Reducing the pressure of neutron-star cores via IBP

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based on recent work with
Pablo Navarrete, Miguel Vaez, Emilio Viacava

and work in progress with
A. Kärkkäinen, M. Nurmela, R. Paatelainen, K. Seppänen

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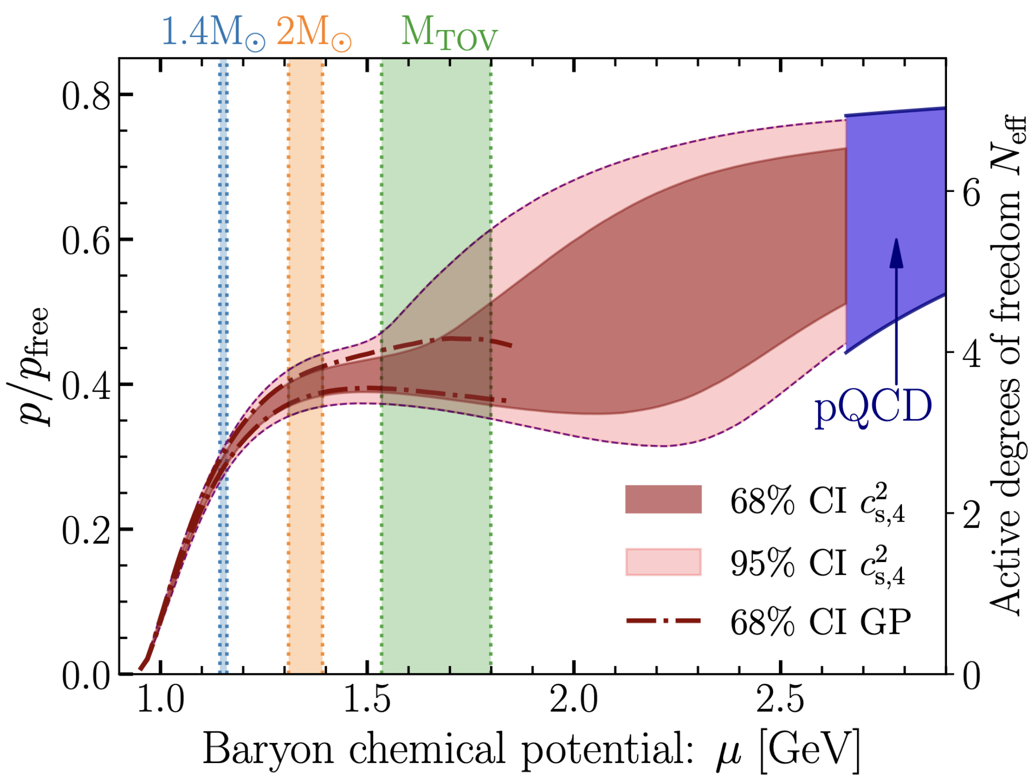
Motivation

- dense (but unpaired) quark matter argued to be relevant in neutron-star physics
  - ▷ weak-coupling expansions at high density provide useful EoS constraints
  - ▷ current bottleneck: cold+dense vacuum Feynman integrals
  - ▷ import techniques from 'ordinary' QFT and hot QCD?



[figure stolen from E.Annala et al. NatureCommun2023]

[see talks by K.Seppänen; O.Komoltsev]

Motivation

- Observable of interest: pressure of cold quark matter

$$p_{\mathrm{QCD}}(\mu) = \lim_{\beta,V\to\infty} \frac{1}{\beta V} \ln \int \mathcal{D}[A^a_\mu, \psi, \bar\psi] \exp\Big(-\frac{1}{\hbar}\int_0^{\hbar\beta} d\tau \int d^{3-2\epsilon}x \left[\mathcal{L}^E_{\mathrm{QCD}} - \mu\bar\psi\gamma_0\psi\right]\Big)$$

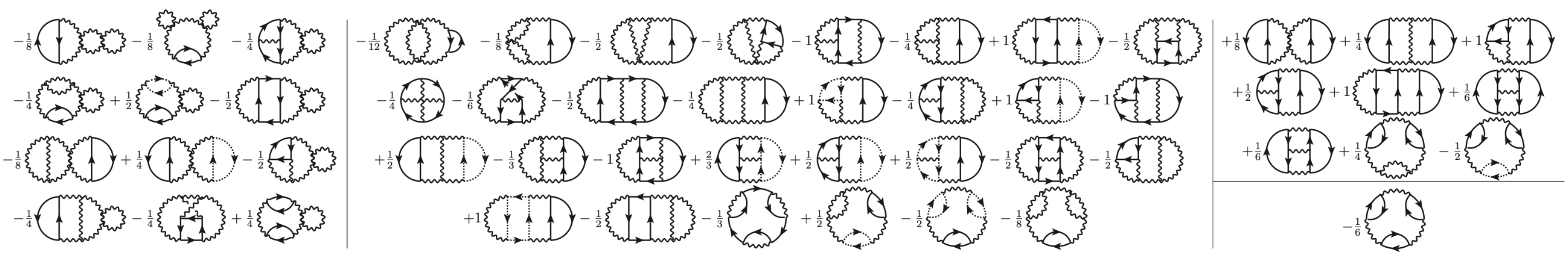
- standard thermal QFT treatment
  - ▷ dynamical scales generated by long-distance physics
  - ▷ resummations (HTL/HDL, EFT) needed to evade IR problems
  - ▷ e.g. pole of full gluon propagator $P^2 + \Pi(P) = 0$
  - ▷ where in the IR an effective mass appears: $m_{\mathrm{E}}^2 = \Pi(P \to 0) \sim g^2 \sum \mu^2 \neq 0$
  - ▷ 'hard' gluons $P \gg m_{\mathrm{E}}$ via naive loop expansion
  - ▷ 'soft' gluons $P \lesssim m_{\mathrm{E}}$ need modified weak-coupling expansion

