A non-perturbative definition of Magnetostatic QCD

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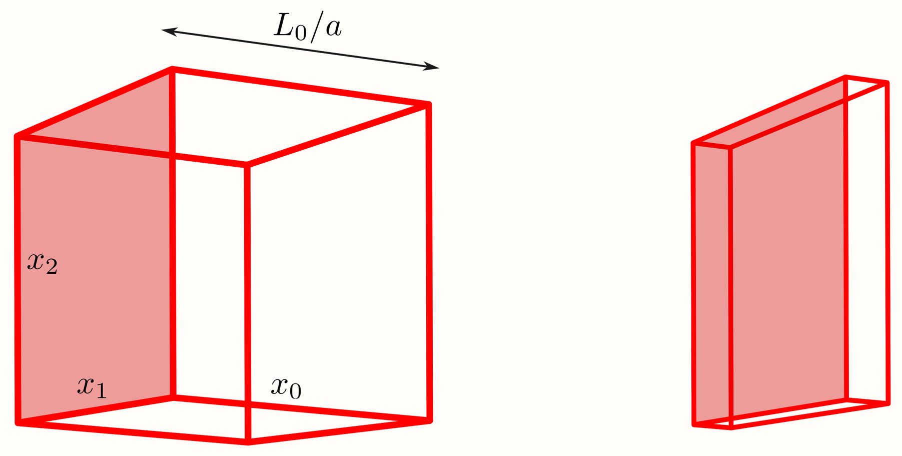
Why are we doing this?

Thermal QCD can be described by 3D effective bosonic theories due to a process of **dimensional reduction** [1]. These effective field theories (EFT) describe the long range dynamics of QCD at temperature T, and the information about the integrated higher energy scales is contained in the **temperature dependent 3D couplings**. The knowledge of such couplings comes from the **matching** of the theories, which is known only perturbatively. We have devised a strategy to perform the **first non-perturbative computation of the matching**, using gradient flow quantities in a finite volume setup, allowing us to keep all lattice systematics under control, and perform the matching over a large range of temperatures with high precision.

Dimensional Reduction



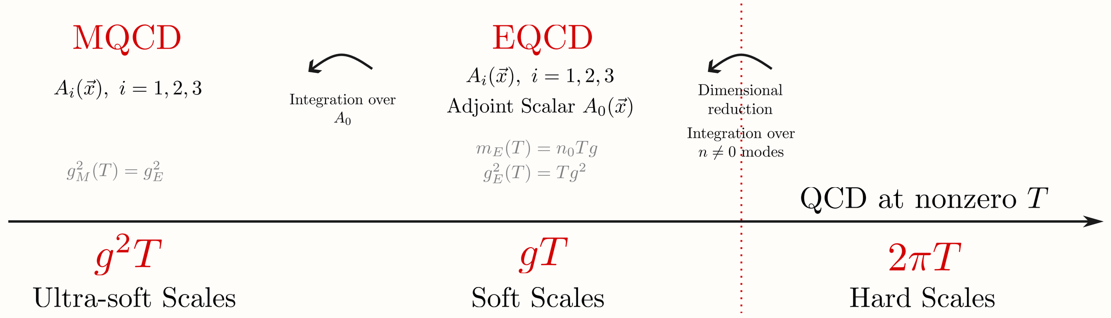
$T = 0$ $\qquad$ $T \neq 0$

High T leads to the decoupling of the *hard* Matsubara modes, $n \neq 0$:

$$A_\mu(x) = T\sum_n \int_{\vec{p}} e^{i(\omega_n x_0 + \vec{p}\cdot\vec{x})}\tilde{A}_\mu(\omega_n,\vec{p}), \quad \omega_n = 2\pi T n$$

$$S_{\mathrm{YM}}^{\mathrm{free}} = \frac{1}{2g_0^2} T\sum_n \int_{\vec{p}} \tilde{A}_\mu^a(\omega_n,\vec{p})\left((2\pi n T)^2 + \vec{p}^2\right)\tilde{A}_\mu^a(\omega_n,\vec{p}).$$

