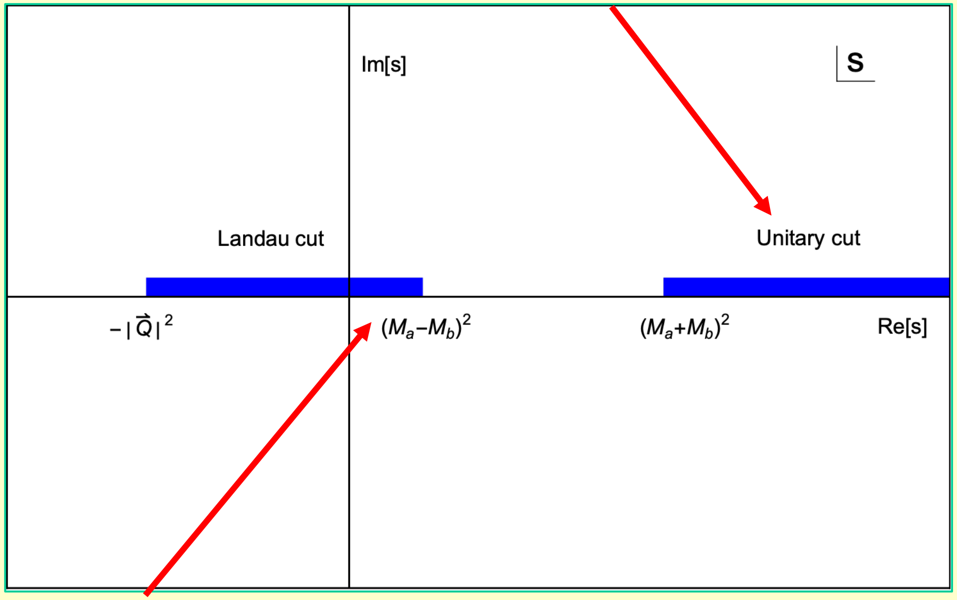
$q_0 = \omega_n \equiv 2\pi i n T$**,** $Q_0 = \omega_m$ $n, m \in \mathbb{Z}$**,** $\omega_m \to -i(Q_0 + i\epsilon)$ **AC to** $Q_0 \in \mathbb{R}$