- upshot: $p_{\mathrm{QCD}} = p_{\mathrm{hard}} + p_{\mathrm{mixed}} + p_{\mathrm{soft}}$ [T.Gorda et al. PRL2021]
  - ▷ going beyond the 'classic' 3-loop result [Freedman/McLerran PRD1977; A.Vuorinen PRD2003]
    (which involves a soft $g^4 \ln g$ at 3 loops from resumming ring diags)
  - ▷ recent advances: 4-loop soft+mixed [T.Gorda et al. PRL(s)2018-2023; Fernandez/Kneur PRL2022]
  - ▷ mainly by HDL effective theory; fixing 4-loop terms: $g^6 \ln^2 g$ and $g^6 \ln g$

Motivation

- so how about the bottleneck $p^{\mathrm{bare}}_{\mathrm{hard}}(\mu) = \mu^4 \Big\{ \frac{N_c N_f}{12\pi^2} + g^2 + g^4 + g^6 + \ldots \Big\}$
  - ▷ strict loop expansion in terms of 1PI vacuum Feynman diagrams
  - ▷ further grading wrt N_f and color factors [keeping gauge group $SU(N_c)$]
  - ▷ Dirac propagators modified; $P_0 \to P_0 + i\mu$ in momentum space
  - ▷ 2-loop: g^2, 1 diagram only, quick homework exercise
  - ▷ 3-loop: g^4, 7 diagrams, homework exercise in Hki
  - ▷ 4-loop: g^6, 52 diagrams, need algorithmic technology

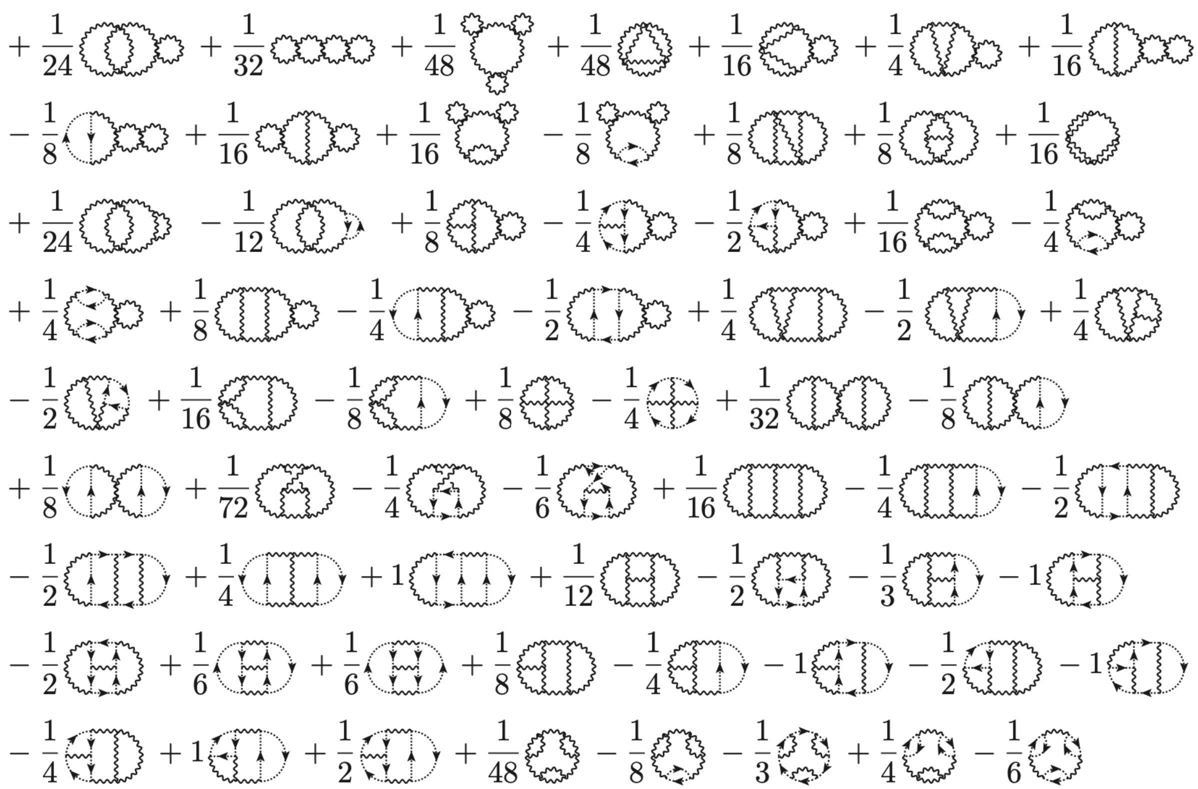
- fresh air due to recent feasibility study from Hki [A.Kärkkäinen et al. PRL2025]
  - ▷ complexity reduction: 150k integrals condensed to 110 (which contrib to g.i. sets)
  - ▷ demonstrating applicability of new dense LTD integration method [⇐ see Kaapo's talk]