- Infrared modes are captured by 3D bosonic theories



From the effective field theory perspective we build the most general 3D Lagrangian with the $n = 0$ d.o.f.

$$\mathscr{L}_{\mathrm{3D}} = \sum_i c_i(g(T)) f_i\left(\vec{A}, A_0\right)\Big|_{n=0},$$
$$\langle \mathscr{O} \rangle_{\mathrm{3D}} = \langle \mathscr{O} \rangle_{\mathrm{4D}} + \mathscr{O}(1/T).$$

Magnetostatic QCD (MQCD) consists of a confining 3D pure gauge theory, that captures the *magnetic* d.o.f. of thermal QCD

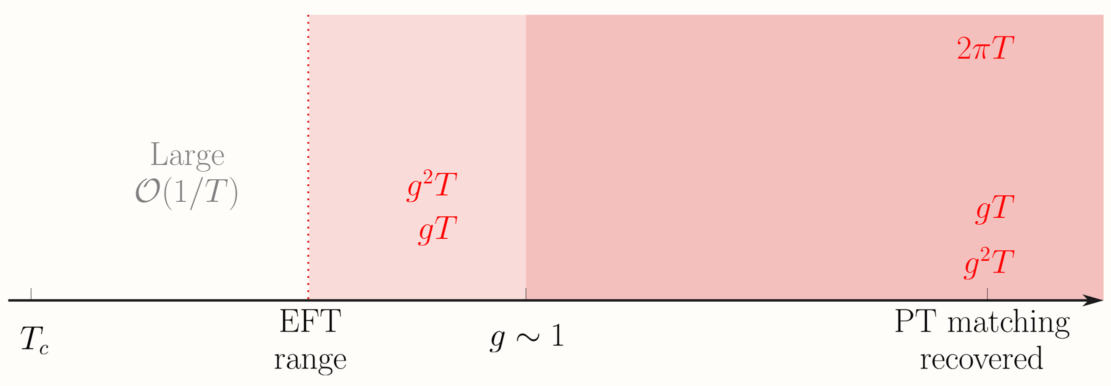
$$\mathscr{L}_{\mathrm{MQCD}} = -\frac{1}{2g_M^2}\mathrm{Tr}\{F_{ij}F_{ij}\} + \mathscr{O}(1/T).$$

- $[g_M^2] = 1$: $\Lambda_{\mathrm{MQCD}} \to g_M^2$, super-renormalizable
- Lattice: $g_M^2 \longrightarrow ag_3^2 = 6/\beta_3$
  - does not run but has known $\mathscr{O}(a)$ cutoff effects [2]

Status of 3D EFT

- $g_M^2(T)$ known to 2-loops [3] (similarly for g_E^2, λ_E)
- Infrared perturbative ambiguities & divergent term uncancelled in the 3-loop computation [4, 5]
- NP contributions to mesonic screening masses [6]
- Mixed perturbative and non-perturbative (NP) effects
  - Perturbative range mixes with the **EFT range of applicability**:

$$\alpha_{\overline{MS}}(2\pi T) \ll 1 \longrightarrow T/T_c \gtrsim 10\ldots20$$



The **non-perturbative matching** answers to:
- Where do we find separation of scales?
  - $g^2T \ll gT \ll 2\pi T$
  - range of validity of the EFT
- Validity of the perturbative matching

Lattice matching strategy

Avoid conventional matching (σ_s, screening masses):
- Loss of predictability & worsened systematics

