Color superconductivity under neutron-star conditions at next-to-leading order

A. Geißel, TG, J. Braun, PRL 135, 211901 (2025) [2504.03834]

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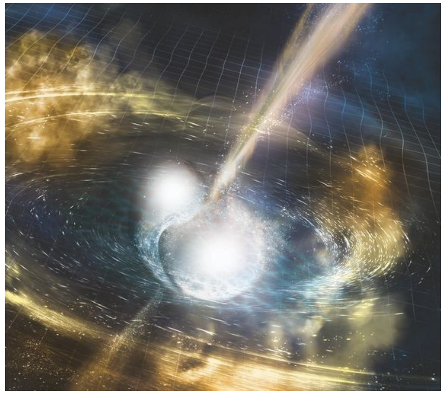
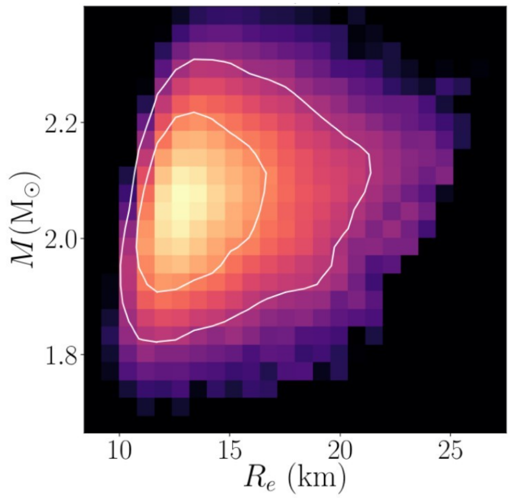
18 August 2026

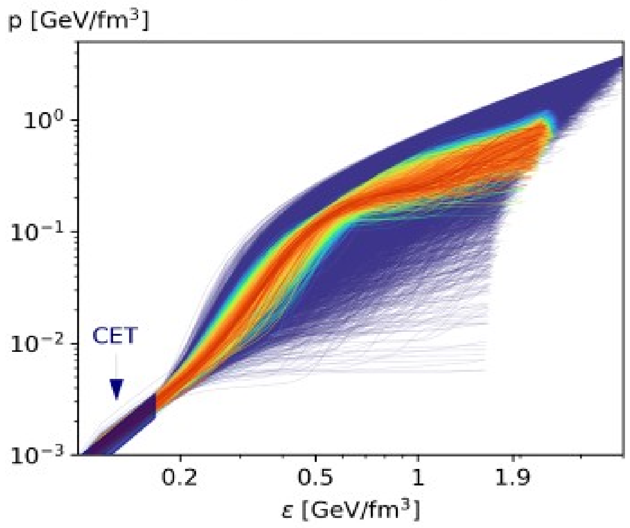
Interdisciplinary approach to learn about dense matter