[figure stolen from A.Kärkkäinen et al. 'hardships' PRL2025]

Dissecting the 4-loop task

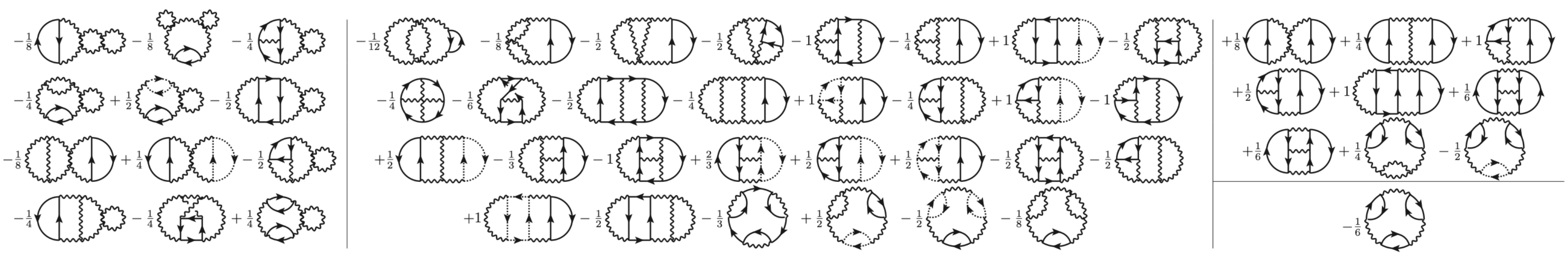
- separate gauge-invariant groups by N_f–power
- N_f^0 is trivial
  - ▷ 65 diags: gluons+ghosts only
  - ▷ scale-free Feynman ints $\Rightarrow$ vanish in dim.reg.



Dissecting the 4-loop task

- left panel: 12 trivial diags
  - ▷ 11 diags: contain a scale-free factor ⇒ vanish in dim.reg.
  - ▷ 1 non-planar diag: color trace evaluates to zero

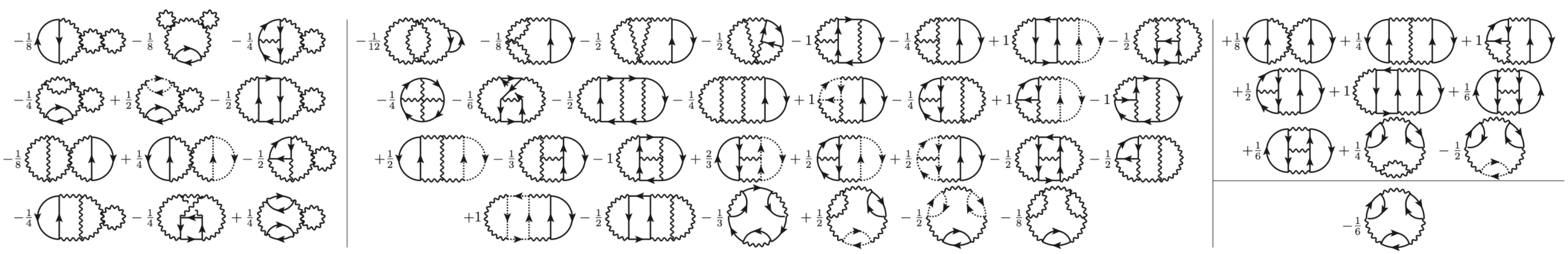
- N_f^3: bottom-right panel
  - ▷ 1 (ring-type) diag; IR sensitive
  - ▷ result can be extracted from cold+dense QED [T.Gorda et al. PRD2023]
  - ▷ re-evaluation provides welcome validation



Dissecting the 4-loop task

- N_f^2: top-right panel
  - ▷ 9 diags; IR sensitive
  - ▷ split into two g.i. groups, $\propto C_\mathrm{A},\ C_\mathrm{F}$
  - ▷ need genuinely new evaluation methods [⇐ see Kaapo's talk]

- N_f^1: center panel
  - ▷ 30 diags; IR finite
  - ▷ split into three g.i. groups, $\propto C_\mathrm{A}^2,\ C_\mathrm{A}C_\mathrm{F},\ C_\mathrm{F}^2$
  - ▷ contains cases with potential for direct analytic evaluations: quark loop + 2-loop YM self-energy insertion [⇐ see pg 12 below]