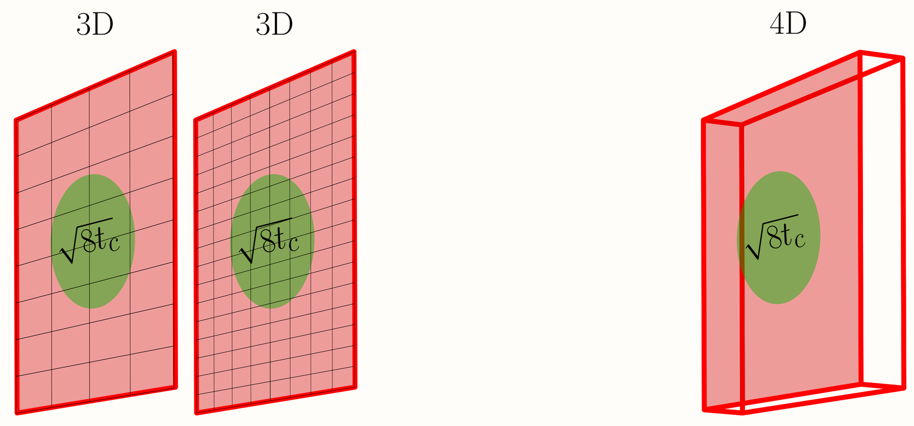
Instead, we use **Gradient flow** quantities [7]

$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}, \qquad B_\mu(x,0) = A_\mu(x)$$

- Deterministic diffusion equation along t
- Reduced systematics
- Simple renormalization [8]
- Coarse-graining transformation – integrates hard modes

Idea: Match the 3D & 4D flowed magnetic action density at the scale $\mu_t = 1/\sqrt{8t}$

$$\langle E_m(t)\rangle = -\frac{1}{4}\langle G_{ij}^a(t)G_{ij}^a(t)\rangle, \quad i = 1,2,3$$



Avoid finite volume (FV) effects $\longrightarrow$ window problem:

$$1/L \qquad \mu_{\mathrm{m}} = 1/\sqrt{8t} \qquad T \qquad 1/a$$

$$\overbrace{1 \ll}^{\text{cutoff effects}} \underbrace{L_0/a \ll \sqrt{8t}/a}_{\text{matching condition}} \overbrace{\ll L/a}^{\text{FV effects}}$$

Finite volume matching scheme

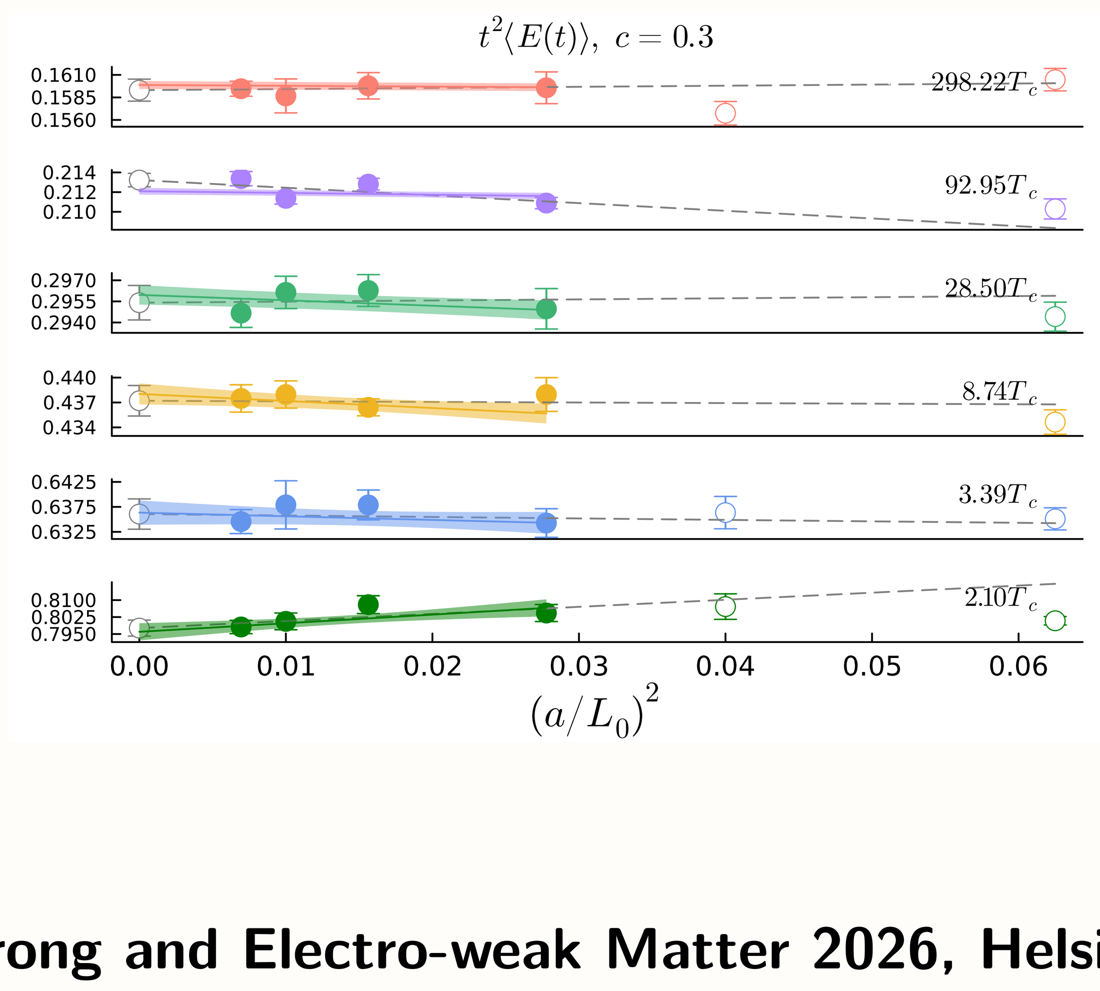
$$\sqrt{8t_c} = cL, \quad c = 0.3, \quad R = L/L_0 = 10$$