Scattering and Resonances within finite-T Unitarized ChPT

- **Unitarity modified from physical thermal bath processes:**

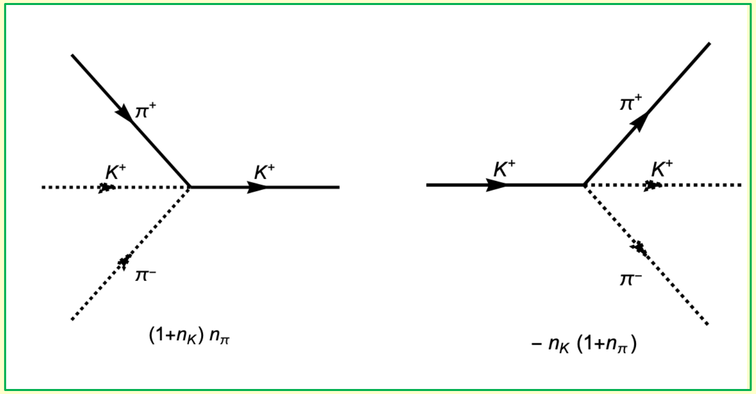


Correction to standard unitary cut



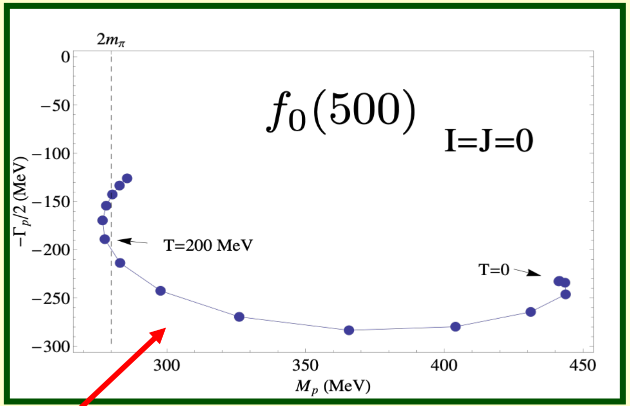
thermal bath particles

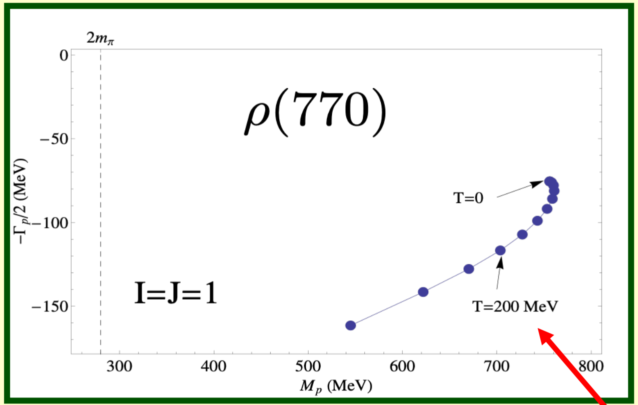
$n_i \equiv n_B(E_i) = \frac{1}{e^{E_i/T}-1}$ **BE distrib.**



Landau cut
(purely thermal, vanishes for $T=0$ or $M_a = M_b$)

$\pi\pi$ scattering





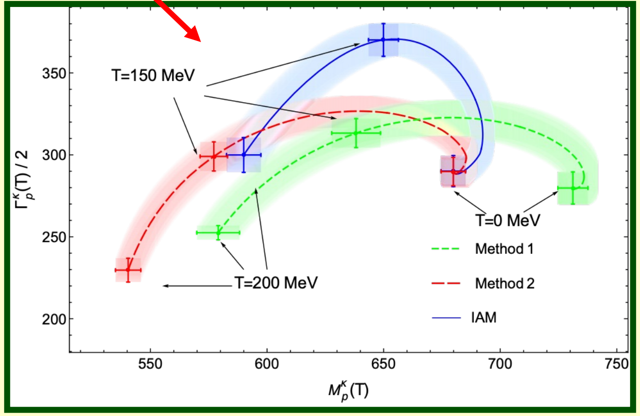
M_σ, Γ_σ **consistent with CSR**

Strong $\Gamma_\rho \uparrow$**, mild** $M_\rho \downarrow$ **compatible with other analyses & dilepton spectrum**

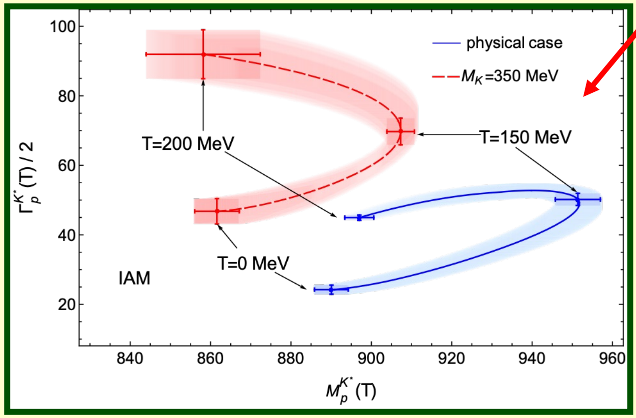
$K\pi$ scattering

Similar pattern as $f_0(500)$

Softer T**-dependence than** ρ **compatible with HIC.**
Approaches ρ **behaviour as** $M_K \downarrow$