Astronomy +Astrophysics

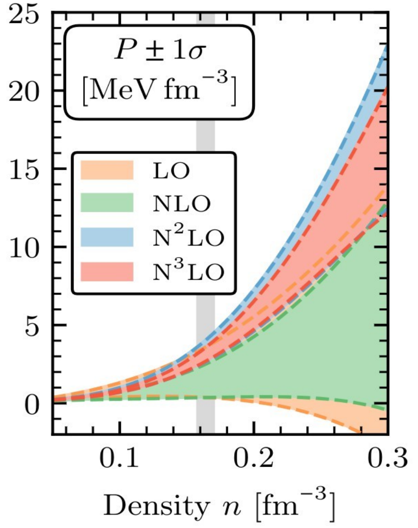


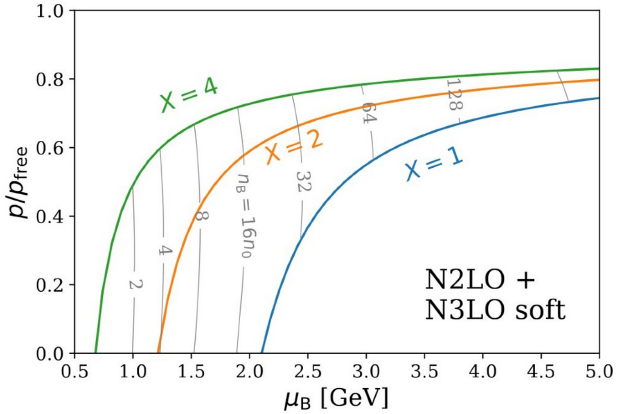


Equation-of-state (EoS) Inference & Properties of dense matter



Nuclear + Particle theory



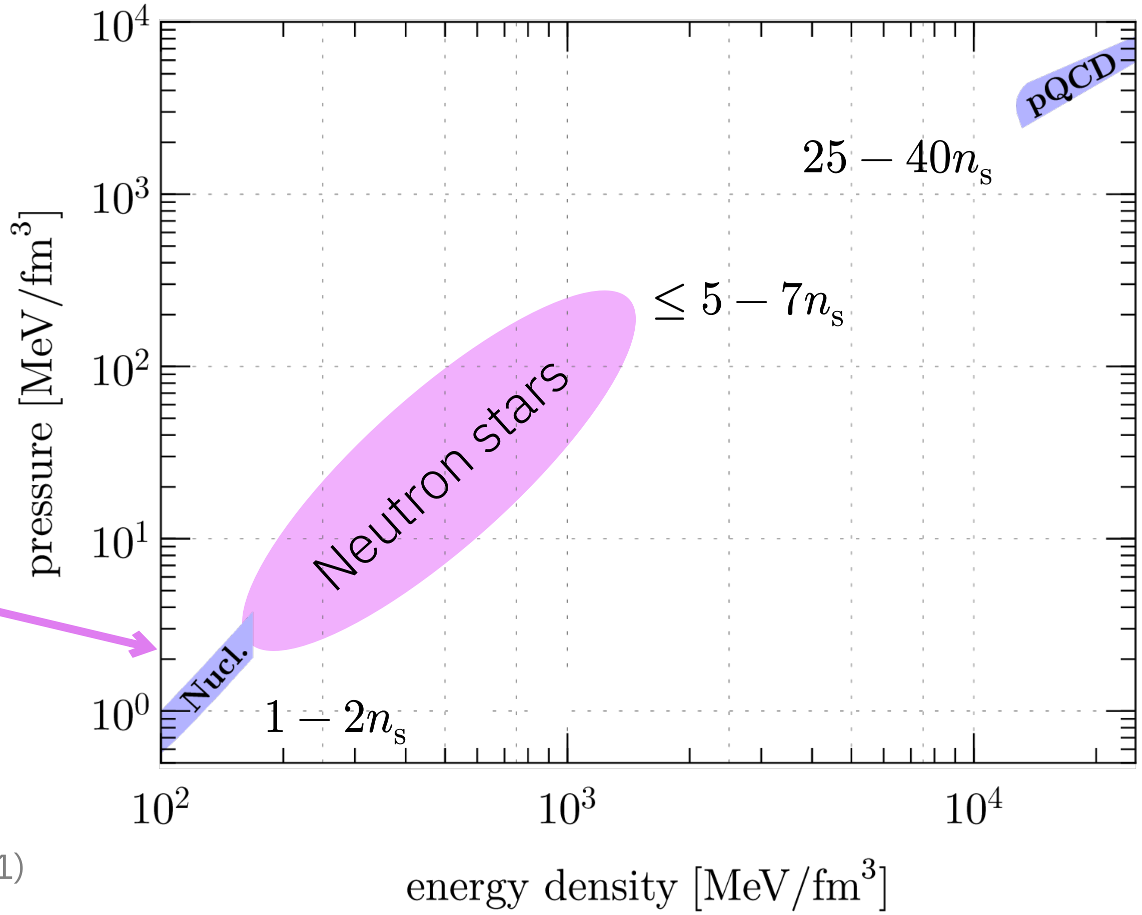


Different inputs constrain different parts of the EoS

Perturbative QCD

Freedman & McLerran PRD 16 (1977)
Kurkela, Romatschke+ PRD 81 (2010),
TG, Kurkela+ PRL 121 (2018),
TG, Kurkela+ PRL 127 (2021)
& PRD 104 (2021),
TG, Paatelainen+ PRL 131 (2023)

K. Seppänen talk (Tue)



Chiral EFT

Hebeler, Lattimer+. ApJ. 773 (2013);
Tews, Krüger+, PRL 110 (2013)
Lynn, Tews, Carlson+ PRL 116 (2016),
Drischler, Hebeler+ PRL 122 (2019),
Drischler, Furnstahl+ PRL 125 (2020),
Drischler, Han, Lattimer, PRC 103 (2021)
Keller, Hebeler+ PRL 130 (2023).

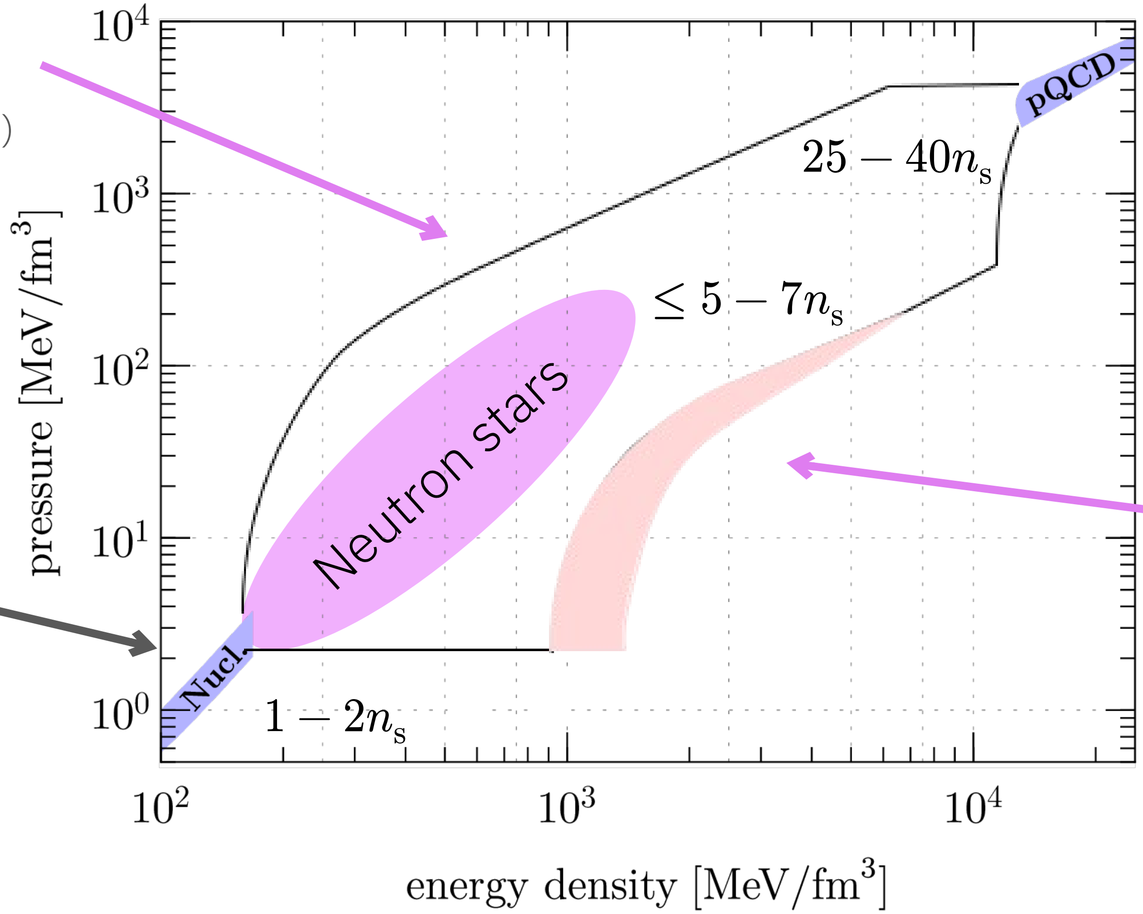
Different inputs constrain different parts of the EoS