$$g_M^2, a/L \text{ such that: } t_c^2\left\langle E(t_c)\right\rangle\Big|_{\mathrm{3D}}^{g_M^2, a/L} = t_c^2\left\langle E_m(t_c)\right\rangle\Big|_{\mathrm{4D}}^{T\neq 0}$$

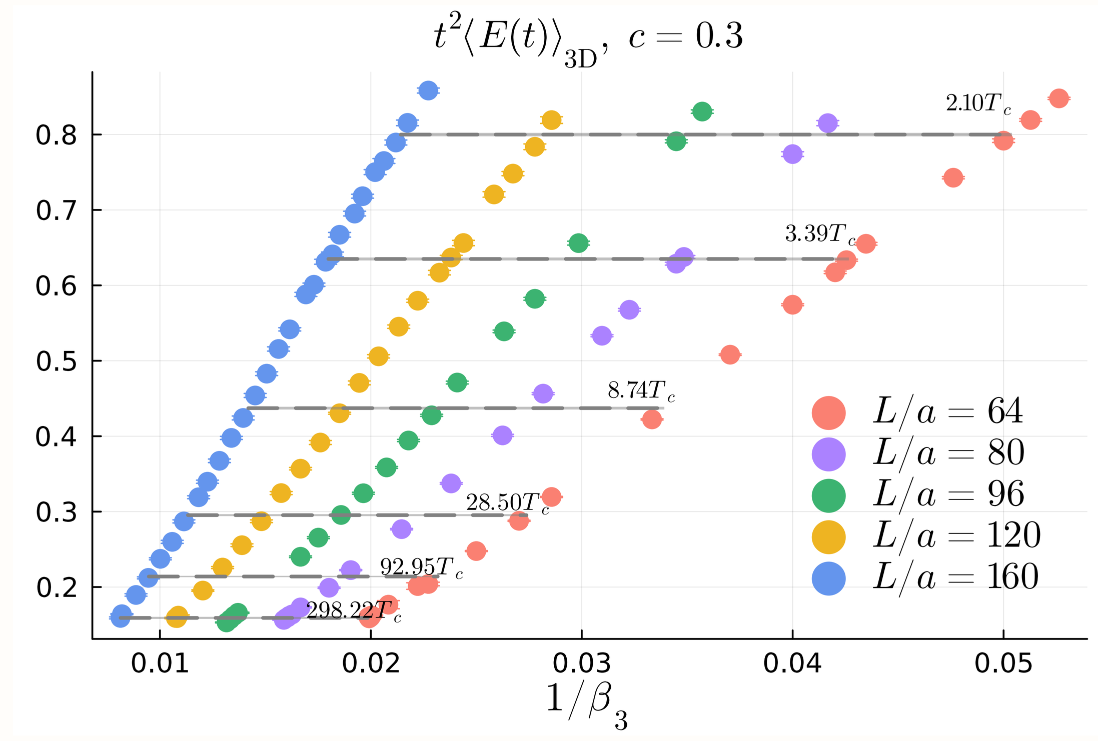
- Requires a study of systematics $cR \gg 1$
- Lines of constant physics [9]