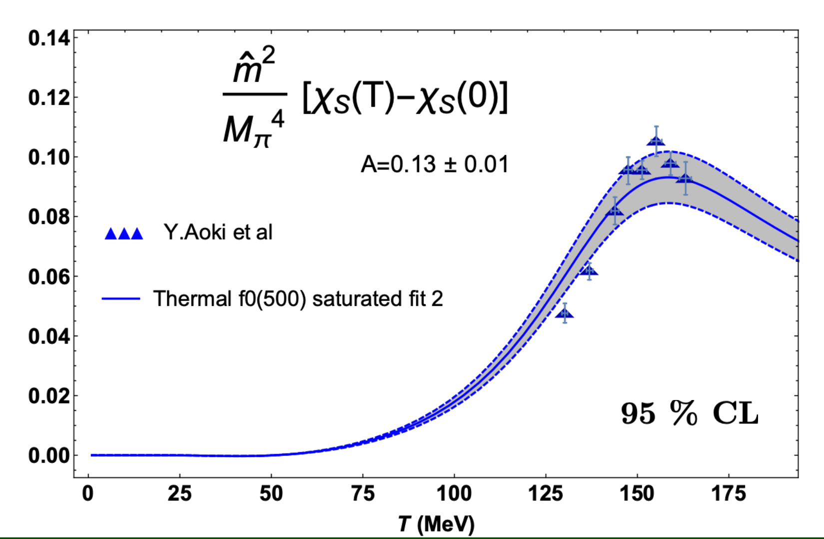


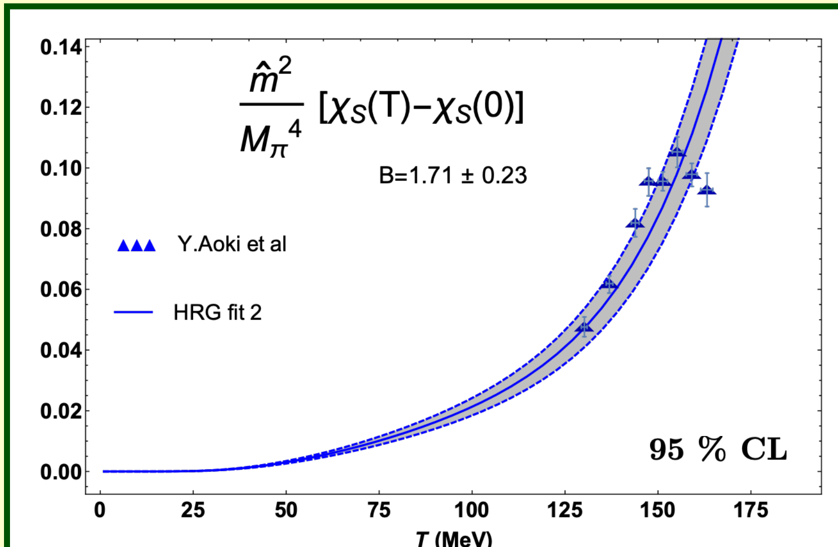
$K_0^*(700)/\kappa\ (I = 1/2, J = 0)$



$K^*(892)\ (I = 1/2, J = 1)$

Saturating scalar susceptibilities with light thermal resonances

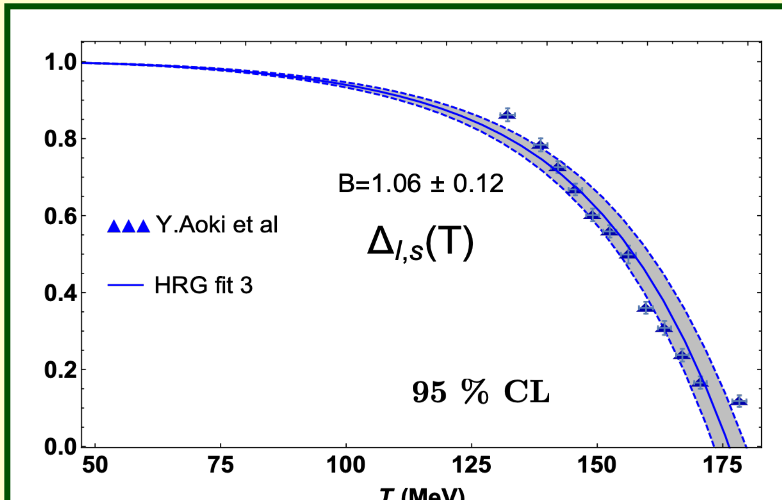




$$\chi_S(T) = A\frac{m_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)} \quad (A_{ChPT} \simeq 0.14)$$

HRG Jankowski et al PRD87, 105018 (2013)
Fitted with free-energy normalization B

Fit	A	B	χ^2/dof	T_{max}(MeV)
Thermal f_0 fit 1	0.13± 0.02	—	6.25	155
Thermal f_0 fit 2	0.13± 0.01	—	4.93	165
HRG fit 1	—	1.90± 0.02	1.33	155
HRG fit 2	—	1.71± 0.23	10.30	165
HRG fit 3	—	1.06± 0.12	3.77	155



- **Thermal $f_0(500)$ fits better around T_c (peak behaviour)**
- **HRG increasing χ_S, in conflict with condensate fit**