Thermodynamics
(causality & consistency)

Komoltsev, Kurkela PRL 128 (2022)

O. Komoltsev talk (Fri)

Chiral EFT



Perturbative QCD

Lattice (Isospin QCD)

Abbot, Detmold+ PRD 108 (2023),

Brandt, Cuterib+, JHEP 07 (2023),
Fujimoto, Reddy, PRD 109 (2024),

(see also Phase-quenched QCD
Moore, TG, JHEP 12 (2023),
Navarrete+, PRD 110, (2024),
TG, Navarrete+ 2511.09627)

State-of-the-art unpaired pQCD EoS

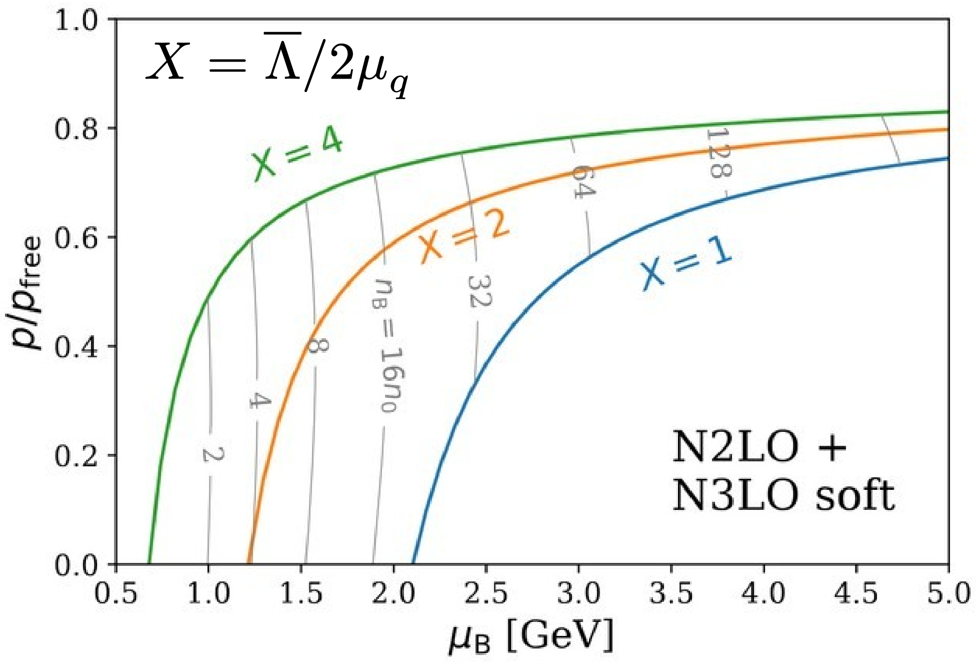
- Describes **(nearly) massless quarks, gluons** interacting. Quarks are weakly interacting with $O(20\%)$ corrections when expanding results in the strong coupling α_s

Freedman, McLerran, PRD 16 (1977)

$$\frac{p}{p_{\mathrm{free}}} = 1 + a_1\alpha_s\left(\overline{\Lambda}\right) + a_2\alpha_s^2\left(\overline{\Lambda}\right) + a_3\alpha_s^3\left(\overline{\Lambda}\right) + \ldots$$

free quark gas*

TG, Kurkela+ PRL 121 (2018) – **ongoing**



TG, Kurkela+ PRL 127 (2021), PRD 104 (2021)
TG, Paatelainen, Säppi, Seppänen, PRL 131 (2023)

- Calibrated by **collider data**
- State-of-the-art results converge well above 25-40n_s