The main actors

- setup and preparation: diagrams $\to$ integrals
  - ▷ generate complete set of (connected, 1PI, vacuum, QCD) diagrams
  - ▷ map all propagator momenta onto complete momentum family $\{P_1,\dots,P_{10}\} = \{K_1, K_2, K_3, K_4, K_{1-4}, K_{2-4}, K_{3-4}, K_{1-2}, K_{1-3}, K_{1-2-3}\}$ made from four loop integration momenta $K_1^\mu \dots K_4^\mu$
  - ▷ insert Feynman rules (quarks with μ, gluons with ξ)
  - ▷ perform Lorentz algebra, Dirac and color traces: generates prefactors $\delta_{\mu\mu} = D = 4-2\varepsilon$, $N_{\rm f}$, $C_{\rm A}$, $C_{\rm F}$, ...
  - ▷ re-write scalar products as (LCs of) inverse propagators

- notation for propagators and bos/fer momenta
  - ▷ lowercase $j = 1\dots4$; uppercase $A = 1\dots10$, greek $\alpha = 1\dots D$
  - ▷ read above family as matrix $\times$ vector: $P_A = F_{Aj}K_j$

$$D_A = \sum_\alpha \tilde{P}_A^\alpha \tilde{P}_A^\alpha\ ,\quad \tilde{P}_A^\alpha = \sum_j F_{Aj}\tilde{K}_j^\alpha\ ,\quad \tilde{K}_j^\alpha = K_j^\alpha + c_j i\mu U^\alpha\ ,\quad U^\alpha = (1,\vec{0})$$

- each of our ints is then parametrized by a list of integers $(c_1\dots c_4, a_1\dots a_4, n_1\dots n_{10})$

$$\mathcal{I}(\vec{c},\vec{a},\vec{n}) = \Big(\prod_j \int_{\mathbb{R}^D} d^D K_j\Big)\frac{\prod_k [U^\alpha \tilde{K}_k^\alpha]^{a_k}}{\prod_A D_A^{n_A}}$$

The main actors

- first trivial observations about the $\mathcal{I}(\vec{c}, \vec{a}, \vec{n}; \mu)$
  - ▷ in our special case of $p_{\rm hard}(\mu)$ all $a_i = 0$ from the outset; but allowing for $a_i \geq 0$ might prove a useful extension of the integral class
  - ▷ Scale-free ints vanish in dim reg $\mathcal{I}(\vec{c} = \vec{0}, \vec{a}, \vec{n}; \mu) = 0 = \mathcal{I}(\vec{c}, \vec{a}, \vec{n}; \mu = 0)$
  - ▷ from structure of $\mathcal{I}$ and invariance under sign flips of all loop momenta get $\mathcal{I}(\vec{c}, \vec{a}, \vec{n}; -\mu) = \mathcal{I}^*(\vec{c}, \vec{a}, \vec{n}; \mu) = \mathcal{I}(-\vec{c}, \vec{a}, \vec{n}; \mu) = (-1)^{\sum a_j}\mathcal{I}(\vec{c}, \vec{a}, \vec{n}; \mu)$ $\Rightarrow \mathcal{I}$ is <u>real</u> (imaginary) if the sum of numerator indices $\sum_j a_j$ is <u>even</u> (odd)

- analyze properties of the $\mathcal{I}(\vec{c}, \vec{a}, \vec{n})$
  - ▷ have (many tens of) thousands of such $\mathcal{I}$ to evaluate
  - ▷ not all are independent: $\exists$ large set of <u>linear rels</u> $0 = \sum c \cdot \mathcal{I}$ [⇐ see below]
  - ▷ use these systematically to eliminate redundancies, reduce int evals

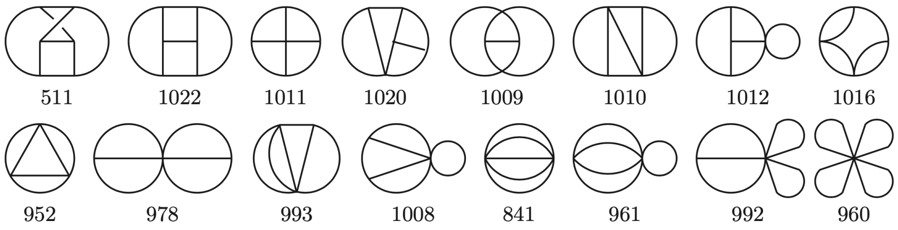
- key underlying mechanism: ordering rel $\mathcal{I}_1 \prec \mathcal{I}_2$
  - ▷ assign list of integers to each $\mathcal{I}$, such as `loop`, `nrPropags`, `xtraPowers`, ...
  - ▷ compare any two $\mathcal{I}$ by comparing those lists from the left, until an element differs

$$\mathcal{I}(\vec{c}, \vec{a}, \vec{n}) = \Big(\prod_j \int_{\mathbb{R}^D} d^D K_j\Big) \frac{\prod_k [U^\alpha \tilde{K}_k^\alpha]^{a_k}}{\prod_A D_A^{n_A}}$$

Linear relations: shifts

- $\mathcal{I}$ are invariant under change of integration variables, e.g. $K_1 \leftrightarrow K_2$
  - ▷ parametrize by $(4{\times}4)$ matrices S as $K_j \to S_{jk} K_k$
  - ▷ all shifts S with $\det S = \pm 1$ are permissible
  - ▷ determine useful classes of S that induce relations among $\mathcal{I}$
  - ▷ key action: effect on the propagator momenta corresponding to positive entries in $\vec{n}$