Saturated Scalar susceptibility in the LSM

S.Ferreres, AGN, A.Vioque, 2018

$$\mathcal{L}_{LSM} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \frac{\lambda}{4}\left[\Phi^T\Phi - v_0^2\right]^2 + h\sigma,$$

$$\chi_S(T) = \left(\frac{d^2h}{dm_q^2}\right) v(T) + \left(\frac{dh}{dm_q}\right)^2 \Delta_\sigma(k=0;T),$$

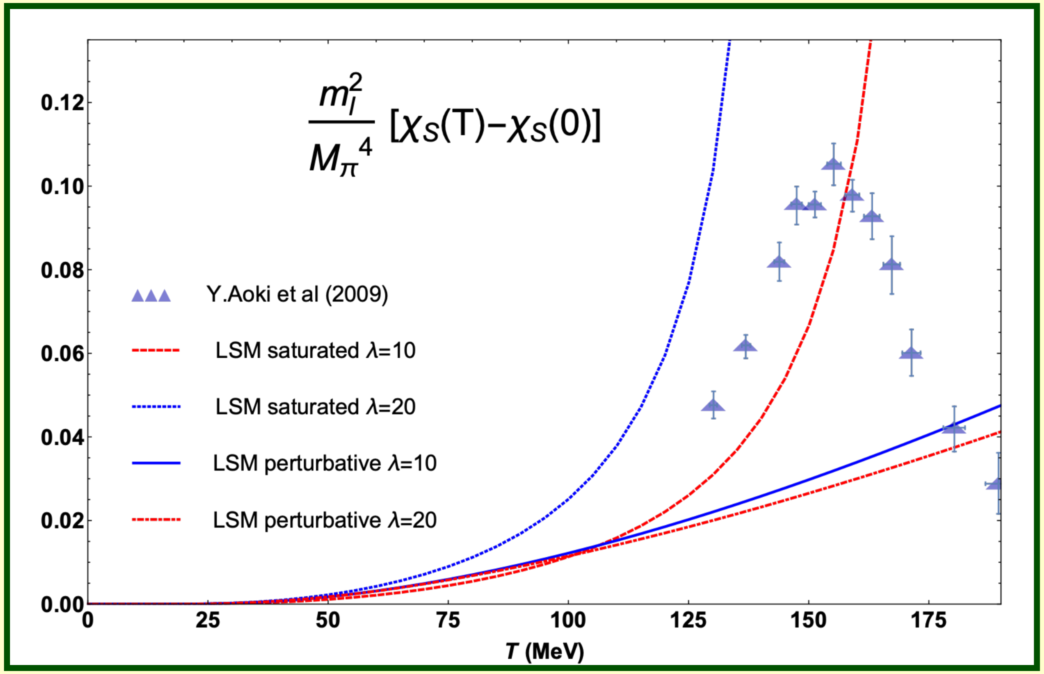
suppressed near T_c

$$M_{0\pi}^2 = \frac{h}{v} = \lambda(v^2 - v_0^2) \quad , \quad M_{0\sigma}^2 = M_{0\pi}^2 + 2\lambda v^2, \quad v = \langle\sigma\rangle$$

calculating the self-energy Σ to one loop:

M_π (MeV)	M_p (MeV)	Γ_p (MeV)	λ
0	450.0	172.5	8.4
0	775.1	550.0	20.0
140	450.0	159.2	9.6
140	750.1	550.0	21.2

$$\frac{\chi_S(T)}{\chi_S(0)} \simeq \frac{M_{0\sigma}^2 + \Sigma(k=0;T=0)}{M_{0\sigma}^2 + \Sigma(k=0;T)}$$



Chiral Imbalance in ChPT

D.Espriu, AGN, A. Vioque-Rodríguez, JHEP06 (2020)
AGN, P.Roa-Bravo, A. Vioque-Rodríguez, PRD109 (2024)

- μ_5 **chem. pot. for approx. (local) conservation of chiral charge** Q_5
  (characteristic time $t_{L-R} \sim m_q^{-1} \gg t_{fireball}$ **for light quarks)**