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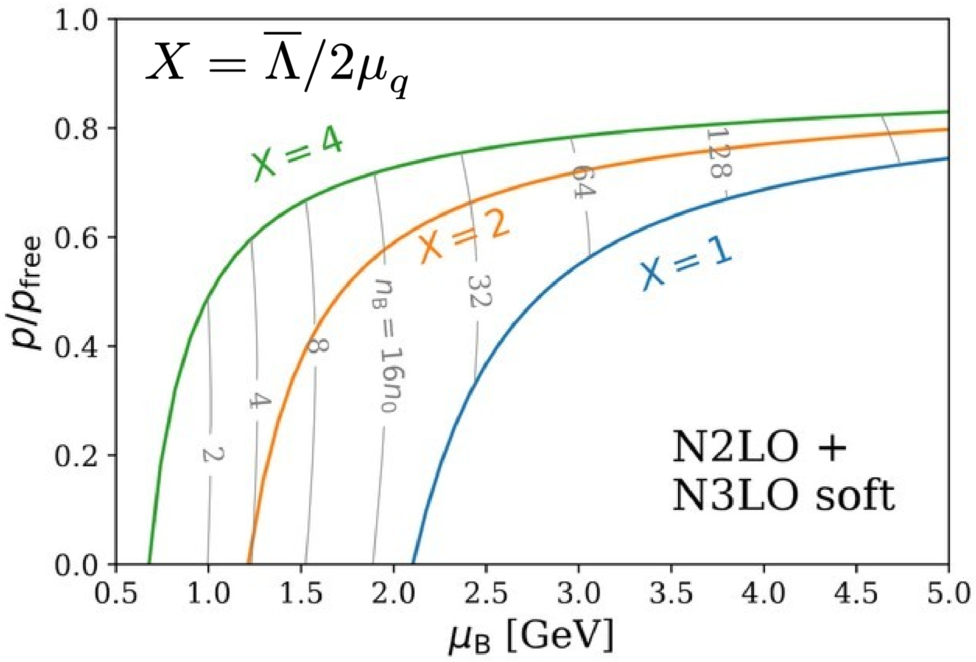

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***but this is not the whole story...**

Ground state of high-density quark matter is *paired*

- Gluonic attraction between quarks with opposite 3-momenta leads to **instability of the quark Fermi surface** and creates a nonzero diquark expectation value (condensate):

$$\Delta^{ij}_{ab} \sim \overline{\psi}^i_a \gamma^5 \mathcal{C} \overline{\psi}^j_b{}^T \qquad \text{(f = flavor, c = color, rest is Dirac)}$$

Barrois, Nucl. Phys. B129 (1977),
Bailin, Love, Phys. Rep. 107 (1984),
Alford, Rajagopal, Wilczek, PLB 422 (1998),
Rapp, Schäfer, Shuryak+, PRL 81 (1998),
Alford+, Rev. Mod. Phys. 80, (2008)

- This leads to an **energy gap** in the quark excitations on Fermi surface

- Leads to additional pressure $\sim \Delta^2 \mu_{\mathrm{B}}^2$; typically $\Delta \sim 100\ \mathrm{MeV}$

- **Complication**: flavor breaking from strange quark $\sim -m_{\mathrm{s}}^2 \mu_{\mathrm{B}}^2$, with $m_{\mathrm{s}} \sim 100\ \mathrm{MeV}$ (as long as m_{s} small enough, pairing still happens)

- **Goal**: Compute **NLO corrections** to QCD in the **paired vacuum**, assuming the coupling and the gap are both small (NLO correction to γ_1):

$$p = p_{\mathrm{free}} \left[\gamma_0(\alpha_{\mathrm{s}}, \overline{m}_{\mathrm{s}}^2) + \gamma_1(\alpha_{\mathrm{s}}, \overline{m}_{\mathrm{s}}^2) \overline{\Delta}_{\mathrm{CFL}}^2 + \ldots \right], \qquad \text{with } \overline{x} := x/(\mu_{\mathrm{B}}/3)$$

Color-flavor-locked (CFL) matter under NS conditions

- CFL matter pattern **couples color and flavor** $\langle \psi_a^{i\,T} \gamma^5 \mathcal{C} \psi_b^i \rangle \sim \Delta_{\mathrm{CFL}} \varepsilon_{abA} \varepsilon^{ijA}$
- NS matter neutral under
  - i. color
  - ii. flavor

  $\Big\}$ **locked & broken by explicit m_s**
  - iii. charge

- Means that general chemical potential is of the form

Alford & Rajagopal, JHEP 06, 031 (2002)

$$\boxed{\mu_{\mathrm{f,c}} = \mu_{\mathrm{B}}/3 - \mu_{\mathrm{e}} Q_{\mathrm{f,c}} + \mu_3 T^3_{\mathrm{f,c}} + \mu_8 T^8_{\mathrm{f,c}}}$$

$$Q_{\mathrm{f,c}} := \mathrm{diag}_{\mathrm{f}}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right) \otimes 1_{\mathrm{c}}$$

$$T^8_{\mathrm{f,c}} := 1_{\mathrm{f}} \otimes \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$$

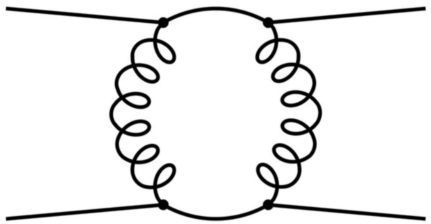
$$T^3_{\mathrm{f,c}} := 1_{\mathrm{f}} \otimes \mathrm{diag}\left(\frac{1}{2}, -\frac{1}{2}, 0\right)$$

- **Important: μ_e, μ_3, μ_8 parametrically small**, so we can expand **in them** (they **vanish** when $m_s = 0$)

Deriving Effective Action in the gapped phase

Geißel, TG, Braun, PRD 110, 014034 (2024)