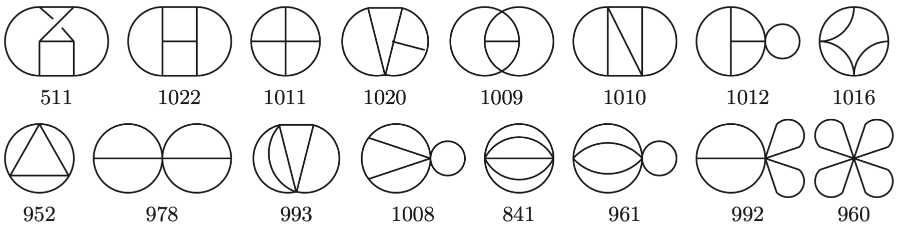
- class 1: sectorShifts
  - ▷ a <u>sector</u> is a subset of the momentum family
  - ▷ label a sector by the binary rep of momenta present: `sectorID`
    e.g. $\{K_1, K_2, K_4, K_{3-4}, K_{1-2-3}\} = \{P_1, P_2, P_4, P_7, P_{10}\} \to 1101001001_2 = 841$
  - ▷ different `sectorID` can correspond to the same underlying graph, e.g. $841 = 31$
  - ▷ note: isomorphic graphs have same Symanzik polynomial [standard QFT technique]
  - ▷ note: polynomials can be normal ordered [A.Pak 2011; J.Hoff 2015]
  - ▷ identify all S that induce such graph isomorphisms, use to e.g. maximize `sectorID`
  - ▷ $\boxed{0 = \mathcal{I} - (S \circ \mathcal{I})}$
  - ▷ results in 16 (out of $2^{10} = 1024$) unique sector reps for our 4-loop vacuum ints



Linear relations: shifts

- class 2: symmShifts
  - ▷ momentum shifts S that do not change the `sectorID`
  - ▷ induce sector automorphisms (leave form of denominator of $\mathcal{I}$ invariant)
  - ▷ can change the index order, and the numerator structure
  - ▷ shifts form group under multiplication (det$=\pm1$); need only generators
  - ▷ efficient: e.g. all 192 symmShifts of sector 960 generated by just two S
  - ▷ $\boxed{0 = \mathcal{I} - (S \circ \mathcal{I})}$

- applying sectorShifts and symmShifts (iteratively), arrive at canonical form
  - ▷ powerful analytic mapping method
  - ▷ only integer algebra, no rational functions (in D)
  - ▷ for $p_{\rm hard}(\mu)$ the canonical representation of its $\mathcal{I}$ leads to
    (a) explicit gauge parameter independence
    (b) a small set of $\mathcal{O}(100)$ remaining cold+dense integrals to be evaluated
  - ▷ the same had been observed for $p_{\rm hard}(T)$ [P.Navarrette/YS JHEP2024; for a sketch see backup]



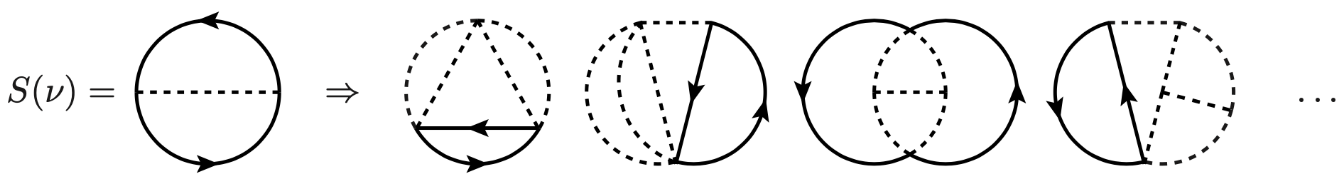
Linear relations: IBP

- additional classes of linear relations among the (now canonical) $\mathcal{I}(\vec{c}, \vec{a}, \vec{n})$
  - ▷ useful since very few $\mathcal{I}$ known analytically
  - ▷ e.g. N_f^1: massless fermionic 2-loop cold+dense vacuum sunset [J.Österman et al. JHEP2023]

$$S(\nu) = \int_{\{Q\}} \int_P \frac{1}{[Q^2]\,[(Q-P)^2]\,[P^2]^\nu} = \frac{(4\mu^2)^{D-2-\nu}\,\Gamma(\frac{1}{2})\,\Gamma(\frac{D}{2}-1-\nu)}{2^{D-3}\,\Gamma(\frac{D-1}{2})\,(D-2-\nu)\,\Gamma(D-1-\nu)}$$

  - ▷ replace massless bosonic propag $[P^2]^\nu$ by another massless bosonic 2-pt fct $\Pi(P)$
  - ▷ dimensionality of Π fixes ν, use above analytic result
  - ▷ $\Pi(P)$ are well-studied, so-called p-integrals; a world of analytic knowledge