$\Rightarrow \mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \mu_5 \bar{\psi}\gamma^0\gamma_5\psi$ **in that region**

$$Q_5 = \int_{vol} d^3\vec{x}\ \bar{\psi}\gamma^0\gamma_5\psi$$

- **GOAL: Construct the most general meson eff. lagr. for** $\mu_5 \neq 0$

- **"local" chiral invariant ChPT with axial** $U(1)$ **external source**

D_μ **& explicit source-dep new terms**
$N_f = 2$

$$\mathcal{L}_2 \to \mathcal{L}_2 + 2\mu_5^2 F^2 (1+\kappa_0)$$

κ_i **new LEC to be fixed**

$$\mathcal{L}_4 \to \mathcal{L}_4 + \kappa_1 \mu_5^2 \mathbf{tr}\left(\partial^\mu U^\dagger \partial_\mu U\right) + \kappa_2 \mu_5^2 \mathbf{tr}\left(\partial_0 U^\dagger \partial^0 U\right) + \kappa_3 M_\pi^2 \mu_5^2 \mathbf{tr}\left(U + U^\dagger\right) + \kappa_4 \mu_5^4$$

- **NLO dispersion relation** $\Longrightarrow$

No lattice data

$$v_\pi(\mu_5) = \left.\frac{|\vec{p}|}{p_0}\right|_{M=0} = 1 + 2\kappa_2\frac{\mu_5^2}{F_\pi^2} \qquad (\kappa_2 < 0)$$

$$\left[M_\pi^2\right]^{pole}(\mu_5) = M_\pi^2\left[1 - 4(\kappa_1+\kappa_2-\kappa_3)\frac{\mu_5^2}{F^2}\right]$$

$$\left[M_\pi^2\right]^{scr}(\mu_5) = M_\pi^2\left[1 - 4(\kappa_1-\kappa_3)\frac{\mu_5^2}{F^2}\right]$$

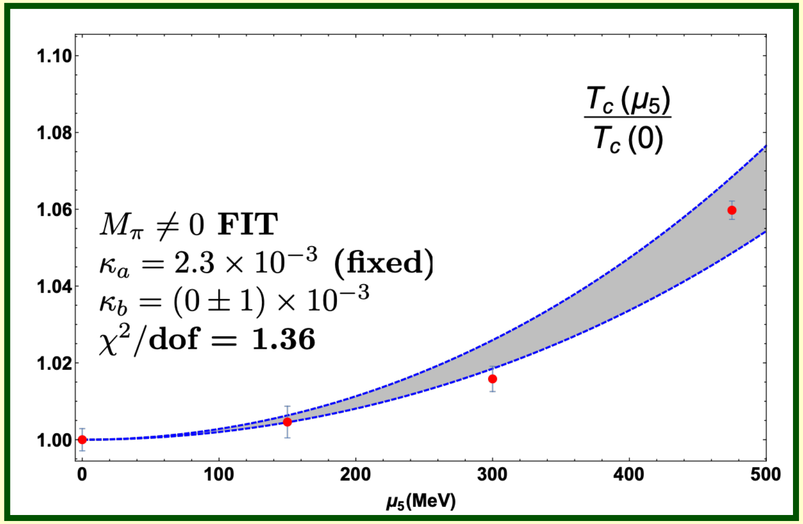
- **NNLO quark condensate** $\Longrightarrow$

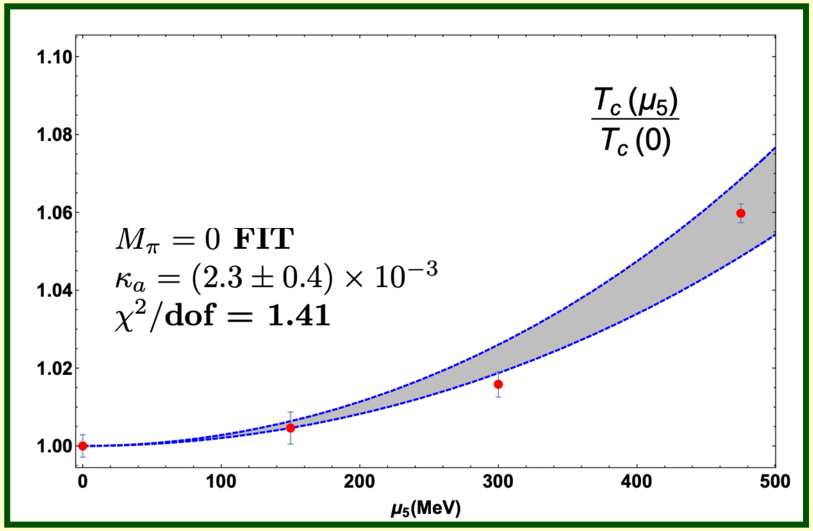
$$\left.\frac{\langle\bar{q}q\rangle_l(T,\mu_5)}{\langle\bar{q}q\rangle_l(0,\mu_5)}\right|_{M=0} = 1 - \frac{T^2}{8F^2}\left[1 - 2\kappa_a\frac{\mu_5^2}{F^2}\right] - \frac{T^4}{384F^4}$$