- Motivated by Renormalization-group (RG) approach, we consider **infinitesimal RG step** down in energy scale from the Euclidean QCD action *S*,
- This generates an infinite set of new interactions, **including 4-quark interaction** from



- Leads to nonzero expectation value of the diquark operator $\Delta^{ij}_{ab} \sim \overline{\psi}^{i}_{a}\gamma^{5}\mathcal{C}\overline{\psi}^{j\,T}_{b}$
- Make diquark explicit by using exact **Hubbard-Stratonovich transformation** to exchange 4-quark interaction for quark-quark-diquark interaction + mass term $m^2\Delta^2$
- In principle, *m* must be determined self-consistently. Here **simplify by taking Δ a constant background field**, whose form is fixed to results from the literature. i.e., $\partial\Gamma/\partial\Delta|_{\Delta=\Delta_{\text{gap}}} = 0$ fixes the mass *m* (Γ is the effective action we compute)

Computing the correction (1)

Geißel, TG, Braun, PRL 135, 211901 (2025)

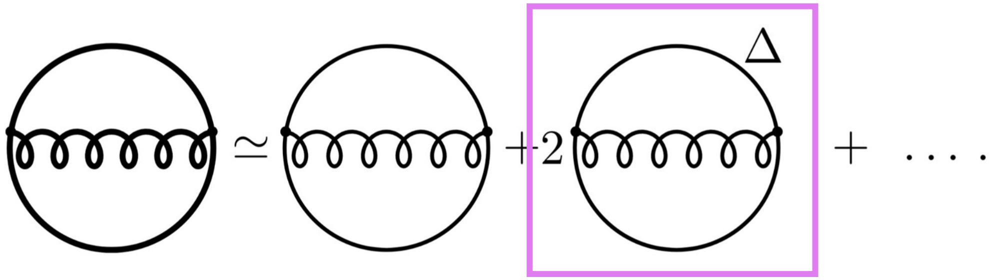
- To compute the term in the pressure, we only need the following modified action:

$$S_{\mathrm{eff}} := \int_x \left\{ \overline{\psi}\left(\mathrm{i}\not{D} - \mathrm{i}\hat{\mu}_{\mathrm{f,c}}\gamma_0 - \widehat{M}_{\mathrm{f,c}}\right)\psi + m^2\Delta^2 + \frac{1}{2}\left(\psi^T \mathcal{C}\gamma_5 \Delta \varepsilon_{\mathrm{a}}^{\mathrm{f}} \varepsilon_{\mathrm{a}}^{\mathrm{c}} \psi + \mathrm{h.c.}\right) \right\} + S_{\mathrm{gf}} + S_{\mathrm{gh}}$$

- Using S_{eff}, compute 2x2 propagators for (ψ, ψ^T) and the gauge field
- Expand each propagator in two parts to make gap part explicit, **power count in the gap**

$$\mathcal{P} = \mathcal{P}^0 + \left(\mathcal{P} - \mathcal{P}^0\right) := \mathcal{P}^0 + \mathcal{P}^\Delta$$

- Must compute corrections to following diagram(s)! Unfortunately must be done numerically...



$O(\alpha_{\mathrm{s}})$ $\qquad$ $O(\alpha_{\mathrm{s}}\Delta^2)$

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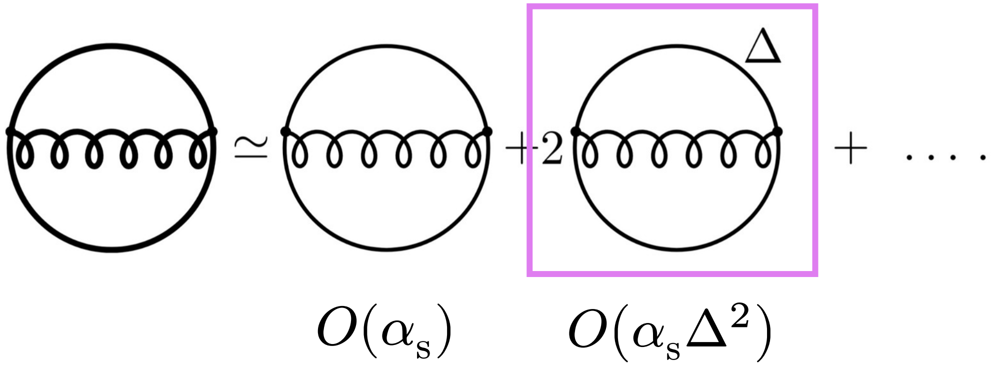


- note this is not a **full** NLO result: double Δ insertions also have a NLO contribution at $O(\alpha_s \Delta^2)$

Computing the correction (2)

Geißel, TG, Braun, PRL 135, 211901 (2025)

- Numerically evaluate this diagram with a high cutoff Λ. **Vary large Λ and small Δ and use fitting function** to evaluate the effective action Γ for these different values

$$\frac{\Gamma^{2-\mathrm{loop}}_{\mathrm{quark}}}{V_4} = 128\pi\alpha_s\Delta^2\mu^2\left[-2.44 + 0.0078\ln\left(\frac{\Delta^2}{\mu^2}\right)\right]$$

- Minimize the effective action wrt Δ to fix *m* (introduce counterterm to remove divergences) and convert the diquark field Δ into the known gaps from the literature

$$\Delta_{\mathrm{gap}} \propto \frac{\mu}{\alpha_{\mathrm{s}}^{5/2}}\exp(-A/\sqrt{\alpha_{\mathrm{s}}})$$