- systematic reduction of such $\Pi(P)$ (in the $\mu = 0$ world) via IBP
  - ▷ key idea: $\boxed{0 = \partial_{K_\mu} Q_\mu \circ \int_K \ldots}$ (huge in collider physics) [Chetyrkin/Tkachov NPB1981]
  - ▷ links with systematics from above: families, sectors, ordering rel [S.Laporta IJMPA2000]
  - ▷ full recursive solution for 2-loop p-ints, `Tarcer` [O.Tarasov NPB1997; Mertig/Scharf CPC1998]



Linear relations: IBP

- after this appetizer, take fresh look into denseIBP reductions to general $\mathcal{I}(\vec{c},\vec{a},\vec{n})$
  - ▷ acting with a derivative on the integrand (notation $\hat{O}_K \circ \int_K f(K) \equiv \int_K \hat{O}_K\, f(K)$)
  - ▷ form standard 16 IBP rels $\boxed{0 = \partial_{K_j^\alpha} \tilde{K}_k^\alpha \circ \mathcal{I}(\vec{c},\vec{a},\vec{n})}$
  - ▷ $0 = \left\{ \delta_{jk}\, D + a_j \frac{[U\cdot\tilde{K}_k]}{[U\cdot\tilde{K}_j]} - \sum_{A,B,i} \frac{n_A}{D_A}\, F_{Aj}\, F_{Ai}\, c_{ikB}\, D_B \right\} \circ \mathcal{I}(\vec{c},\vec{a},\vec{n})$
  - ▷ where c_{jkA} map scalar prods: $2\sum_\alpha \tilde{K}_j^\alpha \tilde{K}_k^\alpha = \sum_A c_{jkA} D_A$

- restriction: avoid to hit the $P_0 + i\mu$ shifted integration contours with derivatives ∂_{K_j}
  - ▷ otherwise boundary terms are not zero [T.Gorda et al. PRD2022]
  - ▷ could be systematized (e.g. with $T \to$ regulator) [J.Österman et al. JHEP2023]
  - ▷ generates rels beyond closed class of ints $\mathcal{I}$; $\exists$ basis?

- reproduces (and avoids use of) above `Tarcer`–assisted reductions
  - ▷ note that our $\mathcal{I}(\vec{c},\vec{a}=\vec{0},\vec{n})$ form a closed set under these <u>fullIBP</u> rels

$$\mathcal{I}(\vec{c},\vec{a},\vec{n}) = \left(\prod_j \int_{\mathbb{R}^D} d^D K_j \right) \frac{\prod_k [U^\alpha \tilde{K}_k^\alpha]^{a_k}}{\prod_A D_A^{n_A}}$$

Linear relations: IBP

- .. but wait, there is more ..
  - ▷ can hit all $\mathcal{I}$ with spatial-only derivatives $\boxed{0 = \partial_{K_j^\alpha}(g^{\alpha\beta} - U^\alpha U^\beta)\tilde{K}_k^\beta \circ \mathcal{I}(\vec{c},\vec{a},\vec{n})}$

$$\Rightarrow 0 = \left\{\delta_{jk}\,(D{-}1) + \sum_{A,i}\frac{n_A}{D_A}\,F_{Aj}\,F_{Ai}\left(2[U{\cdot}\tilde{K}_i][U{\cdot}\tilde{K}_k] - c_{ikB}\,D_B\right)\right\}\circ\mathcal{I}(\vec{c},\vec{a},\vec{n})$$

  - ▷ dimensional analysis gives $\boxed{0 = \left(4D + \sum_j a_j - 2\sum_A n_A - \mu\partial_\mu\right)\circ\mathcal{I}(\vec{c},\vec{a},\vec{n})}$

$$\Rightarrow 0 = \left\{4D + \sum_j a_j - 2\sum_A n_A - \sum_j a_j\frac{\eta\,c_j i\mu}{[U{\cdot}\tilde{K}_j]} + 2\sum_{A,j,k}\frac{n_A}{D_A}\,F_{Aj}\,F_{Ak}\,c_j i\mu[U\cdot\tilde{K}_k]\right\}\circ I(\vec{c},\vec{a},\vec{n})$$

- note that the 16 <u>spatialIBP</u> rels do not close on $\mathcal{I}(\vec{c},\vec{a}=\vec{0},\vec{n})$
  - ▷ give penalty to $\vec{a}\neq\vec{0}$ in ordering rel to eliminate those with priority

- note that the 1 <u>dimRel</u> mixes the even/odd (real/imaginary) worlds
  - ▷ together with spatialIBP reduces all dense 1-loop ints to 1 master

$$\mathcal{I}(\vec{c},\vec{a},\vec{n}) = \left(\prod_j\int_{\mathbb{R}^D} d^D K_j\right)\frac{\prod_k [U^\alpha\tilde{K}_k^\alpha]^{a_k}}{\prod_A D_A^{n_A}}$$