$$\kappa_a = 2\kappa_1 - \kappa_2, \qquad \kappa_b = \kappa_1+\kappa_2-\kappa_3$$

Well acommodated by chiral limit ChPT:

Lattice data Braguta et al 2015 ($N_c = 2$)

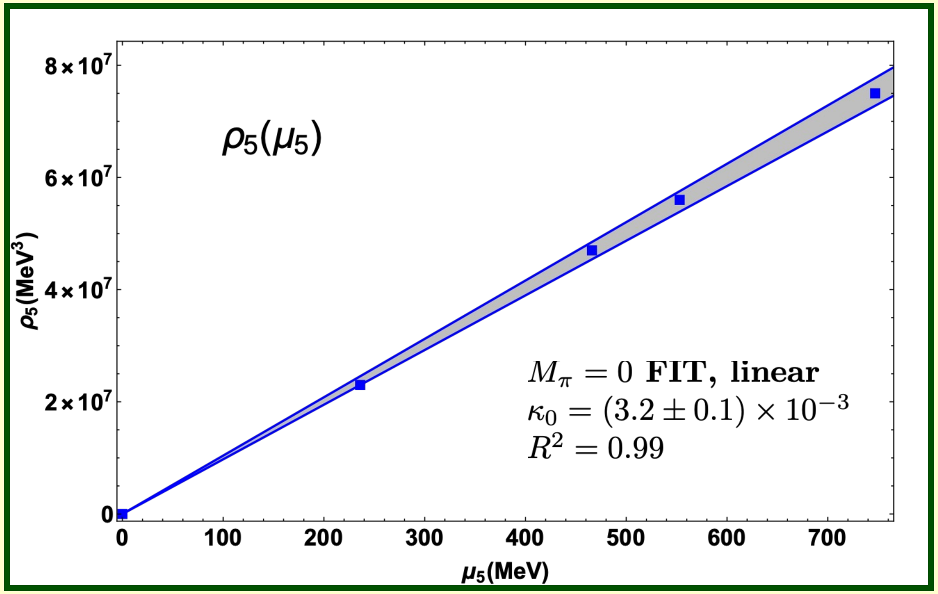




- **NLO chiral charge density** $\Longrightarrow$

  Linear $\mathcal{O}(\mu_5)$ dominant for moderate μ_5

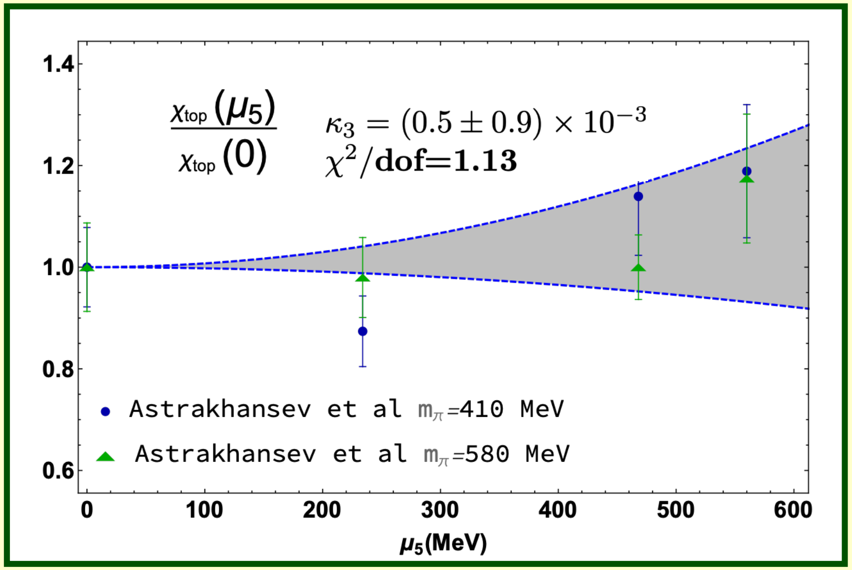
  Quite insensitive to M_π



Lattice data Astrakhantsev et al 2019

- **NLO topological susceptibility** $\Longrightarrow$

$$\boxed{\frac{\chi_{top}(\mu_5)}{\chi_{top}(0)} = 1 + 4\frac{\kappa_3 \mu_5^2}{F^2}}$$