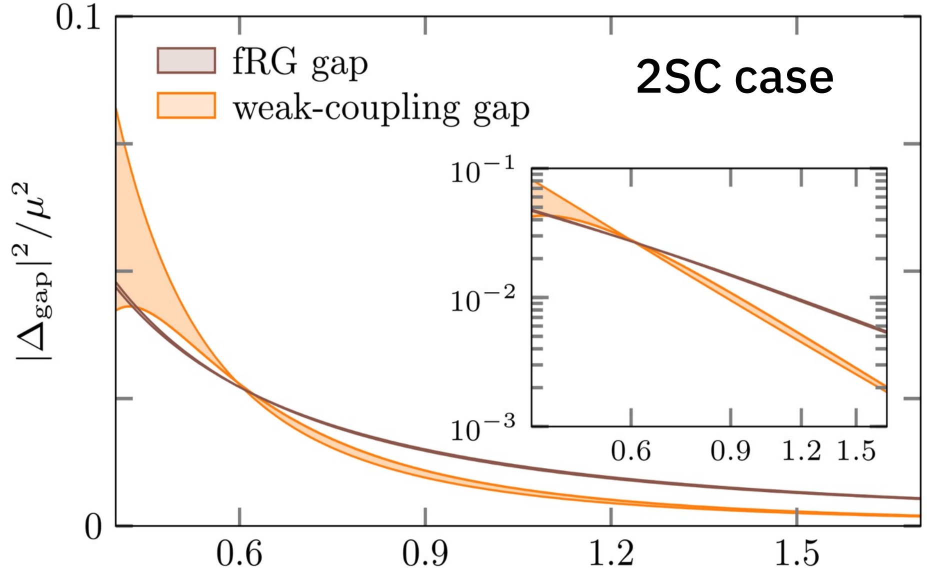
weak coupling
Son PRD 59 (1999)

$$\Delta_{\mathrm{gap}} \sim \Lambda_{\mathrm{QCD}}\exp[-B/(\alpha_{\mathrm{s}}^2\mu^2)]$$

fRG
Braun, Schallmo, PRD 105 (2022)

Both approximately power laws

Geißel, TG, Braun, PRD 110, 014034 (2024)



Results under NS conditions (1)

Geißel, TG, Braun, PRL 135, 211901 (2025)

- NS conditions give $\overline{\mu}_{\mathrm{e}} = \overline{\mu}_3 = 0, \overline{\mu}_8 = -\overline{m}_{\mathrm{s}}^2/2$, which gives the pressure

  Alford & Rajagopal, JHEP 06, 031 (2002)

$$\boxed{p_{\mathrm{CFL}}^{\mathrm{NS}} = p_{\mathrm{NQM}}^{\mathrm{NS}} + p_{\mathrm{free}} \left[\gamma_1(\alpha_{\mathrm{s}}, \overline{m}_{\mathrm{s}}^2) \overline{\Delta}_{\mathrm{CFL}}^2 - \frac{\overline{m}_{\mathrm{s}}^4}{4} \right]}$$

- CFL favored when 2nd term is positive: $\overline{\Delta}_{\mathrm{CFL}} > \frac{\overline{m}_{\mathrm{s}}^2}{4} \underbrace{\left[1 - 5.11\alpha_s + \frac{\overline{m}_{\mathrm{s}}^2}{6} \right]}_{<\,1 \text{ when converged}}$

  ⇨ CFL made **more stable by loop corrections**

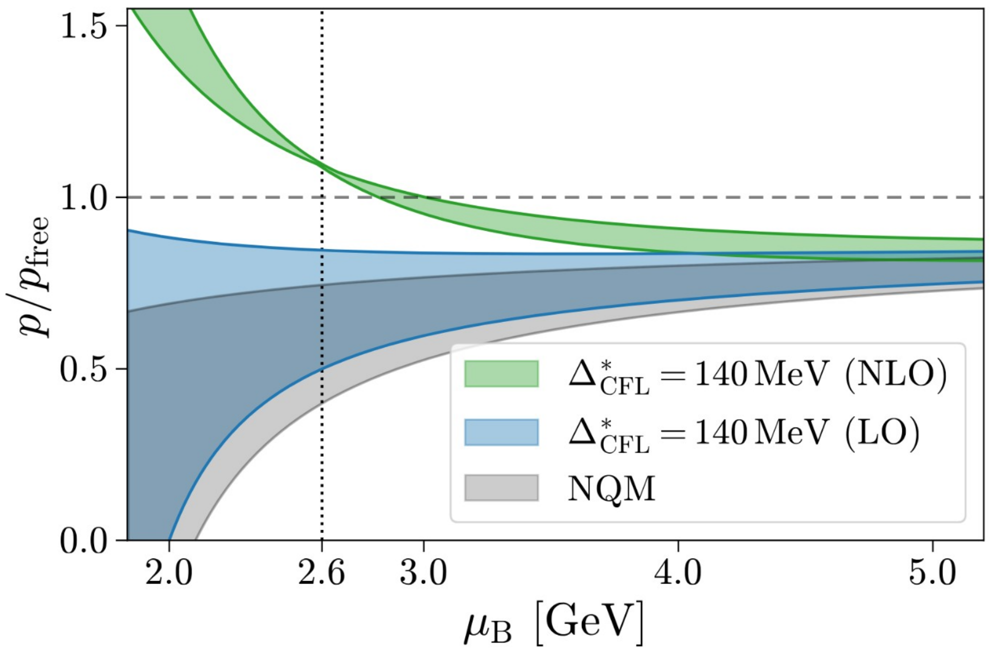
- Also find that **CFL more favored over 2SC phase at NLO**