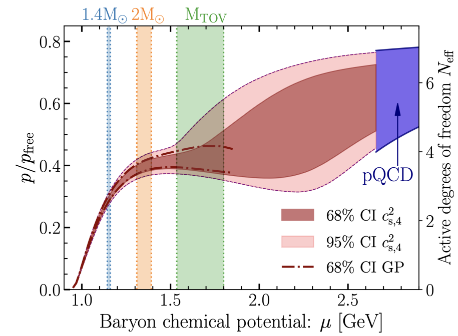
Linear relations: case study

- consider dense 1-loop ints: $\mathcal{I}(c,a,n)=\int_{\mathbb{R}^D}d^DK\,\frac{[U\cdot(K+ci\mu U)]^a}{[(K+ci\mu U)^2]^n}$
- have spatialIBP and dimRel from previous slide as
  - ▷ $0=(D-1-2n)\mathcal{I}(c,a,n)+2n\mathcal{I}(c,a+2,n+1)$
  - ▷ $0=(D-2n+a)\mathcal{I}(c,a,n)+ci\mu\Big(-a\mathcal{I}(c,a-1,n)+2n\mathcal{I}(c,a+1,n+1)\Big)$
- a useful combination is (allowing to recursively reduce $a\to 0$)
  - ▷ $0=\mathrm{dimRel}|_{\{a\to a+1\}}-ci\mu\,\mathrm{spatialIBP}$
  - ▷ $0=ci\mu(2n-a-D)\mathcal{I}(c,a,n)+(1-2n+D+a)\mathcal{I}(c,a+1,n)$

    $\Rightarrow\ \mathcal{I}(c,a,n)=(ci\mu)^{a-\bar{a}}\frac{D-2n+\bar{a}}{D-2n+a}\mathcal{I}(c,\bar{a},n)$
- another useful combination is (allowing to recursively reduce $n\to 1$)
  - ▷ $0=(1+2n-D)(2n-D-a)\mathcal{I}(c,a,n)-2n(ci\mu)^2(2+2n-D-a)\mathcal{I}(c,a,n+1)$

    $\Rightarrow\ \mathcal{I}(c,a,n)=(ci\mu)^{2\bar{n}-2n}\frac{\Gamma(n+1/2-D/2)}{\Gamma(\bar{n}+1/2-D/2}\frac{\Gamma(\bar{n})}{\Gamma(n)}\frac{D-2\bar{n}+a}{D-2n+a}\mathcal{I}(c,a,\bar{n})$
- together these rels imply the full 1loop reduction to a single dense master integral
  - ▷ (bosonic ints) $\quad\mathcal{I}(0,a,n)=0$
  - ▷ (fermionic ints) $\quad\mathcal{I}(c,a,n)=(ci\mu)^{2-2n+a}\frac{\Gamma(n+1/2-D/2)}{\Gamma(n)\,\Gamma(3/2-D/2)}\frac{D-2}{D-2n+a}\mathcal{I}(c,0,1)$

Summary



- EoS of cold+dense quark matter relevant for neutron star physics
  - ▷ inference methods rely on input from χEFT and pQCD

- QCD weakly coupled at very high density $\mu \gg$ GeV: $\alpha_{\mathrm{s}}(90\mathrm{GeV}) \approx 0.12$
  - ▷ simplest scenario (pQCD with unpaired quarks) argued to be applicable
  - ▷ gap $\Delta \propto \mu e^{-1/\sqrt{\alpha_{\mathrm{s}}}}$ and pressure gain $p_\Delta \propto \Delta^2 \mu^2$ exp suppressed
  - ▷ IR accounting organized by HDL effective theory
  - ▷ leaving a clean mathematical problem: bare cold+dense QCD in strict loop expansion

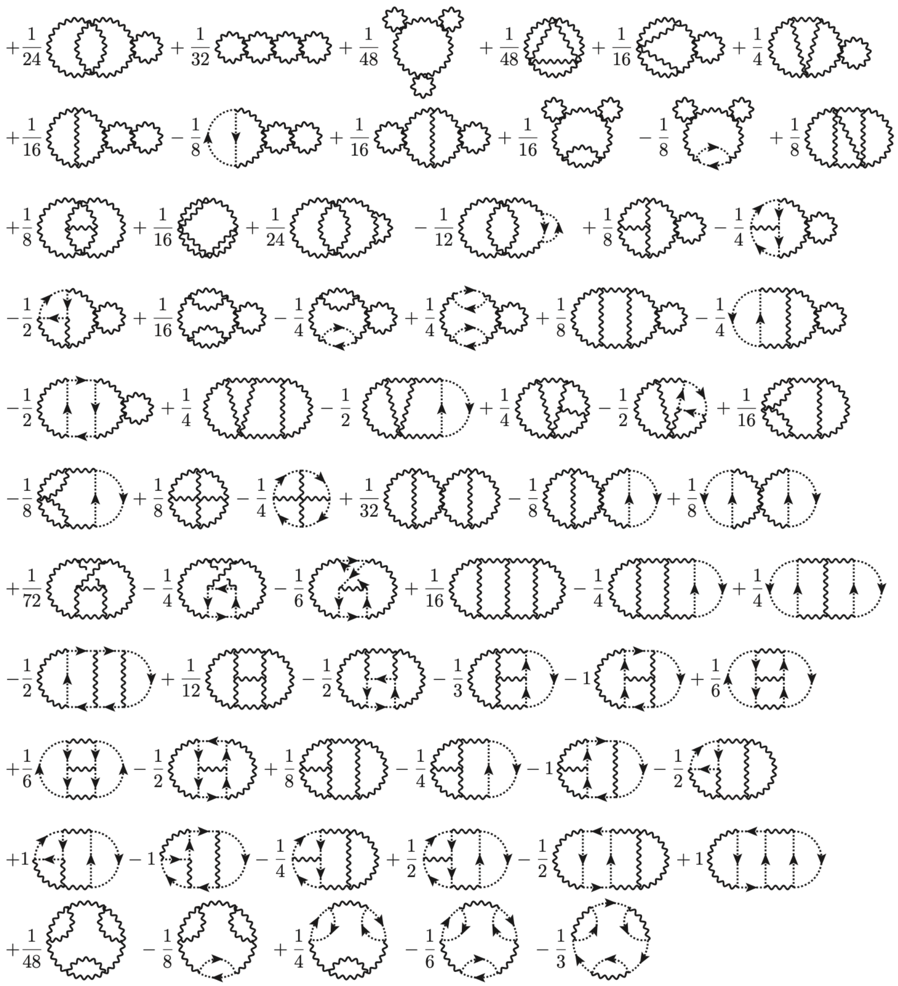
- 'cold' allows to import powerful concepts from collider physics
  - ▷ classification, graph polys, sector mapping, lex ordering
  - ▷ isometries, canonical forms of integrands
  - ▷ IBP reduction to master integrals, optimized treatments of large linear systems