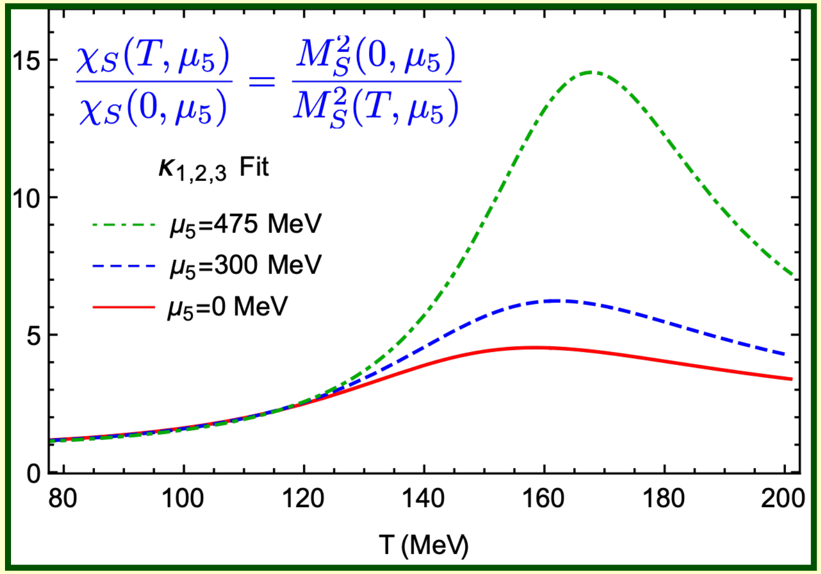


Lattice data Astrakhantsev et al 2019

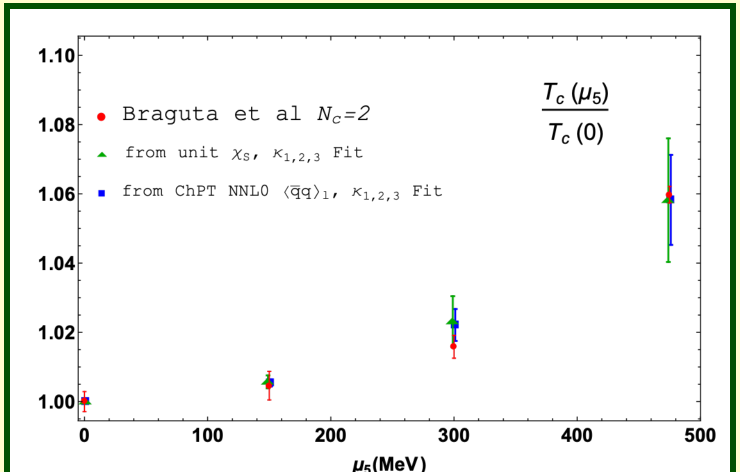
• Pion scattering and light thermal resonances: μ_5 and T dependence

μ_5 NLO corrections:

- **Tree level from $\mathcal{L}_4$**
- **Dispersion relation mod.**
- **LSZ residue**



$T_c(\mu_5) \uparrow$ compatible with lattice and different T_c determinations



Combined fit of χ_{top}, $T_c^{\chi_S}$, $T_c^{\langle \bar{q}q \rangle_l}$

$\kappa_1 \times 10^4$	$\kappa_2 \times 10^4$	$\kappa_3 \times 10^4$	χ^2/dof
$9.4^{+1.1}_{-1.3}$	$-4.5^{+1.5}_{-1.4}$	$3.6^{+9.1}_{-8.7}$	1.37