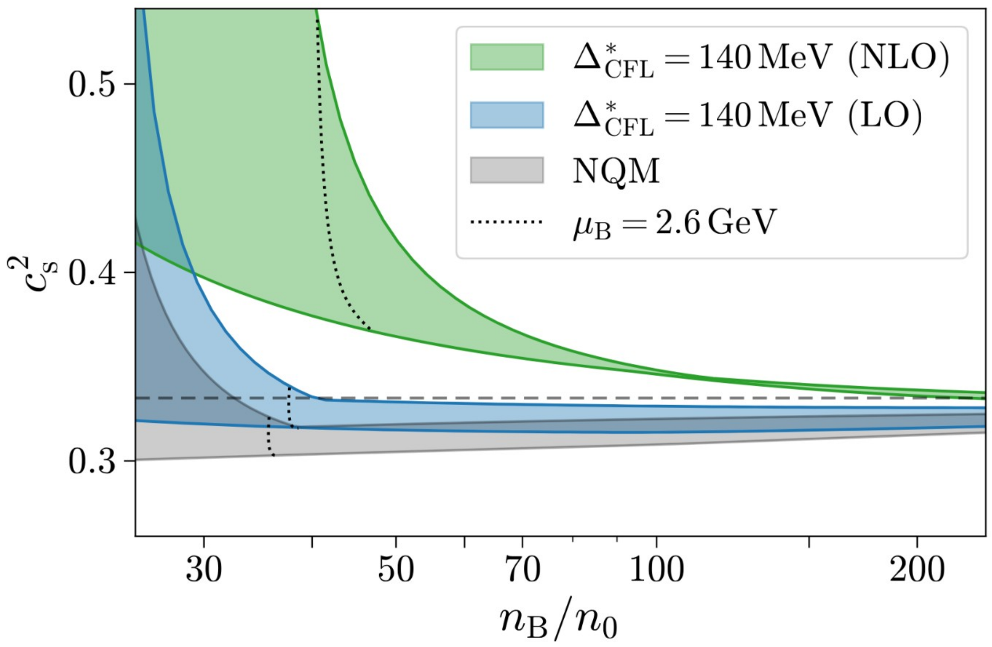
Results under NS conditions (2)

Geißel, TG, Braun, PRL 135, 211901 (2025)

$$\boxed{\gamma_1(\alpha_s, \overline{m}_s^2) = 4(1 - \overline{m}_s^2/3 + 10.2\alpha_s)}$$

- Term shifts the pressure and the speed of sound **towards larger values**





- NLO shift is similar size as LO shift. ***Worrisome???***

Constraining the gap with NS data

Geißel, TG, Braun, PRL 135, 211901 (2025)

- Shown previously that CEFT + Astro place upper bound on the gap at high densities
  Reason:

Komoltsev, Kurkela PRL 128 (2022),
Kurkela, Rajagopal, Steinhorst, PRL 132, 262701 (2024)

$$p_{\mathrm{H}} - p_{\mathrm{L}} \leq \frac{n_{\mathrm{H}}}{2}\left(\mu_{\mathrm{H}} - \frac{\mu_{\mathrm{L}}^2}{\mu_{\mathrm{H}}}\right) \& \ \mathrm{EoS_L\ constrained} \Rightarrow \mathrm{upper\ bound\ on}\ \Delta_{\mathrm{gap}}$$

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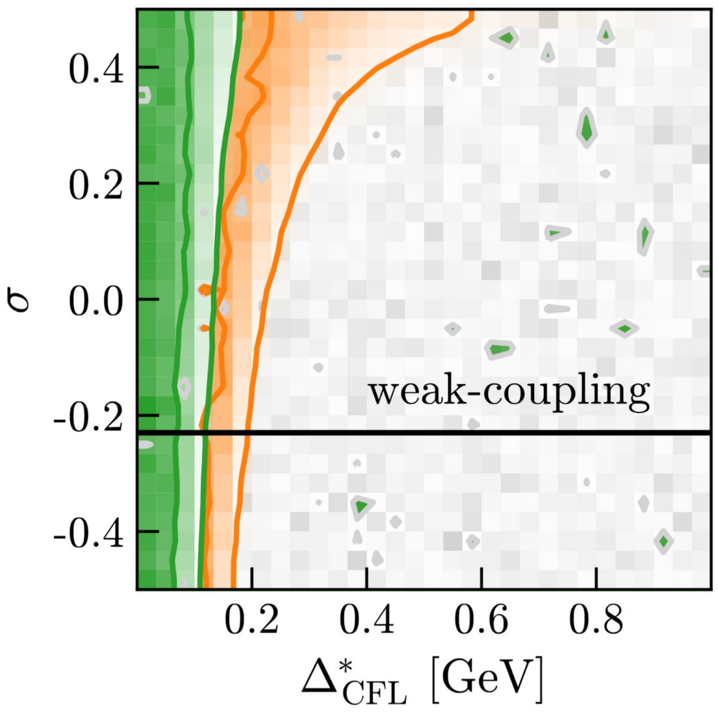
- Assuming power law scaling, with $\mu_B{}^*$=2.6 GeV

$$\Delta_{\mathrm{gap}} = \Delta^*_{\mathrm{CFL}}(\mu_{\mathrm{B}}/\mu^*_{\mathrm{B}})^{\sigma}$$

$$\sigma \approx -0.23 \text{ (weak coupling)}$$

$$\sigma \approx +0.45 \text{ (fRG)}$$

- At 95% credence, $\Delta^*_{\mathrm{CFL}} \lesssim 140$ MeV for the "symmetric" ensemble, after marginalizing over the unknown exponent (green)