- 'dense' enforces crucial adaptations
  - ▷ restricted use of IBPs; provide benchmarks+crosschecks
  - ▷ novel methods to (semi-)numerically evaluate dense masters

- take-home: hugely advantageous to stay analytic as far as possible
  - ▷ massive cancellations, gauge invariance, reduced int evaluation work, ...

The missing hard contribution for $p(T)$

- need p_{E} to 4 loops for physical LO pressure, in (4D, hot) QCD
- look first at gauge sector; SU(N), covariant R_ξ gauge [P.Navarrete/YS JHEP2024]
- QGRAF $\longrightarrow$ 65 diags
- mapping onto family + FORM
- 24M $= 2^9\, 6^6$ terms in hardest diag
- 176k indep sum-ints
- 25k after shifts to sector reps
- 1k after symmetries
- 21 after summing all diags
- gauge parameter drops out
- ≤ 18 after (thermal) IBP



Master sum-integral challenge for p(T)

- have reduced $p_{\rm E}$ at 4 loops from 176k $\Rightarrow$ 21 sum-integrals [P.Navarrete/YS JHEP2024]
  - ▷ all integrands are in canonical form
  - ▷ momentum shifts exhausted
  - ▷ IBP relations might be applied; but enlarge space of target sum-ints
  - ▷ 11/21 sum-ints are already known

- 10/21 sum-ints left as a challenge [⇐ see K.Seppänen's talk; L.Sandbote's poster]
  - ▷ def Euclidean 4-momenta $K = (K_0, \mathbf{k})$, $K_0 = 2n_k\pi T$, $n_k \in \mathbb{Z}$
  - ▷ let $\{P_1, \ldots, P_{10}\} = \{K_1, K_2, K_3, K_4, K_{1-4}, K_{2-4}, K_{3-4}, K_{1-2}, K_{1-3}, K_{1-2-3}\}$
  - ▷ eval massless scalar bosonic vacuum sum-integrals $\big($with propags $1/(P\cdot P) = 1/(P_0^2+\mathbf{p}^2)\big)$

$$
\begin{aligned}
\mathcal{I}(s_1,\ldots,s_{10}) &\equiv \sum\!\!\!\!\!\!\!\int_{K_1}\sum\!\!\!\!\!\!\!\int_{K_2}\sum\!\!\!\!\!\!\!\int_{K_3}\sum\!\!\!\!\!\!\!\int_{K_4} \frac{1}{(P_1\cdot P_1)^{s_1}\ldots(P_{10}\cdot P_{10})^{s_{10}}}\\
\mathcal{I}_6 &= \mathcal{I}(1,1,1,0,1,1,1,0,0,0)\,, & \mathcal{I}_{17} &= \mathcal{I}(1,1,1,1,1,1,-1,-1,1,1)\\
\mathcal{I}_8 &= \mathcal{I}(1,1,1,1,1,0,0,0,0,1)\,, & \mathcal{I}_{18} &= \mathcal{I}(1,1,1,1,1,1,0,-2,1,1)\\
\mathcal{I}_{14} &= \mathcal{I}(1,1,1,1,1,1,0,0,-1,1)\,, & \mathcal{I}_{19} &= \mathcal{I}(1,1,1,1,1,1,1,1,-2,0)\\
\mathcal{I}_{15} &= \mathcal{I}(1,1,1,2,1,1,1,0,0,-2)\,, & \mathcal{I}_{20} &= \mathcal{I}(1,1,1,1,1,1,1,1,0,-2)\\
\mathcal{I}_{16} &= \mathcal{I}(1,1,1,3,1,1,1,0,0,-3)\,, & \mathcal{I}_{21} &= \mathcal{I}(1,1,1,1,1,1,1,1,1,-3)
\end{aligned}
$$