$$\{\beta, L'/a\} \text{ tuned from } g_{\mathrm{GF,m}}^2(\beta, L') = \text{const.}$$

- We take $L_0 = L'/2 \longrightarrow L_0/a = 4,\ldots,12$
- $T/T_c \sim 2,\ldots,300$
- Controlled 4D continuum extrapolations

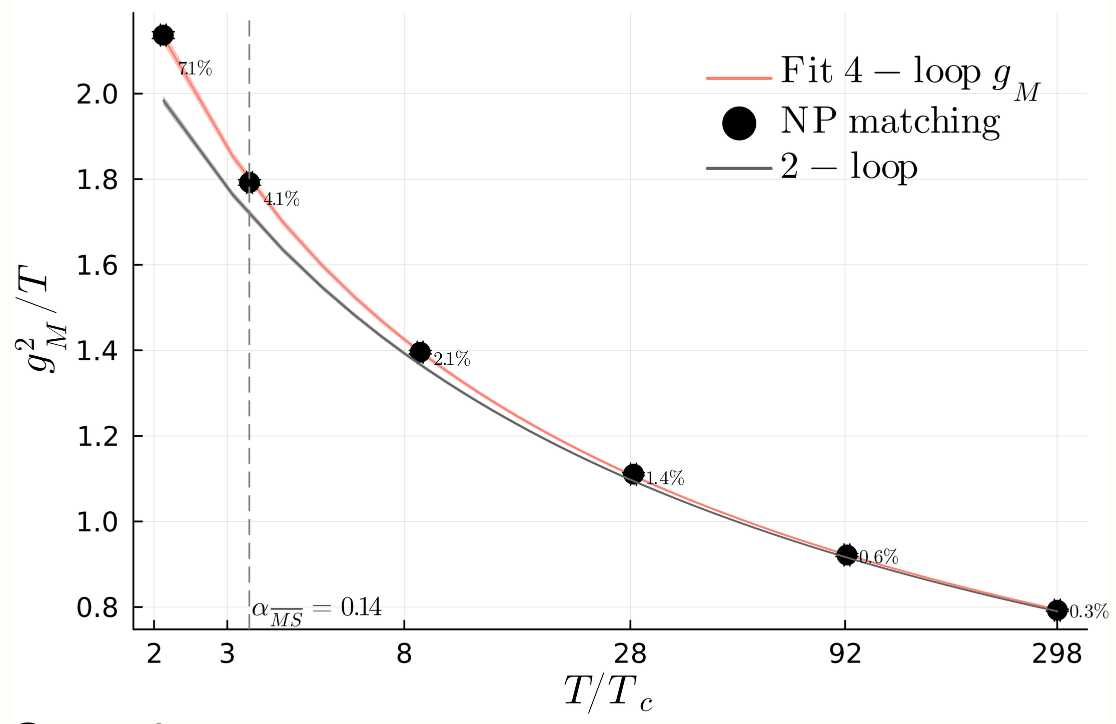


- Tune 3D ag_3^2 to reproduce 4D at each T



Results

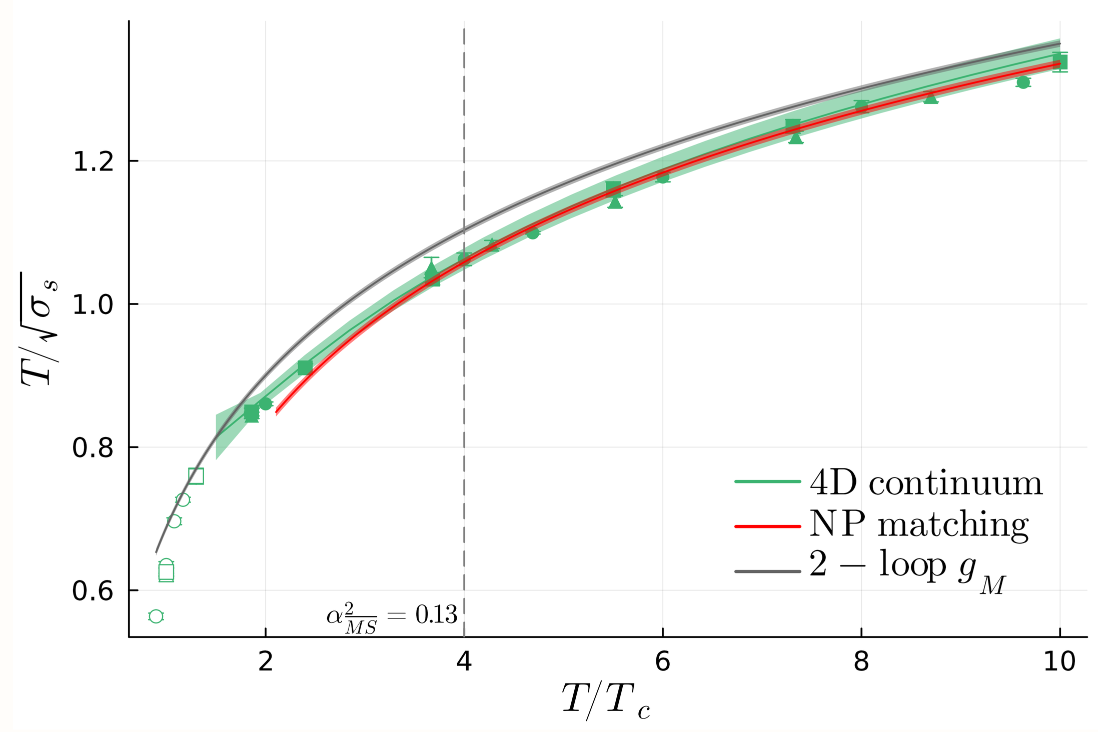
$$\frac{g_M^2(T)}{T} = \frac{1}{R}\lim_{ag_3^2\to 0}(L/a)\left(ag_3^2\right)$$



Spatial string tension σ_s

$$\frac{\sqrt{\sigma_s}}{T}\Bigg|_{\mathrm{4D}} = \underbrace{\frac{\sqrt{\sigma_s}}{g_M^2}}_{\text{3D obs.}} \times \overbrace{\frac{g_M^2}{T}}^{\text{matching}} + \mathscr{O}(1/T)$$

- Non-perturbative 3D factor $\sqrt{\sigma_s}/g_M^2$ from [10]
- Continuum extrapolated 4D data from [11]



Conclusion & Outlook

- **First Non-Perturbative matching of MQCD**
  - Gradient Flow + Finite Volume – precise definition of $g_M(T)$
  - Leading-order MQCD seems to work non-perturbatively $\gtrsim 4T_c$
  - $T/T_c > 4$ – statistically significant NP effects (below $\sim 4\%$)

Future:
- Study of the systematics (cR, observables)
- Consider other 4D test observables
- **Match EQCD & full QCD**

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Strong and Electro-weak Matter 2026, Helsinki