Orange:
1. existence of $2M_\odot$,
2. causal connection to pQCD

Green:
1. NS prior to M_{TOV},
2. $c_s^2 < 2/3$ for extensions

Summary and Outlook

✓ **Computed NLO correction to pressure, speed of sound of paired quark matter under NS conditions at $O(\alpha_s \Delta^2)$**

- Assumed small, constant diquark background field
- Derived effective action S_{eff} to capture pairing effects at this order
- Result is numerical and small compared to free pressure, but large correction to LO correction
- **Suggests that perturbative treatment of "gapped pQCD" regime may only be applicable at extremely high densities (or assumption needs improvement, *or need the full NLO result*?)**

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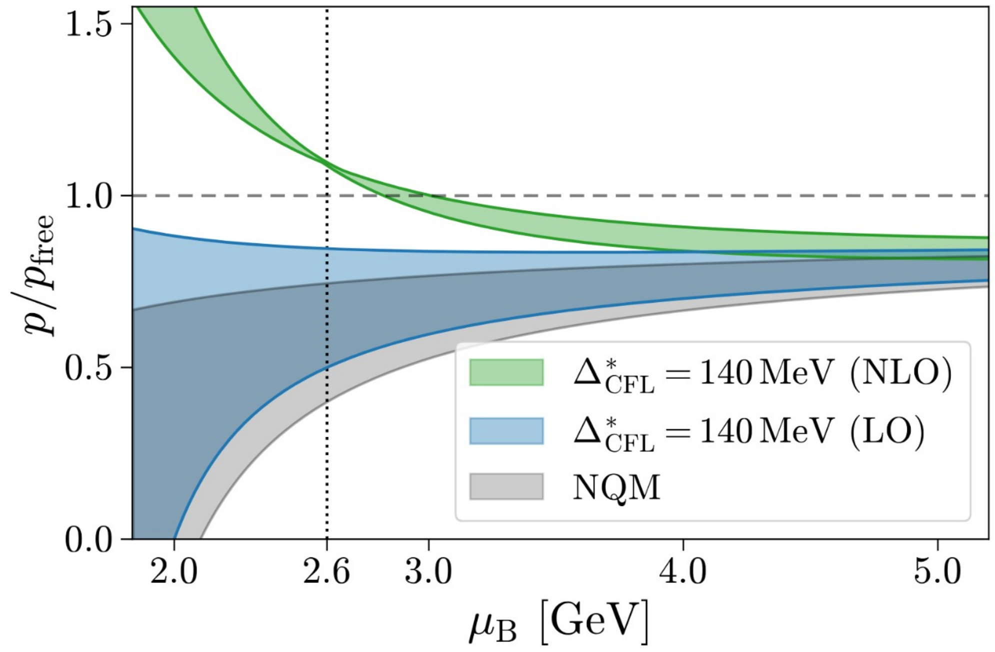

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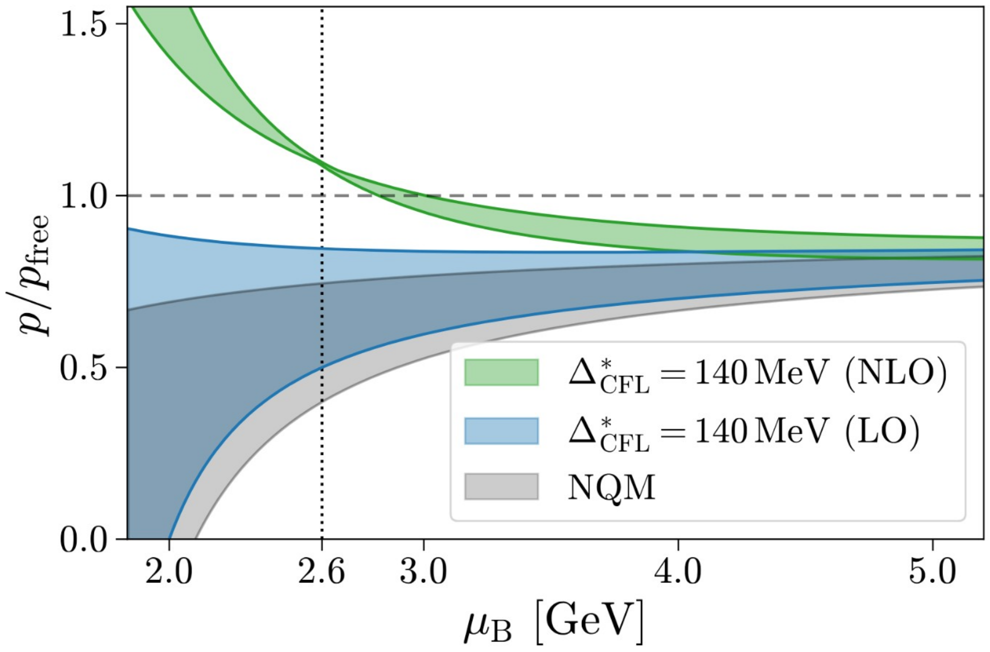

Thanks for your attention!